A Comparison of NSGA II and MOSA for Solving

Multi-depots Time-dependent Vehicle Routing Problem with Heterogeneous Fleet

Arian Razmi Farooji

CM3 Conference

27 May 2015
Jyväskylä, Finland

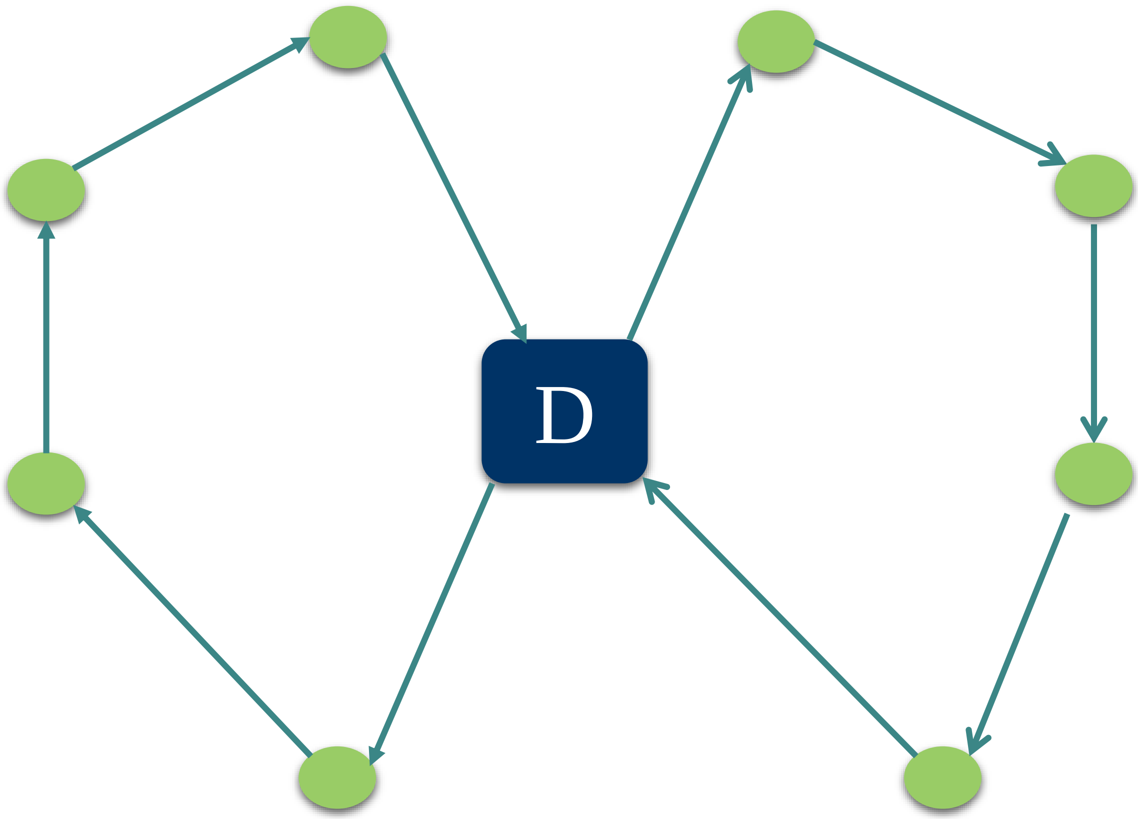
© Tiia Monto

List of Contents

Introduction

Introduction

Vehicle Routing Problem



Introduction

Importance of Vehicle Routing Problem (Toth and Vigo 2002)

- Reducing 5 to 20% of transportation costs

Importance of considering time-dependency (Figliozzi 2012)

- Assuring the optimal solutions
- Reducing transportation and logistics costs
- Reducing air and noise pollution

Previous Studies

Malandraki (1989), Malandraki and Daskin (1992)	Proposing Time-dependent Vehicle Routing Problem for the fist time
Ahn and Shin (1992)	Adding FIFO to TDVRP
Tailard (1999)	Introducing Vehicle Routing Problem with heterogeneous fleet (VRPHF)
Ichoua et al. (2003)	Modification of Time-dependency based on FIFO
Dondo et al. (2004)	Vehicle Routing Problem with multiple depots and time-windows
Omar et al. (2005)	Considering the role of car accidents in traffic congestions
Van Woensel (2008)	Queuing Theory in traffic congestions
Figliozzi (2012)	TDVRP with Hard Time Windows

Problem Definition

Problem Definition

A Vehicle Routing Problem with ,

- Time-dependency
  - dependency of speed to departure time
- Heterogeneous Fleet
  - different types but infinite numbers
- Multiple Depots
- Hard Time-windows
- Possibility for vehicles to return to depots except their origin depots

Problem Definition

Objectives:

- Minimizing the number of routes
- Minimizing the total costs
  - Travel costs
  - Vehicle utilization costs

Problem Definition

Constraints:

- Serving all customers
- Allocating just one vehicle to each customer
- Serving between hard time-windows
- Vehicles’ capacity
- Initiating a route from one depot and terminating it to the same or different depot

Problem Formulation

Parameters and Notations			
q_i	Demand of customer i	t_{ij}^{ku}	Travel time between i and j for vehicle type k in interval u
s_i	Service Time customer i	c_{ij}^{k}	Travel cost between i and j for vehicle type k
a_i	Earliest time of serving customer i	u	Time interval u
b_i	Latest time of serving customer i	T_u	Upper limit for time interval u
C_k	Capacity of vehicle type k	k	Vehicle type k
cf_k	Fixed cost of vehicle type k	**Decision Variables**	
cv_k	Variable cost of vehicle type k	x_{ij}^{ku}	Binary variable indication travel of vehicle type k from i to j in time interval u
d_{ij}	Distance between i and j	y_i^k	Real variable indicating beginning of serving to customer's place

Problem Formulation

Objectives:

$$\min \sum_{k=1}^{\mathcal{K}} \sum_{i=1}^{m} \sum_{j=m+1}^{n} \sum_{u=1}^{\mathcal{U}} x_{ij}^{ku} \qquad (1)$$

$$\min \sum_{k=1}^{\mathcal{K}} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{u=1}^{\mathcal{U}} c_{ij}^{k}\, x_{ij}^{ku} + \sum_{k=1}^{\mathcal{K}} \sum_{i=1}^{m} \sum_{j=m+1}^{n} \sum_{u=1}^{\mathcal{U}} c_{f_k} x_{ij}^{ku} \qquad (2)$$

Problem Formulation

Constraints:

$$\sum_{k=1}^{\mathcal{K}} \sum_{i=1}^{n} \sum_{u=1}^{\mathcal{U}} x_{ij}^{ku} = 1 \qquad \forall\, j = m+1, \dots, n \qquad (3)$$

$$\sum_{k=1}^{\mathcal{K}} \sum_{j=1}^{n} \sum_{u=1}^{\mathcal{U}} x_{ij}^{ku} = 1 \qquad \forall\, i = m+1, \dots, n \qquad (4)$$

$$\sum_{i=1}^{m} \sum_{j=m+1}^{n} \sum_{u=1}^{\mathcal{U}} x_{ij}^{ku} \le 1 \qquad \forall\, k = 1, \dots, \mathcal{K} \qquad (5)$$

$$\sum_{i=m+1}^{n} \sum_{j=1}^{m} \sum_{u=1}^{\mathcal{U}} x_{ij}^{ku} \le 1 \qquad \forall\, k = 1, \dots, \mathcal{K} \qquad (6)$$

Problem Formulation

Constraints:

$$\sum_{i=1}^{n}\sum_{u=1}^{\mathcal{U}} x_{ir}^{ku} - \sum_{j=1}^{n}\sum_{u=1}^{\mathcal{U}} x_{rj}^{ku} = 0 \qquad \forall\, k = 1, \dots, \mathcal{K} \quad \forall\, r = m+1, \dots, n \tag{7}$$

$$a_i \sum_{j=1}^{n} x_{ij}^{ku} \le y_i^k \le b_i \sum_{j=1}^{n} x_{ij}^{ku} \qquad \forall\, k = 1, \dots, \mathcal{K} \quad \forall\, u = 1, \dots, \mathcal{U} \tag{8}$$

$$y_i^k + s_i + t_{ij}^{ku}\left(y_i^k + s_i\right) - y_j^k \le M(1 - x_{ij}^{ku}) \qquad \forall\, k = 1, \dots, \mathcal{K} \quad \forall\, u = 1, \dots, \mathcal{U} \tag{9}$$

$$y_i^k + s_i - T_u \le M(1 - x_{ij}^{k}) \qquad \forall\, k = 1, \dots, \mathcal{K} \quad \forall\, u = 1, \dots, \mathcal{U} \qquad \forall\, i,j \in A \tag{10}$$

Problem Formulation

Constraints:

$y_i^k + s_i \geq T_{u-1}\, x_{ij}^{ku}$	$\forall\, k = 1, \dots, \mathcal{K}$ $\forall\, u = 1, \dots, \mathcal{U}$	$\forall\, i, j \in A$	(11)
$\sum_{i=1}^{m} q_i \sum_{j=1}^{n} \sum_{u=1}^{\mathcal{U}} x_{ij}^{ku} \leq C_k$	$\forall\, k = 1, \dots, \mathcal{K}$		(12)

Solution Approach

NSGA II

1. Solution Representation

Vehicle Type

Customers

1	3	3	2	3	1
0,95	0,1	0,55	0,7	0,18	0,3

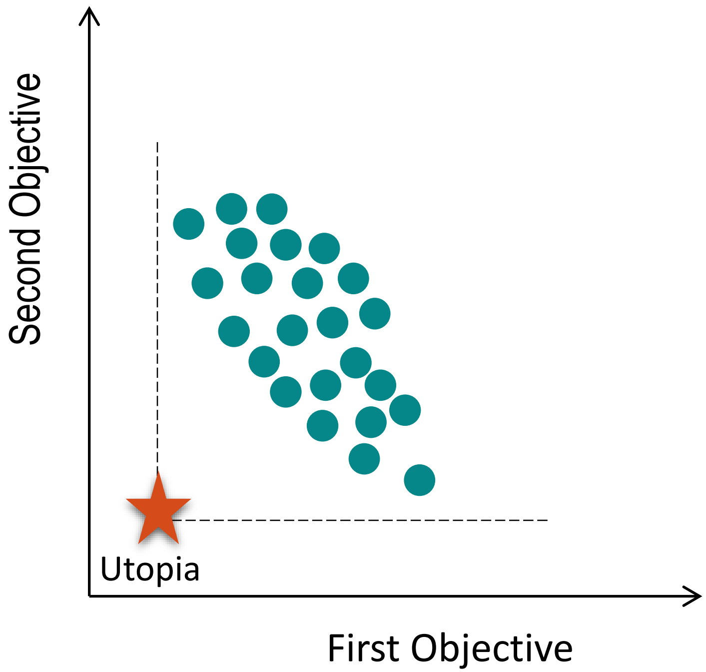
1	3	3	2	3	1
0,1	0,18	0,3	0,5	0,7	0,95

1	3	3	2	3	1
6	5	4	3	2	1

NSGA II

2. Parents Selection

1. Lower Front
2. Less Crowded Distance



NSGA II

3. Mutation

1	3	3	2	3	1
0,95	0,1	0,55	0,7	0,18	0,3

$r_1 = 0{,}38$

$r_2 = 0{,}25$

$\longrightarrow$

1	3	3	2	3	1
0,95	0,038	0,55	0,175	0,18	0,3

NSGA II

4. Crossover

1st Parent

1	3	3	2	3	1
0,95	0,1	0,55	0,7	0,18	0,3

2	2	1	3	3	1
0,38	0,19	0,86	0,49	0,08	0,67

2nd Parent

1st Offspring

1	3	1	3	3	1
0,95	0,1	0,86	0,49	0,18	0,3

2	2	3	2	3	1
0,38	0,19	0,55	0,7	0,08	0,67

2nd Offspring

NSGA II

5. Objective Function

Start

$\boldsymbol{S} \leftarrow$ Set of customers which have not been served yet.

$\boldsymbol{while\ S \neq \{\}}$

$\boldsymbol{for}$ each vehicle $\in K$

one depot is selected randomly

customers are selected randomly and assigned to the vehicle

checking the vehicle's capacity

checking the customers' time windows

checking depots' time windows

end of $\boldsymbol{for}$

deleting served customers from $\boldsymbol{S}$

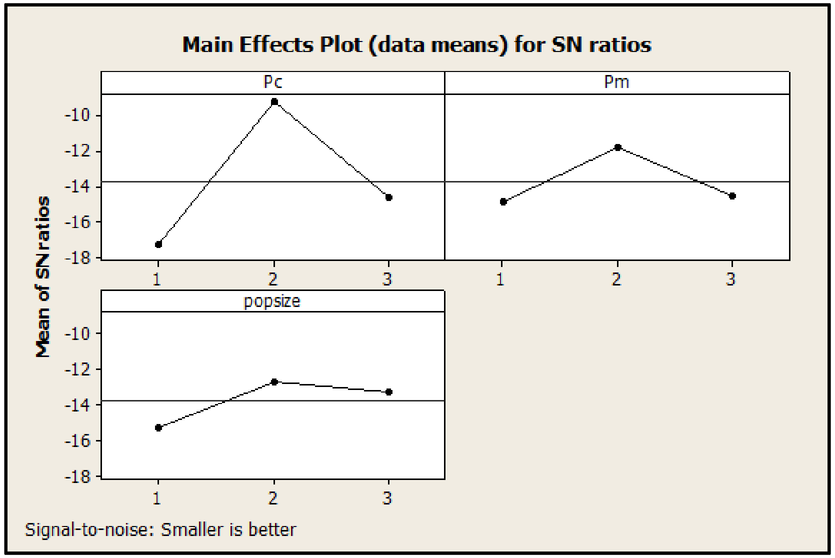
End

NSGA II

6. Parameter Tuning

$$S/N = -10 \ln\left(\frac{\sum_{i=1}^{48} {y_i}^2}{n}\right)$$

	Levels of Experiment		
	Low	Middle	Up
$PopSize$	50	100	200
P_c	0.8	0.85	0.9
P_m	0.025	0.05	0.075



MOSA

1. Neighborhood generating and Annealing

$$T_k = \alpha^k T_0 \qquad 0 < \alpha < 1$$

2. Transition Probability

$$P_t(i,j) = \min\{e^{(-\frac{C(i,j)}{T})}, 0\}$$

MOSA

3. Deciding whether to stay or move to a non-dominated situation

Start

S=S_0

T=T_0

Repeat

Generate a neighbor S'=N(S)

If C(S') dominates C(S)

move to S'

else if C(s) dominates C(S')

move to S' with the transition probability $P_t(C(S), C(S'), T)$

else if C(S) and C(S') do not dominate each other

move to S'

end if

T= annealing (T)

End repeat (until the termination is satisfied)

MOSA

4. Solution Representation

Vehicle Type

Customers

1	3	3	2	3	1
0,95	0,1	0,55	0,7	0,18	0,3

1	3	3	2	3	1
0,1	0,18	0,3	0,5	0,7	0,95

1	3	3	2	3	1
6	5	4	3	2	1

Results

Results

Generating Random Problems

	Number of Problems	Number of Customers	Number of Depots	Time (Seconds)
Small Sized	10	15	2	18
Medium Sized	10	45	3	81
Large Sized	10	75	4	180

$$* \ \theta = \frac{180}{4\ x\ 75} = 0.6$$

$$t = v.m.\theta$$

Results

Small size problems

	NSGA II Vs. Model	Model Vs. NSGA II
$gap\ 1_{ave}$ (%)	17.71	15.00
$gap\ 2_{ave}$ (%)	15.24	0.58
$gap\ 3_{ave}$ (%)	16.48	7.79

Average of gaps for small size test problems for NSGA II and Model

$$RPD^* = \frac{Alg_1 - \min(Alg_1, Alg_2)}{\min(Alg_1, Alg_2)} \text{x}100$$

* Naderi et al. (2011)

	MOSA Vs. Model	Model Vs MOSA
$gap\ 1_{ave}$ (%)	23.19	0.00
$gap\ 2_{ave}$ (%)	4.56	5.68
$gap\ 3_{ave}$ (%)	13.88	2.84

Average of gaps for small size test problems for MOSA and Model

$$Gap\ 1 = \frac{46447 - 46357}{46357} \text{x } 100 = 0{,}19$$

$$Gap = \frac{0{,}19 + 0}{2} \cong 0{,}1$$

Results

Spacing Metric

Test Problems	NSGA II	MOSA
Small Size	3,95E+05	3,78E+05
Medium Size	4,30E+05	3,60E+05
Large Size	4,24E+05	3,69E+05

The average of Spacing Metric for NSGA II and MOSA

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\bar{d} - d_i)^2}$$

$$d_i = min_j(|f_1^i(\vec{x}) - f_1^j(\vec{x})|+|f_2^i(\vec{x}) - f_2^j\left(\overrightarrow{x)|}\right)$$

Results

Generational Distance

Test Problems	NSGA II	MOSA
Small Size	64.15	64.77
Medium Size	52.13	64.13
Large Size	50.44	61.70

The average of Generational Distance Metric for NSGA II and MOSA

$$GD = \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n}}$$

Conclusion and Future Research

Conclusion

- Proposing and formulating a new variant of Vehicle Routing Problems
- Proposing solution approaches based on meta-heuristic algorithms
- Robustness of Mathematical Model
- Comparing NSGA II and MOSA
  - MOSA → Less Spacing Metric
  - NSGA II → Less Generational Distance

Future Research

- Exact and heuristic solution approaches
- Considering the traffic restrictions
- On-line Vehicle Routing Problem

References

Balseiro, S. R., Loiseau, I., Ramonet, J. (2011). An Ant Colony algorithm hybridized with insertion heuristics for the Time Dependent Vehicle Routing Problem with Time Windows, Computers and Operations Research, 38, 957-966.

Dantzig, G.B., Ramser, J.H., (1959). The truck dispatching problem, Management Science, 6-80.

Deb, K., Agrawal, S., Pratap, A., Meyarivan, T. (2002). A fast Elitist Non-dominated Sorting Genetic Algorithm for multi-objective Optimization (NSGA II), IEEE Transactions on Evolutionary Algorithms, 6, 182-197.

Figliozzi, M.A. (2012). The time dependent vehicle routing problem with time windows: Benchmark problems, an efficient solution algorithm, and solution characteristics, Transportation Research Part E, 48, 616-636.

Fleischmann, B., Gietz, M., Gnutzmann, S. (2004). Time-varying travel times in vehicle routing, Transportation Science, 38, 160–173.

Ichoua, S., Gendreau, M., Potvin, J.Y. (2003). Vehicle dispatching with time-dependent travel times, European Journal of Operational Research, 144, 379–396.

Jagiełło, S., Żelazny, D., (2013). Solving Multi-criteria Vehicle Routing Problem by Parallel Tabu

Search on GPU, International Conference on Computational Science, 2529-2532.

Kok, A.L. (2010). Congestion Avoidance and Break Scheduling within Vehicle Routing, PhD. Thesis, University of Twente, Enschede, the Netherlands.

Malandraki, C. (1989). Time Dependent Vehicle Routing Problems: Formulations, Solution Algorithms and Computational Experiments, Ph.D. Dissertation, Northwestern University, Evanston, Illinois.

Malandraki, C., Daskin, M.S. (1992). Time-dependent vehicle-routing problems – formulations, properties and heuristic algorithms, Transportation Science, 26, 185–200.

Naderi, B., Fatemi Ghomi, S.M.T., Aminyari, M., Zandieh, M. (2011). Scheduling open shops with parallel machines to minimize total completion times, Journal of Computational and Applied Mathematics235, 1275-1287.

References

Nam, D., Park, C., (2000). Multiobjective simulated annealing: A comparative study to Evolutionary algorithms. International Journal of Fuzzy Systems, 2, 87–97.

Schott J. (1995). Fault tolerant design using single and multicriteria genetic algorithms optimization, .Department of Aeronautics and Astronautics. Cambridge: Master's thesis, Massachusetts

Institute of Technology.

Serafini, P., (1992). Simulated Annealing for Multiobjective Optimization Problems in Proc. of the 10th International Conference on Multiple Criteria Decision Making, Taipei Taiwan, 87-96.

Solomon Test Problems: http://w.cba.neu.edu/~msolomon/problems.htm

Srinivas, N., Deb, K. (1994). Multi-Objective function Optimization using non-dominated genetic algorithms', Evolutionary Computation, 2, 221-248.

Taguchi, G. (1986). Introduction to quality engineering, White Plains: Asian Productivity Organization, UNIPUB.

Tasgetiren, M., Faith, Suganthan, P., Gencyilmaz G., Chen, Angela H-L. (2009). A smallest position value approach, Invited book chapter in Differential Evolution: A handbook for Global Permutation-Based Combinatirial Optimization edited by Godfrey Onwubolu and Donald Davendra, Springer-Verlag, 121-138.

Toth, P., Vigo, D., (2002). The Vehicle Routing Problem, Society for Industrial and Applied Mathematics Philadelphia.

E. Ulungu, J. Teghem, P. Fortemps, D. Tuyttens, (1999). Mosa method: A tool for solving moco problems, Journal of Multi-Criteria Decision Analysis, 8, 221-236.

Van Veldhuizen, D., Lamont G., B., (1998), Evolutionary Computation and Convergence to a Pareto Front. In John R. Koza, editor, Late Breaking Papers at the Genetic Programming Conference, Stanford University, California, Stanford University Bookstore. 221-228,

Van Woensel, T., Kerbache, L., Peremans, H., Vandaele, N. (2008). Vehicle routing with dynamic travel times: a queueing approach, European Journal of Operational Research, 186, 990–1007

Żelazny, D., (2012). Multicriteria optimization in vehicle routing problem, in: National Conference of Descrite Processes Automation, 157-163.

Thank You for Your Attention

Questions and Answers

A Comparison of NSGA II and MOSA for Solving

Multi-depots Time-dependent Vehicle Routing Problem with Heterogeneous Fleet

Arian Razmi Farooji

CM3 Conference

27 May 2015
Jyväskylä, Finland

© Tiia Monto