Problem-set 1

Questions on A* search

1. Figure 1 shows a map of the second floor of the Kilburn Building. Suppose you wanted to find the shortest route between room 2.100 and the nearest women's toilet, using A* search. Suppose you did not have measurements of the corridors and rooms. What could you use as a heuristic? How would you use the algorithm to find the path to the closest women's toilet?

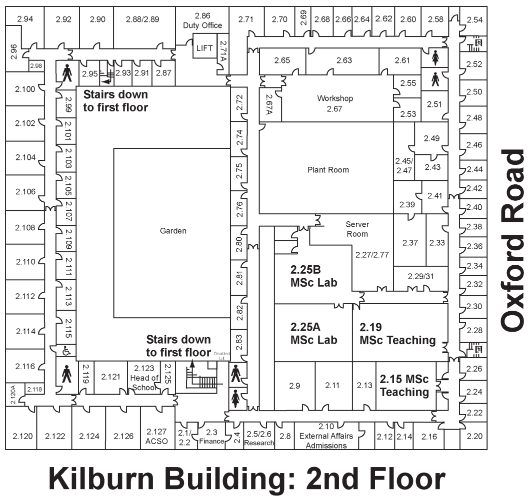

Figure 1: The second floor of the Kilburn Building. The figure refers to question 1. You want to find the shortest route from Kilburn 2.100 to the nearest women's toilet.

2. What would be a good heuristic for the Towers of Hanoi problem? If you do not know the Towers of Hanoi problem, look it up, perhaps here.

Game trees

Discussion on game trees is in Schalk's "Lecture Notes in Game Theory" starting on page 6.

NimN Consider the game called *NimN*[1] The rules are as follows.
Start with N matches (some integer N).

1. Players take turns.
2. At its turn, a player can take 1, 2, or 3 matches.
3. Whoever takes the last match *loses*

1. Consider NimN with $N = 4$. I.e. starting with 4 matches, each player in turns takes 1, 2, or 3 matches. Player taking the last match *loses*.
   (a) Draw the game tree for NimN with $N = 4$. Can either side guarantee a win? Which side?
   (b) There is a winning strategy for Player 1 or for Player 2 (but not both) for any positive integer value of N? See if you can find it. Try a few other values of N For example, $N = 5, 6, 8, 9$. (Don't draw out the game tree for these. Just think which player could force a win from each starting number of matches.)

2. Consider an auction for a work of art. This is an *English Auction* where the bids increase in value. The item goes to the highest bidder. The rules are: the starting bid must be £10, and all subsequent bids must be raised by exactly £10.

   There are only two bidders at the auction: Alice and Bob. Alice values the artwork at £25, but she is willing to bid £30 for it, but no higher. Bob values the artwork at £20, and is not willing to bid higher than that. Draw the game tree for this auction, taking Alice as player 1.

   Each player has two possible actions: raise the bid by £10, and pass. If a player passes after its opponent has made a bid, the opponent gets the item at the cost of its bid. The exception is at the beginning of the game. If no bid has yet been made, then if Player 1 passes followed by Player 2 passing, Player 1 can make the opening bid.

   At the terminal positions, write down the value of the result. A player who did not win the auction gets a value of 0 they did not get the artwork, but they did not spend any money. The winner of the auction gets the artwork, but pays money for it, so the value they get is

$$\text{(The value of the artwork to them)} - \text{(the amount they paid for it)}\ .$$

Games in Normal Form

Normal form games are discussed in Schalk's "Lecture Notes in Game Theory" in section 1.3, starting on page 20.

1. Bill is falling in love with Amy, and Amy is likewise falling for Bill, but neither has told the other. They both know that Amy loves basketball, and is happiest at the basketball center; Bill loves jazz, and is happiest

[1]Totally different than the game called *Nim* in the lecture notes. The version here is what we called Nim when I was growing up.

at the jazz club. They both decide simultaneously to go to one of the venues in the hopes of meeting up, but they have not discussed with each other where they will be. Express this as a normal form game with the two players, Bill and Amy, and the two actions being the two venues they could go to. You will need to think of numbers for the payoffs to express the situation: they are both happy if they might up, both unhappy if they go to different venues, and there is an extra degree of happiness if they go to their preferred venues.