# Math 151A

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### Today...

- Newton's Method
  - Basics
  - Implementation
- Special Topics on Newton's Method
  - Complex Roots and Fractals
  - Fast Inverse Square Root Algorithm
- Exercise Problems

Homework Q&A



#### Newton's Method

Algebraic Derivation

• Geometric Interpretation



#### Newton's Method

• MATLAB Implementation



#### Complex Roots and Newton Fractals

- Note that for fixed point methods, it doesn't matter if the domain is real or complex (or even a higher dimensional space)
  - as long as it is a contraction

- However, we can't yet tell which point converges to which root when it's not close to any roots
- There are deep results on this topic. We will not cover this, but search for "Newton Fractal" if interested
- Some examples: (MATLAB)



# Fast Inverse Square Root Algorithm

• In computer graphics, we use a technique called "trianglulization"

 For each triangle, to represent reflections of light, we need to find "normalized vector"

- We need to compute inverse square roots to do this
- We have sometimes billions, or trillions(!) of triangles
- So a fast computation of inverse sqrt is required

• (YT)

Surface normals are used
extensively in lighting and shading
calculations, requiring the calculation of
norms for vectors. A field of vectors
normal to a surface is shown here.



#### Exercise Problems

• Discuss when Newton's method gives linear/quadratic convergence for  $f(x) = x^2(x-1)$ 



## HW Q&A

- Need hints for Q7?
- Other Questions?

