Sectoral Labor Reallocation, Inflation and Monetary Policy *

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Abstract

We study how frictional labor reallocation shapes the inflation dynamics and monetary policy using multi-sector New Keynesian economy. We derive analytical expressions for the Phillips curve and welfare, demonstrating that the frictional labor mobility influences sectoral inflation dynamics through two sufficient statistics: a mobility elasticity and the steady-state labor transition matrix. Using a spectral decomposition, we show that the persistence of sectoral shocks is determined by the interaction between frictions in labor mobility and nominal rigidity. When labor reallocation across sticky sectors is costly, the effect of sectoral shocks become more persistent. In response to sectoral shocks, monetary policy balances stabilization of standard aggregate output gaps with persistent labor and price misallocation. When these sticky sectors face a negative productivity shock, optimal monetary policy has an extra incentive to tighten to decrease the relative demand and thereby incentivize labor outflows.

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1 Introduction

Sectoral shocks are pervasive: energy prices spike, supply chains break, demand rotates across goods and services, and technologies reshape which tasks are valuable. These events call for relative adjustment—prices realign and workers move toward expanding activities. In a frictionless benchmark, both margins operate efficiently. In practice, both margins are sluggish. On the nominal side, sticky prices delay relative price adjustments, generating distortions that misallocate expenditure. On the real side, worker reallocation unfolds gradually through a complex network of bilateral flows shaped by skills, geography, and institutional barriers.

These two frictions are individually well established. Price stickiness is the foundation of the New Keynesian framework, and dynamic labor reallocation has been extensively studied in the trade and labor literature. In reality, the two margins are deeply intertwined: price distortions contaminate the signals that guide reallocation, while worker flow frictions feed back into the cost pressures that drive inflation. Analyzing these margins in isolation misses a feedback loop that governs the economy's ability to absorb structural shocks.

Because sectors differ in price rigidity, monetary policy inevitably reshapes the relative prices that drive labor reallocation. This introduces a trade-off between aggregate stabilization and cross-sector labor misallocation. The severity of this trade-off depends on how the network of labor flows aligns with sectoral price rigidities. We show how this alignment governs shock transmission, inflation persistence, and optimal policy design.

In this paper, we develop a multi-sector New Keynesian model with frictional labor reallocation and establish three main results. First, labor mobility dampens the initial inflation response to sectoral shocks but slows convergence, as forward-looking firms anticipate future cost relief from worker inflows. Second, shock persistence depends not on aggregate labor mobility per se, but on the price flexibility of the network's reallocation bottlenecks. If restoring equilibrium requires workers to reallocate through a sticky-to-sticky corridor, labor and nominal frictions become a "double trap," and sectoral shocks persist longer. Third, when structural labor bottlenecks coincide with asymmetric price flexibility, optimal policy deviates from standard target rules: the planner deliberately over- or under-heats the economy in order to generate the relative price changes needed to redirect worker flows in the labor mobility bottlenecks.

In Section 2, we build a multi-sector New Keynesian model that incorporates frictional labor reallocation via a Artuç et al. (2010) dynamic discrete choice block. Workers choose sectors based on forward-looking continuation values, subject to bilateral mobility costs and idiosyncratic preference shocks. Despite the richness of the underlying micro structure, we show that the entire reallocation block enters the linearized equilibrium and the policy problem through just two sufficient statistics: a scalar mobility elasticity ϕ that governs how responsive aggregate flows are to sectoral value differentials, and the steady-state labor transition matrix $\bar{\mu}$ that encodes the baseline pattern of worker flows.

Section 3 analyzes a static benchmark with within-period reallocation, collapsing the network structure into a single scalar sufficient statistic that enters the sectoral Phillips curve and welfare loss function. In this limit, the mobility elasticity determines how labor adjustment splits between worker flows (the extensive margin) and hours (the intensive margin). Lower mobility forces adjustment onto the intensive margin, increasing the welfare cost of relative price distortions. This yields closed-form solutions that clarify the optimal targeting rule. Immobile workers cannot easily relocate to absorb relative price wedges, forcing policy toward strict sticky-price targeting to prevent misallocation at the source. Conversely, high mobility breaks these bottlenecks: as workers flow from sectors that raise prices to those that do not, sectoral marginal costs become less sensitive. This prompts the central bank to shift its targeting weight toward a broader inflation index.

Section 4 extends the analysis to the general dynamic economy, where sectoral employment shares act as state variables. Using homogeneous price stickiness as a benchmark, we show the system cleanly separates into a policy-controlled aggregate block and an autonomous relative block. Using a spectral decomposition of the labor transition matrix, we can decompose this relative block into independent eigenvectors or "reallocation corridors," each characterized by its own mixing eigenvalue and cross-variable interactions. We derive a closed-form characteristic polynomial whose roots pin down shock persistence along each corridor. Comparing the price-adjustment and labor-mixing rates yields a sharp diagnostic for whether nominal or real frictions dictate shock persistence. As a natural limiting case, fully integrated labor markets demonstrate that while anticipated worker inflows dampen impact inflation, they mechanically slow overall convergence—highlighting an inverse relationship between impact amplitude and shock persistence.

Heterogeneous price stickiness breaks the spectral separation of the economy, coupling previously independent reallocation corridors and generating two distinct consequences. The first consequence is that the persistence of a sectoral shock is determined by the stickiness of the sectors along the specific path workers must take to reallocate. We formalize this via the *effective rigidity* of each labor reallocation corridor ($\Delta\kappa_k$). If a labor bottleneck connects sectors that are both price-sticky, the economy falls into a "double trap": workers cannot move because relative prices haven't shifted, and prices don't shift because workers haven't moved. In these corridors, labor mobility fails to dissipate misallocation and instead propagates the shock, causing inflation differentials to persist. While derived analytically as a local perturbation, our quantitative analysis evaluates the robustness of this diagnostic under the large, global dispersion of price stickiness observed in the data.

The second consequence is that aggregate demand now exerts a relative effect across sectors, captured by the *spillover coefficient* d_k. This statistic measures whether sticky and flexible sectors sit on opposite sides of a corridor, allowing aggregate demand to actively tilt relative prices along that dimension. Importantly, persistence and spillover can move independently. A corridor connecting two sticky sectors, for instance, may suffer from extreme persistence (negative $\Delta\kappa_k^{\text{eff}}$) while leaving the central bank powerless to intervene (a near-zero d_k). Both sufficient statistics are directly computable from the transition matrix of worker flows and sectoral price adjustment frequencies.

A dynamic Ramsey problem formalizes how these price-network interactions shape optimal policy. We derive the full dynamic second-order welfare loss, which penalizes cross-sector employment misallocation and relative price distortions—costs that compound over time when labor reallocation is slow. We derive a targeting criterion that nests the textbook rule when stickiness is uniform, but introduces a reallocation wedge when aggregate demand can influence relative prices. The planner deviates from the textbook rule when the two network statistics described above are large along the same reallocation corridor. The effective rigidity $\Delta\kappa_k^{\text{eff}}$ captures the necessity of intervention: sticky corridors create persistent welfare costs. The spillover d_k determines the direction of the deviation: if the recessionary side of a bottleneck corridor is sticky $(d_k > 0)$, its relative price must fall to stimulate demand; expansionary policy achieves this by disproportionately raising flexible-sector prices. The bias reverses when the sticky side instead needs to shed workers.

Section 5 brings the model to U.S. sectoral data, estimating the transition matrix $\bar{\mu}$ from worker flows and measuring the alignment between price rigidity and the network's slowest-mixing corridors to assess the quantitative relevance of these channels.

Related Literature

Our paper relates to three main strands of literature. The first is the study of optimal monetary policy in multi-sector New Keynesian economies. Existing work shows how heterogeneous price rigidity Aoki (2001) and production networks La'O and Tahbaz-Salehi (2022); Rubbo (2023); Afrouzi and Bhattarai (2025) shape optimal policy and inflation persistence. However, these models typically assume labor is either perfectly mobile or permanently segmented across sectors. We contribute to this literature by introducing frictional, network-based labor reallocation, demonstrating how moving away from these polar assumptions generates new theoretical channels for policy and aggregate persistence.

Second, we build on the trade and labor literature on dynamic worker reallocation, originating from Lilien (1982) and formalized via dynamic discrete choice by Artuç et al. (2010), and applied in general equilibrium settings in Caliendo et al. (2019); Allen et al. (2020); Seo and Oh (2025); Kim et al. (2026). We contribute by embedding this micro-foundation into a multi-sector New Keynesian model. This allows us to analytically map the sufficient statistics directly into the New Keynesian Phillips curve and the welfare loss function. Methodologically, we adapt spectral decomposition techniques recently used in spatial and input-output settings Kleinman et al. (2023); Liu and Tsyvinski (2024) to characterize cross-sector worker flows.

Third, we add to the empirical and quantitative literature studying how heterogeneous price rigidity Bils and Klenow (2004); Nakamura and Steinsson (2008) and sectoral linkages Pasten et al. (2020); Pastén et al. (2024) shape aggregate dynamics. While prior work shows that heterogeneous propagation causes inflation persistence Altissimo et al. (2009) and highlights the importance of labor segmentation Carvalho et al. (2021), our contribution is to show that the specific topology of worker flows, interacting with price stickiness, can serve as a distinct, complementary source of aggregate persistence.

Finally, our work relates closely to studies of monetary policy during structural reallocation, such as Guerrieri et al. (2021), who study a two-sector model with downward wage rigidity, and Ferrante et al. (2023), who quantitatively estimate a multi-sector model with reallocation costs. Relative to these papers, our contribution is analytical: by generalizing to an arbitrary number of sectors with a rich reallocation network and standard Calvo pricing, we obtain a fully tractable Ramsey problem. This allows us to derive observable sufficient statistics that diagnose exactly when the reallocation margin dictates a shift in optimal policy.

2 Model

Time is discrete, $t = 0, 1, \ldots$ The economy consists of N sectors indexed by $i, j \in \mathcal{N} = \{1, \ldots, N\}$. We describe the production side, household consumption and labor supply, labor reallocation across sectors, and close the model with equilibrium conditions and their log-linear approximations.

2.1 Production and Pricing

Final and sectoral aggregation. The economy produces a final good Y_t which is a Cobb-Douglas aggregate of sectoral bundles $\{Y_{it}\}_{i \in \mathcal{N}}$:

$$Y_t = \prod_{i \in \mathcal{N}} (Y_{it}/\alpha_{it})^{\alpha_i}, \qquad \sum_{i \in \mathcal{N}} \alpha_i = 1, \tag{1}$$

with associated consumer price index $P_t^C = P_{it}^{\alpha_i}$, where P_{it} is the price index for sector i. Each sectoral bundle is a CES aggregate of differentiated varieties $f \in [0,1]$ with elasticity of substitution $\epsilon_i > 1$:

$$Y_{it} = \left(\int_0^1 Y_{it}(f)^{\frac{\epsilon_i - 1}{\epsilon_i}} \, df \right)^{\frac{\epsilon_i}{\epsilon_i - 1}}, \tag{2}$$

yielding the standard demand for variety f, $Y_{it}(f) = (P_{it}(f)/P_{it})^{-\epsilon_i} Y_{it}$, and the sectoral price index $P_{it} = \left(\int_0^1 P_{it}(f)^{1-\epsilon_i} \, df \right)^{\frac{1}{1-\epsilon_i}}$.

Technology. Each firm f in sector i operates a linear production technology:

$$Y_{it}(f) = A_{it} L_{it}(f), \tag{3}$$

where A_{it} is sector-specific productivity and $L_{it}(f)$ is labor input. Nominal marginal cost is $MC_{it} = W_{it}/A_{it}$, where W_{it} is the nominal wage in sector i.

Pricing. Firms set prices subject to Calvo frictions. In each period, a fraction $1-\delta_i \in (0,1)$ of firms in sector i can reset their prices. The government provides a time-invariant ad-valorem revenue subsidy of rate $\frac{1}{\epsilon_i - 1}$ to each sector, financed by lump-sum taxes, that offsets the monopolistic markup and ensures the flexible-price steady state is efficient. Under this subsidy, a re-optimizing firm chooses $P^*_{it}(f)$ to maximize:

$$\max_{P^*_{it}(f)} \mathbb{E}_t \sum_{s=0}^{\infty} \delta_i^s Q_{t,t+s} \left(\frac{P^*_{it}(f)}{P_{it+s}} \right)^{-\epsilon_i} Y_{it+s} \left[\frac{\epsilon_i}{\epsilon_i - 1} P^*_{it}(f) - MC_{it+s} \right], \tag{4}$$

where $MC_{it} = W_{it}/A_{it}$ is nominal marginal cost and $Q_{t,t+s} = \beta^s (C_{t+s}/C_t)^{-\gamma} (P^C_t / P^C_{t+s})$ is the stochastic discount factor.

2.2 Households

A representative family consists of a unit mass of workers distributed across sectors. Let N_{it} denote the fraction of workers in sector i at the start of period t, with $\sum_{i \in \mathcal{N}} N_{it} = 1$. Each worker in sector i supplies L_{it} hours during period t. The distribution $\{N_{it}\}$ is predetermined at the start of each period; it evolves through a reallocation process described in Section 2.3.

Preferences. The family maximizes:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \sum_{i \in \mathcal{N}} N_{it} \zeta_i \frac{L_{it}^{1+\varphi}}{1+\varphi} - \Psi_t \right], \tag{5}$$

where $\beta \in (0,1)$ is the discount factor, $\gamma > 0$ is the coefficient of relative risk aversion, $\varphi > 0$ is the inverse Frisch elasticity, $\zeta_i > 0$ are sector-specific disutility shifters, and Ψ_t is the cost of labor reallocation incurred at period t (defined in Section 2.3).

Budget constraint. The family faces the nominal budget constraint:

$$P^C_t C_t + B_t \le (1 + i_{t-1}) B_{t-1} + \sum_{i \in \mathcal{N}} W_{it} L_{it} N_{it} + \Pi_t + T_t, \tag{6}$$

where B_t are one-period nominal bonds paying gross interest $1 + i_t$ set by the central bank, Π_t are aggregate profits, and T_t are net lump-sum transfers from the government.

Optimality conditions. Maximizing (5) subject to (6), taking $\{N_{it}\}$ as given, yields the intratemporal labor supply condition for each sector i:

$$\frac{W_{it}}{P_t^C} = \zeta_i L_{it}^{\varphi} C_t^{\gamma}, \tag{7}$$

and the intertemporal Euler equation:

$$1 = \beta(1 + i_t)\, \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t^C}{P_{t+1}^C} \right]. \tag{8}$$

2.3 Labor Reallocation

We now describe how workers move across sectors between periods. At the end of period t, after production and consumption decisions have been made, the representative family coordinates the reallocation of its workers for the start of period $t + 1$.[1]

Reallocation cost. The per-period reallocation cost in the family's objective (5) takes the form:

$$\Psi_t = \sum_{i \in \mathcal{N}} N_{it}\, \psi_i \Big(\{\mu_t^{ij}\}_{j \in \mathcal{N}} \Big), \tag{9}$$

where $\psi_i(\cdot)$ is a sector-specific cost function that is strictly convex in the transition probabilities $\{\mu_t^{ij}\}$ satisfying $\mu_t^{ij} \geq 0, \sum_{j \in \mathcal{N}} \mu_t^{ij} = 1$. We adopt the following parametric specification, standard in the dynamic discrete choice literature (Anderson et al., 1988; Artuç et al., 2010; Caliendo et al., 2019):

$$\psi_i \Big(\{\mu_t^{ij}\}_j \Big) = \sum_{j \in \mathcal{N}} \mu_t^{ij} \tau^{ij} + \frac{1}{\theta} \sum_{j \in \mathcal{N}} \mu_t^{ij} \log \mu_t^{ij}, \tag{10}$$

where $\tau^{ij} \geq 0$ are bilateral reallocation costs (with $\tau^{ii} = 0$). As is standard in the literature, this aggregate penalty perfectly mimics a decentralized setting where individual workers draw idiosyncratic type-I extreme value preference shocks with dispersion $1/\theta$. The parameter $\theta > 0$ governs the elasticity of labor reallocation: as θ falls, flows become less responsive to continuation-value differentials. In the limit $\theta \to 0$, sectoral employment shares do not respond

[1] The timing is: (1) period t begins with employment distribution $\{N_{it}\}$ and last period's prices $\{P_{it-1}\}$ given; (2) shocks are realized, firms set prices under Calvo frictions, and workers supply hours L_{it}; (3) at the end of period t, workers choose which sector to enter for period $t + 1$; (4) this determines $\{N_{it+1}\}$.

to shocks at first order. Conversely, as $\theta \to \infty$, the family sorts workers deterministically toward high-value destinations, approaching a frictionless benchmark.

Value functions and optimal flows. The family chooses the flows $\{\mu_t^{ij}\}$ to maximize welfare. Let V_{it} denote the marginal continuation value to the family of having an additional worker in sector i at time t. Under the functional form for ψ_i, optimal transition probabilities take the logit form:

$$\mu_t^{ij} = \frac{\exp\left(\theta[\beta\,\mathbb{E}_t V_{jt+1} - \tau^{ij}]\right)}{\sum_{j'\in\mathcal{N}} \exp\left(\theta[\beta\,\mathbb{E}_t V_{j't+1} - \tau^{ij'}]\right)}. \tag{11}$$

Substituting these optimal flows back into the family's objective yields a recursive expression for the marginal value of a worker:

$$V_{it} = U_{it}^{\text{flow}} + \frac{1}{\theta}\log\left(\sum_{j\in\mathcal{N}} \exp\left(\theta[\beta\mathbb{E}_t V_{jt+1} - \tau^{ij}]\right)\right), \tag{12}$$

where the flow utility from working in sector j at time t is:

$$U_{jt}^{\text{flow}} \equiv \frac{W_{jt}}{P_t^C} C_t^{-\gamma} L_{jt} - \zeta_j \frac{L_{jt}^{1+\varphi}}{1+\varphi}. \tag{13}$$

Using the intratemporal labor supply condition (7), this simplifies to:

$$U_{jt}^{\text{flow}} = \zeta_j \frac{\varphi}{1+\varphi} L_{jt}^{1+\varphi}. \tag{14}$$

Steady-state balance. We impose a mild restriction on bilateral reallocation costs:

Assumption 1 (Quasi-symmetric reallocation costs)**.** *The bilateral reallocation costs admit a decomposition $\tau^{ij} = \tau_i^{\text{out}} + \tau_j^{\text{in}} + \hat{\tau}^{ij}$, where $\hat{\tau}^{ij} = \hat{\tau}^{ji}$ for all i, j.*

Quasi-symmetry is standard in the dynamic discrete choice literature, imposed both to reduce the dimensionality of bilateral cost parameters and to simplify the equilibrium system (e.g., Artuç et al., 2010; Allen et al., 2020; Seo and Oh, 2025). It nests the common special cases of symmetric costs ($\tau^{ij} = \tau^{ji}$) and additive origin-destination costs with zero diagonals ($\tau^{ij} = \tau_1^i + \tau_2^j$). A corollary is that the steady-state transition matrix $\bar{\mu} = \{\bar{\mu}^{ij}\}$ satisfies the detailed balance condition:

$$\bar{N}_i \bar{\mu}^{ij} = \bar{N}_j \bar{\mu}^{ji}, \qquad \forall\, i, j \in \mathcal{N}, \tag{15}$$

where $\bar{N}_i$ is the steady-state employment share. This ensures that gross worker flows between any pair of sectors are balanced in steady state. Detailed balance is used in Section 4 to symmetrize the transition matrix and decompose the dynamic system into independent components.

Sufficient statistics. While equations (11) and (12) provide the microfoundations for labor reallocation, the subsequent analysis of the linearized equilibrium (Section 2.5) reveals that the specific bilateral costs τ^{ij} and micro-parameters θ, ζ do not enter the dynamics independently. Instead, the labor block is summarized by two sufficient statistics:

- **The steady-state transition matrix** $\bar{\mu} \equiv \{\bar{\mu}^{ij}\}$, which disciplines the baseline worker flows. Under Assumption 1, $\bar{\mu}$ satisfies detailed balance, which we use in Section 4 to decompose the dynamic system.
- **The mobility elasticity** $\phi \equiv \varphi\theta\zeta\bar{L}^{1+\varphi}$, a composite parameter that governs how relative continuation values tilt aggregate flows.

2.4 Equilibrium

Market clearing. In equilibrium, the goods market for each variety and the aggregate economy must clear:

$$Y_{it}(f) = C_{it}(f), \quad Y_t = C_t, \tag{16}$$

where C_t is household consumption. Combined with Cobb-Douglas demand, this implies that nominal sectoral output equals the expenditure share of total nominal consumption:

$$P_{it}Y_{it} = \alpha_i P_t^C C_t. \tag{17}$$

Labor market clearing in each sector requires:

$$\int_0^1 L_{it}(f)\,df = N_{it}L_{it}. \tag{18}$$

The government finances the output subsidies through lump-sum taxes, balancing its budget each period:

$$T_t + \sum_{i\in\mathcal{N}} \frac{1}{\epsilon_i - 1} P_{it}Y_{it} = 0. \tag{19}$$

The central bank sets the nominal interest rate i_t according to a policy rule specified below.

Definition 1 (Equilibrium)**.** *Given an initial employment distribution $\{N_{i0}\}$ and a sequence of productivity shocks $\{A_{it}\}$, an equilibrium is a collection of prices $\{P_{it}(f), P_{it}, P_t^C, W_{it}, i_t\}$, quantities*

$\{C_t, Y_{it}, L_{it}(f), L_{it}, B_t\}$, employment shares $\{N_{it}\}$, transition probabilities $\{\mu_t^{ij}\}$, and value functions $\{V_{it}\}$ such that: (i) households optimize consumption, savings, labor supply, and reallocation; (ii) firms optimize pricing subject to Calvo constraints; (iii) all markets clear and the government budget constraint (19) *holds; and (iv) the central bank follows its policy rule.*

Steady state. We impose that steady-state flow surplus is equalized across sectors: $\zeta_i \bar{L}_i^{1+\varphi}$ is constant across i. This is the natural interior condition for the logit reallocation problem: if any sector offered strictly higher surplus, workers would concentrate there as $\theta \to \infty$. We achieve this by normalizing $\bar{L}_i = \bar{L}$ and $\zeta_i = \zeta$ for all i.[2] Combining steady-state labor supply (7) with goods market clearing (17) gives $\alpha_i \bar{C}^{1-\gamma} = \zeta \bar{N}_i \bar{L}^{1+\varphi}$, so equal surplus implies $\bar{N}_i = \alpha_i$: steady-state employment shares equal consumption expenditure shares. This simplification is maintained throughout the paper.

2.5 Log-Linearized Equilibrium

Standard log-linearization of the equilibrium conditions yields the following system; derivations are in Appendix A.

Notation. For any variable X_t, we write $x_t \equiv \log X_t - \log \bar{X}$ for its log-deviation from steady state. We use the following conventions throughout:

- *Natural (flexible-price) levels*: x_t^{nat} denotes the value under flexible prices.
- *Gaps*: $\tilde{x}_t \equiv x_t - x_t^{\text{nat}}$ denotes the deviation from the natural level.
- *Relative prices*: $\hat{p}_{it} \equiv p_{it} - p_t^C$ is the log sectoral price relative to the CPI.
- *Relative price distortion*: $\chi_{it} \equiv \hat{p}_{it} + \hat{a}_{it}$, where $\hat{a}_{it} \equiv a_{it} - \sum_j \alpha_j a_{jt}$. In the static economy this equals $\hat{p}_{it} - \hat{p}_{it}^{\text{nat}}$ (Lemma 2). In the dynamic economy with predetermined n_{it}, the true flexible-price relative price also depends on inherited employment (Lemma 4), so χ_{it} measures the deviation from the *static* natural benchmark throughout; the distinction is discussed after Lemma 4.

We write $\overset{(1)}{=}$ for equalities that hold up to a first-order approximation, and $\overset{(2)}{=}$ for second-order. Plain $=$ denotes exact relationships or definitions.

The sectoral New Keynesian Phillips curve is, for each i:

$$\pi_{it} \overset{(1)}{=} \kappa_i \left[mc_{it} - p_{it} \right] + \beta \, \mathbb{E}_t \pi_{it+1}, \tag{20}$$

[2] The normalization $\bar{L}_i = \bar{L}$ is without loss of generality: N_{it} measures the mass of workers in sector i, and L_{it} can be interpreted as either per-worker hours or the fraction of workers who supply labor (Galí, 2011). The condition $\zeta_i = \zeta$ then ensures equal surplus.

where $\pi_{it} = p_{it} - p_{it-1}$ is sectoral inflation and $\kappa_i \equiv (1-\delta_i)(1-\beta\delta_i)/\delta_i$. Marginal cost can be expressed using the production technology (3), labor supply (7), goods market clearing ($y_{it} = a_{it} + n_{it} + l_{it}$), and sectoral demand ($y_{it} - y_t = -\hat{p}_{it}$) as:

$$mc_{it} - p_{it} \overset{(1)}{=} (\gamma + \varphi) y_t - (1+\varphi)(\hat{p}_{it} + a_{it}) - \varphi n_{it}. \tag{21}$$

The log-linearized Euler equation gives the IS curve:

$$\tilde{y}_t \overset{(1)}{=} \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\gamma}\left(i_t - \mathbb{E}_t \pi^C_{t+1} - r^{\text{nat}}_t\right), \tag{22}$$

where $\pi^C_t = \sum_i \alpha_i \pi_{it}$ is CPI inflation and $r^{\text{nat}}_t = -\ln\beta + \gamma(\mathbb{E}_t y^{\text{nat}}_{t+1} - y^{\text{nat}}_t)$ is the natural real rate, characterized in Sections 3–4. Relative prices evolve according to:

$$\hat{p}_{it} = \hat{p}_{it-1} + \pi_{it} - \pi^C_t. \tag{23}$$

The labor reallocation block links employment dynamics to forward-looking value functions. We use bold to distinguish N-vectors from scalars: $\mathbf{n}_t$, $\mathbf{v}_t$, $\mathbf{u}_t$ have entries n_{it}, v_{it}, u_{it}. Normalizing value-function and flow-utility deviations by $\varphi\zeta\bar{L}^{1+\varphi}$, the reallocation block is:

$$\mathbf{n}_{t+1} = \bar{\mu}\,\mathbf{n}_t + \phi\beta(I - \bar{\mu}^2)\,\mathbb{E}_t[\mathbf{v}_{t+1}], \tag{24}$$

$$\mathbf{v}_t = \mathbf{u}_t + \beta\bar{\mu}\,\mathbb{E}_t[\mathbf{v}_{t+1}], \tag{25}$$

$$u_{it} = y_t - \hat{p}_{it} - a_{it} - n_{it}, \tag{26}$$

where $\phi \equiv \varphi\theta\zeta\bar{L}^{1+\varphi}$ is the composite mobility parameter. Because we work with log-deviations $n_{it} \equiv \log(N_{it}/\bar{N}_i)$ and impose detailed balance, the linearized operator governing $\mathbf{n}_t$ is written in terms of $\bar{\mu}$ (see Appendix A). The normalized deviations are $\mathbf{v}_t \equiv \Delta\mathbf{V}_t/(\varphi\zeta\bar{L}^{1+\varphi})$ and $\mathbf{u}_t \equiv \Delta\mathbf{U}^{\text{flow}}_t/(\varphi\zeta\bar{L}^{1+\varphi})$, subject to the aggregate constraint $\sum_i \alpha_i n_{it} = 0$.[3]

Finally, the central bank sets the nominal interest rate according to:

$$i_t = \psi_\pi \pi_t(\omega) + \psi_y \tilde{y}_t, \tag{27}$$

for some inflation index $\pi_t(\omega) = \sum_i \omega_i \pi_{it}$ with weights summing to one.

Discussion. The system (20)–(27) constitutes the full linearized dynamic model. It nests two benchmarks. As $\phi \to 0$ (segmented labor markets), we have $n_{it} = 0$ and the model reverts to a multi-sector New Keynesian setup with sector-specific labor. As $\phi \to \infty$ (frictionless mobility), flows equalize utilities across sectors, reducing the model to a single-labor-market economy

[3]Combining steady-state labor supply (7) with goods market clearing (17) gives $\zeta\bar{L}^{1+\varphi} = \bar{C}^{1-\gamma}$, so $\phi = \varphi\theta\bar{C}^{1-\gamma}$.

(e.g., Rubbo, 2023).

We analyze this system in two steps. First, a static formulation (Section 3) where n_{it} adjusts within the period, collapsing the reallocation block into a single sufficient statistic. Second, a dynamic formulation (Section 4) where n_{it} is a predetermined state variable. This distinction allows us to isolate how the speed of reallocation—governed by ϕ and $\bar{\mu}$—shapes the persistence of sectoral inflation and the central bank's optimal trade-off between output and relative price stability.

3 Static Economy: Mobility Elasticity as Sufficient Statistic

We first analyze the case where labor reallocates within the period. This collapses the dynamic system into a static problem where the impact of mobility on the Phillips curve, welfare loss, and optimal policy is summarized by a single scalar sufficient statistic. This limiting case provides sharp analytical intuition for how mobility dampens sectoral cost pressures and shifts the central bank's optimal inflation-targeting weights.

3.1 The static limit

In a static model, because workers can reoptimize their sector each period and today's sector choice does not constrain tomorrow's feasible set, the continuation component is common across choices and cancels from the logit shares: workers choose sectors based on current-period flow utility alone. The logit transition (11) reduces to a static choice anchored by the steady-state baseline shares $\bar{N}_i$:

$$N_{it} = \frac{\bar{N}_i \exp(\theta U_{it}^{\text{flow}})}{\sum_{j \in \mathcal{N}} \bar{N}_j \exp(\theta U_{jt}^{\text{flow}})}. \tag{28}$$

where $U_{it}^{\text{flow}} = \zeta \frac{\varphi}{1+\varphi} L_{it}^{1+\varphi}$ from (14). The employment distribution $\{N_{it}\}$ is now a jump variable that adjusts contemporaneously to shocks, rather than a predetermined state. Log-linearizing (28) around the symmetric steady state yields:

$$n_{it} = \phi(l_{it} - l_t^{\alpha}), \qquad l_t^{\alpha} \equiv \sum_i \alpha_i l_{it}, \tag{29}$$

where $\phi \equiv \varphi\theta\zeta\bar{L}^{1+\varphi}$ is the composite mobility parameter. Workers flow toward sectors with above-average hours (and hence above-average wages), with elasticity ϕ. The constraint $\sum_i \alpha_i n_{it} = 0$ is satisfied by construction.

In the static limit, employment n_{it} is no longer a state variable. By substituting (29) into the Phillips curve, the extensive margin (reallocation) and intensive margin (hours) aggregate into a single labor supply response. This collapses the dynamic system into a standard New Keynesian framework where sectoral dynamics are fully characterized by relative prices and the output gap.

Lemma 1 (Extensive Margin Aggregation)**.** *Under (29), the extensive margin aggregates out of the output gap. Specifically, aggregate output satisfies $y_t = \sum_i \alpha_i a_{it} + \sum_i \alpha_i (n_{it} + l_{it})$, and since $\sum_i \alpha_i n_{it} = 0$:*

$$\tilde{y}_t = \sum_i \alpha_i \tilde{l}_{it} = \tilde{l}_t^{\alpha}.$$

The output gap depends only on the weighted average of intensive-margin labor gaps, not on employment reallocation.

3.2 Flexible-Price Allocation

We characterize the natural (flexible-price) allocation. Since the output subsidy offsets the monopolistic markup, the flexible-price competitive equilibrium is efficient, and we can characterize it as the solution to a social planner's problem.

In the static limit, workers reallocate instantaneously and their choices are independent of their origin sector. The dynamic adjustment friction from Section 2.3 therefore collapses to a static relative-entropy penalty that anchors employment to the steady-state baseline shares $\{\bar{N}_i\}$. Because no state variables carry across periods, the planner's intertemporal problem reduces to the following per-period problem: for each t, choose $\{L_{it}, N_{it}\}$ to maximize

$$\frac{C^{1-\gamma}}{1-\gamma} - \sum_i \zeta N_{it} \frac{L_{it}^{1+\varphi}}{1+\varphi} - \frac{1}{\theta} \sum_i N_{it} \log \frac{N_{it}}{\bar{N}_i} \tag{30}$$

subject to the technology constraint $C_t = \prod_{i \in \mathcal{N}} (A_{it} N_{it} L_{it})^{\alpha_i}$ and the employment constraint $\sum_{i \in \mathcal{N}} N_{it} = 1$. The final term in (30) is exactly the penalty that rationalizes the decentralized static logit choice (28).

The first-order condition for L_{it} uses $\partial C / \partial L_{it} = \alpha_i C / L_{it}$ and gives:

$$\alpha_i C^{1-\gamma} = \zeta N_{it} L_{it}^{1+\varphi}. \tag{31}$$

The first-order condition for N_{it} reproduces the decentralized extensive margin condition; log-linearized, it yields (29).

Combining these, we can solve for the natural allocation. Log-linearizing (31) gives $(1-\gamma) y_t = n_{it} + (1+\varphi) l_{it}$. Weighting by α_i and summing, using $\sum_i \alpha_i n_{it} = 0$:

$$l_t^{\alpha} = \frac{1-\gamma}{1+\varphi} y_t. \tag{32}$$

Substituting back, the intensive margin FOC becomes $(1+\varphi)(l_{it} - l_t^\alpha) = -n_{it}$, which combined with $n_{it} = \phi(l_{it} - l_t^\alpha)$ implies $l_{it} = l_t^\alpha$ and $n_{it} = 0$ for all i. That is, the planner equalizes real wage (and hours) across sectors and does not reallocate workers away from steady state.

Lemma 2 (Natural Allocation Invariance)**.** *The flexible-price planner allocation features uniform sectoral hours and zero reallocation:*

$$l_{it}^{nat} = \frac{1-\gamma}{1+\varphi} y_t^{nat}, \qquad n_{it}^{nat} = 0.$$

Natural output and natural relative prices are:

(i) $y_t^{nat} = \frac{1+\varphi}{\gamma+\varphi} \sum_i \alpha_i a_{it}$,

(ii) $\hat{p}_{it}^{nat} = \sum_j \alpha_j a_{jt} - a_{it}$,

and are both independent of the mobility parameter ϕ.

Proof. Natural output follows from substituting $l_t^\alpha = \frac{1-\gamma}{1+\varphi} y_t$ and $\sum_i \alpha_i n_{it} = 0$ into $y_t = \sum_i \alpha_i a_{it} + \sum_i \alpha_i (n_{it} + l_{it})$ and solving. For natural relative prices, goods market clearing gives $\hat{p}_{it}^{\text{nat}} = y_t^{\text{nat}} - a_{it} - l_{it}^{\text{nat}} - n_{it}^{\text{nat}}$. Substituting $n_{it}^{\text{nat}} = 0$ and $l_{it}^{\text{nat}} = \frac{1-\gamma}{1+\varphi} y_t^{\text{nat}}$ yields $\hat{p}_{it}^{\text{nat}} = \frac{\gamma+\varphi}{1+\varphi} y_t^{\text{nat}} - a_{it} = \sum_j \alpha_j a_{jt} - a_{it}$. Neither expression involves ϕ. □

The invariance result reflects a simple economic intuition: under flexible prices, the planner achieves the efficient allocation solely through the intensive margin (l_{it}^{nat}), leaving no incentive for sectoral reallocation. Consequently, the mobility parameter ϕ—which governs the cost of moving away from steady-state employment—is irrelevant for the natural allocation. Mobility only becomes a relevant margin for welfare and dynamics once sticky prices drive the economy away from this first-best benchmark.

3.3 Phillips Curve with Mobility

We now derive the key first-order result in the static model: labor mobility enters the Phillips curve through a single sufficient statistic.

Marginal cost in gaps. From the log-linearized marginal cost (21) and the natural allocation (Lemma 2), the real marginal cost gap is:

$$mc_{it} - p_{it} = \gamma \tilde{y}_t + \varphi \tilde{l}_{it} - \chi_{it}, \tag{33}$$

where $\chi_{it} \equiv \hat{p}_{it} - \hat{p}_{it}^{\text{nat}}$ is the relative price distortion and we have used $mc_{it}^{\text{nat}} = p_{it}^{\text{nat}}$ (which holds due to the efficient output subsidy). To close this expression, we need to eliminate $\tilde{l}_{it}$ using the reallocation condition.

Margin decomposition. Total sectoral labor gaps satisfy $\tilde{h}_{it} = \tilde{n}_{it} + \tilde{l}_{it}$, and goods market clearing requires $\tilde{h}_{it} = \tilde{y}_t - \chi_{it}$. Combined with the static reallocation condition (29), a one-unit relative price distortion χ_{it} induces a total labor adjustment of -1, split between margins as:

$$\text{intensive:} \quad \tilde{l}_{it} = \tilde{y}_t - \frac{1}{1+\phi}\chi_{it}, \tag{34}$$

$$\text{extensive:} \quad \tilde{n}_{it} = -\frac{\phi}{1+\phi}\chi_{it}. \tag{35}$$

Higher mobility (larger ϕ) shifts the adjustment from hours to worker flows. Substituting (34) into the marginal cost gap (33) and collecting terms:

$$mc_{it} - p_{it} = (\gamma + \varphi)\,\tilde{y}_t - \sigma\,\chi_{it}, \qquad \sigma \equiv 1 + \frac{\varphi}{1+\phi} \in [1,\, 1+\varphi). \tag{36}$$

The coefficient φ in the marginal cost response arises from the labor supply margin, but only a fraction $1/(1+\phi)$ of the sectoral adjustment occurs through hours; the rest is absorbed by worker flows. So the effective response is $1 + \varphi/(1+\phi) = \sigma$, not $1+\varphi$. Full mobility ($\phi \to \infty$) gives $\sigma = 1$; segmented markets ($\phi = 0$) gives $\sigma = 1+\varphi$.

Sectoral NKPC in reduced form. Labor reallocation is static, but price setting remains forward-looking under Calvo. In this section, we characterize the NKPC by treating the expectations $\{\mathbb{E}_t \pi_{it+1}\}$ as given; the full dynamic solution is deferred to Section 4. Assuming the economy starts from steady state at $t = -1$ (such that $\chi_{i,-1} = 0$ and $\hat{a}_{i,-1} = 0$), we can express the relative price distortion as:

$$\chi_{it} = (\pi_{it} - \pi_t^C) + \chi_{i,t-1} + \Delta\hat{a}_{it}, \tag{37}$$

where $\hat{a}_{it} \equiv a_{it} - \sum_j \alpha_j a_{jt}$ is demeaned productivity. Substituting (36) into (20) and solving for CPI inflation (Appendix A), we obtain:

$$\begin{aligned} \pi_{it} = b_i\,\tilde{y}_t - \frac{\bar{\kappa}_i}{1-\bar{\kappa}^\alpha}\left[(1-\bar{\kappa}^\alpha)(\chi_{i,t-1} + \Delta\hat{a}_{it}) + \textstyle\sum_j \alpha_j \bar{\kappa}_j (\chi_{j,t-1} + \Delta\hat{a}_{jt})\right] \\ + \frac{\beta}{1+\sigma\kappa_i}\mathbb{E}_t\pi_{it+1} + \frac{\beta\bar{\kappa}_i}{1-\bar{\kappa}^\alpha}\textstyle\sum_j \frac{\alpha_j}{1+\sigma\kappa_j}\mathbb{E}_t\pi_{jt+1} \end{aligned} \tag{38}$$

where we define the *adjusted Calvo coefficient*

$$\bar{\kappa}_i \equiv \frac{\sigma\kappa_i}{1+\sigma\kappa_i} \in (0,1), \qquad \bar{\kappa}^\alpha \equiv \sum_i \alpha_i \bar{\kappa}_i, \tag{39}$$

and the *sectoral output gap slope*

$$b_i \equiv \frac{\gamma + \varphi}{\sigma} \frac{\bar{\kappa}_i}{1 - \bar{\kappa}^\alpha} = \frac{(\gamma + \varphi)\, \kappa_i}{(1 + \sigma\kappa_i)(1 - \bar{\kappa}^\alpha)}. \tag{40}$$

The coefficient $\bar{\kappa}_i$ measures how responsive sector-i inflation is to contemporaneous cost pressures after accounting for the feedback between mobility and relative prices: it is increasing in both the Calvo slope κ_i and the mobility-adjusted elasticity σ.

Corollary 1. *When $\kappa_i = \kappa$ for all i, $1 - \bar{\kappa}^\alpha = (1 + \sigma\kappa)^{-1}$, so $b_i = (\gamma + \varphi)\kappa$ for all i, independent of σ. Mobility then affects the Phillips curve only through cost-push shocks and relative price dynamics, not through the output gap slope.*

3.4 Monetary Policy Problem and Optimal Rule

We now formalize the central bank's problem and characterize optimal monetary policy. Because the efficient steady state is supported by sectoral subsidies that neutralize monopolistic distortions, the first-order terms in the welfare expansion vanish, justifying a linear-quadratic (LQ) approach.

Proposition 1 (Discretionary LQ Problem)**.** *In each period t, the central bank chooses the output gap $\tilde{y}_t$ to minimize the second-order approximation of the welfare loss $\mathcal{L}_t$ subject to the sectoral Phillips curve:*

$$\min_{\tilde{y}_t} \mathcal{L}_t \overset{(2)}{=} \frac{\gamma + \varphi}{2}\, \tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i}\, \pi_{it}^2 + \frac{\sigma}{2} \sum_i \alpha_i \chi_{it}^2, \tag{41}$$

subject to the sectoral Phillips curve

$$\pi_{it} = b_i\, \tilde{y}_t + u_{it}, \tag{42}$$

where $\sigma = 1 + \frac{\varphi}{1+\phi}$ is the mobility sufficient statistic and χ_{it} is the relative price distortion.

Proof. Derived in Appendix B. □

The three terms in (41) identify distinct economic inefficiencies. The first two—the aggregate output gap and within-sector Calvo price dispersion—are standard. The third term is the novel contribution: it penalizes cross-sector relative price distortions, which induce inefficient reallocation of expenditure and labor. Under segmented markets ($\sigma = 1 + \varphi$), relative price distortions are fully absorbed by hours, generating large welfare costs; as mobility increases ($\sigma \to 1$), worker flows partially offset these distortions, reducing the welfare penalty to residual expenditure misallocation (Rubbo, 2023). That σ appears both as the marginal cost slope on relative price distortions (36) and as the welfare weight here is not a coincidence: both reflect the same margin decomposition (34)–(35), making σ a *sufficient statistic* for sectoral reallocation in the static problem.

The compact Phillips curve (42) makes clear that the central bank's only instrument is $\tilde{y}_t$. Each unit of aggregate demand shifts sector-i inflation by the slope b_i and, by the identity (37), relative price distortions by $(b_i - b^C)$, where $b^C \equiv \sum_j \alpha_j b_j$ is the weighted average slope. All other terms—initial price wedges, productivity shocks, and inflation expectations—are embedded in u_{it} and taken as given under discretion.

Lemma 3 (Output-Gap-Targeting Inflation Index)**.** *Define the α/κ-weighted inflation index $\pi_t^R \equiv \sum_i \frac{\alpha_i}{\kappa_i}\pi_{it}$. Then*

$$\sum_i \frac{\alpha_i}{\kappa_i}(\pi_{it} - \beta \mathbb{E}_t \pi_{it+1}) = (\gamma + \varphi)\tilde{y}_t. \tag{43}$$

In particular, if policy implements $\pi_t^R = 0$ for all t (so that $\mathbb{E}_t \pi_{t+1}^R = 0$), then $\tilde{y}_t = 0$.

Proof. Summing the NKPC (20) with weights α_i/κ_i gives $\sum_i \frac{\alpha_i}{\kappa_i}(\pi_{it} - \beta\,\mathbb{E}_t \pi_{it+1}) = \sum_i \alpha_i(mc_{it} - p_{it})$. From (33), $mc_{it} - p_{it} = \gamma\tilde{y}_t + \varphi\tilde{l}_{it} - \chi_{it}$. Using $\chi_{it} = \tilde{y}_t - \tilde{h}_{it}$ and $\tilde{h}_{it} = \tilde{n}_{it} + \tilde{l}_{it}$, then substituting the static reallocation condition $\tilde{n}_{it} = \phi(\tilde{l}_{it} - \tilde{y}_t)$:

$$mc_{it} - p_{it} = (\gamma - 1 - \phi)\,\tilde{y}_t + (1 + \varphi + \phi)\,\tilde{l}_{it}.$$

Weighting by α_i and summing:

$$\sum_i \alpha_i(mc_{it} - p_{it}) = (\gamma - 1 - \phi)\,\tilde{y}_t + (1 + \varphi + \phi)\,\tilde{l}_t^{\alpha} = (\gamma + \varphi)\,\tilde{y}_t,$$

where the last equality uses $\tilde{l}_t^{\alpha} = \tilde{y}_t$ (Lemma 1). All terms involving ϕ cancel. □

Lemma 3 demonstrates that an α/κ-weighted inflation index perfectly tracks the output gap regardless of the mobility parameter ϕ. This extends the aggregation insight of Rubbo (2023) to economies with frictional labor mobility: because the intensive and extensive margin adjustments perfectly offset each other upon aggregation, all mobility terms cancel. If the central bank could perfectly stabilize π_t^R, the output gap would be closed.

However, this policy leaves the dispersion terms in (41) nonzero, since $\pi_t^R = 0$ does not imply $\pi_{it} = 0$ or $\chi_{it} = 0$ for each i. The optimal policy must therefore trade off output gap stabilization against cross-sector misallocation; we formalize this trade-off in the following targeting rule, obtained by the first-order condition of the planner's linear-quadratic problem:

Proposition 2 (Optimal Discretionary Targeting Rule)**.** *The optimal discretionary policy satisfies*

$$0 = (\gamma + \varphi)\,\tilde{y}_t + \sum_i \alpha_i\omega_i\,\pi_{it} + c_t, \tag{44}$$

with sectoral inflation weights

$$\omega_i = \underbrace{\frac{\epsilon_i}{\kappa_i} b_i}_{\text{within-sector (Calvo)}} + \underbrace{\sigma(b_i - b^C)}_{\text{cross-sector (dispersion)}}, \tag{45}$$

and cost-push term $c_t = \sigma \sum_i \alpha_i (b_i - b^C)(\chi_{i,t-1} + \Delta \hat{a}_{it})$.

Proof. The central bank's instrument is $\tilde{y}_t$. From (42), $\partial \pi_{it} / \partial \tilde{y}_t = b_i$. From (37), $\partial \chi_{it} / \partial \tilde{y}_t = b_i - b^C$, where $b^C \equiv \sum_j \alpha_j b_j$. Differentiating (41) with respect to $\tilde{y}_t$:

$$0 = (\gamma + \varphi)\,\tilde{y}_t + \sum_i \frac{\alpha_i \epsilon_i}{\kappa_i}\,\pi_{it}\,b_i + \sigma \sum_i \alpha_i\,\chi_{it}\,(b_i - b^C).$$

Substituting $\chi_{it} = \chi_{i,t-1} + \pi_{it} - \pi_t^C + \Delta \hat{a}_{it}$ and collecting the period-t inflation terms (using π_{is} for $s < t$ as given under discretion) yields (44) with $\omega_i = \frac{\epsilon_i}{\kappa_i} b_i + \sigma(b_i - b^C)$ and $c_t = \sigma \sum_i \alpha_i (b_i - b^C)(\chi_{i,t-1} + \Delta \hat{a}_{it})$. □

The weight ω_i on sector-i inflation comprises two channels. The Calvo channel, $\frac{\epsilon_i}{\kappa_i} b_i$, weighs sticky sectors: high ϵ_i / κ_i means that inflation in sector i generates large within-sector price dispersion, so the central bank prioritizes stabilizing these sectors. This channel is standard and appears in any New Keynesian model.

The dispersion channel, $\sigma(b_i - b^C)$, is novel and depends on mobility. It gives positive (negative) weight to sectors whose Phillips curve is steeper (flatter) than average. Intuitively, these are the sectors where a unit change in $\tilde{y}_t$ produces the largest relative price movement, generating cross-sector misallocation. The magnitude of this channel is scaled by σ: under segmented markets ($\sigma = 1 + \varphi$), cross-sector misallocation is costly and the dispersion channel is large; under mobile labor ($\sigma \to 1$), workers absorb relative price distortions and the dispersion channel shrinks.

When $\kappa_i = \kappa$ for all i, $b_i = b^C$ and the dispersion channel vanishes entirely. The optimal index then weights sectoral inflation solely through within-sector dispersion costs, proportional to ϵ_i. If we further assume uniform substitutability ($\epsilon_i = \epsilon$), optimal policy perfectly reduces to stabilizing the CPI, consistent with Corollary 1 under constant κ_i. More generally, the optimal policy puts weight on both sticky and flexible sectors—sticky sectors through the Calvo channel, and flexible sectors (sectors with high b_i) through the dispersion channel.

A back-of-the-envelope illustration. To gauge the quantitative importance of the two channels, consider a simple two-sector calibration. We approximate the distribution of sectoral price rigidities documented in Pastén et al. (2024) by grouping sectors into a flexible sector (food and energy, $\alpha_1 = 0.5$, average monthly adjustment frequency of 40%) and a sticky sector (remaining goods, $\alpha_2 = 0.5$, average monthly frequency of 12%), implying quarterly Calvo parameters

	Segmented Markets ($\sigma = 3$)			Integrated Markets ($\sigma = 1$)		
	Calvo	Disp.	Total (ω_i)	Calvo	Disp.	Total (ω_i)
Flexible ($i = 1$)	7.0	+2.1	**9.1**	11.1	+1.6	**12.7**
Sticky ($i = 2$)	41.0	−2.1	**38.9**	36.9	−1.6	**35.3**

Table 1: Two-sector experiment with segmented vs integrated markets.

$\delta_1 = 0.2$ ($\kappa_1 \approx 3.2$) and $\delta_2 = 0.6$ ($\kappa_2 \approx 0.27$). Set $\gamma = 1$, $\varphi = 2$, $\beta = 0.99$, and $\epsilon_i = 8$ for both sectors, following Rubbo (2023).

To illustrate the role of labor mobility, we compare the optimal targeting weights under segmented markets ($\sigma = 1 + \varphi = 3$) and fully integrated markets ($\sigma \to 1$). Moving from segmented to integrated markets drastically alters the relative steepness of the sectoral Phillips curves. Under segmented markets, the output gap slopes are $b_1 \approx 2.8$ (flexible) and $b_2 \approx 1.4$ (sticky). Under integrated markets, the flexible sector can freely absorb workers, causing its slope to steepen to $b_1 \approx 4.5$, while the sticky sector flattens slightly to $b_2 \approx 1.2$. This shift in responsiveness reshapes the optimal weights:

The results are in Table 1. Under standard calibrations ($\epsilon = 8$), the Calvo channel dominates, prioritizing the sticky sector in both regimes. However, the exact targeting weights are sensitive to labor mobility. As labor markets become more mobile ($\sigma \to 1$), the flexible sector's ability to easily hire workers makes it far more responsive to aggregate demand. This amplified slope (b_1) significantly raises the Calvo penalty for flexible-sector inflation ($\frac{\epsilon_1}{\kappa_1} b_1$). Consequently, the optimal index shifts *toward* the flexible sector, with the sticky-to-flexible weight ratio falling from roughly 4.3x under segmented markets to 2.8x under integrated markets.

The resulting targeting rule confirms that incorrect assumptions about labor mobility will generically lead to suboptimal policy. A central bank that falsely assumes integrated markets will under-weight the sticky sector. When labor cannot reallocate easily, the central bank must tilt its focus heavily toward stabilizing sticky prices to prevent severe misallocation.

These forces are enriched in the dynamic economy, which we present next. When employment is a predetermined state variable, labor misallocation cannot be corrected within the period, raising the intertemporal cost of relative price distortions. But predetermined employment also creates a new policy margin absent from the static model: by deliberately tilting relative prices today, the planner can redirect worker flows and improve tomorrow's employment distribution. The dynamic optimal policy balances these two forces.

4 Dynamic Economy: Flows as Sufficient Statistic

We now turn to the general case in which labor reallocation is gradual: employment shares $\{N_{it}\}$ are predetermined state variables that evolve over time through worker transitions. The key new feature relative to the static economy of Section 3 is that sectoral employment cannot jump to absorb shocks within the period. Instead, reallocation occurs through forward-looking reallocation decisions that depend on the expected path of sectoral conditions. This generates richer dynamics—in particular, an interaction between the speed of price adjustment and the speed of labor reallocation—but the model remains tractable: the entire micro reallocation structure enters through just two objects, the mobility elasticity ϕ and the steady-state transition matrix $\bar{\mu}$.

4.1 Timing and the Reallocation Block

We now work with the full dynamic reallocation block (24)–(26) from Section 2.5. Employment shares $\mathbf{n}_t$ are predetermined: workers choose transitions at the end of period t, but arrive in their new sector at $t+1$. The key contrast with the static economy of Section 3 is that $\mathbf{n}_t$ can no longer be substituted out of marginal cost using a contemporaneous reallocation condition. Instead, the path of employment is governed by the forward-looking Bellman recursion (25). Iterating forward:

$$\mathbf{v}_t = \sum_{s=0}^{\infty} \beta^s \bar{\mu}^s \, \mathbb{E}_t[\mathbf{u}_{t+s}], \tag{46}$$

so the continuation value in each sector reflects the entire expected path of flow utilities, discounted by both β and $\bar{\mu}$ (which captures the option value of future reallocation). A sector that is temporarily depressed but expected to recover retains workers through high v_{it}, even if current flow utility u_{it} is low.

Recall that the sectoral NKPC (21) can be written as $\pi_{it} = \kappa_i[(\gamma+\varphi)y_t - (1+\varphi)(\hat{p}_{it}+a_{it}) - \varphi\, n_{it}] + \beta\,\mathbb{E}_t\pi_{it+1}$: employment enters with coefficient $-\varphi\kappa_i$, replacing the sufficient statistic σ of the static Phillips curve 36. The effect of mobility on inflation now operates through the *path* of $\mathbf{n}_t$, not a contemporaneous elasticity. Only two features of the micro reallocation structure enter the linearized system.

Proposition 3 (Dynamic Sufficient Statistics)**.** *With predetermined employment shares and logit reallocation, the linearized dynamic system depends on the labor reallocation structure only through:*

(i) the mobility elasticity $\phi = \varphi\theta\zeta\bar{L}^{1+\varphi}$, and

(ii) the steady-state transition matrix $\bar{\mu}$.

The limiting cases are: $\phi \to 0$ (segmented markets), in which $\mathbf{n}_t = \mathbf{0}$ for all t; and $\phi \to \infty$ (free mobility), in which expected continuation values equalize across sectors ($\mathbb{E}_t[\mathbf{v}_{t+1}] \propto \mathbf{1}$) and, for all $t \geq 1$, employment adjusts to offset relative price distortions ($\mathbf{n}_t = -\chi_t$ in relative terms).

Proof. Inspection of (24)–(26): the transition rates $\{\mu^{ij}\}$ enter only through $\bar{\mu}$ and ϕ. When $\phi = 0$, (24) gives $\mathbf{n}_{t+1} = \bar{\mu}\,\mathbf{n}_t$ with $\mathbf{n}_0 = \mathbf{0}$, so $\mathbf{n}_t = \mathbf{0}$.

When $\phi \to \infty$, bounded $\mathbf{n}_{t+1}$ in (24) requires $(I - \bar{\mu}^2)\mathbb{E}_t[\mathbf{v}_{t+1}] \to \mathbf{0}$. Because $\bar{\mu}$ is a stochastic matrix, the null space of $(I - \bar{\mu}^2)$ is spanned by the constant vector $\mathbf{1}$. Therefore, expected continuation values equalize across sectors: $\mathbb{E}_t[\mathbf{v}_{t+1}] = c_t\mathbf{1}$ for some scalar c_t.

Assuming perfect foresight paths following the initial shock, $\mathbf{v}_t = \mathbb{E}_{t-1}[\mathbf{v}_t] = c_{t-1}\mathbf{1}$ for $t \geq 1$. Substituting this into (25) yields $\mathbf{u}_t = (c_{t-1} - \beta c_t)\mathbf{1}$, meaning flow utilities also equalize. Because we work in relative log-deviations (where aggregate constraints imply cross-sector means are zero), cross-sector equalization requires $\mathbf{u}_t = \mathbf{0}$ for $t \geq 1$. By (26), this implies $\mathbf{n}_t = -\chi_t$ for $t \geq 1$. At impact ($t = 0$), $\mathbf{n}_0$ is predetermined and cannot jump, so $\mathbf{u}_0$ remains un-equalized. □

In the static limit, all reallocation information collapses to the scalar $\sigma = 1 + \frac{\varphi}{1+\phi}$. In the dynamic economy, the matrix $\bar{\mu}$ plays an additional role: its eigenvalues govern the speed at which workers reallocate, creating an interaction between price stickiness and labor mobility. Before analyzing this interaction, we must first characterize the flexible-price benchmark, which now differs from the static case because inherited employment misallocation affects sectoral hours and natural relative prices even when all prices are free to adjust.

4.2 Dynamic Flexible-Price Allocation

When employment shares are predetermined, the natural allocation differs from the static case Lemma 2) because the planner can adjust the intensive margin within the period but must wait for the reallocation block to correct the extensive margin.

Lemma 4 (Dynamic Natural Allocation)**.** *When employment shares $\mathbf{n}_t$ are predetermined, the flexible-price allocation satisfies:*

(i) Natural output $y_t^{\text{nat}} = \frac{1+\varphi}{\gamma+\varphi}\sum_i \alpha_i a_{it}$ and the natural real rate $r_t^{\text{nat}} = -\ln\beta + \gamma\,\mathbb{E}_t(y_{t+1}^{\text{nat}} - y_t^{\text{nat}})$ are unchanged from the static case and independent of $\mathbf{n}_t$.

(ii) Sectoral hours and natural relative prices depend on inherited employment:

$$l_{it}^{\text{nat}} = \bar{l}_t^{\text{nat}} - \frac{1}{1+\varphi}\,n_{it}, \tag{47}$$

$$\hat{p}_{it}^{\text{nat}} = \hat{p}_{it}^{\text{nat,static}} - \frac{\varphi}{1+\varphi}\,n_{it}. \tag{48}$$

An overstaffed sector ($n_{it} > 0$) has lower hours and a lower natural relative price.

(iii) Real marginal cost in gaps decomposes as:

$$mc_{it} - p_{it} = (\gamma+\varphi)\,\tilde{y}_t - (1+\varphi)\,\chi_{it} - \varphi\,n_{it}, \tag{49}$$

where $\chi_{it} \equiv \hat{p}_{it} + \hat{a}_{it}$ *is the deviation of relative prices from their* static *natural benchmark* $\hat{p}_{it}^{nat,static} = \sum_j \alpha_j a_{jt} - a_{it}$.[4]

Proof. See Appendix C. □

Part (iii) is the key equation for what follows. In the static economy, the reallocation condition $n_{it} = -\frac{\phi}{1+\phi}\chi_{it}$ combines the last two terms into $-\sigma\chi_{it}$. With predetermined employment, relative price distortions and employment misallocation enter marginal cost as independent cost shifters. Substituting into the NKPC yields the dynamic Phillips curve (67), in which employment appears with coefficient $-\varphi\kappa_i$. Under flexible prices, overstaffed sectors gradually shed workers as lower flow utility reduces their continuation values; Appendix C characterizes these natural reallocation dynamics explicitly. Sticky prices distort this process: the relative price distortion χ_{it} contaminates the flow-utility signals that drive reallocation, and the resulting employment misallocation feeds back into marginal cost—a two-way interaction absent in the static economy.

4.3 Constant κ: Reallocation Dynamics and Inflation Propagation

In this subsection, we assume homogeneous stickiness: $\kappa_i = \kappa$ for all i. When all sectors share the same Calvo frequency, the linearized system separates cleanly into two independent blocks. An aggregate block recovers the standard one-sector New Keynesian model and is the only block that monetary policy can influence. A relative block governs sectoral inflation differentials and labor reallocation, driven entirely by cross-sectoral productivity gaps and beyond the reach of the interest rate. This separation lets us isolate the core mechanism through which labor mobility shapes the *persistence* of relative inflation.

Lemma 5 (Block Separation)**.** *If* $\kappa_i = \kappa$ *for all* i*, the linearized system decomposes into:*

(i) An aggregate block *in* (π_t^C, y_t)*, equivalent to a standard one-sector New Keynesian model:*

$$\pi_t^C = \kappa[(\gamma+\varphi)\, y_t - (1+\varphi)\, \bar{a}_t] + \beta\, \mathbb{E}_t \pi_{t+1}^C, \tag{50}$$
$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\gamma}(i_t - \mathbb{E}_t \pi_{t+1}^C - r_t^{nat}), \tag{51}$$

where $\bar{a}_t = \sum_j \alpha_j a_{jt}$*, closed by the Taylor rule* $i_t = \psi_\pi \pi_t^C + \psi_y \tilde{y}_t$.

[4] Note that the true dynamic natural relative price is $\hat{p}_{it}^{\text{nat}} = \hat{p}_{it}^{\text{nat,static}} - \frac{\varphi}{1+\varphi}\, n_{it}$ (part (ii)), so the deviation from the contemporaneous flexible-price benchmark is $\hat{p}_{it} - \hat{p}_{it}^{\text{nat}} = \chi_{it} + \frac{\varphi}{1+\varphi}\, n_{it}$. We retain the static-benchmark χ_{it} because it and n_{it} then enter as *independent* cost shifters in (49), making the two margins of adjustment—price gaps versus employment gaps—transparent.

(ii) A relative block *in* $(\hat{\boldsymbol{\pi}}_t, \hat{\mathbf{p}}_t, \mathbf{n}_t, \hat{\mathbf{v}}_t)$ *that is orthogonal to* y_t *at first order:*

$$\hat{\boldsymbol{\pi}}_t = -\kappa[(1+\varphi)\,\boldsymbol{\chi}_t + \varphi\,\mathbf{n}_t] + \beta\,\mathbb{E}_t\hat{\boldsymbol{\pi}}_{t+1}, \tag{52}$$

$$\boldsymbol{\chi}_t = \boldsymbol{\chi}_{t-1} + \hat{\boldsymbol{\pi}}_t + \Delta\hat{\mathbf{a}}_t, \tag{53}$$

$$\mathbf{n}_{t+1} = \bar{\mu}\,\mathbf{n}_t + \phi\beta(I-\bar{\mu}^2)\,\mathbb{E}_t[\hat{\mathbf{v}}_{t+1}], \tag{54}$$

$$\hat{\mathbf{v}}_t = -\boldsymbol{\chi}_t - \mathbf{n}_t + \beta\bar{\mu}\,\mathbb{E}_t[\hat{\mathbf{v}}_{t+1}], \tag{55}$$

where $\boldsymbol{\chi}_t \equiv \hat{\mathbf{p}}_t + \hat{\mathbf{a}}_t$ *and hats denote demeaning* ($\hat{x}_{it} = x_{it} - \sum_j \alpha_j x_{jt}$).

As a result, monetary policy (y_t) *affects the aggregate block but cannot directly influence relative inflation or reallocation dynamics.*

Proof. With $\kappa_i = \kappa$, weighting the NKPC (21) by α_i and summing gives (50), using $\sum_i \alpha_i n_{it} = 0$. Subtracting yields the relative NKPC (52), which depends only on $\boldsymbol{\chi}_t$ and $\mathbf{n}_t$. Orthogonality of the reallocation block follows from $(I-\bar{\mu}^2)\mathbf{1} = \mathbf{0}$: since $u_{it} = y_t - \hat{p}_{it} - a_{it} - n_{it}$, the operator $(I-\bar{\mu}^2)$ applied to $\mathbf{u}_t$ annihilates the $y_t\mathbf{1}$ component, so reallocation responds only to cross-sectoral differences. □

The separation has a clean economic interpretation. When all sectors share the same Calvo frequency, aggregate demand y_t shifts marginal cost uniformly across sectors, leaving relative prices and hence reallocation incentives unchanged. Relative inflation dynamics are driven entirely by sectoral productivity differentials $\hat{\mathbf{a}}_t$. This extends Corollary 1 to the dynamic setting: under constant κ, mobility matters for the relative block but is irrelevant for the aggregate Phillips curve.

Within the relative block, two forces drive convergence after a sectoral shock: prices adjust through the Calvo mechanism, and workers reallocate across sectors. Each operates at its own speed, and the slower of the two acts as a bottleneck for the persistence of inflation differentials. To disentangle these margins, we decompose the reallocation network $\bar{\mu}$ into orthogonal components along which both price adjustment and labor reallocation dynamics are self-contained.

Spectral decomposition. Sectoral shocks trigger cascading flows through the reallocation network $\bar{\mu}$. We diagonalize $\bar{\mu}$ to isolate orthogonal combinations of sectors, allowing us to independently characterize relative adjustment dynamics.[5] Detailed balance (Assumption 1) ensures $\bar{\mu}$ is diagonalizable, while a constant κ across sectors guarantees the complete decoupling of the relative block (52)–(55).

[5]This follows the spectral approach to spatial transition matrices in Kleinman et al. (2023) and to input-output networks in Liu and Tsyvinski (2024).

Lemma 6 (Symmetrization)**.** *Under detailed balance, $A\bar{\mu} = \bar{\mu}^T A$ where $A = \mathrm{diag}(\alpha_i)$. The matrix $Q = A^{1/2}\bar{\mu}\, A^{-1/2}$ is therefore symmetric, with eigenvalues $1 = \rho_1 > \rho_2 \geq \cdots \geq \rho_N > -1$ and orthonormal eigenvectors $\{s_k\}$. The coordinate transformation $x^*_{kt} = s_k^T A^{1/2}\hat{\mathbf{x}}_t$ for $x \in \{\chi, n, \hat{v}, \hat{a}\}$ decomposes the relative block into $N-1$ independent three-variable systems in $(\chi^*_{kt}, n^*_{kt}, \hat{v}^*_{kt})$, one for each* spectral component $k \geq 2$.

Proof. See Appendix C. □

Each eigenvector s_k isolates an orthogonal *component*, or "reallocation corridor" whose dynamics are perfectly self-contained (e.g., the relative employment difference in two sectors isolated from the rest of the economy). Transforming back to the original sector basis, define $v_{k,i} \equiv \alpha_i^{-1/2} s_{k,i}$, so that $v_{k,i}$ is sector i's loading on component k and $x^*_{kt} = \sum_i \alpha_i\, v_{k,i}\, \hat{x}_{it}$. Throughout, indices i, j refer to sectors and k to spectral components.

The eigenvalue magnitude, $|\rho_k|$, governs the speed of mean reversion along this corridor. The largest non-unit eigenvalue, $\max_{k\geq 2} |\rho_k|$, therefore identifies the economy's slowest mean-reverting pattern: the structural bottleneck in labor reallocation.[6]

In the transformed coordinates, component k satisfies:

$$\beta\,\mathbb{E}_t[\chi^*_{kt+1}] - [\kappa(1+\varphi) + \beta + 1]\,\chi^*_{kt} + \chi^*_{kt-1} - \kappa\varphi\, n^*_{kt} = -\Delta\hat{a}^*_{kt} + \beta\,\mathbb{E}_t[\Delta\hat{a}^*_{kt+1}], \tag{56}$$

$$n^*_{kt+1} = \rho_k\, n^*_{kt} + \phi\beta(1-\rho_k^2)\,\mathbb{E}_t[\hat{v}^*_{kt+1}], \qquad \hat{v}^*_{kt} = -\chi^*_{kt} - n^*_{kt} + \beta\rho_k\,\mathbb{E}_t[\hat{v}^*_{kt+1}]. \tag{57}$$

The first equation is a decoupled Phillips curve for component k. Uniform price stickiness ($\kappa_i = \kappa$) ensures the NKPC naturally commutes to this dimension, correcting relative price distortions χ^*_{kt} subject to the employment cost-push shifter n^*_{kt}. The second pair governs reallocation: workers flow to close the steady-state gap $-\chi^*_{kt} - n^*_{kt}$ at speeds dictated by ρ_k and ϕ. The persistence of relative inflation depends on the interaction between these price and quantity margins. We build intuition from two limiting cases.

Segmented markets ($\phi = 0$). Consider a permanent relative productivity shock $\hat{a}^*_{kt} = \hat{a}$ for $t \geq 0$. Without worker mobility, employment is fixed ($n^*_{kt} = 0$) and (56) becomes a univariate second-order difference equation in χ^*_{kt}. The unique stable solution is $\chi^*_{kt} = \lambda^t_{\mathrm{seg}}\,\chi^*_{k0}$, where $\lambda_{\mathrm{seg}} \in (0,1)$ solves:

$$\beta\lambda^2 - [\kappa(1+\varphi) + \beta + 1]\lambda + 1 = 0. \tag{58}$$

Evaluating at $t = 0$ yields the initial condition $\chi^*_{k0} = \lambda_{\mathrm{seg}}\hat{a}$ and impact inflation $\hat{\pi}^*_{k0} = (\lambda_{\mathrm{seg}} - 1)\hat{a}$. Convergence occurs at rate λ_{seg}, the speed at which the sectoral NKPC alone closes relative price gaps.

[6] Under the logit specification (11), $\bar{\mu}$ is strictly positive. By the Perron-Frobenius theorem, the dominant eigenvalue $\rho_1 = 1$ is unique and $\rho_N > -1$, ruling out non-decaying periodic oscillations.

Integrated markets ($\phi \to \infty$). When workers move freely, continuation values equalize, requiring $n^*_{kt} = -\chi^*_{kt}$ for $t \geq 1$ (Proposition 3). Substituting into (56), the φ component of the slope cancels—reallocation absorbs the intensive-margin pressure—and the stable solution is $\chi^*_{kt} = \lambda^t_{\text{int}} \chi^*_{k0}$, where $\lambda_{\text{int}} \in (0, 1)$ solves

$$\beta\lambda^2 - (\kappa + \beta + 1)\lambda + 1 = 0. \tag{59}$$

At impact, however, employment has not yet adjusted ($n^*_{k0} = 0$), so the initial condition is determined by the NKPC evaluated at $t = 0$ with expectations:

$$\chi^*_{k0} = \frac{1}{1 + \beta + \kappa(1 + \varphi) - \beta\lambda_{\text{int}}} \hat{a}. \tag{60}$$

Comparing (58) and (59), the only difference is the coefficient on λ: $\kappa(1 + \varphi)$ versus κ. Since $\kappa\varphi > 0$, it follows that $\lambda_{\text{seg}} < \lambda_{\text{int}}$: relative price gaps decay *more slowly* under integrated markets.

Proposition 4 (Impact vs. Persistence Trade-off)**.** *Under a permanent relative productivity shock $\hat{a}^*_{kt} = \hat{a}$:*

(i) Segmented markets *yield larger initial relative inflation ($|\hat{\pi}^*_{k0}|$ higher) but faster subsequent decay (rate λ_{seg}).*

(ii) Integrated markets *yield smaller initial relative inflation but slower subsequent decay ($\lambda_{int} > \lambda_{seg}$).*

In both cases, total cumulative relative inflation is $\sum_{t=0}^{\infty} \hat{\pi}^*_{kt} = -\hat{a}$.

Proof sketch. See Appendix C. Without reallocation, segmented markets face unmitigated intensive-margin cost pressures ($\kappa\varphi$), driving a larger initial inflation jump but faster subsequent decay ($\lambda_{\text{seg}} < \lambda_{\text{int}}$). Conversely, integrated markets anticipate future cost relief via worker flows, which dampens impact inflation but slows convergence. Finally, cumulative inflation telescopes to $-\hat{a}$ because long-run relative prices must fully adjust to the shock regardless of mobility. □

The intuition comes from the forward-looking firm price setting. A price-resetting firm sets its price as a markup over expected discounted future marginal costs, which in component k depend on both the price–productivity gap χ^*_{kt} and the employment cost-push term n^*_{kt}. Under integrated markets, firms anticipate that worker inflows will progressively offset χ^*_{kt} through n^*_{kt}, compressing marginal-cost gaps at future horizons. This anticipated relief reduces the price adjustment firms choose *today*—dampening impact inflation—but simultaneously weakens the cost pressure that drives convergence, raising the decay rate from λ_{seg} to λ_{int}.

Persistence for arbitrary mobility. The two limiting cases are endpoints of a continuous family of dynamics parametrized by ϕ. Conjecturing $\chi_{kt}^* = \lambda^t C$, $n_{kt}^* = \lambda^t N$, $\hat{v}_{kt}^* = \lambda^t V$ in (56)–(57) after shocks have died out, and eliminating V and N, yields the system's characteristic equation which pin down the persistence of inflation.

Proposition 5 (Persistence and Bottleneck Effect)**.** *For each component $k \geq 2$, the persistence λ of the dynamic system satisfies a degree-4 polynomial:*

$$\Big[(\lambda - \rho_k)(1 - \beta\rho_k\lambda) + \phi\beta(1-\rho_k^2)\lambda\Big]\Big[\beta\lambda^2 - (\kappa(1+\varphi) + \beta + 1)\lambda + 1\Big] \\ + \kappa\varphi\phi\beta(1-\rho_k^2)\lambda^2 = 0. \tag{61}$$

This equation yields exactly two stable roots ($|\lambda| < 1$). Let $\lambda^*(\phi)$ denote the maximum absolute value of these stable roots, which governs the persistence of the system along component k. Then $\lambda^*(\phi)$ is continuous in ϕ, with limiting values:

$$\lim_{\phi \to 0} \lambda^*(\phi) = \max\{\lambda_{seg}, |\rho_k|\},$$
$$\lim_{\phi \to \infty} \lambda^*(\phi) = \lambda_{int}.$$

Proof. See Appendix C. □

The polynomial (61) cleanly isolates the two margins of adjustment. The first bracketed term governs labor reallocation, the second recovers the segmented-market Phillips curve (58), and the final additive term captures the cross-equation coupling driven by labor mobility ϕ.

The limit cases formalize the bottleneck effect. Moving from segmented to integrated markets *increases* persistence when price rigidity is the bottleneck ($|\rho_k| < \lambda_{\text{seg}}$), and *decreases* persistence when reallocation is the bottleneck ($|\rho_k| > \lambda_{\text{int}}$). When $\lambda_{\text{seg}} \leq |\rho_k| \leq \lambda_{\text{int}}$, the net effect is ambiguous. At intermediate values of ϕ, the price and reallocation roots interact through the coupling term such that persistence need not vary monotonically.

Ultimately, this nests both limiting cases and makes precise how the eigenvalues of $\bar{\mu}$ determine the structural bottleneck. Estimating $\bar{\mu}$ from worker reallocation data, computing its eigenvalues, and comparing the slowest mixing rate $\max_{k\geq 2} |\rho_k|$ to the segmented price decay λ_{seg} provides a sharp empirical test of whether the bottleneck for sectoral inflation persistence is price rigidity or labor reallocation.

4.4 Heterogeneous κ: The Double Trap and Demand Spillovers

When price stickiness κ_i varies across sectors, the block separation of Lemma 5 breaks down: aggregate demand $\tilde{y}_t$ no longer shifts marginal cost uniformly, and the spectral decomposition of Section 4.3 no longer yields independent components. Two new questions arise. First, does labor mobility *dissipate* or *propagate* sectoral wedges? If displaced workers flow into flexible-price sectors, those neighbors adjust prices on the sticky sector's behalf and misallocation dissipates—persistence is governed by average stickiness $\bar{\kappa}$. If instead workers flow into equally sticky sectors, the receiving sectors cannot adjust prices either. The segmentation of the labor network and price rigidity compound, a *double trap*. Second, does aggregate demand—by moving sticky and flexible prices at different speeds—affect reallocation?

We retain the spectral basis of Section 4.3. While heterogeneous κ_i couples the previously independent components $(\chi^*_{kt}, n^*_{kt}, \hat{v}^*_{kt})$, the $\bar{\mu}$-eigenbasis remains the natural coordinate system because it continues to diagonalize the labor reallocation network. By treating heterogeneous stickiness as a perturbation in this basis, we can capture its primary first-order effect as an *effective slope* for each component, which measures whether the sectors most heavily exposed to that specific reallocation pathway are equipped with flexible prices.

Proposition 6 (Persistence and Reallocation)**.** *Write $\kappa_i = \bar{\kappa} + \varepsilon\,\Delta\kappa_i$ with $\sum_i \alpha_i \Delta\kappa_i = 0$ and $|\varepsilon|$ small.*

(i) Effective slope: For each component $k \geq 2$, define

$$\Delta\kappa_k^{eff} \;\equiv\; \sum_i \alpha_i\,\Delta\kappa_i\,v_{k,i}^2. \tag{62}$$

To first order in ε, the joint dynamics of relative prices and labor reallocation along component k behave as if governed by a uniform Phillips curve with slope $\bar{\kappa} + \varepsilon\,\Delta\kappa_k^{eff}$.

(ii) Persistence: Let λ_k^0 be the dominant stable root of the constant-$\bar{\kappa}$ characteristic polynomial (61) for component k. If λ_k^0 is simple and distinct from the roots of all other components $j \neq k$,

$$\lambda_k^*(\varepsilon) = \lambda_k^0 + \varepsilon\,\Delta\kappa_k^{eff} \cdot \Gamma_k + O(\varepsilon^2), \tag{63}$$

where $\Gamma_k \equiv -\partial_\kappa p(\lambda_k^0; \bar{\kappa}, \rho_k)/\partial_\lambda p(\lambda_k^0; \bar{\kappa}, \rho_k) < 0$ is a constant determined by the baseline solution under constant $\bar{\kappa}$. A negative $\Delta\kappa_k^{eff}$ strictly increases persistence.

Proof. Write $K = \bar{\kappa} I + \varepsilon\ \mathrm{diag}(\Delta\kappa_i)$. Projecting onto the $\bar{\mu}$-eigenbasis (Lemma 6), the perturbation shifts the diagonal entry of the component-k polynomial by $\varepsilon\,\Delta\kappa_k^{\mathrm{eff}}$, while off-diagonal couplings to other components enter only at $O(\varepsilon^2)$. The implicit function theorem applied to $p(\lambda; \bar{\kappa} + \varepsilon\,\Delta\kappa_k^{\mathrm{eff}}, \rho_k) = 0$ at the simple root λ_k^0 yields (63). See Appendix C for the formal argument and explicit formulas for Γ_k. □

When a shock requires labor reallocation along component k, worker mobility along that component depends on the ability of exposed sectors to adjust prices that induce worker flows. If the involved sectors tend to be sticky ($\Delta\kappa_k^{\mathrm{eff}} < 0$), physical labor frictions and nominal rigidities compound into a *double trap*, lowering the effective Phillips curve slope below $\bar{\kappa}$, and pushing the decay rate λ_k^* closer to unity and increasing the duration of relative inflation. This bottleneck is symmetric: a sticky sector delays convergence whether it is shedding workers or absorbing them, explaining the square $v_{k,i}^2$ in equation (62)—the direction of the flow does not matter, only the magnitude of exposure.

Example: identical stickiness, different network position. Consider a symmetric three-sector economy where sectors A and B are sticky ($\kappa_A = \kappa_B < \bar{\kappa}$) and sector C is flexible ($\kappa_C > \bar{\kappa}$). Suppose a shock depresses productivity in A, requiring an outflow of workers. The persistence of this shock depends entirely on which sectors the slowest-mixing—highest non-unit eigenvalue—component k connects.

If component k captures reallocation between A and C (large $|v_{k,A}|$ and $|v_{k,C}|$), the effective slope is near $\bar{\kappa}$. Sector C's price flexibility offsets A's stickiness, dissipating the shock at roughly the baseline rate. Conversely, if the bottleneck lies between A and B (large $|v_{k,A}|$ and $|v_{k,B}|$), the effective slope drops: neither A nor B can adjust prices, and the shock to A persists as if the stickiness is closer to κ_A.

Sector A is equally sticky, but its network position determines the persistence. In the second scenario, the presence of the flexible sector C provides no relief. In fact, in the second (bottleneck) case, even if workers move from A into C along a fast-mixing component, that rapid adjustment only resolves the A–C margin. Sector C clears and drops out of the transition dynamics, leaving the economy trapped in the slow A–B bottleneck and making the residual shock persist as if the flexible sector did not exist.

Aggregate demand spillover into reallocation. Heterogeneous κ_i creates a second, distinct channel: aggregate demand moves relative prices. With constant κ, a change in $\tilde{y}_t$ affects all sectoral Phillips curves uniformly, leaving relative prices and reallocation incentives unchanged. With heterogeneous κ_i, flexible sectors respond more than sticky ones, and the residual—after removing the CPI-weighted mean—is a relative inflation response that affects labor reallocation incentives across sectors, tying aggregate demand to cross-sectional reallocation.

Proposition 7 (Aggregate Demand Spillover)**.** *Suppose $\kappa_i = \bar{\kappa} + \Delta\kappa_i$ with $\sum_i \alpha_i \Delta\kappa_i = 0$. A unit change in $\tilde{y}_t$ shifts relative inflation in component $k \geq 2$ by $(\gamma + \varphi)d_k$, where*

$$d_k \;\equiv\; \sum_i \alpha_i \, \Delta\kappa_i \, v_{k,i} \;=\; \mathrm{Cov}_\alpha(\Delta\kappa_i, v_{k,i}). \tag{64}$$

A persistent change in $\tilde{y}_t$ shifts χ^*_{kt} and $n^*_{k,t+1}$ proportionally to d_k. Under constant κ, $d_k = 0$ for all $k \geq 2$, recovering block separation.

Proof. Project the demeaned Phillips curve response $MK(\gamma + \varphi)\mathbf{1}\,\tilde{y}_t$ onto the $\bar{\mu}$-eigenbasis (Lemma 6). The k-th component receives $v_k^\top A\, MK\, \mathbf{1}\,(\gamma + \varphi)\,\tilde{y}_t = (\gamma + \varphi)d_k\,\tilde{y}_t$, using $\sum_i \alpha_i v_{k,i} = 0$ for $k \geq 2$. □

The spillover coefficient d_k is an exact, global result that holds regardless of the magnitude of dispersion in κ_i. It measures the expenditure-weighted covariance between a sector's price flexibility and its *signed* loading on component k.

The spillover is large when sticky and flexible sectors sit on opposite sides of the reallocation corridor. In this scenario, an aggregate demand expansion moves their prices at different speeds, generating a relative price distortion χ^*_{kt} that acts as a wedge in the labor market. For example, suppose component k captures mobility between energy (flexible, loading positively) and services (sticky, loading negatively). Then $d_k > 0$: a positive output gap raises energy prices faster than service prices. This relative price movement increases the marginal revenue product of labor in the energy sector, actively drawing workers out of sticky services and into flexible energy. Through heterogeneous price transmission, monetary policy inadvertently creates a directed reallocation incentive along this specific corridor.

Two statistics, two channels. The effective slope $\Delta\kappa^{\text{eff}}_k$ and the spillover coefficient d_k capture distinct channels by weighting sectoral stickiness $\Delta\kappa_i$ against different measures of network exposure. The effective slope weights by the squared loading $v^2_{k,i}$, measuring whether heavily *involved* sectors tend to be sticky, and governs the persistence of shocks. In contrast, the spillover coefficient weights by the signed loading $v_{k,i}$, measuring whether sectors on one side of the corridor are systematically stickier than those on the other. It governs whether aggregate demand induces cross-sector labor flows. These statistics are independent: for example, consider a symmetric two-sector economy ($\alpha_1 = \alpha_2 = \frac{1}{2}$) with one flexible and one sticky sector ($\kappa_1 \neq \kappa_2$). The sole relative eigenvector is $v_2 = (1, -1)^\top$. Because the sectors have different stickiness, aggregate demand heavily spills over into the relative block ($d_2 = \frac{1}{2}\Delta\kappa_1 - \frac{1}{2}\Delta\kappa_2 \neq 0$): a positive output gap raises the flexible sector's relative price and actively draws workers toward it. Yet, the effective slope is exactly zero ($\Delta\kappa^{\text{eff}}_2 = \frac{1}{2}\Delta\kappa_1 + \frac{1}{2}\Delta\kappa_2 = 0$).

Observability and Quantitative Validation. Both statistics are directly observable: they require the transition matrix $\bar{\mu}$ estimated from worker flow data together with sectoral price adjustment frequencies. Because empirical dispersion in κ_i is typically large, a natural concern is whether the local perturbation formulas in Propositions 6 and 7 adequately capture the globally heterogeneous system, or if higher-order cross-corridor coupling dominates.

To bridge this gap, in the quantitative section, we compute the exact eigenvalues of the fully coupled system using empirical transition matrices and observed price adjustment frequencies, comparing them to the first-order approximation $\lambda^*_k(\varepsilon)$.[7] We hypothesize that the result holds quantitatively in the data. If, as expected, the cross-corridor coupling is weak enough to preserve the rank-ordering of corridor persistence, then when both statistics are large along the

[7] This part is under construction.

slowest-mixing components, the full heterogeneous model is needed: price stickiness and labor immobility interact, persistence shifts, and—as we show in the next section—monetary policy acquires a nontrivial reallocation margin.

4.5 Optimal Monetary Policy with Commitment

We now formalize the Ramsey problem for monetary policy in the dynamic economy. Unlike the discretionary case, the central bank under commitment chooses a sequence of aggregate demand to minimize the intertemporal loss, accounting for how current policy shapes future labor and price states.

Proposition 8 (Ramsey Problem under Commitment)**.** *Under commitment, the central bank chooses a sequence* $\{\tilde{y}_t, \boldsymbol{\pi}_t, \mathbf{n}_{t+1}, \chi_t\}_{t=0}^{\infty}$ *to minimize the intertemporal welfare loss*

$$\min \quad \mathcal{W} = \sum_{t=0}^{\infty} \beta^t \, \mathcal{L}_t, \tag{65}$$

where the per-period loss is

$$\mathcal{L}_t \overset{(2)}{=} \frac{\gamma+\varphi}{2}\,\tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i}\,\pi_{it}^2 + \sum_i \alpha_i \left[\frac{1+\varphi}{2}\,\chi_{it}^2 + \varphi\,\chi_{it}\,n_{it} + \frac{\varphi}{2}\,n_{it}^2\right], \tag{66}$$

subject to, for each sector i and date t:

$$\pi_{it} = \kappa_i[(\gamma+\varphi)\,\tilde{y}_t - (1+\varphi)\,\chi_{it} - \varphi\,n_{it}] + \beta\,\mathbb{E}_t\pi_{it+1}, \tag{67}$$

$$\chi_{it} = \chi_{it-1} + \pi_{it} - \pi_t^C + \Delta\hat{a}_{it}, \tag{68}$$

$$\mathbf{n}_{t+1} = \bar{\mu}\,\mathbf{n}_t + \phi\beta(I - \bar{\mu}^2)\,\mathbb{E}_t[\hat{\mathbf{v}}_{t+1}], \tag{69}$$

$$\hat{\mathbf{v}}_t = -\boldsymbol{\chi}_t - \mathbf{n}_t + \beta\bar{\mu}\,\mathbb{E}_t[\hat{\mathbf{v}}_{t+1}], \tag{70}$$

taking as given initial conditions $(\hat{\mathbf{p}}_{-1}, \mathbf{n}_0)$ *and the exogenous productivity process* $\{\mathbf{a}_t\}$*.*

Proof. The per-period loss (66) follows from a second-order expansion of household welfare around the efficient steady state (Appendix B). The constraints restate the NKPC (21), relative price identity, and reallocation block from Section 2.5. □

The Ramsey problem differs from its static analogue (Lemma 1) in two ways. First, the state vector now includes predetermined employment $\mathbf{n}_t$ alongside relative prices $\hat{\mathbf{p}}_t$: the central bank inherits both price wedges and labor misallocation from the past. Second, the constraints (69)–(70) create a dynamic linkage absent in the static case. Today's output gap affects relative prices, which serve as signals shifting reallocation incentives, thereby determining tomorrow's employment distribution.

This dynamic structure amplifies the welfare cost of ignoring the reallocation margin. In the static economy, the welfare effects of labor misallocation were confined to a single period, entirely summarized by a contemporaneous penalty on relative price distortions, with the coefficient increasing in mobility frictions. In the dynamic economy, labor misallocation creates a persistent welfare cost.

We can see this directly in the constraints and the loss function. In the dynamic Phillips curve (67), inherited misallocation $-\varphi n_{it}$ acts as a persistent, endogenous cost-push shock. In the intertemporal loss (65), the cross-term $\varphi \chi_{it} n_{it}$ and quadratic term $\frac{\varphi}{2} n_{it}^2$ are no longer absorbed instantly but decay slowly at a rate governed by the transition matrix $\bar{\mu}$. Consequently, current relative price distortions compound inherited employment misallocation, transforming a one-off static penalty into a persistent stream of welfare losses.

Vector notation. To characterize the solution, collect all N sectors into vectors $\boldsymbol{\pi}_t$, $\boldsymbol{\chi}_t$, $\mathbf{n}_t$, $\hat{\mathbf{v}}_t \in \mathbb{R}^N$ and define the diagonal matrices

$$A \equiv \operatorname{diag}(\alpha_i), \qquad K \equiv \operatorname{diag}(\kappa_i), \qquad D_\pi \equiv \operatorname{diag}\left(\tfrac{\alpha_i \epsilon_i}{\kappa_i}\right),$$

together with the *demeaning matrix* $M \equiv I - \mathbf{1}\boldsymbol{\alpha}^\top$: $M\boldsymbol{\pi}_t = \boldsymbol{\pi}_t - \pi_t^C \mathbf{1}$. The constraints (67)–(70) become

$$\boldsymbol{\pi}_t - \beta\,\mathbb{E}_t \boldsymbol{\pi}_{t+1} - K[(\gamma+\varphi)\,\mathbf{1}\,\tilde{y}_t - (1+\varphi)\,\boldsymbol{\chi}_t - \varphi\,\mathbf{n}_t] = \mathbf{0}, \qquad [\boldsymbol{\lambda}_t] \quad \text{(PC)} \tag{71}$$

$$\boldsymbol{\chi}_t - \boldsymbol{\chi}_{t-1} - M\boldsymbol{\pi}_t - \Delta \hat{\mathbf{a}}_t = \mathbf{0}, \qquad [\boldsymbol{\eta}_t] \quad (\chi) \tag{72}$$

$$\mathbf{n}_{t+1} - \bar{\mu}\,\mathbf{n}_t - \phi\beta(I - \bar{\mu}^2)\,\mathbb{E}_t \hat{\mathbf{v}}_{t+1} = \mathbf{0}, \qquad [\boldsymbol{\psi}_t] \quad (n) \tag{73}$$

$$\hat{\mathbf{v}}_t + \boldsymbol{\chi}_t + \mathbf{n}_t - \beta\bar{\mu}\,\mathbb{E}_t \hat{\mathbf{v}}_{t+1} = \mathbf{0}. \qquad [\boldsymbol{\xi}_t] \quad (v) \tag{74}$$

Lagrangian and first-order conditions. Denote by $\boldsymbol{\lambda}_t$, $\boldsymbol{\eta}_t$, $\boldsymbol{\psi}_t$, and $\boldsymbol{\xi}_t$ the vector multipliers attached to the constraints. Each multiplier has a direct economic interpretation: $\boldsymbol{\lambda}_t$ is the shadow cost of price-setting frictions—the marginal welfare gain from relaxing the Phillips curve in sector i; $\boldsymbol{\eta}_t$ captures the welfare value of moving relative prices, and hence reallocation signals; $\boldsymbol{\psi}_t$ measures the cost of employment misallocation over time; and $\boldsymbol{\xi}_t$ captures the welfare benefit of changing workers' expectations about future wages to encourage them to move today. The first-order conditions of the Lagrangian are:

$$(\gamma+\varphi)\,\tilde{y}_t - (\gamma+\varphi)\,\mathbf{1}^\top K\,\boldsymbol{\lambda}_t = 0 \iff \tilde{y}_t = \mathbf{1}^\top K\,\boldsymbol{\lambda}_t, \tag{F$_{\tilde{y}}$}$$

$$D_\pi\,\boldsymbol{\pi}_t + \boldsymbol{\lambda}_t - \boldsymbol{\lambda}_{t-1} - M^\top \boldsymbol{\eta}_t = \mathbf{0}, \tag{F$_\pi$}$$

$$A[(1+\varphi)\,\boldsymbol{\chi}_t + \varphi\,\mathbf{n}_t] + (1+\varphi)\,K\,\boldsymbol{\lambda}_t + \boldsymbol{\eta}_t - \beta\,\mathbb{E}_t\boldsymbol{\eta}_{t+1} + \boldsymbol{\xi}_t = \mathbf{0}, \tag{F$_\chi$}$$

$$A\varphi(\boldsymbol{\chi}_t + \mathbf{n}_t) + \varphi\,K\,\boldsymbol{\lambda}_t + \boldsymbol{\xi}_t - \bar{\mu}^\top \boldsymbol{\psi}_t + \beta^{-1}\boldsymbol{\psi}_{t-1} = \mathbf{0}, \tag{F$_n$}$$

$$\boldsymbol{\xi}_t - \bar{\mu}^\top \boldsymbol{\xi}_{t-1} - \phi\,(I-\bar{\mu}^2)^\top \boldsymbol{\psi}_{t-1} = \mathbf{0}. \tag{F$_v$}$$

Under commitment from date 0, the initial conditions $\boldsymbol{\lambda}_{-1} = \boldsymbol{\psi}_{-1} = \boldsymbol{\xi}_{-1} = \mathbf{0}$ apply.

Reading the FOCs. Condition (F$_{\tilde{y}}$) dictates when the central bank should deviate from a zero output gap. The central bank has expansionary bias if the most distorted sectors ($\boldsymbol{\lambda}_t$) have flexible enough prices (K) to translate that aggregate demand into corrective relative-price shifts.

Condition (F$_\pi$) defines the inflation trade-off. The planner balances stabilizing average prices ($\boldsymbol{\lambda}_{t-1}$) against generating cross-sectoral inflation dispersion ($M^\top \boldsymbol{\eta}_t$) to send reallocation signals.

Condition (F$_\chi$) highlights that relative prices are forward-looking. Through $\boldsymbol{\xi}_t$, the central bank manipulates current prices to shape future wage expectations, actively guiding workers toward labor-scarce sectors to ease tomorrow's misallocation ($\boldsymbol{\psi}_t$).

Conditions (F$_n$)–(F$_v$) formalize how aggregate demand affects reallocation in the labor market. The term $\varphi K \boldsymbol{\lambda}_t$ in (F$_n$) is the link: aggregate demand affects sectoral inflation paths ($\boldsymbol{\lambda}_t$), which shift relative prices and alter workers' reallocation incentives ($\boldsymbol{\xi}_t$). Because migration frictions make this process slow, (F$_v$) shows that the planner sustains these policy-induced wage signals over time to gradually resolve labor bottlenecks ($\boldsymbol{\psi}_t$).

These conditions can be collapsed into one single targeting criterion, which we present next:

Proposition 9 (Monetary Policy Target)**.** *Under commitment, optimal monetary policy is characterized by the targeting rule*

$$\tilde{y}_t - \tilde{y}_{t-1} + \sum_i \alpha_i \epsilon_i\,\pi_{it} \;=\; (MK\mathbf{1})^\top \boldsymbol{\eta}_t = \sum_i (\kappa_i - \bar{\kappa})\eta_{it} \tag{75}$$

where the multiplier $\boldsymbol{\eta}_t$ is determined by the system (F$_\chi$)–(F$_v$)*.*

Proof. Multiply (F$_\pi$) by $\mathbf{1}^\top K$ and use $\mathbf{1}^\top K\,\boldsymbol{\lambda}_t = \tilde{y}_t$ from (F$_{\tilde{y}}$). The left-hand side becomes $\mathbf{1}^\top K D_\pi\,\boldsymbol{\pi}_t + \tilde{y}_t - \tilde{y}_{t-1}$. Since $\mathbf{1}^\top K D_\pi = (\alpha_1\epsilon_1, \ldots, \alpha_N\epsilon_N)$, this yields the left-hand side of (75). The right-hand side follows from $\mathbf{1}^\top K M^\top = (MK\mathbf{1})^\top$. □

This targeting criterion isolates the role of the dispersion of price stickiness. It shows that optimal policy deviates from the textbook New Keynesian target rule if and only if κ-heterogeneity provides a lever to ease labor market bottlenecks.

Constant κ: reduction to one sector. If $\kappa_i = \kappa$ for all i, then $K\mathbf{1} = \kappa\mathbf{1}$ and $MK\mathbf{1} = \kappa(I - \mathbf{1}\boldsymbol{\alpha}^\top)\mathbf{1} = \mathbf{0}$, since $\boldsymbol{\alpha}^\top\mathbf{1} = 1$. The right-hand side of (75) vanishes, and the criterion reduces to

$$\tilde{y}_t - \tilde{y}_{t-1} + \sum_i \alpha_i \epsilon_i \, \pi_{it} = 0. \tag{76}$$

If, additionally, $\epsilon_i = \epsilon$, this simplifies to the textbook one-sector history-dependent rule $\tilde{y}_t - \tilde{y}_{t-1} + \epsilon \, \pi_t^C = 0$. Because uniform stickiness means aggregate demand shifts all sectoral prices equally, the central bank cannot use the output gap to manipulate relative prices. Labor reallocation proceeds autonomously, and the planner simply targets welfare-weighted aggregate inflation, making this the dynamic counterpart to the static non-intervention result (Proposition 2).

Heterogeneous κ: the reallocation wedge. When κ_i varies, the right-hand side of (75) becomes $\sum_i (\kappa_i - \bar{\kappa})\eta_{it}$, it need not be zero. The wedge driving policy away from the one-sector rule is the alignment between a sector's relative price flexibility ($\kappa_i - \bar{\kappa}$) and the shadow value of adjusting its relative price (η_{it}).

If this sum is positive—meaning the sectors that most desperately need a relative price adjustment ($\eta_{it} > 0$) also have above-average price flexibility ($\kappa_i > \bar{\kappa}$)—the planner optimally runs a positive output gap. Aggregate stimulus disproportionately inflates prices in those flexible sectors, effectively engineering the precise relative-price shift needed to draw workers in. When this alignment is exactly zero, aggregate demand offers no directional advantage, and the standard criterion (76) applies exactly.

The multiplier $\boldsymbol{\eta}_t$ is forward-looking, and is pinned down by the system (F_χ)–(F_v), which depends on the employment state $\mathbf{n}_t$, the mobility structure $\bar{\mu}$, and the reallocation elasticity ϕ.

Comparison with gap-closing policy. Before exploring the full Ramsey solution, it is useful to identify a natural benchmark: closing the output gap. The aggregation result of Lemma 3 extends immediately to the dynamic economy. Summing the NKPC (21) with weights α_i/κ_i and using $\sum_i \alpha_i n_{it} = 0$, the α/κ-weighted inflation index $\pi_t^R \equiv \sum_i \frac{\alpha_i}{\kappa_i}\pi_{it}$ satisfies

$$\pi_t^R = (\gamma + \varphi)\,\tilde{y}_t + \beta\,\mathbb{E}_t \pi_{t+1}^R, \tag{77}$$

independent of $\mathbf{n}_t$, $\hat{\mathbf{p}}_t$, or $\bar{\mu}$. Stabilizing $\pi_t^R = 0$ therefore closes the output gap at all dates. However, gap-closing policy is generically suboptimal in the dynamic economy. To see why, compare (77) with the Ramsey criterion (75). Gap-closing policy sets $\tilde{y}_t = 0$ for all t, treating the relative block as exogenous. The Ramsey planner, by contrast, accepts a small output gap cost today whenever $(MK\mathbf{1})^\top \boldsymbol{\eta}_t \neq 0$—that is, whenever tilting aggregate demand can improve tomorrow's labor allocation by enough to offset the contemporaneous welfare loss from $\tilde{y}_t \neq 0$. The costate system (F_χ)–(F_v) makes the trade-off precise: $\boldsymbol{\eta}_t$ encodes the present value of the entire future reallocation path that current relative prices set in motion, discounted at the effective rate governed by $\bar{\mu}$ and ϕ.

Interpreting the wedge: spectral decomposition. The severity of this trade-off is determined by the spectral structure of the wedge. We can expand the term $MK\mathbf{1} = \sum_{k=2}^{N} v_k d_k$ in the $\bar{\mu}$-eigenbasis (Lemma 6) to have

$$\tilde{y}_t - \tilde{y}_{t-1} + \sum_i \alpha_i \epsilon_i \, \pi_{it} \;=\; \sum_{k=2}^{N} d_k \, \eta_{kt}^*, \qquad \eta_{kt}^* \equiv v_k^\top \boldsymbol{\eta}_t, \tag{78}$$

where $d_k = \sum_i \alpha_i (\kappa_i - \bar{\kappa}) \, v_{k,i}$ is the demand spillover coefficient from Proposition 7 and η_{kt}^* is the shadow value of relaxing the relative-price constraint along corridor k. The Ramsey planner deviates from the one-sector rule only when a persistent bottleneck (large $|\eta_{kt}^*|$) overlaps with asymmetric price transmission ($d_k \neq 0$).

The two factors play distinct roles. The projected multiplier η_{kt}^* measures the discounted welfare cost of labor mismatch along corridor k. This cost is amplified when the corridor is persistent: by Proposition 6, sticky sectors on both sides ($\Delta\kappa_k^{\text{eff}} < 0$) make the dominant root λ_k^* larger, making the labor misallocation more persistent and increasing $|\eta_{kt}^*|$. The spillover d_k, by contrast, governs whether the planner can act on this misallocation. When $d_k \approx 0$, aggregate demand cannot tilt relative prices along that corridor, and the standard constant-κ, ϵ Ramsey problem

$$\min_{\{\tilde{y}_t\}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\gamma + \varphi}{2} \, \tilde{y}_t^2 + \frac{\epsilon}{2\kappa} \, (\pi_t^C)^2 \right] \quad \text{s.t.} \quad \pi_t^C = \kappa(\gamma + \varphi) \, \tilde{y}_t + \beta \, \mathbb{E}_t \pi_{t+1}^C + u_t, \tag{79}$$

is a good approximation. The planner deviates from (79) most when the two statistics are correlated: if a persistent bottleneck ($\Delta\kappa_k^{\text{eff}} < 0$) is also controllable via aggregate demand ($|d_k|$ large), the planner deviates from the Ramsey target rule. The sign of the deviation is pinned down by the demand spillover d_k: If workers need to flow toward the sticky (κ small) sectors of a corridor, the planner runs a recessionary bias ($\tilde{y}_t < 0$). This asymmetric transmission effectively "pulls" workers towards higher relative wages. Conversely, if workers must flow towards flexible sectors in the corridor, the planner runs an expansionary bias $\tilde{y}_t > 0$. Overall, the gains from this monetary policy bias are largest when large $|\Delta\kappa_k^{\text{eff}}|$ overlap with high price stickiness asymmetry $|d_k|$. We explore these deviations quantitatively in the next section.

5 Quantitative Illustration

[This section is under construction.]

We illustrate the model's key mechanisms in a two-sector economy calibrated as in Table . The sectors are symmetric in size but differ in price rigidity ($\kappa_1 = 0.02, \kappa_2 = 0.05$.)

5.1 Calibration

Parameter	Value	Description
β_i	[0.5000, 0.5000]	Expenditure shares
θ	0.1000	Migration elasticity
β	0.9900	Discount factor
γ	1.0000	Risk aversion
φ	2.0000	Frisch elasticity
ϕ_π	1.5000	Taylor rule coefficient on inflation
κ_i	[0.0200, 0.0500]	Slope of Phillips curve

Table 2: Parameter values. Two symmetric sectors that differ in price rigidity

5.2 Impulse Responses

Figure 1 plots impulse responses to a permanent relative productivity shock under varying degrees of labor mobility. Higher mobility dampens the initial inflation response but raises persistence, consistent with Proposition 4: forward-looking firms discount future cost pressures when they anticipate worker inflows.

6 Conclusion

We have developed a multi-sector New Keynesian model in which frictional labor reallocation, summarized by a mobility elasticity and a steady-state transition matrix, reshapes the transmission of sectoral shocks and the design of optimal monetary policy. Three results stand out. First, labor mobility creates countervailing effects on sectoral inflation: it dampens impact but slows convergence. Second, the persistence of shocks depends not on average mobility or average stickiness alone, but on whether displaced workers flow into flexible or sticky sectors—the "double trap." Third, optimal policy under commitment deviates from standard sticky-price targeting precisely when price rigidity aligns with the slowest-mixing corridors of the reallocation network, as the planner exploits flexible sectors' price responsiveness to redirect worker flows.

Several extensions are natural. Our model assumes that aggregate demand affects demand for all sectors symmetrically: in practice, interest rate changes have heterogeneous incidence across sectors—durables respond more than services, for instance—which would give the central bank a richer set of relative price levers. Incorporating production networks alongside the labor network would allow both input-output and worker-flow linkages to shape persistence

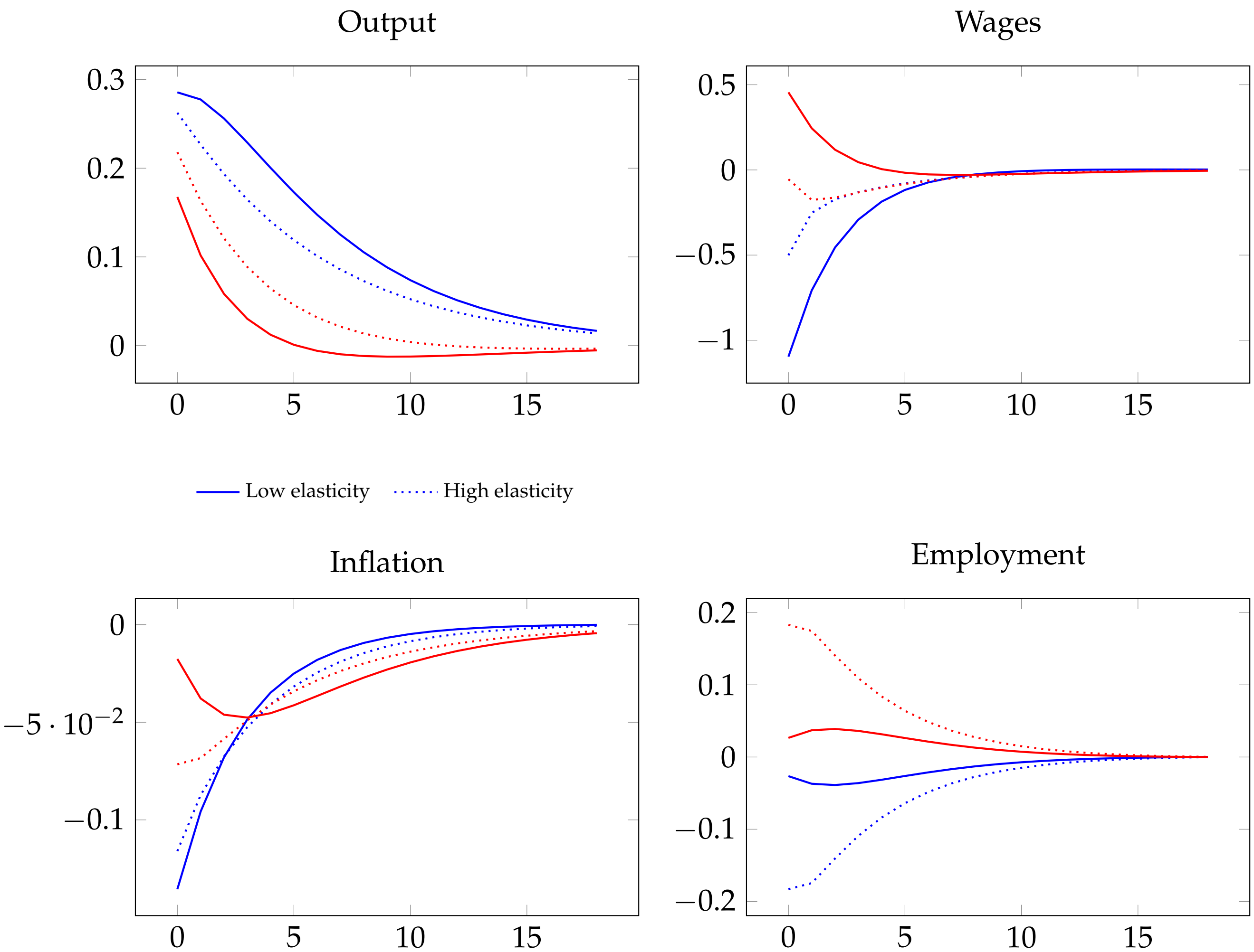

Figure 1: Response to a positive productivity shock at sector 1. Solid lines represent a model with low mobility (low θ), and dotted lines represent a sector with high mobility (low θ)

and policy simultaneously. Finally, estimating the alignment between price rigidity and the reallocation network in disaggregated U.S. data would discipline the quantitative relevance of the double trap and the Ramsey wedge we have characterized analytically.

References

AFROUZI, H. AND S. BHATTARAI (2025): "Inflation and GDP Dynamics in Production Networks: A Sufficient Statistics Approach," *Working Paper*.

ALLEN, T., C. ARKOLAKIS, AND Y. TAKAHASHI (2020): "Universal Gravity," *Journal of Political Economy*, 128, 393–433.

ALTISSIMO, F., B. MOJON, AND P. ZAFFARONI (2009): "Can aggregation explain the persistence of inflation?" *Journal of Monetary Economics*, 56, 231–241.

ANDERSON, S. P., A. DE PALMA, AND J.-F. THISSE (1988): "A Representative Consumer Theory of the Logit Model," *International Economic Review*, 461–466.

AOKI, K. (2001): "Optimal monetary policy responses to relative-price changes," *Journal of Monetary Economics*, 48, 55–80.

ARTUÇ, E., S. CHAUDHURI, AND J. MCLAREN (2010): "Trade Shocks and Labor Adjustment: A Structural Empirical Approach," *American Economic Review*, 100, 1008–1045.

BILS, M. AND P. KLENOW (2004): "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112, 947–985.

CALIENDO, L., M. DVORKIN, AND F. PARRO (2019): "Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock," *Econometrica*, 87, 741–835.

CARVALHO, C., J. W. LEE, AND W. Y. PARK (2021): "Sectoral Price Facts in a Sticky-Price Model," *American Economic Journal: Macroeconomics*, 13, 216–256.

FERRANTE, F., S. GRAVES, AND M. IACOVIELLO (2023): "The inflationary effects of sectoral reallocation," *Journal of Monetary Economics*, 140, S64–S81.

GALÍ, J. (2011): "The Return of the Wage Phillips Curve," *Journal of the European Economic Association*, 9, 436–461, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1542-4774.2011.01023.x.

GUERRIERI, V., G. LORENZONI, L. STRAUB, AND I. WERNING (2021): "Monetary Policy in Times of Structural Reallocation," *Working Paper*.

KIM, B., M. DE LA BARRERA, AND M. FUKUI (2026): "Currency Pegs, Trade Deficits and Unemployment: A Reevaluation of the China Shock," *Working Paper*.

KLEINMAN, B., E. LIU, AND S. J. REDDING (2023): "Dynamic Spatial General Equilibrium," *Econometrica*, 91, 385–424, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA20273.

LA'O, J. AND A. TAHBAZ-SALEHI (2022): "Optimal Monetary Policy in Production Networks," *Econometrica*, 90, 1295–1336, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA18627.

LILIEN, D. M. (1982): "Sectoral Shifts and Cyclical Unemployment," *Journal of Political Economy*, 90, 777–793.

LIU, E. AND A. TSYVINSKI (2024): "A Dynamic Model of Input–Output Networks," *The Review of Economic Studies*, 91, 3608–3644.

NAKAMURA, E. AND J. STEINSSON (2008): "Five Facts about Prices: A Reevaluation of Menu Cost Models," *The Quarterly Journal of Economics*, 123, 1415–1464.

PASTEN, E., R. SCHOENLE, AND M. WEBER (2020): "The propagation of monetary policy shocks in a heterogeneous production economy," *Journal of Monetary Economics*, 116, 1–22.

PASTÉN, E., R. SCHOENLE, AND M. WEBER (2024): "Sectoral Heterogeneity in Nominal Price Rigidity and the Origin of Aggregate Fluctuations," *American Economic Journal: Macroeconomics*, 16, 318–352.

RUBBO, E. (2023): "Networks, Phillips curves, and monetary policy," *Econometrica*, 91, 1417–1455.

SEO, J. AND R. OH (2025): "Sectoral Shocks and Labor Market Dynamics:," *Working Paper*.

Appendix

A Log-Linearization Derivations

This appendix details the log-linearization of the model around the steady state. Because all output is consumed, we impose goods market clearing $y_t = c_t$ and $y_{it} = c_{it}$ throughout, using y to denote both output and consumption.

Notation. For any variable X_t, we write $x_t \equiv \log X_t - \log \bar{X}$ for its log-deviation from steady state. We use the following conventions throughout:

- *Natural (flexible-price) levels*: x_t^{nat} denotes the value under flexible prices.
- *Gaps*: $\tilde{x}_t \equiv x_t - x_t^{\text{nat}}$ denotes the deviation from the natural level.
- *Relative prices*: $\hat{p}_{it} \equiv p_{it} - p_t^C$ is the log sectoral price relative to the CPI.
- *Relative price distortion*: $\chi_{it} \equiv \hat{p}_{it} + \hat{a}_{it}$, the deviation of relative prices from their static natural benchmark. See Section 2.5 for a discussion of how this relates to the dynamic natural relative price when $n_{it} \neq 0$.

We write $\overset{(1)}{=}$ for equalities that hold up to a first-order approximation, and $\overset{(2)}{=}$ for second-order. Plain $=$ denotes exact relationships or definitions.

A.1 The Macro Block

A.1.1 Sectoral Marginal Cost

Real marginal cost in sector i (in log-deviations) is given by $mc_{it} - p_{it} = w_{it} - a_{it} - p_{it}$. Using the intratemporal labor supply condition (7) and substituting aggregate consumption with output ($c_t = y_t$), the sectoral wage satisfies:

$$w_{it} - p_t^C = \varphi l_{it} + \gamma y_t. \tag{A.1}$$

Substituting this into the marginal cost expression yields:

$$mc_{it} - p_{it} = \gamma y_t + \varphi l_{it} - a_{it} - \hat{p}_{it},$$

where $\hat{p}_{it} \equiv p_{it} - p_t^C$ is the relative sectoral price. To eliminate l_{it}, we use the log-linearized production function ($l_{it} = y_{it} - a_{it} - n_{it}$) and substitute out sectoral output y_{it} using the demand curve ($y_{it} = y_t - \hat{p}_{it}$). Grouping terms directly yields Equation (21) from the main text:

$$mc_{it} - p_{it} \overset{(1)}{=} (\gamma + \varphi) y_t - (1 + \varphi)(\hat{p}_{it} + a_{it}) - \varphi n_{it}. \tag{A.2}$$

A.1.2 Phillips Curve

Standard log-linearization of the Calvo pricing problem (4) yields the sectoral New Keynesian Phillips curve. A re-optimizing firm in sector i sets its price as a markup over a weighted average of current and expected future marginal costs. Log-linearizing the optimal reset price around the zero-inflation steady state, and combining with the Calvo price index evolution $P_{it} = [\delta_i P_{it-1}^{1-\epsilon_i} + (1 - \delta_i)(P_{it}^*)^{1-\epsilon_i}]^{1/(1-\epsilon_i)}$, yields:

$$\pi_{it} \overset{(1)}{=} \kappa_i (mc_{it} - p_{it}) + \beta\, \mathbb{E}_t \pi_{it+1}, \tag{A.3}$$

where $\kappa_i \equiv (1 - \delta_i)(1 - \beta\delta_i)/\delta_i$. This is Equation (20).

A.1.3 The IS Curve and Natural Real Rate

Note on convention: As is standard, we make an exception to the log-deviation convention for interest rates. We let i_t and r_t^{nat} denote the levels of the net nominal and natural real interest rates, respectively.

The Euler equation (8) is

$$1 = \beta(1 + i_t)\, \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t^C}{P_{t+1}^C} \right].$$

Taking logs and using the first-order approximation $\log(1 + i_t) \overset{(1)}{=} i_t$:

$$0 \overset{(1)}{=} \log \beta + i_t - \gamma\, \mathbb{E}_t(y_{t+1} - y_t) - \mathbb{E}_t \pi_{t+1}^C,$$

where we have substituted $c_t = y_t$ (goods market clearing) and $\pi_{t+1}^C = p_{t+1}^C - p_t^C$. Rearranging yields:

$$y_t \overset{(1)}{=} \mathbb{E}_t y_{t+1} - \frac{1}{\gamma}(i_t - \mathbb{E}_t \pi_{t+1}^C + \log \beta). \tag{A.4}$$

The same equation holds under flexible prices with y_t^{nat} in place of y_t and i_t^{nat} in place of i_t (and $\pi_t^C = 0$ under flexible prices). We define the natural real rate (in levels) as:

$$r_t^{\text{nat}} \equiv -\log \beta + \gamma(\mathbb{E}_t y_{t+1}^{\text{nat}} - y_t^{\text{nat}}).$$

Subtracting the flexible-price Euler equation from (A.4) gives the IS curve, Equation (22):

$$\tilde{y}_t \overset{(1)}{=} \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\gamma}\left(i_t - \mathbb{E}_t \pi^C_{t+1} - r^{\text{nat}}_t\right). \tag{A.5}$$

A.1.4 Goods Market Clearing and Relative Prices.

Cobb-Douglas demand gives $P_{it}C_{it} = \alpha_i P^C_t C_t$, so sectoral output satisfies $P_{it}Y_{it} = \alpha_i P^C_t C_t$. In logs:

$$y_{it} - y_t = -\hat{p}_{it}. \tag{A.6}$$

Log aggregate output is $y_t = \sum_i \alpha_i a_{it} + \sum_i \alpha_i (n_{it} + l_{it})$, which uses $y_{it} = a_{it} + n_{it} + l_{it}$ (from the production function and labor market clearing) weighted by consumption shares.

The CPI definition $P^C_t = \prod_i P^{\alpha_i}_{it}$ implies $p^C_t = \sum_i \alpha_i p_{it}$, so CPI inflation is $\pi^C_t = \sum_i \alpha_i \pi_{it}$. Relative prices evolve as:

$$\hat{p}_{it} - \hat{p}_{it-1} = (p_{it} - p_{it-1}) - (p^C_t - p^C_{t-1}) = \pi_{it} - \pi^C_t, \tag{A.7}$$

which is Equation (23).

A.2 Dynamic Labor Reallocation Block

The dynamic reallocation block consists of the flow utility, the Bellman equation, and the employment evolution equation.

A.2.1 Flow Utility

Using the intratemporal labor supply condition (7), flow utility from (13) simplifies to $U^{\text{flow}}_{it} = \zeta \frac{\varphi}{1+\varphi} L^{1+\varphi}_{it}$ (Equation (14)). Define the normalized deviation $u_{it} \equiv (U^{\text{flow}}_{it} - \bar{U}^{\text{flow}})/(\varphi \zeta \bar{L}^{1+\varphi})$. Since $U^{\text{flow}}_{it} = \zeta \frac{\varphi}{1+\varphi} L^{1+\varphi}_{it}$, we compute:

$$\frac{U^{\text{flow}}_{it} - \bar{U}^{\text{flow}}}{\varphi \zeta \bar{L}^{1+\varphi}} = \frac{\zeta \frac{\varphi}{1+\varphi}(L^{1+\varphi}_{it} - \bar{L}^{1+\varphi})}{\varphi \zeta \bar{L}^{1+\varphi}} = \frac{L^{1+\varphi}_{it} - \bar{L}^{1+\varphi}}{(1+\varphi)\bar{L}^{1+\varphi}}.$$

A first-order expansion of $L^{1+\varphi}_{it}$ around $\bar{L}$ gives $L^{1+\varphi}_{it} \approx \bar{L}^{1+\varphi}(1 + (1+\varphi) l_{it})$, so:

$$u_{it} \overset{(1)}{=} l_{it}.$$

Substituting $l_{it} = y_t - \hat{p}_{it} - a_{it} - n_{it}$ (from goods market clearing and the production function) yields Equation (26):

$$u_{it} = y_t - \hat{p}_{it} - a_{it} - n_{it}. \tag{A.8}$$

A.2.2 Value Function Recursion

The Bellman equation (12) is:

$$V_{it} = U_{it}^{\text{flow}} + \frac{1}{\theta} \log \left(\sum_{j \in \mathcal{N}} \exp\left(\theta[\beta\, \mathbb{E}_t V_{jt+1} - \tau^{ij}]\right) \right).$$

In steady state, $\bar{V}_i = \bar{U}^{\text{flow}} + \frac{1}{\theta} \log\left(\sum_j \exp(\theta[\beta \bar{V}_j - \tau^{ij}])\right)$. Define the log-sum-exp term as $\Lambda_i(\mathbf{z}) \equiv \frac{1}{\theta} \log\left(\sum_j \exp(\theta z_j)\right)$ evaluated at $z_j = \beta\, \mathbb{E}_t V_{jt+1} - \tau^{ij}$. The first-order Taylor expansion of Λ_i around the steady-state values $\bar{z}_j = \beta \bar{V}_j - \tau^{ij}$ uses the standard logit property:

$$\left.\frac{\partial \Lambda_i}{\partial z_j}\right|_{\bar{z}} = \frac{\exp(\theta \bar{z}_j)}{\sum_{j'} \exp(\theta \bar{z}_{j'})} = \bar{\mu}^{ij},$$

where $\bar{\mu}^{ij}$ is the steady-state transition probability from sector i to sector j (Equation (11) evaluated at steady state). Therefore:

$$\Lambda_i(\mathbf{z}_t) - \Lambda_i(\bar{\mathbf{z}}) \overset{(1)}{=} \sum_j \bar{\mu}^{ij}\, \beta(\mathbb{E}_t V_{jt+1} - \bar{V}_j).$$

Subtracting the steady-state Bellman from the time-t Bellman and normalizing by $\varphi \zeta \bar{L}^{1+\varphi}$:

$$v_{it} = u_{it} + \beta \sum_j \bar{\mu}^{ij}\, \mathbb{E}_t[v_{jt+1}].$$

In matrix notation, this is Equation (25):

$$\mathbf{v}_t = \mathbf{u}_t + \beta \bar{\mu}\, \mathbb{E}_t[\mathbf{v}_{t+1}]. \tag{A.9}$$

A.2.3 Employment Evolution

Employment evolves according to $N_{jt+1} = \sum_i \mu_t^{ij} N_{it}$, where the transition probabilities are given by the logit form (11):

$$\mu_t^{ij} = \frac{\exp\left(\theta[\beta\, \mathbb{E}_t V_{jt+1} - \tau^{ij}]\right)}{\sum_{j'} \exp\left(\theta[\beta\, \mathbb{E}_t V_{j't+1} - \tau^{ij'}]\right)}.$$

Linearizing μ_t^{ij}. The transition probability has the logit form, so its first-order expansion around the steady state $(\bar{\mu}^{ij}, \bar{V}_j)$ is:

$$\begin{aligned}\mu_t^{ij} - \bar{\mu}^{ij} &\overset{(1)}{=} \bar{\mu}^{ij}\, \theta\beta \left(\mathbb{E}_t[V_{jt+1} - \bar{V}_j] - \sum_{j'} \bar{\mu}^{ij'}\, \mathbb{E}_t[V_{j't+1} - \bar{V}_{j'}] \right) \\ &= \bar{\mu}^{ij}\, \theta\beta\, \varphi\zeta\bar{L}^{1+\varphi} \left(\mathbb{E}_t[v_{jt+1}] - \sum_{j'} \bar{\mu}^{ij'}\, \mathbb{E}_t[v_{j't+1}] \right),\end{aligned}$$

where the second line uses the normalization $v_{jt} = (V_{jt} - \bar{V}_j)/(\varphi\zeta\bar{L}^{1+\varphi})$ and defines $\phi \equiv \varphi\theta\zeta\bar{L}^{1+\varphi}$. Noting that $\sum_{j'} \bar{\mu}^{ij'}\mathbb{E}_t[v_{j't+1}]$ is the $\bar{\mu}$-weighted average from origin i, we have:

$$\mu_t^{ij} - \bar{\mu}^{ij} \overset{(1)}{=} \phi\beta\, \bar{\mu}^{ij} \left(\mathbb{E}_t[v_{jt+1}] - \sum_{j'} \bar{\mu}^{ij'}\, \mathbb{E}_t[v_{j't+1}] \right). \tag{A.10}$$

Deriving the employment law of motion. Log-linearize $N_{jt+1} = \sum_i \mu_t^{ij} N_{it}$ around $\bar{N}_j = \alpha_j$. Since $\bar{N}_j = \sum_i \bar{\mu}^{ij}\bar{N}_i$, taking deviations:

$$\bar{N}_j\, n_{jt+1} \overset{(1)}{=} \sum_i \bar{\mu}^{ij}\bar{N}_i\, n_{it} + \sum_i \bar{N}_i(\mu_t^{ij} - \bar{\mu}^{ij}).$$

We handle the two terms in turn. For the first term, detailed balance (15) gives $\bar{N}_i\bar{\mu}^{ij} = \bar{N}_j\bar{\mu}^{ji}$, so:

$$\sum_i \bar{\mu}^{ij}\bar{N}_i\, n_{it} = \bar{N}_j \sum_i \bar{\mu}^{ji} n_{it}.$$

Reading this entry-wise, the j-th component is $\bar{N}_j[\bar{\mu}^\top \mathbf{n}_t]_j$. However, because we work with the log-deviations $n_{it} = \log(N_{it}/\bar{N}_i)$ and impose detailed balance, the symmetrization $\bar{N}_i\bar{\mu}^{ij} = \bar{N}_j\bar{\mu}^{ji}$ allows us to write this equivalently (dividing through by $\bar{N}_j$) as:

$$(\text{first term}) = \sum_i \bar{\mu}^{ji} n_{it} = [\bar{\mu}\, \mathbf{n}_t]_j,$$

where the last equality exploits the fact that under detailed balance with symmetric steady state ($\bar{N}_i = \alpha_i$), the operator acting on $\mathbf{n}_t$ can be written as $\bar{\mu}$ without transposes (see Remark below).

For the second term, substituting (A.10):

$$\sum_i \bar{N}_i(\mu_t^{ij} - \bar{\mu}^{ij}) = \phi\beta \sum_i \bar{N}_i\bar{\mu}^{ij} \left(\mathbb{E}_t[v_{jt+1}] - \sum_{j'} \bar{\mu}^{ij'}\mathbb{E}_t[v_{j't+1}] \right).$$

Using detailed balance ($\bar{N}_i \bar{\mu}^{ij} = \bar{N}_j \bar{\mu}^{ji}$), dividing by $\bar{N}_j$, and writing the result in matrix form:

$$\begin{aligned}(\text{second term})/\bar{N}_j &= \phi\beta\left(\mathbb{E}_t[v_{jt+1}] \sum_i \bar{\mu}^{ji} - \sum_i \bar{\mu}^{ji} \sum_{j'} \bar{\mu}^{ij'} \mathbb{E}_t[v_{j't+1}] \right) \\ &= \phi\beta\left(\mathbb{E}_t[v_{jt+1}] - [\bar{\mu}^2 \, \mathbb{E}_t \mathbf{v}_{t+1}]_j \right),\end{aligned}$$

where we used $\sum_i \bar{\mu}^{ji} = 1$ and the fact that $\sum_i \bar{\mu}^{ji} \sum_{j'} \bar{\mu}^{ij'} \mathbb{E}_t[v_{j't+1}]$ is the j-th entry of $\bar{\mu}^2 \, \mathbb{E}_t \mathbf{v}_{t+1}$ (again using detailed balance to avoid transposes).

Combining both terms:

$$n_{jt+1} = [\bar{\mu} \, \mathbf{n}_t]_j + \phi\beta \Big[(I - \bar{\mu}^2) \, \mathbb{E}_t \mathbf{v}_{t+1} \Big]_j. \tag{A.11}$$

In vector form, this is Equation (24):

$$\mathbf{n}_{t+1} = \bar{\mu} \, \mathbf{n}_t + \phi\beta (I - \bar{\mu}^2) \, \mathbb{E}_t[\mathbf{v}_{t+1}]. \tag{A.12}$$

Remark 1 (Transpose convention under detailed balance)**.** *In general, $N_{jt+1} = \sum_i \mu_t^{ij} N_{it}$ would give the operator $\bar{\mu}^\top$ acting on $\mathbf{N}_t$ (since the sum is over the* origin *i). However, we work with log-deviations $n_{it} = \log(N_{it}/\bar{N}_i)$ rather than levels. Detailed balance implies $\bar{N}_i \bar{\mu}^{ij} = \bar{N}_j \bar{\mu}^{ji}$, or equivalently $A\bar{\mu} = \bar{\mu}^\top A$ where $A = \operatorname{diag}(\alpha_i)$. This conjugacy, combined with dividing the evolution equation by $\bar{N}_j$, converts $\bar{\mu}^\top$ into $\bar{\mu}$ when acting on the proportional deviations $\mathbf{n}_t$. The same symmetrization applies to the $\bar{\mu}^2$ term.*

Limiting cases. When $\phi = 0$ (segmented markets), the second term vanishes and $\mathbf{n}_{t+1} = \bar{\mu} \, \mathbf{n}_t$. Starting from $\mathbf{n}_0 = \mathbf{0}$, we have $\mathbf{n}_t = \mathbf{0}$ for all t: no reallocation occurs. When $\phi \to \infty$, boundedness of $\mathbf{n}_{t+1}$ requires $(I - \bar{\mu}^2) \, \mathbb{E}_t[\mathbf{v}_{t+1}] \to \mathbf{0}$. Since $I - \bar{\mu}^2$ has a nontrivial null space only along the constant vector $\mathbf{1}$ (which is already removed by the constraint $\sum_i \alpha_i n_{it} = 0$), this implies $\mathbb{E}_t[\mathbf{v}_{t+1}] \to \mathbf{1} c_t$. Iterating the value recursion (25) then gives $\mathbf{u}_t = \mathbf{1} c_t$, so $n_{it} = -(y_t - \hat{p}_{it} - a_{it}) = -\chi_{it}$ from (26) and $\sum_i \alpha_i n_{it} = 0$: employment fully offsets relative price distortions.

A.3 Static Economy and the Reduced-Form Sectoral NKPC

In the static limit, continuation values are common across destinations and cancel from the logit choice, so reallocation occurs within the period.

A.3.1 Static Reallocation Condition

In the static model, the dynamic transition friction is replaced by a within-period discrete choice problem. Workers choose their sector contemporaneously to maximize current flow utility, subject to idiosyncratic preference shocks that anchor aggregate shares to the steady-state baseline $\bar{N}_i$. As established in Section 3, this yields the standard static logit choice rule for the cross-sectional employment distribution:

$$N_{it} = \frac{\bar{N}_i \exp(\theta U_{it}^{\text{flow}})}{\sum_j \bar{N}_j \exp(\theta U_{jt}^{\text{flow}})}.$$

Taking logs:

$$\log N_{it} = \log \bar{N}_i + \theta U_{it}^{\text{flow}} - \log\left(\sum_j \bar{N}_j \exp(\theta U_{jt}^{\text{flow}})\right).$$

Subtracting $\log \bar{N}_i$ and expanding the log-share expression in deviations of U^{flow}, we get:

$$n_{it} \stackrel{(1)}{=} \theta(U_{it}^{\text{flow}} - \bar{U}^{\text{flow}}) - \sum_j \alpha_j \theta(U_{jt}^{\text{flow}} - \bar{U}^{\text{flow}}).$$

The second term arises from log-linearizing the denominator: $\log\left(\sum_j \bar{N}_j \exp(\theta U_{jt}^{\text{flow}})\right) \stackrel{(1)}{=} \theta\bar{U}^{\text{flow}} + \theta \sum_j \alpha_j (U_{jt}^{\text{flow}} - \bar{U}^{\text{flow}})$. Using the result from Appendix A.2.1 that $(U_{it}^{\text{flow}} - \bar{U}^{\text{flow}})/(\varphi\zeta\bar{L}^{1+\varphi}) \stackrel{(1)}{=} l_{it}$, and defining $\phi \equiv \varphi\theta\zeta\bar{L}^{1+\varphi}$:

$$n_{it} = \phi(l_{it} - l_t^{\alpha}), \qquad l_t^{\alpha} \equiv \sum_i \alpha_i l_{it}, \tag{A.13}$$

which is Equation (29). The constraint $\sum_i \alpha_i n_{it} = 0$ is satisfied by construction, since $\sum_i \alpha_i (l_{it} - l_t^{\alpha}) = 0$.

A.3.2 Derivation of the Reduced-Form Sectoral NKPC

We derive the reduced-form Phillips curve (38) in five steps, starting from the NKPC with the sufficient statistic representation of marginal cost.

1: Express the NKPC in terms of σ and χ_{it}. Substituting the marginal cost expression into the sectoral NKPC yields:

$$\pi_{it} = \kappa_i \left[(\gamma + \varphi)\tilde{y}_t - \sigma\chi_{it}\right] + \beta\mathbb{E}_t\pi_{it+1}. \tag{A.14}$$

2: Express χ_{it} dynamically for any t. The relative price distortion χ_{it} evolves according to:

$$\chi_{it} = \chi_{it-1} + (\pi_{it} - \pi_t^C) + \Delta\hat{a}_{it}, \tag{A.15}$$

where $\Delta\hat{a}_{it}$ represents the change in demeaned productivity. We define the generic cost-push term $\hat{c}_{it} \equiv \chi_{it-1} + \Delta\hat{a}_{it}$, which captures the inherited price misalignment and the current productivity shock. Substituting this into (A.14):

$$\pi_{it} = \kappa_i(\gamma + \varphi)\tilde{y}_t - \sigma\kappa_i(\pi_{it} - \pi_t^C + \hat{c}_{it}) + \beta\mathbb{E}_t\pi_{it+1}.$$

3: Isolate sectoral inflation π_{it}. Collecting π_{it} terms and dividing by $(1 + \sigma\kappa_i)$, and utilizing the definition $\bar{\kappa}_i \equiv \frac{\sigma\kappa_i}{1+\sigma\kappa_i}$:

$$\pi_{it} = \frac{\gamma + \varphi}{\sigma}\bar{\kappa}_i\tilde{y}_t + \bar{\kappa}_i\pi_t^C - \bar{\kappa}_i\hat{c}_{it} + \frac{\beta}{1 + \sigma\kappa_i}\mathbb{E}_t\pi_{it+1}. \tag{A.16}$$

4: Solve for aggregate inflation π_t^C. Aggregating (A.16) using sectoral weights α_i (where $\sum_i \alpha_i\pi_{it} = \pi_t^C$ and $\bar{\kappa}^\alpha = \sum_i \alpha_i\bar{\kappa}_i$):

$$\pi_t^C = \frac{\gamma + \varphi}{\sigma}\bar{\kappa}^\alpha\tilde{y}_t + \bar{\kappa}^\alpha\pi_t^C - \sum_j \alpha_j\bar{\kappa}_j\hat{c}_{jt} + \beta\sum_j \frac{\alpha_j}{1 + \sigma\kappa_j}\mathbb{E}_t\pi_{jt+1}.$$

Solving for π_t^C:

$$\pi_t^C = \frac{\gamma + \varphi}{\sigma}\frac{\bar{\kappa}^\alpha}{1 - \bar{\kappa}^\alpha}\tilde{y}_t - \frac{1}{1 - \bar{\kappa}^\alpha}\sum_j \alpha_j\bar{\kappa}_j\hat{c}_{jt} + \frac{\beta}{1 - \bar{\kappa}^\alpha}\sum_j \frac{\alpha_j}{1 + \sigma\kappa_j}\mathbb{E}_t\pi_{jt+1}. \tag{A.17}$$

5: Reduced-form Sectoral NKPC. Plugging (A.17) back into (A.16) and grouping terms:

$$\begin{aligned}\pi_{it} = b_i\tilde{y}_t - \frac{\bar{\kappa}_i}{1 - \bar{\kappa}^\alpha}\left[(1 - \bar{\kappa}^\alpha)\hat{c}_{it} + \sum_j \alpha_j\bar{\kappa}_j\hat{c}_{jt}\right] \\ + \frac{\beta}{1 + \sigma\kappa_i}\mathbb{E}_t\pi_{it+1} + \frac{\beta\bar{\kappa}_i}{1 - \bar{\kappa}^\alpha}\sum_j \frac{\alpha_j}{1 + \sigma\kappa_j}\mathbb{E}_t\pi_{jt+1},\end{aligned} \tag{A.18}$$

where the slope coefficient is $b_i = \frac{(\gamma+\varphi)\kappa_i}{(1+\sigma\kappa_i)(1-\bar{\kappa}^\alpha)}$.

Verification: constant κ. When $\kappa_i = \kappa$ for all i, we have $\bar{\kappa}_i = \bar{\kappa} \equiv \frac{\sigma\kappa}{1+\sigma\kappa}$ and $\bar{\kappa}^\alpha = \bar{\kappa}$, so $1 - \bar{\kappa}^\alpha = \frac{1}{1+\sigma\kappa}$. The output gap slope simplifies to $b_i = (\gamma + \varphi)\kappa$, independent of σ. This confirms Corollary 1: with uniform price stickiness, mobility affects the Phillips curve only through the cost-push terms and relative price dynamics, not through the output gap slope.

B Second-Order Expansion Details

This appendix derives the per-period welfare loss functions stated in Proposition 1 and 8. We expand household utility around the efficient (natural) allocation to second order, term by term.

B.1 Proposition 1: Static LQ

We derive Equation 41:

$$\mathcal{L}_t \overset{(2)}{=} \frac{\gamma+\varphi}{2}\,\tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i}\,\pi_{it}^2 + \frac{\sigma}{2}\sum_i \alpha_i \chi_{it}^2,$$

B.1.1 Setup

The per-period utility is

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \sum_i N_{it}\,\zeta\,\frac{L_{it}^{1+\varphi}}{1+\varphi} - \Psi_t, \tag{B.1}$$

where $\Psi_t = \frac{1}{\theta}\sum_i N_{it}\log(N_{it}/\bar{N}_i)$ is the static reallocation cost from (10). We expand each of the three terms around the natural allocation $(C_t^{\text{nat}}, L_{it}^{\text{nat}}, N_{it}^{\text{nat}})$, normalizing throughout by $U_C\bar{C}^{\text{nat}} = (\bar{C}^{\text{nat}})^{1-\gamma}$. The planner first-order condition on L_{it} (Equation (31)) gives $\alpha_i(\bar{C}^{\text{nat}})^{1-\gamma} = \zeta\,\alpha_i\,\bar{L}^{1+\varphi}$, or equivalently $\zeta\bar{L}^{1+\varphi} = U_C\bar{C}^{\text{nat}}$, a normalization used repeatedly below.

B.1.2 Consumption

A standard second-order expansion of the CRRA utility gives:

$$\frac{1}{U_C\bar{C}^{\text{nat}}}\frac{C_t^{1-\gamma} - (C_t^{\text{nat}})^{1-\gamma}}{1-\gamma} \overset{(2)}{=} \tilde{y}_t + \frac{1-\gamma}{2}\,\tilde{y}_t^2, \tag{B.2}$$

where $\tilde{y}_t = \log(C_t/C_t^{\text{nat}})$ is the output gap (using goods market clearing $c_t = y_t$).

B.1.3 Labor Disutility

This term requires care because both N_{it} and L_{it} deviate from their natural levels simultaneously.

Expanding N_{it}. By definition, $n_{it} \equiv \log(N_{it}/\alpha_i)$, which implies $N_{it} = \alpha_i e^{n_{it}}$. The exact adding-up constraint is $\sum_i \alpha_i e^{n_{it}} = 1$. Expanding the exponential to second order yields $\sum_i \alpha_i(1 + n_{it} +$

$\frac{1}{2}n_{it}^2) \overset{(2)}{=} 1$, which provides the correct second-order relationship for the employment shares:

$$\sum_i \alpha_i n_{it} \overset{(2)}{=} -\frac{1}{2}\sum_i \alpha_i n_{it}^2$$

We can therefore expand N_{it} directly:

$$N_{it} \overset{(2)}{=} \alpha_i \left(1 + n_{it} + \frac{1}{2}n_{it}^2\right)$$

Expanding $L_{it}^{1+\varphi}$. Writing $L_{it} = L_{it}^{\text{nat}} e^{\tilde{l}_{it}}$:

$$L_{it}^{1+\varphi} \overset{(2)}{=} (L_{it}^{\text{nat}})^{1+\varphi} \left[1 + (1+\varphi)\,\tilde{l}_{it} + \frac{(1+\varphi)^2}{2}\,\tilde{l}_{it}^2\right].$$

Combining. Multiplying the expansions for N_{it} and $L_{it}^{1+\varphi}$ and retaining terms up to second order:

$$N_{it}\,L_{it}^{1+\varphi} \overset{(2)}{=} \alpha_i (L_{it}^{\text{nat}})^{1+\varphi} \left[1 + n_{it} + \frac{1}{2}n_{it}^2 + (1+\varphi)\,\tilde{l}_{it} + \frac{(1+\varphi)^2}{2}\,\tilde{l}_{it}^2 + (1+\varphi)\,\tilde{l}_{it}\,n_{it}\right].$$

The planner FOC gives $\frac{\zeta\,\alpha_i\,(L_{it}^{\text{nat}})^{1+\varphi}}{(1+\varphi)\,U_C \bar{C}^{\text{nat}}} = \frac{\alpha_i}{1+\varphi}$. Summing the normalized labor disutility deviation over sectors:

$$\begin{aligned} &\frac{1}{U_C \bar{C}^{\text{nat}}} \sum_i \zeta \left[\frac{N_{it}\,L_{it}^{1+\varphi}}{1+\varphi} - \frac{\alpha_i\,\bar{L}^{1+\varphi}}{1+\varphi}\right] \\ &\overset{(2)}{=} \sum_i \frac{\alpha_i}{1+\varphi}\left[n_{it} + \tfrac{1}{2}\left(n_{it}^2 - \textstyle\sum_j \alpha_j\,n_{jt}^2\right) + (1+\varphi)\,\tilde{l}_{it} + \tfrac{(1+\varphi)^2}{2}\,\tilde{l}_{it}^2 + (1+\varphi)\,\tilde{l}_{it}\,n_{it}\right]. \end{aligned} \tag{B.3}$$

We simplify each group of terms using the second-order constraint $\sum_i \alpha_i n_{it} \overset{(2)}{=} -\frac{1}{2}\sum_i \alpha_i n_{it}^2$:

- The n_{it} and n_{it}^2 terms perfectly cancel: $\sum_i \frac{\alpha_i}{1+\varphi}\left(n_{it} + \frac{1}{2}n_{it}^2\right) \overset{(2)}{=} 0$.
- The $\tilde{l}_{it}$ terms: $\sum_i \alpha_i\,\tilde{l}_{it} = \tilde{l}_t^{\alpha}$.
- The $\tilde{l}_{it}^2$ terms: decompose as $\sum_i \alpha_i\,\tilde{l}_{it}^2 = (\tilde{l}_t^{\alpha})^2 + \mathcal{M}_t$, where $\mathcal{M}_t \equiv \sum_i \alpha_i(\tilde{l}_{it} - \tilde{l}_t^{\alpha})^2$ is cross-sector labor dispersion.
- The cross terms: using $n_{it} = \phi(\tilde{l}_{it} - \tilde{l}_t^{\alpha})$ from (29),

$$\sum_i \alpha_i\,\tilde{l}_{it}\,n_{it} = \sum_i \alpha_i\,n_{it}(\tilde{l}_{it} - \tilde{l}_t^{\alpha}) = \phi\sum_i \alpha_i(\tilde{l}_{it} - \tilde{l}_t^{\alpha})^2 = \phi\,\mathcal{M}_t.$$

Collecting:

$$\frac{1}{U_C \bar{C}^{\text{nat}}} \sum_i \zeta \left[\frac{N_{it}\, L_{it}^{1+\varphi}}{1+\varphi} - \frac{\alpha_i\, \bar{L}^{1+\varphi}}{1+\varphi} \right] \overset{(2)}{=} \tilde{l}_t^{\alpha} + \frac{1+\varphi}{2} \Big((\tilde{l}_t^{\alpha})^2 + \mathcal{M}_t \Big) + \phi\, \mathcal{M}_t. \tag{B.4}$$

B.1.4 Mobility Cost

The static reallocation cost is $\Psi_t = \frac{1}{\theta} \sum_i N_{it} \log(N_{it}/\bar{N}_i) = \frac{1}{\theta} \sum_i N_{it}\, n_{it}$. We expand $N_{it}\, n_{it}$ to second order using $N_{it} \overset{(1)}{=} \alpha_i (1 + n_{it})$:

$$\sum_i N_{it}\, n_{it} \overset{(2)}{=} \sum_i \alpha_i (1 + n_{it}) n_{it} = \sum_i \alpha_i\, n_{it} + \sum_i \alpha_i\, n_{it}^2$$

Substituting the second-order constraint $\sum_i \alpha_i n_{it} \overset{(2)}{=} -\frac{1}{2} \sum_i \alpha_i n_{it}^2$ simplifies this directly:

$$\Psi_t \overset{(2)}{=} \frac{1}{\theta} \left(-\frac{1}{2} \sum_i \alpha_i\, n_{it}^2 + \sum_i \alpha_i\, n_{it}^2 \right) = \frac{1}{2\theta} \sum_i \alpha_i\, n_{it}^2$$

Substituting $n_{it} = \phi(\tilde{l}_{it} - \tilde{l}_t^{\alpha})$ and normalizing by $U_C \bar{C}^{\text{nat}}$:

$$\frac{\Psi_t}{U_C \bar{C}^{\text{nat}}} \overset{(2)}{=} \frac{\phi^2}{2\theta\, U_C \bar{C}^{\text{nat}}}\, \mathcal{M}_t = \frac{\phi\varphi}{2}\, \mathcal{M}_t, \tag{B.5}$$

where the last step uses $\phi = \varphi\theta\zeta \bar{L}^{1+\varphi} = \varphi\theta\, U_C \bar{C}^{\text{nat}}$ (by the planner FOC), so $\phi^2/(\theta\, U_C \bar{C}^{\text{nat}}) = \phi\varphi$.

B.1.5 Assembly: Raw Loss

Combining (B.2), (B.4), and (B.5):

$$\frac{U_t - U_t^{\text{nat}}}{U_C \bar{C}^{\text{nat}}} \overset{(2)}{=} \left(\tilde{y}_t + \frac{1-\gamma}{2} \tilde{y}_t^2 \right) - \left(\tilde{l}_t^{\alpha} + \frac{1+\varphi}{2} (\tilde{l}_t^{\alpha})^2 + \left(\frac{1+\varphi}{2} + \phi \right) \mathcal{M}_t \right) - \frac{\phi\varphi}{2}\, \mathcal{M}_t. \tag{B.6}$$

Since $\tilde{l}_t^{\alpha} \overset{(1)}{=} \tilde{y}_t$ (Lemma 1), we can replace $(\tilde{l}_t^{\alpha})^2 \overset{(2)}{=} \tilde{y}_t^2$ and combine the squared terms:

$$\frac{U_t - U_t^{\text{nat}}}{U_C \bar{C}^{\text{nat}}} \overset{(2)}{=} (\tilde{y}_t - \tilde{l}_t^{\alpha}) - \frac{\gamma+\varphi}{2} \tilde{y}_t^2 - \left(\frac{1+\varphi}{2} + \phi + \frac{\phi\varphi}{2} \right) \mathcal{M}_t. \tag{B.7}$$

B.1.6 The Output–Labor Gap

The remaining first-order term $\tilde{y}_t - \tilde{l}_t^{\alpha}$ vanishes at first order but has a nonzero second-order component arising from two sources of misallocation: within-sector price dispersion and across-sector employment dispersion.

Within-sector. Define the sectoral dispersion index $\Delta_{it} \equiv \int_0^1 (P_{it}(f)/P_{it})^{-\epsilon_i} df \geq 1$ and $\xi_{it} \equiv -\log \Delta_{it}$. This captures misallocation across varieties within sector i: effective production satisfies $Y_{it} = A_{it}\, e^{\xi_{it}}\, N_{it}\, L_{it}$. Under Calvo pricing, price dispersion evolves recursively: to second order, its law of motion is $\xi_{it} \overset{(2)}{=} \delta_i \xi_{i,t-1} - \frac{\epsilon_i \delta_i}{2(1-\delta_i)} \pi_{it}^2$.As is standard in New Keynesian welfare derivations, assuming the economy starts from a steady state with zero price dispersion ($\xi_{i,-1} = 0$), we can rewrite the discounted sum of these dispersion terms over the infinite horizon. The present-value equivalence is:

$$\sum_{t=0}^{\infty} \beta^t \xi_{it} \overset{(2)}{=} -\sum_{t=0}^{\infty} \beta^t \frac{\epsilon_i}{2\kappa_i} \pi_{it}^2 \tag{B.8}$$

Exact decomposition. Cobb-Douglas aggregation gives $C_t = \prod_i (A_{it}\, e^{\xi_{it}}\, N_{it}\, L_{it})^{\alpha_i}$. Taking logs and expressing in gaps from the natural allocation:

$$\tilde{y}_t = \sum_i \alpha_i\, \tilde{l}_{it} + \sum_i \alpha_i\, \xi_{it} + \sum_i \alpha_i \log \frac{N_{it}}{N_{it}^{\text{nat}}} \tag{B.9}$$

This is exact. The first term is $\tilde{l}_t^{\alpha}$. For the third term, since $\log(N_{it}/N_{it}^{\text{nat}}) = n_{it}$, we can apply the second-order constraint directly:

$$\sum_i \alpha_i \log \frac{N_{it}}{N_{it}^{\text{nat}}} = \sum_i \alpha_i n_{it} \overset{(2)}{=} -\frac{1}{2} \sum_i \alpha_i n_{it}^2$$

Substituting (B.8) and $n_{it} = \phi(\tilde{l}_{it} - \tilde{l}_t^{\alpha})$:

$$\tilde{y}_t - \tilde{l}_t^{\alpha} \overset{(2)}{=} -\sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i} \pi_{it}^2 - \frac{\phi^2}{2} \mathcal{M}_t \tag{B.10}$$

in discounted present value terms.

B.1.7 Final Loss Function

Substituting (B.10) into (B.7):

$$\mathcal{L}_t \equiv -\frac{U_t - U_t^{\text{nat}}}{U_C \bar{C}^{\text{nat}}} \overset{(2)}{=} \frac{\gamma + \varphi}{2} \tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i} \pi_{it}^2 + \underbrace{\left(\frac{1+\varphi}{2} + \phi + \frac{\phi\varphi}{2} + \frac{\phi^2}{2} \right)}_{\equiv\ \Omega} \mathcal{M}_t. \tag{B.11}$$

Expressing $\mathcal{M}_t$ in terms of χ_{it}. From the margin decomposition (34), $\tilde{l}_{it} - \tilde{l}_t^{\alpha} = -\frac{1}{1+\phi}\,\chi_{it}$, so:

$$\mathcal{M}_t = \sum_i \alpha_i (\tilde{l}_{it} - \tilde{l}_t^{\alpha})^2 = \frac{1}{(1+\phi)^2} \sum_i \alpha_i\, \chi_{it}^2. \tag{B.12}$$

Simplifying the coefficient.

$$\begin{aligned} \frac{\Omega}{(1+\phi)^2} &= \frac{1}{(1+\phi)^2}\left(\frac{1+\varphi}{2} + \phi + \frac{\phi\varphi}{2} + \frac{\phi^2}{2}\right) = \frac{1}{(1+\phi)^2} \cdot \frac{1+\varphi+2\phi+\phi\varphi+\phi^2}{2} \\ &= \frac{(1+\phi)(1+\phi+\varphi)}{2(1+\phi)^2} = \frac{1+\phi+\varphi}{2(1+\phi)} = \frac{\sigma}{2}, \end{aligned} \tag{B.13}$$

where $\sigma \equiv 1 + \frac{\varphi}{1+\phi} = \frac{1+\phi+\varphi}{1+\phi}$.

Result. Substituting (B.12) and (B.13) into (B.11):

$$\mathcal{L}_t \overset{(2)}{=} \frac{\gamma+\varphi}{2}\,\tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i}\,\pi_{it}^2 + \frac{\sigma}{2}\sum_i \alpha_i\, \chi_{it}^2, \tag{B.14}$$

which is Equation (41) in Proposition 1. The coefficient $\sigma \in [1, 1+\varphi)$ ranges from $1+\varphi$ (segmented markets, $\phi = 0$) to 1 (frictionless mobility, $\phi \to \infty$), confirming that σ is a sufficient statistic for mobility in the welfare loss.

Remark 2 (Limiting cases)**.** *When $\phi = 0$, the mobility cost and employment dispersion terms vanish, and $\Omega\,\mathcal{M}_t = \frac{1+\varphi}{2}\sum_i \alpha_i\, \chi_{it}^2$: all adjustment occurs through the intensive margin (hours). When $\phi \to \infty$, workers fully absorb relative price distortions ($\mathcal{M}_t \to 0$), and the residual coefficient $\sigma \to 1$ reflects only expenditure misallocation from relative price gaps, recovering the welfare loss in Rubbo (2023).*

B.2 Proposition 8: Dynamic LQ

We derive Equation (66):

$$\mathcal{L}_t \overset{(2)}{=} \frac{\gamma+\varphi}{2}\,\tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i}\,\pi_{it}^2 + \sum_i \alpha_i \left[\frac{1+\varphi}{2}\,\chi_{it}^2 + \varphi\,\chi_{it}\,n_{it} + \frac{\varphi}{2}\,n_{it}^2\right].$$

The key differences from the static derivation are: (i) the employment distribution $\mathbf{n}_t$ is a predetermined state variable, so we can no longer impose $n_{it} = \phi(\tilde{l}_{it} - \tilde{l}_t^{\alpha})$; (ii) the relevant first-order relationship is $\tilde{l}_{it} - \tilde{l}_t^{\alpha} \overset{(1)}{=} -(\chi_{it} + n_{it})$; and (iii) the dynamic mobility cost telescopes when discounted over the infinite horizon and drops out of the per-period loss.

B.2.1 Telescoping of the Mobility Cost

The per-period mobility cost for workers currently in sector i is

$$\Psi_{it} = \sum_j \mu_t^{ij} \tau^{ij} + \frac{1}{\theta} \sum_j \mu_t^{ij} \log \mu_t^{ij}.$$

At the optimum, the logit choice probability satisfies $\log \mu_t^{ij} = \theta[\beta \mathbb{E}_t V_{j,t+1} - \tau^{ij}] - \log Z_{i,t}$, with $\log Z_{i,t} = \theta(V_{it} - U_{it}^{\text{flow}})$. Substituting back, the τ^{ij} terms cancel:

$$\Psi_{it} = \beta \sum_j \mu_t^{ij} \, \mathbb{E}_t V_{j,t+1} - V_{it} + U_{it}^{\text{flow}}. \tag{B.15}$$

Multiplying by N_{it}, summing over i, and using $N_{j,t+1} = \sum_i N_{it} \, \mu_t^{ij}$:

$$\Psi_t \equiv \sum_i N_{it} \, \Psi_{it} = \beta \sum_j N_{j,t+1} \, \mathbb{E}_t V_{j,t+1} - \sum_i N_{it} \, V_{it} + \sum_i N_{it} \, U_{it}^{\text{flow}}. \tag{B.16}$$

Forming the discounted sum and relabeling $t+1 \to t$ in the first term:

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \, \Psi_t &= \sum_{t=1}^{\infty} \sum_j \beta^t \, N_{jt} \, \mathbb{E}_{t-1} V_{jt} - \sum_{t=0}^{\infty} \sum_i \beta^t \, N_{it} \, V_{it} + \sum_{t=0}^{\infty} \beta^t \sum_i N_{it} \, U_{it}^{\text{flow}} \\
&= -\sum_i N_{i0} \, V_{i0} + \sum_{t=0}^{\infty} \beta^t \sum_i N_{it} \, U_{it}^{\text{flow}},
\end{aligned} \tag{B.17}$$

where the telescoping uses the transversality condition $\lim_{T\to\infty} \beta^T \sum_i N_{iT} \, V_{iT} = 0$. This identity is exact.

B.2.2 Implementability Representation and the Boundary Term

Substituting (B.17) into the household's discounted welfare $\mathcal{W} = \sum_{t=0}^{\infty} \beta^t [u(C_t) - \sum_i N_{it} \, v(L_{it}) - \Psi_t]$:

$$\mathcal{W} = \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - \sum_i N_{it} \big(v(L_{it}) + U_{it}^{\text{flow}} \big) \right] + \sum_i N_{i0} \, V_{i0}. \tag{B.18}$$

Since $v(L_{it}) + U_{it}^{\text{flow}} = u'(C_t) \frac{W_{it}}{P_t^C} L_{it}$, the implementability representation yields:

$$\mathcal{W} = \sum_{t \geq 0} \beta^t \left[u(C_t) - u'(C_t) \sum_i N_{it} \frac{W_{it}}{P_t^C} L_{it} \right] + \sum_i N_{i0} V_{i0}$$

Note that goods market clearing $C_t = Y_t$ does not imply the real wage bill equals C_t under monopolistic competition/Calvo pricing, because aggregate profits (net of lump-sum taxes financing subsidies) generally differ from zero away from the steady state. Rather than substituting the wage bill for consumption, we proceed by expressing aggregate output using the sectoral production identities with within-sector price dispersion Δ_{it}, which generates the standard second-order correction $\xi_{it} = -\log \Delta_{it}$ and yields the familiar π_{it}^2 welfare term. The boundary term $\sum_i N_{i0} V_{i0}$ affects only the initial variables and does not alter the per-period quadratic loss under the timeless perspective.

The boundary term is policy-dependent. Because the continuation values V_{i0} depend on the entire future allocation, $\mathcal{B} = \sum_i N_{i0} V_{i0}$ is *not* constant for the Ramsey planner. We now show that $\mathcal{B}$ nonetheless does not contribute an independent quadratic term to the per-period loss; its welfare effect enters the Ramsey problem exclusively through the initial costate conditions.

Consider the welfare gap $\mathcal{W} - \mathcal{W}^{\text{nat}}$, where $\mathcal{W}^{\text{nat}}$ is welfare under the natural (flexible-price) allocation starting from the same initial state $(\hat{\mathbf{p}}_{-1}, \mathbf{n}_0)$. The implementability representation (**??**) holds for both allocations (since the telescoping is exact), so the welfare gap decomposes as

$$\mathcal{W} - \mathcal{W}^{\text{nat}} = \left(\mathcal{W}^{\text{per}} - \mathcal{W}^{\text{per,nat}}\right) + \sum_i N_{i0}\left(V_{i0} - V_{i0}^{\text{nat}}\right). \tag{B.19}$$

Because the natural allocation is efficient (the output subsidy eliminates markups), the first-order component of $\mathcal{W} - \mathcal{W}^{\text{nat}}$ vanishes. The first-order component of $V_{i0} - V_{i0}^{\text{nat}}$ is generically nonzero (it is first-order in the gaps $\tilde{y}_t, \chi_{it}$), but must be exactly cancelled by the first-order component of the per-period sum. The second-order welfare gap is therefore

$$\left[\mathcal{W} - \mathcal{W}^{\text{nat}}\right] \overset{(2)}{=} \left[\mathcal{W}^{\text{per}} - \mathcal{W}^{\text{per,nat}}\right] + \left[\textstyle\sum_i N_{i0}\left(V_{i0} - V_{i0}^{\text{nat}}\right)\right] \tag{B.20}$$

We define the per-period loss $\mathcal{L}_t$ as the second-order expansion of the per-period terms in $\mathcal{W}^{\text{per}}$—i.e., the terms inside the sum in (**??**)—so that $[\mathcal{W}^{\text{per}} - \mathcal{W}^{\text{per,nat}}]^{(2)} = -\sum_{t=0}^{\infty} \beta^t \mathcal{L}_t$.

Why $\mathcal{B}$ does not affect the per-period loss or targeting criterion. In the Lagrangian formulation of the Ramsey problem (Section 4.5), the value functions $\hat{\mathbf{v}}_t$ appear as choice variables subject to the recursion constraint (70), with associated multiplier $\boldsymbol{\xi}_t$. The boundary term $\mathcal{B}$ depends on $\hat{\mathbf{v}}_0$ but not on $\hat{\mathbf{v}}_t$ for $t \geq 1$. Differentiating $\mathcal{B}$ with respect to $\hat{\mathbf{v}}_0$ therefore modifies only the date-0 first-order condition for $\hat{\mathbf{v}}_0$—equivalently, it shifts the effective initial costate $\boldsymbol{\xi}_{-1}$ from $\mathbf{0}$ to a nonzero value that depends on $\mathbf{n}_0$.

For $t \geq 1$, the costate recursion (F_v) and all other first-order conditions are unchanged. In particular, the Ramsey targeting criterion (Proposition 9) holds for all $t \geq 1$. This is the standard "start-up" issue in commitment problems: the date-0 policy accounts for the boundary term through the modified initial costates, while the ongoing targeting rule depends only on the per-period loss $\mathcal{L}_t$. Under the timeless perspective, one directly imposes the ergodic costate conditions, and the boundary term plays no role.

The per-period loss $\mathcal{L}_t$ therefore involves only the consumption utility, labor disutility, and the output–labor gap correction—with n_{it} and χ_{it} as independent state variables. The intertemporal welfare effects of the mobility cost are encoded in the Ramsey constraints (69)–(70) and correctly priced by the costates $(\boldsymbol{\psi}_t, \boldsymbol{\xi}_t)$.

Remark 3 (Consistency with the static loss)**.** *In the static limit, the boundary term $\mathcal{B} - \mathcal{B}^{nat}$ contributes a per-period second-order welfare effect equal to $\frac{\varphi\phi}{2}\sum_i \alpha_i \mathcal{M}_t$, where $\mathcal{M}_t = \sum_i \alpha_i(\tilde{l}_{it} - \tilde{l}_t^{\alpha})^2$. Adding this to the dynamic per-period loss $\mathcal{L}_t$ and imposing $n_{it} = -\frac{\phi}{1+\varphi}\chi_{it}$ recovers the static loss $\frac{\sigma}{2}\sum_i \alpha_i \chi_{it}^2$ of Proposition 1, confirming that the two derivations are consistent.*

B.2.3 Consumption

Identical to the static case:

$$\frac{1}{U_C \bar{C}^{\text{nat}}} \frac{C_t^{1-\gamma} - (C_t^{\text{nat}})^{1-\gamma}}{1-\gamma} \overset{(2)}{=} \tilde{y}_t + \frac{1-\gamma}{2}\,\tilde{y}_t^2. \tag{B.21}$$

B.2.4 Labor Disutility

The expansions of N_{it} and $L_{it}^{1+\varphi}$ in the static case carry over unchanged. The only difference arises in the cross terms: since n_{it} is now a state variable, we can no longer substitute $n_{it} = \phi(\tilde{l}_{it} - \tilde{l}_t^{\alpha})$. Starting from the raw expansions:

$$\begin{aligned} &\frac{1}{U_C \bar{C}^{\text{nat}}} \sum_i \zeta \left[\frac{N_{it}\, L_{it}^{1+\varphi}}{1+\varphi} - \frac{\alpha_i\, \bar{L}^{1+\varphi}}{1+\varphi} \right] \\ &\quad \overset{(2)}{=} \sum_i \frac{\alpha_i}{1+\varphi} \left[n_{it} + \tfrac{1}{2} n_{it}^2 + (1+\varphi)\,\tilde{l}_{it} + \tfrac{(1+\varphi)^2}{2}\,\tilde{l}_{it}^2 + (1+\varphi)\,\tilde{l}_{it}\, n_{it} \right]. \end{aligned} \tag{B.22}$$

We simplify each group of terms using the second-order constraint $\sum_i \alpha_i\, n_{it} \overset{(2)}{=} -\frac{1}{2}\sum_i \alpha_i\, n_{it}^2$:

- The n_{it} and n_{it}^2 terms perfectly cancel: $\sum_i \frac{\alpha_i}{1+\varphi}\left(n_{it} + \frac{1}{2} n_{it}^2\right) \overset{(2)}{=} 0$.
- The $\tilde{l}_{it}$ terms: $\sum_i \alpha_i\, \tilde{l}_{it} = \tilde{l}_t^{\alpha}$.
- The $\tilde{l}_{it}^2$ terms: $\sum_i \alpha_i\, \tilde{l}_{it}^2 = (\tilde{l}_t^{\alpha})^2 + \mathcal{M}_t$.
- The cross terms: since $\sum_i \alpha_i\, n_{it} \overset{(1)}{=} 0$, the first-order cross term simplifies to

$$\sum_i \alpha_i\, \tilde{l}_{it}\, n_{it} = \sum_i \alpha_i\, n_{it}\,(\tilde{l}_{it} - \tilde{l}_t^{\alpha}).$$

  This expression is left in general form (no substitution for n_{it}).

Collecting, and using $(\tilde{l}_t^\alpha)^2 \overset{(2)}{=} \tilde{y}_t^2$:

$$\frac{1}{U_C \bar{C}^{\text{nat}}} \sum_i \zeta \left[\frac{N_{it}\, L_{it}^{1+\varphi}}{1+\varphi} - \frac{\alpha_i\, \bar{L}^{1+\varphi}}{1+\varphi} \right] \overset{(2)}{=} \tilde{l}_t^\alpha + \frac{1+\varphi}{2} \left(\tilde{y}_t^2 + \mathcal{M}_t \right) + \sum_i \alpha_i\, n_{it}\, (\tilde{l}_{it} - \tilde{l}_t^\alpha). \tag{B.23}$$

B.2.5 Expressing in Terms of (χ_{it}, n_{it})

Using the first-order relationship $\tilde{l}_{it} - \tilde{l}_t^\alpha \overset{(1)}{=} -(\chi_{it} + n_{it})$:

$$\mathcal{M}_t = \sum_i \alpha_i\, (\tilde{l}_{it} - \tilde{l}_t^\alpha)^2 \overset{(2)}{=} \sum_i \alpha_i\, (\chi_{it} + n_{it})^2, \tag{B.24}$$

$$\sum_i \alpha_i\, n_{it}\, (\tilde{l}_{it} - \tilde{l}_t^\alpha) \overset{(2)}{=} -\sum_i \alpha_i\, n_{it}\, (\chi_{it} + n_{it}). \tag{B.25}$$

Substituting into (B.23):

$$\frac{\mathcal{L}_t^{\text{labor}}}{U_C \bar{C}^{\text{nat}}} \overset{(2)}{=} \tilde{l}_t^\alpha + \frac{1+\varphi}{2}\, \tilde{y}_t^2 + \frac{1+\varphi}{2} \sum_i \alpha_i\, (\chi_{it} + n_{it})^2 - \sum_i \alpha_i\, n_{it}\, (\chi_{it} + n_{it}). \tag{B.26}$$

Expanding sector by sector:

$$\begin{aligned} \frac{1+\varphi}{2} (\chi_{it} + n_{it})^2 - n_{it} (\chi_{it} + n_{it}) &= \frac{1+\varphi}{2}\, \chi_{it}^2 + (1+\varphi)\, \chi_{it}\, n_{it} + \frac{1+\varphi}{2}\, n_{it}^2 - n_{it}\, \chi_{it} - n_{it}^2 \\ &= \frac{1+\varphi}{2}\, \chi_{it}^2 + \varphi\, \chi_{it}\, n_{it} + \frac{\varphi-1}{2}\, n_{it}^2. \end{aligned}$$

Therefore:

$$\frac{\mathcal{L}_t^{\text{labor}}}{U_C \bar{C}^{\text{nat}}} \overset{(2)}{=} \tilde{l}_t^\alpha + \frac{1+\varphi}{2}\, \tilde{y}_t^2 + \sum_i \alpha_i \left[\frac{1+\varphi}{2}\, \chi_{it}^2 + \varphi\, \chi_{it}\, n_{it} + \frac{\varphi-1}{2}\, n_{it}^2 \right]. \tag{B.27}$$

B.2.6 Assembly: Raw Loss

Combining (B.21) and (B.27) (with no separate mobility cost term, by the telescoping argument):

$$\frac{U_t - U_t^{\text{nat}}}{U_C \bar{C}^{\text{nat}}} \overset{(2)}{=} (\tilde{y}_t - \tilde{l}_t^\alpha) - \frac{\gamma+\varphi}{2}\, \tilde{y}_t^2 - \sum_i \alpha_i \left[\frac{1+\varphi}{2}\, \chi_{it}^2 + \varphi\, \chi_{it}\, n_{it} + \frac{\varphi-1}{2}\, n_{it}^2 \right]. \tag{B.28}$$

B.2.7 The Output–Labor Gap

The exact Cobb–Douglas decomposition (B.9) carries over unchanged:

$$\tilde{y}_t - \tilde{l}_t^\alpha = \sum_i \alpha_i\, \xi_{it} + \sum_i \alpha_i \log \frac{N_{it}}{N_{it}^{\text{nat}}}. \tag{B.29}$$

Within-sector price dispersion contributes $\zeta_{it} \overset{(2)}{=} -\frac{\epsilon_i}{2\kappa_i}\pi_{it}^2$ as before. For the employment dispersion term, the algebra is identical to the static case—the only difference is that n_{it} is no longer a function of χ_{it}:

$$\sum_i \alpha_i \log \frac{N_{it}}{N_{it}^{\text{nat}}} \overset{(2)}{=} -\frac{1}{2}\sum_i \alpha_i\, n_{it}^2. \tag{B.30}$$

Therefore:

$$\tilde{y}_t - \tilde{l}_t^{\alpha} \overset{(2)}{=} -\sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i}\pi_{it}^2 - \frac{1}{2}\sum_i \alpha_i\, n_{it}^2. \tag{B.31}$$

B.2.8 Final Loss Function

Substituting (B.31) into (B.28):

$$-\frac{U_t - U_t^{\text{nat}}}{U_C \bar{C}^{\text{nat}}} \overset{(2)}{=} \frac{\gamma+\varphi}{2}\tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i}\pi_{it}^2 + \frac{1}{2}\sum_i \alpha_i\, n_{it}^2 + \sum_i \alpha_i \left[\frac{1+\varphi}{2}\chi_{it}^2 + \varphi\,\chi_{it}\, n_{it} + \frac{\varphi-1}{2} n_{it}^2\right]. \tag{B.32}$$

Collecting the n_{it}^2 terms: $\frac{1}{2} + \frac{\varphi-1}{2} = \frac{\varphi}{2}$. Therefore:

$$\mathcal{L}_t \equiv -\frac{U_t - U_t^{\text{nat}}}{U_C \bar{C}^{\text{nat}}} \overset{(2)}{=} \frac{\gamma+\varphi}{2}\tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i}\pi_{it}^2 + \sum_i \alpha_i \left[\frac{1+\varphi}{2}\chi_{it}^2 + \varphi\,\chi_{it}\, n_{it} + \frac{\varphi}{2} n_{it}^2\right], \tag{B.33}$$

which is Equation (66) in Proposition 8.

Remark 4 (Sources of the n_{it}^2 coefficient)**.** *The coefficient $\frac{\varphi}{2}$ on n_{it}^2 combines two distinct sources. The labor disutility expansion contributes $\frac{\varphi-1}{2}$, reflecting the convexity of $v(L)$ and the mechanical effect of uneven employment on aggregate hours. The remaining $\frac{1}{2}$ comes from the output–labor gap* (B.31)*: the concavity of the Cobb–Douglas aggregator means that a nonuniform distribution of workers across sectors reduces aggregate output, even holding total labor input fixed.*

C Additional Proofs of Propositions and Lemmas

C.1 Section 4

Proof of Lemma 4. We prove each statement sequentially.

Parts (i)–(ii): Natural output and sectoral hours. Under flexible prices, the efficient subsidy eliminates markups and the intratemporal first-order condition (31) gives, in log-deviations:

$$(1-\gamma)\, y_t^{\text{nat}} = n_{it} + (1+\varphi)\, l_{it}^{\text{nat}}, \qquad \forall\, i. \tag{C.1}$$

Since $\sum_i \alpha_i n_{it} = 0$, weighting by α_i and summing yields $y_t^{\text{nat}} = \frac{1+\varphi}{\gamma+\varphi} \sum_i \alpha_i a_{it}$, identical to the static case and independent of $\mathbf{n}_t$. Solving (C.1) for sectoral hours gives $l_{it}^{\text{nat}} = \bar{l}_t^{\text{nat}} - \frac{1}{1+\varphi} n_{it}$, where $\bar{l}_t^{\text{nat}} \equiv \frac{1-\gamma}{1+\varphi} y_t^{\text{nat}}$.

Part (ii) continued: Natural relative prices. Goods market clearing gives $\hat{p}_{it}^{\text{nat}} = y_t^{\text{nat}} - a_{it} - l_{it}^{\text{nat}} - n_{it}$. Substituting (47):

$$\hat{p}_{it}^{\text{nat}} = \frac{\gamma+\varphi}{1+\varphi}\, y_t^{\text{nat}} - a_{it} - \frac{\varphi}{1+\varphi}\, n_{it} = \left(\sum_j \alpha_j a_{jt} - a_{it}\right) - \frac{\varphi}{1+\varphi}\, n_{it},$$

where the second equality uses $\frac{\gamma+\varphi}{1+\varphi}\, y_t^{\text{nat}} = \sum_j \alpha_j a_{jt}$.

Part (i) continued: Natural real rate. Since y_t^{nat} is independent of $\mathbf{n}_t$, the Euler equation (8) gives $r_t^{\text{nat}} = -\ln\beta + \gamma\, \mathbb{E}_t(y_{t+1}^{\text{nat}} - y_t^{\text{nat}})$, identical to the static case.

Part (iii): Marginal cost decomposition. The log-linearized marginal cost (21) is $mc_{it} - p_{it} = (\gamma+\varphi) y_t - (1+\varphi)(\hat{p}_{it} + a_{it}) - \varphi\, n_{it}$. Defining $\chi_{it} \equiv \hat{p}_{it} + \hat{a}_{it}$ and using $y_t^{\text{nat}} = \frac{1+\varphi}{\gamma+\varphi} \sum_i \alpha_i a_{it}$, the natural-level marginal cost satisfies $mc_{it}^{\text{nat}} = p_{it}^{\text{nat}}$ (by the efficient subsidy). Taking the gap $\tilde{y}_t = y_t - y_t^{\text{nat}}$ and noting that $\hat{p}_{it} + a_{it} = \chi_{it} - \hat{a}_{it} + a_{it} = \chi_{it} + \sum_j \alpha_j a_{jt}$:

$$\begin{aligned} mc_{it} - p_{it} &= (\gamma+\varphi)\, \tilde{y}_t + (\gamma+\varphi)\, y_t^{\text{nat}} - (1+\varphi)\Big(\chi_{it} + \textstyle\sum_j \alpha_j a_{jt}\Big) - \varphi\, n_{it} \\ &= (\gamma+\varphi)\, \tilde{y}_t - (1+\varphi)\, \chi_{it} - \varphi\, n_{it}, \end{aligned}$$

where the second line uses $(\gamma+\varphi)\, y_t^{\text{nat}} = (1+\varphi) \sum_j \alpha_j a_{jt}$.

Natural reallocation dynamics. We additionally characterize the law of motion for $\mathbf{n}_t$ under flexible prices. The demeaned flow utility at the natural allocation is:

$$\hat{u}_{it}^{\text{nat}} = l_{it}^{\text{nat}} - \bar{l}_t^{\text{nat}} = -\frac{1}{1+\varphi}\, n_{it},$$

using (47) and the fact that the aggregate component cancels upon demeaning. Workers in overstaffed sectors receive lower flow utility and hence have incentives to leave.

Substituting $\hat{\mathbf{u}}_t^{\text{nat}} = -\frac{1}{1+\varphi}\,\mathbf{n}_t^{\text{nat}}$ into the value recursion (25) and iterating forward, the continuation value under flexible prices satisfies:

$$\hat{\mathbf{v}}_t^{\text{nat}} = -\frac{1}{1+\varphi}\sum_{s=0}^{\infty}(\beta\bar{\mu})^s\,\mathbb{E}_t[\mathbf{n}_{t+s}^{\text{nat}}]. \tag{C.2}$$

Suppose the natural law of motion takes the linear form $\mathbf{n}_{t+1}^{\text{nat}} = F\,\mathbf{n}_t^{\text{nat}}$ for some operator F to be determined. Then $\mathbb{E}_t[\mathbf{n}_{t+s}^{\text{nat}}] = F^s\,\mathbf{n}_t^{\text{nat}}$ and:

$$\hat{\mathbf{v}}_t^{\text{nat}} = -\frac{1}{1+\varphi}(I - \beta\bar{\mu}\,F)^{-1}\,\mathbf{n}_t^{\text{nat}}.$$

Substituting into the employment evolution (24):

$$\mathbf{n}_{t+1}^{\text{nat}} = \bar{\mu}\,\mathbf{n}_t^{\text{nat}} - \frac{\phi\beta(I-\bar{\mu}^2)}{1+\varphi}(I-\beta\bar{\mu}\,F)^{-1}F\,\mathbf{n}_t^{\text{nat}} \equiv F\,\mathbf{n}_t^{\text{nat}}. \tag{C.3}$$

which is a fixed-point characterization of F. To verify that F implies mean-reversion, diagonalize $\bar{\mu}$. Under detailed balance (Assumption 1), $\bar{\mu}$ is similar to a symmetric matrix and hence has real eigenvalues $1 = \lambda_1 > \lambda_2 \geq \cdots \geq \lambda_N > -1$ (cannot equal -1 by Perron-Frobenius Theorem). Along the k-th eigenvector (for $k \geq 2$), the fixed-point condition (C.3) reduces to the scalar equation:

$$f_k = \lambda_k - \frac{\phi\beta(1-\lambda_k^2)f_k}{(1+\varphi)(1-\beta\lambda_k f_k)}. \tag{C.4}$$

where f_k is the eigenvalue of F along component k. Rearranging, f_k solves the quadratic:

$$\beta\lambda_k f_k^2 - \left[1 + \beta\lambda_k^2 + \frac{\phi\beta(1-\lambda_k^2)}{1+\varphi}\right] f_k + \lambda_k = 0. \tag{C.5}$$

The product of roots is $\lambda_k/(\beta\lambda_k) = 1/\beta > 1$, so one root is inside and one outside the unit circle. The stable root $f_k \in (0, \lambda_k)$ gives the natural rate of employment mean-reversion along component k. That $f_k < \lambda_k$ confirms that directed reallocation (governed by ϕ) accelerates convergence beyond what passive baseline transitions (governed by $\bar{\mu}$) alone would achieve. $\square$

Proof of Lemma 6. The proof proceeds in three steps: symmetrization, spectral properties, and decoupling.

Step 1: Symmetrization. Detailed balance (15) states $\alpha_i \bar{\mu}^{ij} = \alpha_j \bar{\mu}^{ji}$ for all i, j, which in matrix form is $A\bar{\mu} = \bar{\mu}^\top A$ where $A = \text{diag}(\alpha_i)$. Define $Q \equiv A^{1/2}\bar{\mu}\, A^{-1/2}$. Then:

$$Q^\top = A^{-1/2}\bar{\mu}^\top A^{1/2} = A^{-1/2}(A\bar{\mu}\, A^{-1})A^{1/2} = A^{1/2}\bar{\mu}\, A^{-1/2} = Q,$$

where the second equality uses $\bar{\mu}^\top = A\bar{\mu}\, A^{-1}$ (from $A\bar{\mu} = \bar{\mu}^\top A$). So Q is real symmetric.

Step 2: Spectral properties. Since Q is real symmetric, it has real eigenvalues $\rho_1 \geq \rho_2 \geq \cdots \geq \rho_N$ and orthonormal eigenvectors $\{s_k\}_{k=1}^N$. We establish the claimed ordering.

Dominant eigenvalue. Since $\bar{\mu}$ is a row-stochastic matrix, $\bar{\mu}\,\mathbf{1} = \mathbf{1}$. Therefore $Q(A^{1/2}\mathbf{1}) = A^{1/2}\bar{\mu}\,\mathbf{1} = A^{1/2}\mathbf{1}$, so $s_1 = A^{1/2}\mathbf{1}/\|A^{1/2}\mathbf{1}\|$ is an eigenvector with eigenvalue $\rho_1 = 1$.

Strict bounds. Under the logit specification (11), $\bar{\mu}^{ij} > 0$ for all i, j (the exponential is always positive). Therefore $\bar{\mu}$ is a strictly positive stochastic matrix. By the Perron–Frobenius theorem, $\rho_1 = 1$ is a simple eigenvalue and all other eigenvalues satisfy $|\rho_k| < 1$ for $k \geq 2$, which gives $1 = \rho_1 > \rho_2 \geq \cdots \geq \rho_N > -1$.

Stochastic bounds. Since Q is similar to the stochastic matrix $\bar{\mu}$ (via the conjugation $A^{\pm 1/2}$), they share the same eigenvalues. All eigenvalues of a stochastic matrix have modulus at most one, confirming $|\rho_k| \leq 1$.

Step 3: Decoupling the relative block. Define the coordinate transformation $x^*_{kt} \equiv s_k^\top A^{1/2}\hat{\mathbf{x}}_t$ for each variable $x \in \{\chi, n, \hat{v}, \hat{a}\}$. Since $\hat{\mathbf{x}}_t = M\mathbf{x}_t$ where $M = I - \mathbf{1}\boldsymbol{\alpha}^\top$, we have $\boldsymbol{\alpha}^T\hat{x}_t = 0$. Then

$$x^*_{1t} = s_1^T A^{1/2}\hat{x}_t \propto (A^{1/2}\mathbf{1})^T A^{1/2}\hat{\mathbf{x}}_t = 1^T A\hat{\mathbf{x}}_t = \alpha^T\hat{\mathbf{x}}_t = 0$$

So only components $k \geq 2$ are nontrivial.

We now show that each equation in the relative block (52)–(55) decouples. The key identity is that whenever $\bar{\mu}$ or $\bar{\mu}^2$ appears, the transformation diagonalizes it:

$$s_k^\top A^{1/2}\bar{\mu}\,\hat{\mathbf{x}}_t = s_k^\top A^{1/2}\bar{\mu}\, A^{-1/2}A^{1/2}\hat{\mathbf{x}}_t = s_k^\top Q\, A^{1/2}\hat{\mathbf{x}}_t = \rho_k\, x^*_{kt},$$

using $Q\, s_k = \rho_k\, s_k$ and $s_k^\top s_j = \delta_{kj}$. Similarly, $s_k^\top A^{1/2}\bar{\mu}^2\hat{\mathbf{x}}_t = \rho_k^2\, x^*_{kt}$ and $s_k^\top A^{1/2}(I - \bar{\mu}^2)\hat{\mathbf{x}}_t = (1 - \rho_k^2)\, x^*_{kt}$.

Applying $s_k^\top A^{1/2}$ to each equation in the relative block (with constant κ):

Relative NKPC (52): $s_k^\top A^{1/2}\hat{\boldsymbol{\pi}}_t = -\kappa[(1+\varphi)\,\chi^*_{kt} + \varphi\, n^*_{kt}] + \beta\,\mathbb{E}_t[\pi^*_{kt+1}]$, where we used $\kappa_i = \kappa$ to factor κ out of the projection.

Relative price evolution (53): $\chi^*_{kt} = \chi^*_{k,t-1} + \pi^*_{kt} + \Delta\hat{a}^*_{kt}$, which is immediate since M and the identity commute with the projection.

Employment evolution (54): $n^*_{k,t+1} = \rho_k\, n^*_{kt} + \phi\beta(1 - \rho_k^2)\,\mathbb{E}_t[\hat{v}^*_{k,t+1}]$, using the diagonalization identities above.

Value recursion (55): $\hat{v}^*_{kt} = -\chi^*_{kt} - n^*_{kt} + \beta\rho_k \mathbb{E}_t[\hat{v}^*_{k,t+1}]$.

Each projected equation for component k involves only $(\chi^*_{kt}, n^*_{kt}, \hat{v}^*_{kt})$ and the exogenous forcing $\hat{a}^*_{kt}$, with no coupling to any $k' \neq k$. The constant-κ assumption is used only in the NKPC projection: with heterogeneous κ_i, the term $s_k^\top A^{1/2}\,\mathrm{diag}(\kappa_i)\hat{\chi}_t$ would involve off-diagonal entries $s_k^\top A^{1/2} K A^{-1/2} s_{k'}$ for $k' \neq k$, coupling the components. □

Proof of Proposition 4. We evaluate the persistence, impact, and cumulative inflation in turn.

Persistence. Let $f(\lambda;\Gamma) = \beta\lambda^2 - \Gamma\lambda + 1$. The stable root $\lambda \in (0,1)$ decreases strictly in Γ. Comparing (58) and (59), the segmented coefficient $\Gamma_{\text{seg}} = 1 + \beta + \kappa(1+\varphi)$ is strictly larger than the integrated coefficient $\Gamma_{\text{int}} = 1 + \beta + \kappa$ because $\kappa\varphi > 0$. Therefore, $\lambda_{\text{seg}} < \lambda_{\text{int}}$, proving that segmented markets decay faster.

Impact. The characteristic equation (58) implies $\lambda_{\text{seg}} = (\Gamma_{\text{seg}} - \beta\lambda_{\text{seg}})^{-1}$. Therefore, the initial distortion under segmented markets is $\chi^*_{j0,\text{seg}} = \hat{a}/(\Gamma_{\text{seg}} - \beta\lambda_{\text{seg}})$. Comparing this to the integrated initial condition in (60):

$$\chi^*_{j0,\text{int}} = \frac{1}{\Gamma_{\text{seg}} - \beta\lambda_{\text{int}}}\,\hat{a}$$

Because $\lambda_{\text{seg}} < \lambda_{\text{int}}$, the denominator for the integrated market is smaller, meaning $\chi^*_{j0,\text{seg}} < \chi^*_{j0,\text{int}}$. Since impact inflation is $\hat{\pi}^*_{j0} = \chi^*_{j0} - \hat{a}$, its magnitude is $|\hat{\pi}^*_{j0}| = \hat{a} - \chi^*_{j0}$. The smaller initial distortion under segmented markets thus yields a strictly larger initial inflation impact: $|\hat{\pi}^*_{j0,\text{seg}}| > |\hat{\pi}^*_{j0,\text{int}}|$.

Cumulative. By definition, relative inflation is the change in the relative price level, $\hat{\pi}^*_{jt} = \hat{p}^*_{jt} - \hat{p}^*_{jt-1}$. The cumulative sum telescopes: $\sum_{t=0}^{\infty} \hat{\pi}^*_{jt} = \lim_{t\to\infty} \hat{p}^*_{jt} - \hat{p}^*_{j,-1}$. Starting from steady state ($\hat{p}^*_{j,-1} = 0$), prices must eventually clear the market to eliminate the distortion ($\lim_{t\to\infty} \chi^*_{jt} = 0$). Since $\chi^*_{jt} = \hat{p}^*_{jt} + \hat{a}$, the relative price level must converge to $\lim_{t\to\infty} \hat{p}^*_{jt} = -\hat{a}$, yielding $\sum_{t=0}^{\infty} \hat{\pi}^*_{jt} = -\hat{a}$ regardless of ϕ. □

Proof of Proposition 5. The proof proceeds in four steps: derivation of the characteristic polynomial, root counting, and verification of the two limiting cases.

Step 1: Characteristic polynomial. After shocks have died out ($\hat{a}^*_{kt} = 0$), conjecture $\chi^*_{kt} = \lambda^t C$, $n^*_{kt} = \lambda^t N$, $\hat{v}^*_{kt} = \lambda^t V$ for constants C, N, V. Substituting into the component-k system (56)–(57):

$$\Big[\beta\lambda^2 - (\kappa(1+\varphi) + \beta + 1)\lambda + 1\Big]\, C = \kappa\varphi\lambda\, N, \tag{C.6}$$

$$(\lambda - \rho_k)\, N = \phi\beta(1-\rho_k^2)\lambda\, V, \tag{C.7}$$

$$V = -(C+N) + \beta\rho_k\lambda\, V. \tag{C.8}$$

Solving (C.8) for V:

$$V = -\frac{C+N}{1-\beta\rho_k\lambda}. \tag{C.9}$$

Substituting (C.9) into (C.7):

$$(\lambda - \rho_k)\, N = -\frac{\phi\beta(1-\rho_k^2)\lambda}{1-\beta\rho_k\lambda}\,(C+N).$$

Multiplying through by $(1-\beta\rho_k\lambda)$ and collecting:

$$\Big[(\lambda-\rho_k)(1-\beta\rho_k\lambda) + \phi\beta(1-\rho_k^2)\lambda\Big]\, N = -\phi\beta(1-\rho_k^2)\lambda\, C. \tag{C.10}$$

Using (C.10) to express N in terms of C and substituting into (C.6) yields, after cancelling $C \neq 0$:

$$\Big[(\lambda-\rho_k)(1-\beta\rho_k\lambda) + \phi\beta(1-\rho_k^2)\lambda\Big]\Big[\beta\lambda^2 - (\kappa(1+\varphi)+\beta+1)\lambda + 1\Big] \\ + \kappa\varphi\phi\beta(1-\rho_k^2)\lambda^2 = 0,$$

which is (61). Expanding, the first bracket is quadratic in λ (with leading term $-\beta\rho_k\lambda^2$) and the second bracket is quadratic (with leading term $\beta\lambda^2$), so the product is degree four. The final additive term is degree two, preserving the degree. Hence (61) is a degree-4 polynomial $p(\lambda) = 0$.

Step 2: Root count. The system has three variables $(\chi^*_{kt}, n^*_{kt}, \hat{v}^*_{kt})$ and one lag from the NKPC (through $\chi^*_{k,t-1}$), giving a total state dimension of four. By the Blanchard–Kahn condition, a unique bounded solution requires exactly two stable roots ($|\lambda| < 1$) and two unstable roots ($|\lambda| > 1$).[8]

[8] The degree-4 polynomial (61) can be verified numerically to have exactly two roots inside the unit circle across the economically relevant parameter space.

Step 3: Limiting case $\phi \to 0$. Setting $\phi = 0$ in (61):

$$(\lambda - \rho_k)(1 - \beta\rho_k\lambda)\left[\beta\lambda^2 - (\kappa(1+\varphi) + \beta + 1)\lambda + 1\right] = 0.$$

This factors into four explicit roots. The first factor gives $\lambda = \rho_k$ (stable, since $|\rho_k| < 1$). The second gives $\lambda = 1/(\beta\rho_k)$ (unstable, since $|\rho_k| < 1$). The quadratic is the segmented-market characteristic equation (58), whose roots have product $1/\beta > 1$: one stable root $\lambda_{\text{seg}} \in (0, 1)$ and one unstable root $1/(\beta\lambda_{\text{seg}}) > 1$. The two stable roots are therefore $\{\lambda_{\text{seg}}, \rho_k\}$, and the spectral radius of the stable block is $\lambda^*(0) = \max\{\lambda_{\text{seg}}, |\rho_k|\}$.

Step 4: Limiting case $\phi \to \infty$. Divide (61) by ϕ and take $\phi \to \infty$. All terms not proportional to ϕ vanish, leaving:

$$\beta(1-\rho_k^2)\lambda\left[\beta\lambda^2 - (\kappa(1+\varphi) + \beta + 1)\lambda + 1\right] + \kappa\varphi\beta(1-\rho_k^2)\lambda^2 = 0.$$

Since $\beta(1-\rho_k^2)\lambda \neq 0$ for $\lambda \neq 0$ and $|\rho_k| < 1$, divide through:

$$\beta\lambda^2 - (\kappa(1+\varphi) + \beta + 1)\lambda + 1 + \kappa\varphi\lambda = 0,$$

which simplifies to

$$\beta\lambda^2 - (\kappa + \beta + 1)\lambda + 1 = 0.$$

This is the integrated-market characteristic equation (59), whose stable root is λ_{int}. The fourth root at $\lambda = 0$ (from the factor λ extracted above) confirms that one degree of freedom collapses as $\phi \to \infty$: employment fully tracks relative prices ($n^*_{kt} = -\chi^*_{kt}$), eliminating one independent state variable.

Continuity. The roots of (61) depend continuously on ϕ (as roots of a polynomial with coefficients that are continuous in ϕ). Since the dominant stable root $\lambda^*(\phi)$ remains bounded away from the unit circle for all finite $\phi > 0$ (by Blanchard–Kahn), $\lambda^*(\phi)$ is a continuous function of ϕ on $[0, \infty)$, interpolating between the two limiting values. □

Proof of Proposition 6. We proceed in steps.

Part (i): Effective slope. Write $K = \text{diag}(\kappa_i) = \kappa I + \varepsilon\,\text{diag}(\Delta\kappa_i)$. Under constant κ, the NKPC projection onto component k (Lemma 6) involves $s_k^\top\,\text{diag}(\kappa_i)\,s_{k'} = \kappa\,\delta_{kk'}$. Under the perturbation,

$$s_k^\top\,\text{diag}(\kappa_i)\,s_{k'} = \kappa\,\delta_{kk'} + \varepsilon\sum_i \Delta\kappa_i\,s_{k,i}\,s_{k',i}.$$

Using $s_{k,i} = \alpha_i^{1/2} v_{k,i}$, the diagonal entry is $\kappa + \varepsilon\, \Delta\kappa_k^{\text{eff}}$. Off-diagonal entries are $O(\varepsilon)$ but affect the eigenvalue only at $O(\varepsilon^2)$ by standard non-degenerate perturbation theory. To first order, the component-k polynomial is therefore obtained by replacing κ with $\kappa + \varepsilon\, \Delta\kappa_k^{\text{eff}}$ in (61).

Part (ii): Persistence shift. The dominant stable root λ_k^0 satisfies $p(\lambda_k^0;\, \kappa,\, \rho_k) = 0$. By Part (i), the perturbed polynomial is $p(\lambda;\, \kappa + \varepsilon\, \Delta\kappa_k^{\text{eff}},\, \rho_k) + O(\varepsilon^2) = 0$. The implicit function theorem at the simple root λ_k^0 gives (63) with $\Gamma_k = -\partial_\kappa p / \partial_\lambda p$ evaluated at $(\lambda_k^0, \kappa, \rho_k)$.

Formula and Sign of Γ_k. Decompose (61) as $p = R \cdot S + T$ where

$$
\begin{aligned}
R(\lambda) &\equiv (\lambda - \rho_k)(1 - \beta\rho_k\lambda) + \phi\beta(1 - \rho_k^2)\lambda, \\
S(\lambda;\kappa) &\equiv \beta\lambda^2 - [\kappa(1+\varphi) + \beta + 1]\lambda + 1, \\
T(\lambda;\kappa) &\equiv \kappa\varphi\phi\beta(1 - \rho_k^2)\lambda^2.
\end{aligned}
$$

Since T is linear in κ, $\partial_\kappa T = T/\kappa$. Using $p(\lambda_k^0) = 0$, i.e. $T(\lambda_k^0) = -R(\lambda_k^0)S(\lambda_k^0)$, the κ-derivative simplifies to

$$
\partial_\kappa\, p\big|_{\lambda_k^0} = -(1+\varphi)\lambda_k^0\, R(\lambda_k^0) + \tfrac{1}{\kappa}\, T(\lambda_k^0) = -\frac{R(\lambda_k^0)}{\kappa}\Big[\kappa(1+\varphi)\lambda_k^0 + S(\lambda_k^0)\Big].
$$

Substituting the definition of S collapses the bracket:

$$
\kappa(1+\varphi)\lambda_k^0 + S(\lambda_k^0) = \beta(\lambda_k^0)^2 - (\beta+1)\lambda_k^0 + 1 = (\lambda_k^0 - 1)(\beta\lambda_k^0 - 1) > 0,
$$

where the inequality uses $\lambda_k^0 < 1$ and $\beta < 1$. For $\rho_k \in (0,1)$, $T(\lambda_k^0) > 0$ and $RS = -T$ imply $R(\lambda_k^0) > 0$ and $S(\lambda_k^0) < 0$. Together, $\partial_\kappa p|_{\lambda_k^0} < 0$. Finally, the degree-4 polynomial satisfies $p(0) > 0$ and $p(1) < 0$, so it crosses zero from above at the dominant stable root: $\partial_\lambda p|_{\lambda_k^0} < 0$. Therefore

$$
\Gamma_k = -\frac{\partial_\kappa\, p}{\partial_\lambda\, p}\bigg|_{\lambda_k^0} < 0:
$$

a higher effective slope strictly accelerates convergence. $\square$