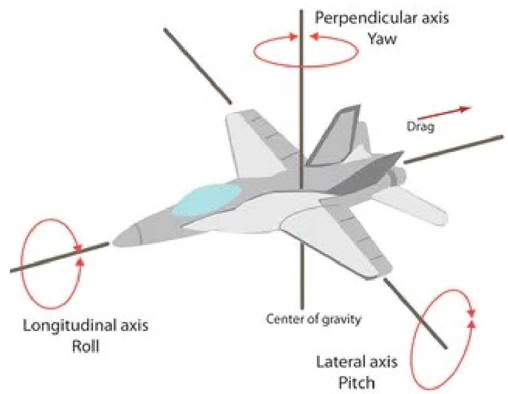
6-Axis IMU Fusion Algorithm

This algorithm utilized both the acceleretometer and gyroscope readings to calculate roll and pitch angles. Gyroscope data is used to minimize the accelerometer jiggle in the measurements. Orientation calculation makes use of quaternions described by the Irish mathematician William Rowan Hamilton in 1843.



Quaternions are very efficient for analyzing situations where rotations in three dimensions are involved. A quaternion is a 4-tuple, which is a more concise representation than a rotation matrix. Its geometric meaning is also more obvious as the rotation axis and angle can be trivially recovered.

Let's define a quaternion vector,

$$\vec{q} = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \qquad q = q_0 + q_1 \cdot i + q_2 \cdot j + q_3 \cdot k \qquad i^2 = j^2 = k^2 = i \cdot j \cdot k = -1$$

It is important to know this vector notation for quaternions,when we discuss the fusion algorithm this will become handy.

$$\overrightarrow{g_{measured,0}} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

Start with an initial condition where the unity gravitational vector is assumed to be in purely z-axis. Comma deliminator represents the measurement take.

$$\overrightarrow{\omega_{gyro,i}} = \begin{pmatrix} \omega_{x,i} \\ \omega_{y,i} \\ \omega_{z,i} \end{pmatrix} \frac{deg}{s}$$

The first thing to do after setting initial condition to define, or get, sensor data. Chronogically this is the initial take of measurements but for sake of notation it will be denoted as i'th measurement.

$$\overrightarrow{a_{accel,i}} = \begin{pmatrix} a_{x,i} \\ a_{y,i} \\ a_{z,i} \end{pmatrix} \cdot mg$$

$$\overrightarrow{q_{gyro,i}} = \begin{bmatrix} 1 - \dfrac{\left(\int \omega_{x,i}\,dt\right)^2 \cdot \left(\int \omega_{y,i}\,dt\right)^2 \cdot \left(\int \omega_{z,i}\,dt\right)^2}{2} \\ \dfrac{-\int \omega_{x,i}\,dt}{2} \\ \dfrac{-\int \omega_{y,i}\,dt}{2} \\ \dfrac{-\int \omega_{z,i}\,dt}{2} \end{bmatrix}$$

Then define the quaternion vector for the gyroscope reading. We take the discrete integrals for defining the vector. Such that, 'dt' denotes the sampling period and the quaternion consisting change in angle.

Using gyro quaternion rotating the initial gravitational vector, result first measure of the gravitational vector.

$$\overrightarrow{q_{g,measured,i}} = \begin{pmatrix} 0 \\ g_{x\_measured,i} \\ g_{y\_measured,i} \\ g_{z\_measured,i} \end{pmatrix}$$

This is the quaternion of the measured gravitational vector. chronogically this is the initial condition that is defined above but for the notations sake it will be referred as i'th measurement.

$$\left(\overrightarrow{q_{g,measured,i}} \otimes \overrightarrow{q_{gyro,i}} \right) \otimes \overline{\overrightarrow{q_{gyro,i}}} = \overrightarrow{q_{g,measured,gyro,i}}$$

There is three types of vector multiplication operations to mention, dot product, cross product and quaternion product. Quaternion product is denoted with cross in a circle.

By multiplying the initial condition or resulting measured gravity quaternion with the gyroscope quaternion and its conjugate, it gives the quaternion of the rotated gravity vector. By the calculated angles using the gyro.

$$quaternion\_product = cross\_product - dot\_product$$

Using the gyro rotated gravity vector quaternion extract the 3D unit vector.

$$\vec{g}_{measured,gyro,i} = \begin{bmatrix} \left(\vec{q}_{g,measured,gyro,i}^{\,T}\right)^{\langle 2 \rangle} \\ \left(\vec{q}_{g,measured,gyro,i}^{\,T}\right)^{\langle 3 \rangle} \\ \left(\vec{q}_{g,measured,gyro,i}^{\,T}\right)^{\langle 4 \rangle} \end{bmatrix}$$

Then the accelerometer data is normalized and fused with the gyroscope vector with pre-defined fusion coefficents

$$\vec{a}_{accel,normalized,i} = \left[\frac{\left[\vec{a}_{accel,i} = \begin{pmatrix} a_{x,i} \\ a_{y,i} \\ a_{z,i} \end{pmatrix} \right]}{\left| \vec{a}_{accel,i} = \begin{pmatrix} a_{x,i} \\ a_{y,i} \\ a_{z,i} \end{pmatrix} \right|} \right]$$

Fusion coefficent is calculated by the angle difference of the accelerometer imposed gravity vector and gyroscope imposed gravity vector.

$$\xi = 40 \quad \text{if} \quad \left(\vec{a}_{accel,normalized,i} \cdot \vec{g}_{measured,gyro,i} = 1 \cdot 1 \cdot \cos\beta \right) \le 0.96$$

$$\xi = 10 \quad \text{if} \quad \left(\vec{a}_{accel,normalized,i} \cdot \vec{g}_{measured,gyro,i} = 1 \cdot 1 \cdot \cos\beta \right) \ge 0.96$$

Fusing the measured vectors from accel and gyro, results the next iteration of the measured gravitational vector.

$$\vec{g}_{measured,i+1} = \frac{\xi \cdot \vec{g}_{measured,gyro,i} + \vec{a}_{accel,normalized,i}}{\left| \xi \cdot \vec{g}_{measured,gyro,i} + \vec{a}_{accel,normalized,i} \right|} \qquad \text{(It is an unit vector)}$$

To obtain the angle between the measured gravitational vector and the reference gravitational vector, again the vectors converted to quaternions. This time, quaternion is calculated between a reference and measurement

$$\left[\overrightarrow{g_{reference}} = \begin{pmatrix} 0 \\ 0 \\ -9.81 \end{pmatrix}\right]\frac{m}{s^2} \qquad \overrightarrow{g_{reference,normalized}} = \frac{\overrightarrow{g_{reference,normalized}}}{\left|\overrightarrow{g_{reference,normalized}}\right|}$$

Calculate the Euler angle and Euler axis,

$$\overrightarrow{g_{measured,i+1}} \cdot \frac{\overrightarrow{g_{reference,normalized}} + \overrightarrow{g_{measured,i+1}}}{\left|\overrightarrow{g_{reference,normalized}} + \overrightarrow{g_{measured,i+1}}\right|} = \varepsilon_{angle}$$

$$\overrightarrow{g_{measured,i+1}} \times \frac{\overrightarrow{g_{reference,normalized}} + \overrightarrow{g_{measured,i+1}}}{\left|\overrightarrow{g_{reference,normalized}} + \overrightarrow{g_{measured,i+1}}\right|} = \varepsilon_{axis}$$ (It is an unit 3D vector)

Construct quaternion in between reference and measurement,

$$\overrightarrow{q_{in\_between}} = \begin{bmatrix} \varepsilon_{angle} \\ \left(\varepsilon_{axis}{}^{T}\right)^{\langle 1\rangle} \\ \left(\varepsilon_{axis}{}^{T}\right)^{\langle 2\rangle} \\ \left(\varepsilon_{axis}{}^{T}\right)^{\langle 3\rangle} \end{bmatrix}$$

Calculate euler angles (rotation angles) using the quaternion between measured and reference,

$$\phi = \frac{180deg}{\pi}\cdot\arctan\left[\frac{\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 1\rangle}\cdot\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 2\rangle}+\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 3\rangle}\cdot\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 4\rangle}}{1-2\cdot\left[\left[\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 2\rangle}\right]^{2}+\left[\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 3\rangle}\right]^{2}\right]}\right]$$

$$\theta = \frac{180deg}{\pi}\cdot\arcsin\left[\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 1\rangle}\cdot\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 3\rangle}-\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 4\rangle}\cdot\left(\overrightarrow{q_{in\_between}}^{T}\right)^{\langle 2\rangle}\right]$$