OULUN YLIOPISTO
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Vehicle Routing Problem

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10 February 2015

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1. Introduction

- The role and importance of transportation in economic growth

    1. Transporting fuel, raw materials and machineries
    2. Balancing strong and weak economic regions
    3. Guaranteeing manufacturing and production of goods

- Transportation and the end price

2. Vehicle Routing Problem (VRP)

- Introduced by Dantzig and Ramser in 1959
- Goal:

> *“ To plan a set of routes for some vehicles which are located in one or more depot(s) in order to serve some customers in less possible Travel Time, Travel Costs or Travel distance ”*

3. VRP Components

1. Depots
2. Customers
3. Vehicles
4. Routes

3-1. Depots

- A Depot is the starting point and the finishing point of VRP.
- A VRP can have one or more depots.

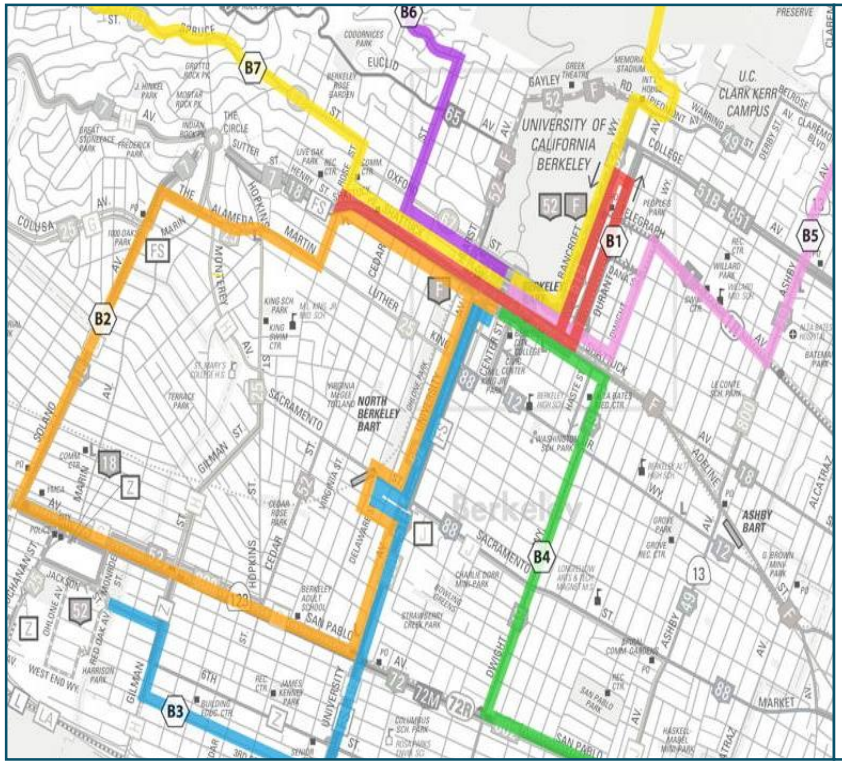
3-2. Customers

- The goal of VRP is to service the customers.
- Customers are spread around the Depot(s).
- Their demands can be definitive or stochastic.

3-3. Vehicles

- Vehicles' characteristics are:
  1. Capacity
  2. Cost
  3. Departure
  4. Maximum traveling time
  5. Number of Vehicles

3-4. Routes



- Routes
  1. Connect customers together.
  2. Can have different travel time and cost.
  3. Can be one-or two-way.

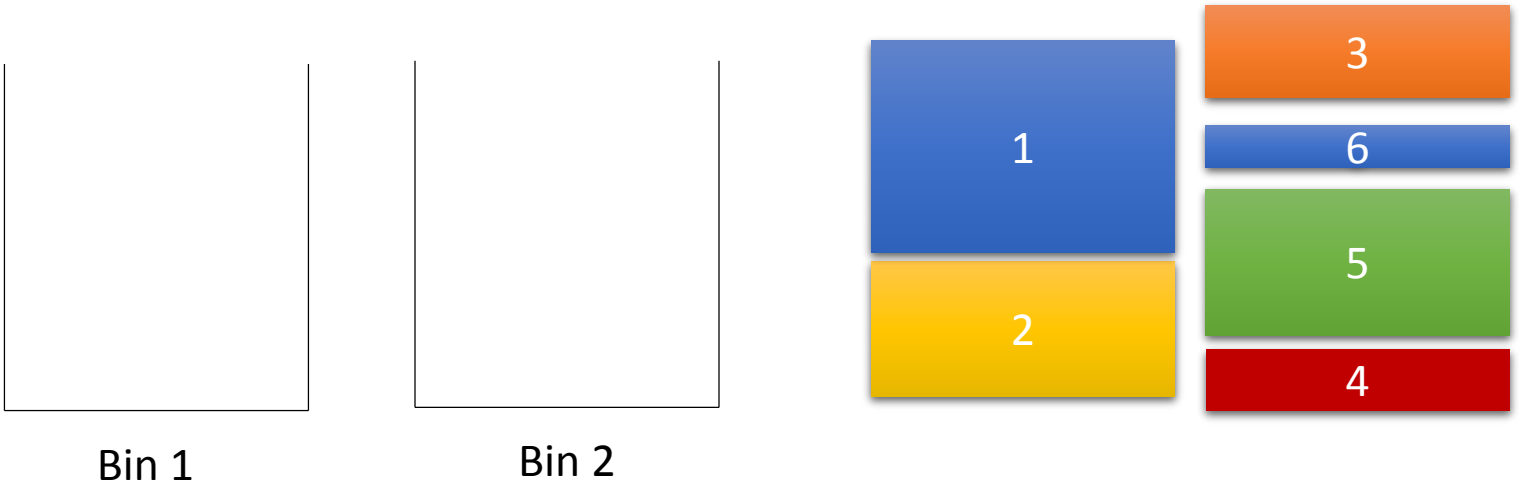
4. VRP Characteristics

- An NP-Hard Combinatorial Problem
- Based on two well-studied following problems:

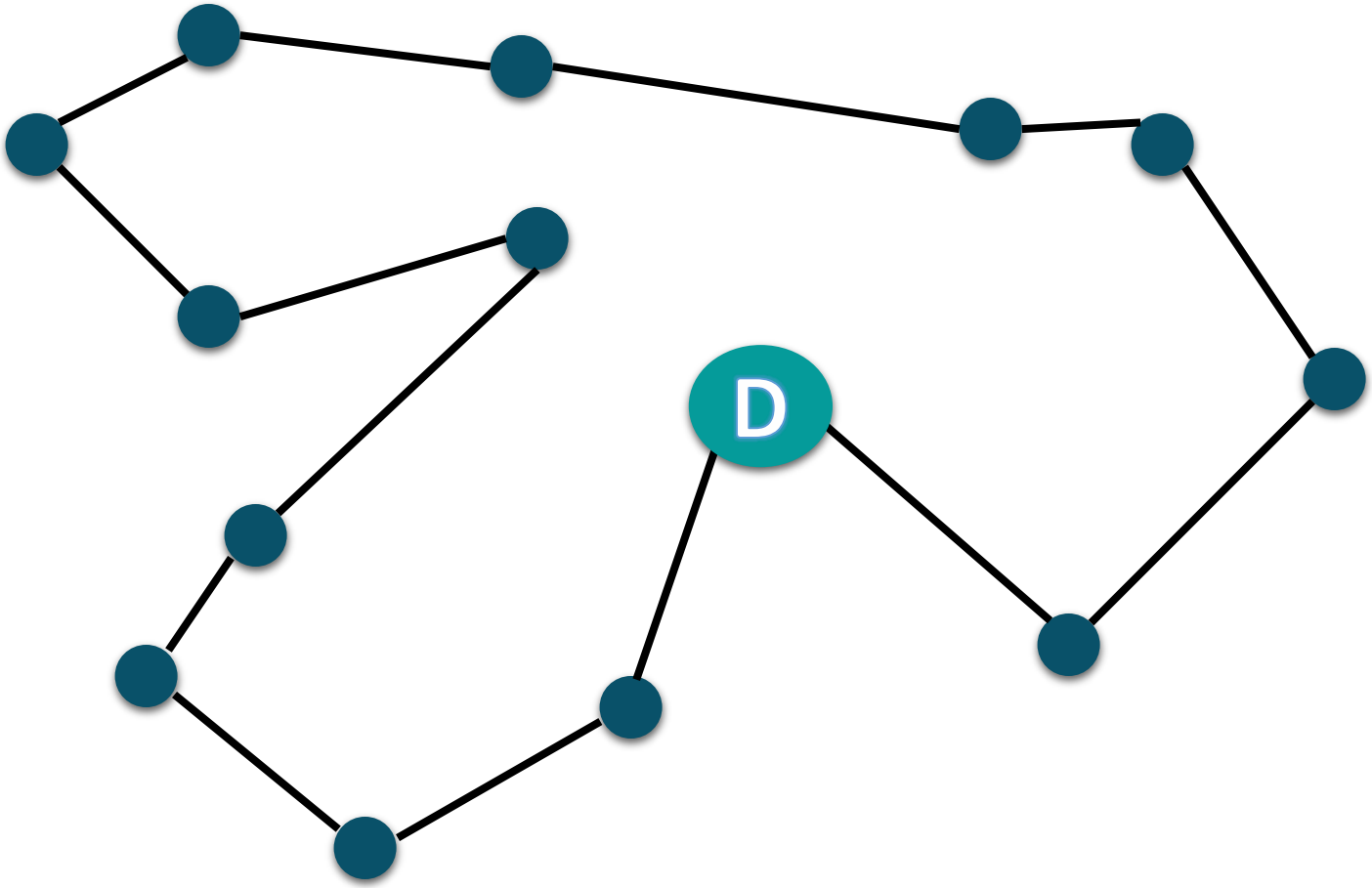
    1. Bin Packing Problem
    2. Travelling Salesman Problem

4-1. Bin Packing Problem (BPP)

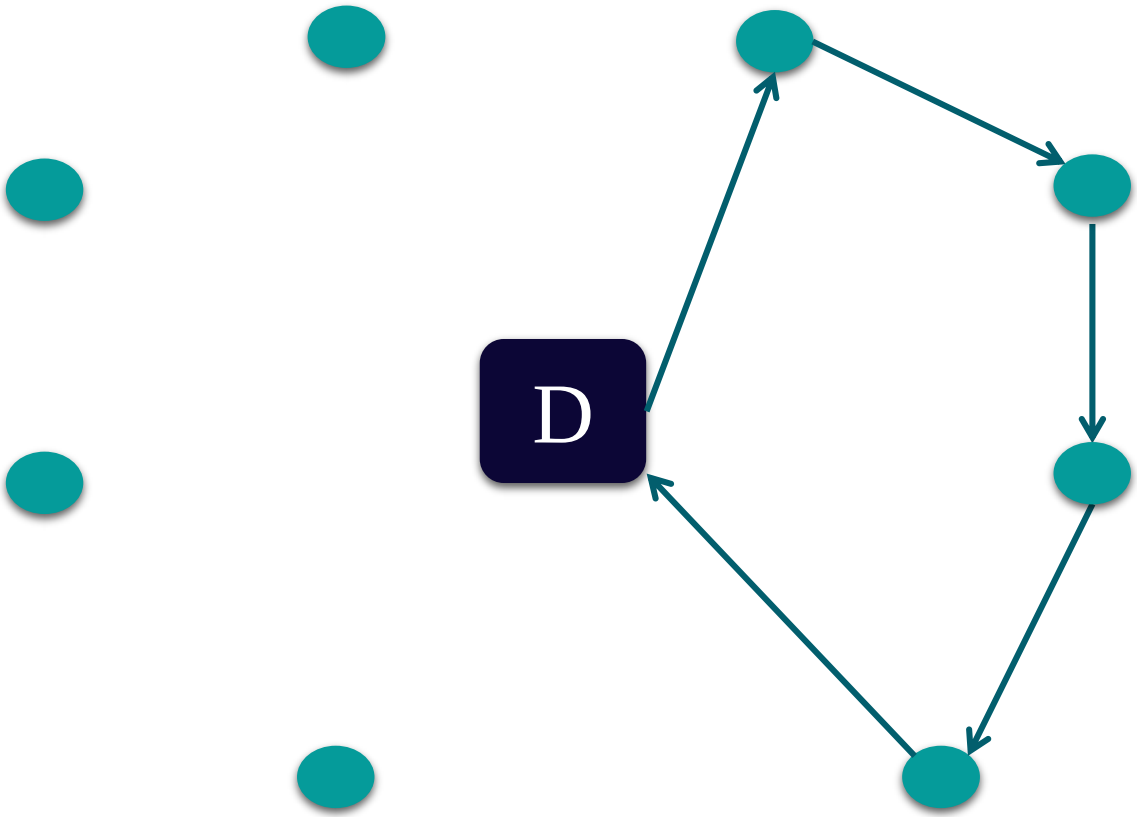
- BPP is about how to put *n* items in *m* bins, to the degree in which volume and weight of the items must not exceed the maximum limit of bins.



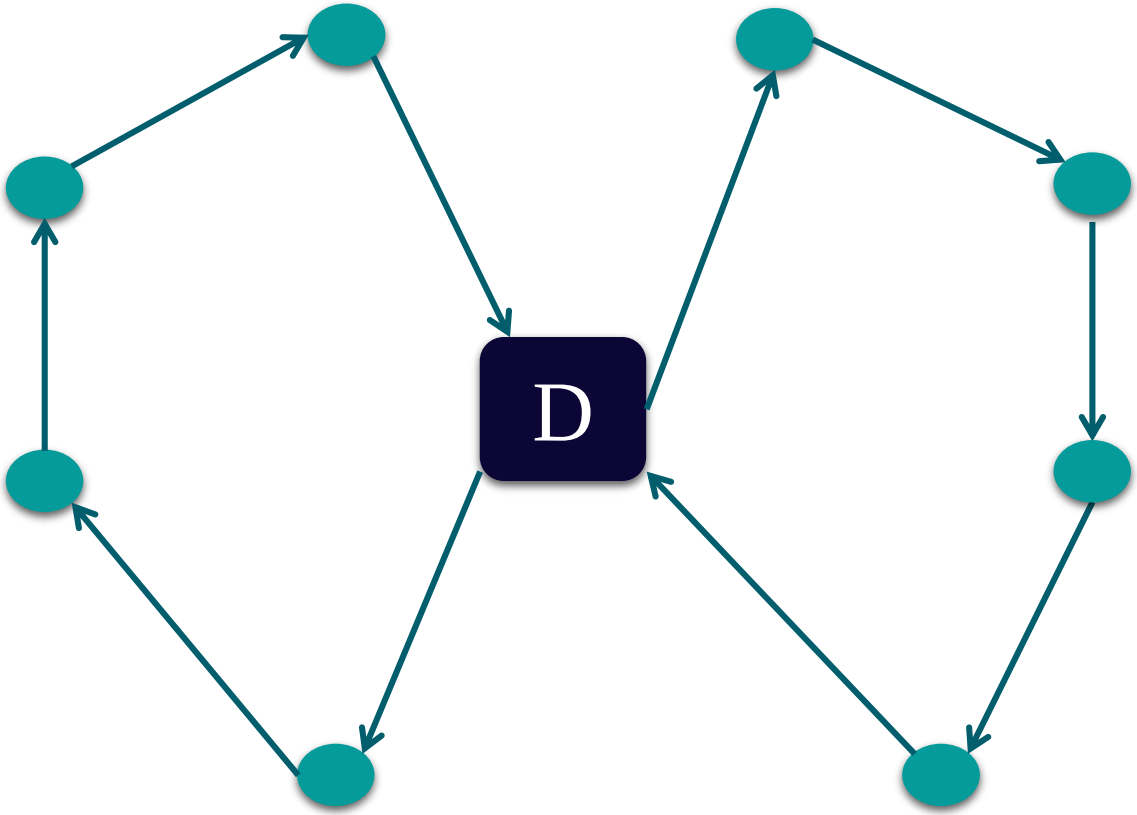
4-2. Travelling Salesman Problem (TSP)



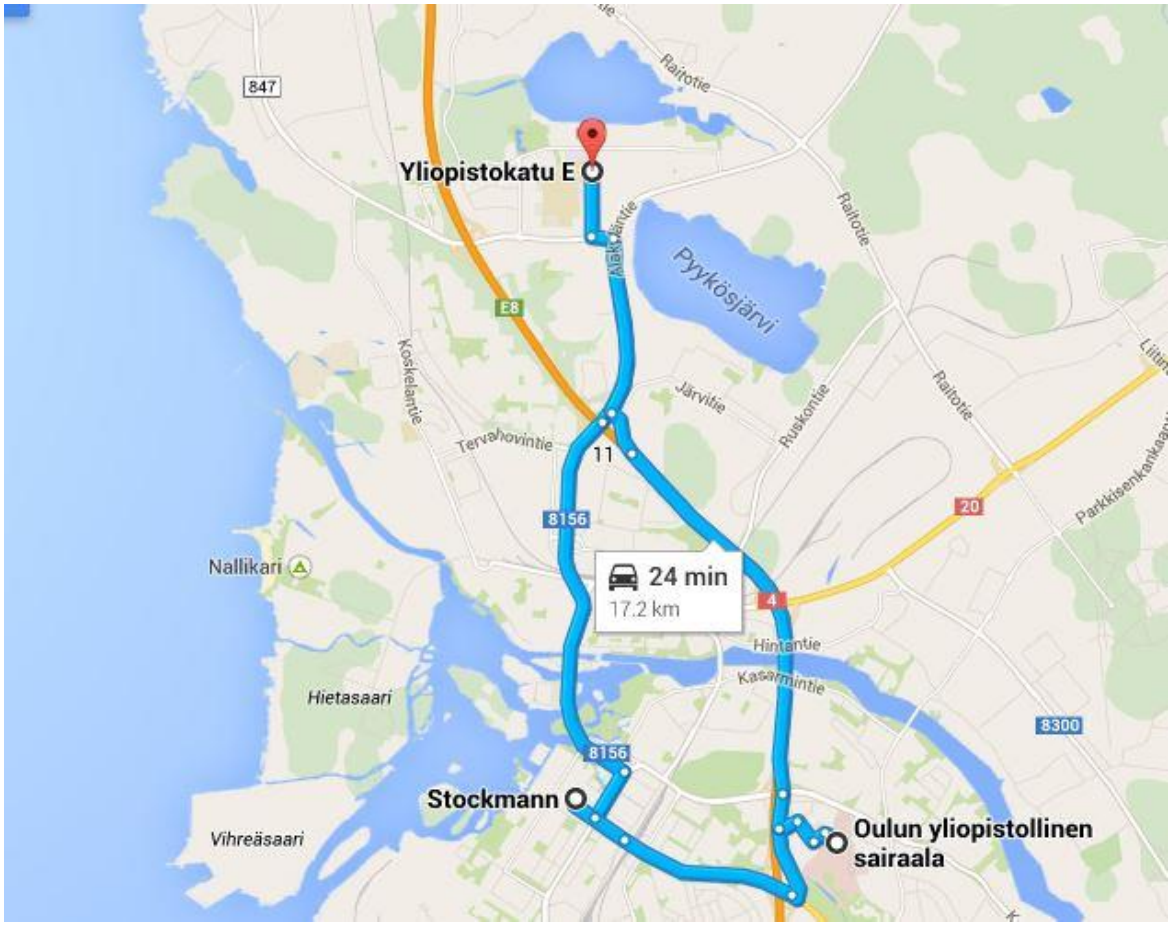
Vehicle Routing Problem in a glance



Vehicle Routing Problem in a glance



VRP- an example of everyday life



5. Importance and Applications

- Reducing between 5 to 20% of transportation cost
- Reduction of 10 to 20% of products end price
- Avoiding delivery delays and increasing customers satisfaction
- Saving fuel and environmentally related issues

6. VRP Assumptions

1. Each customer must be served once and by one of the available vehicles.

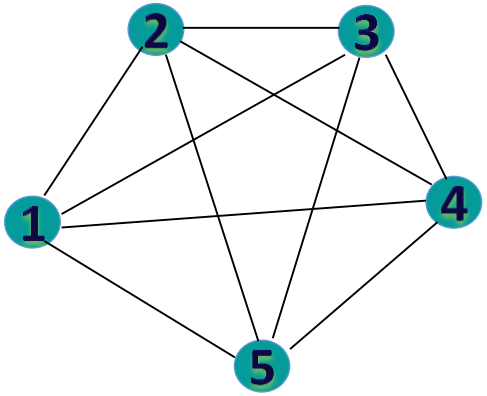
2. Each tour starts from and ends to one depot.

3. Sum of customer demands of each tour must not exceed the vehicles' capacity.

4. Total travelling distance or time must not exceed their maximum limit.

7. VRP Mathematical Formulation

- We assume that $G(V, A)$ is a complete graph.



- Decision Variable:

$$x_{ij} \begin{cases} \text{1 if vehicle travels from } i \text{ to } j \\ \text{0 if vehicle does not travel from } i \text{ to } j \end{cases}$$

7. VRP Mathematical Formulation

- Objective Function

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij}\, x_{ij} \qquad (1)$$

Subject to

$$\sum_{\boldsymbol{i \in V}} \boldsymbol{x_{ij}} = \mathbf{1} \qquad \forall\, \boldsymbol{j} \in \backslash \mathrm{V}\{\mathbf{0}\} \qquad (2)$$

$$\sum_{j \in V} x_{ij} = \mathbf{1} \qquad \forall\, \boldsymbol{i} \in \backslash \mathrm{V}\{\mathbf{0}\} \qquad (3)$$

7. VRP Mathematical Formulation

$$\sum_{i \in V} x_{i0} = K \tag{4}$$

$$\sum_{j \in V} x_{0j} = K \tag{5}$$

$$\sum_{i \notin S} \sum_{j \in S} x_{ij} \geq r(S) \qquad \forall\, S \subset V\{0\}, S \neq \theta \tag{6}$$

$$x_{ij} \in \{0,1\} \qquad \forall\, i,j \in V \tag{7}$$

8. VRP Variants

- There are many VRP variants based on problems we face in real world.
  1. Capacitated Vehicle Routing Problem (CVRP)
  2. Vehicle Routing Problem with Time-windows (VRPTW)
  3. Vehicle Routing Problem with Heterogeneous Fleet (HFVRP)
  4. Time-dependent Vehicle Routing Problem (TDVRP)
  5. Vehicle Routing Problem with Multiple Depots (MDVRP)
  6. …
- Most of the time a new variant based on the real-world needs is made.

8-1. CVRP

1. The most studied VRP
2. Customers' demands are definitive and known in advance.
3. Vehicles are identical.
4. Sum of customers' demands must not exceed the vehicles' capacity.

- Goal:

  " service all the customers in which the total travel costs will be minimized."

8-2. VRPTW

1. An extension to CVRP
2. Each customer has a time-window and that customer should be served during his or her own time-window.
3. Time windows can be :
   - Soft→ delays will be punished
   - Hard → no allowance for delays

- Goal:

“ servicing all the customers in which the total travel costs, waiting times and delays will be minimized.”

8-3. HFVRP

1. Different Vehicles, different capacities
2. Each vehicle type has its own fixed and variable cost.
3. There can be some restrictions for some vehicles

- Goal:

“ servicing all the customers in which the total travel costs (fixed + variable costs) will be minimized.”

8-4. TDVRP

1. Less-studied VRP
2. Relates to urban logistics
3. Considers that the vehicles travel speed is not constant during serving customers.
4. Can be also applied to green logistics

- Goal:

“ serving all the customers in which number of routes and the total travel costs will be minimized.”

Assignment

- Write down a short summary about what already was discussed during this session.

- Research the answer of the following question:

  “ What are the relationships of Vehicle Routing Problem with Travelling Salesman Problem and Bin Packing Problem?”

9. References

- Dantzig, G.B., Ramser, J.H., 1959, The truck dispatching problem, Management Science, 6-80.
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- Malandraki, C., Daskin, M.S., 1992, Time-dependent vehicle-routing problems – formulations, properties and heuristic algorithms. Transportation Science, 26 (3), 185–200.
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