COMP 34111 — Artificial Intelligence and Games

Theory of Games I - representation of games

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Announcements

- Reading for this week: Andrea's "Notes On Game Theory", available electronically (PDF) on Blackboard.
- **Reading:** pages 16 – 29 for the rest of the week (6 – 16 for today).

- Examples classes - **Log in with UoM email and UoM Zoom account!**
  - Else Zoom does not know what room to put you in.
  - I spend half the time putting you in the correct rooms, when I could be helping you.

Announcements (cont)

- If you have questions about the course, ask on the *Question and discussion Forum* so everyone can get the benefit of the answer.
- Subscribe to the announcement forum.

Sign up to the Announcement forum

Please subscribe so you receive important announcements about the course

Subscribe to the Question and Discussion forum

To ask questions and participate in discussions.

What is a game?

- A finite set of players
- A finite game tree.
- Each node of the tree is associated with a possible move that player can move from that position.

Example: Chomp

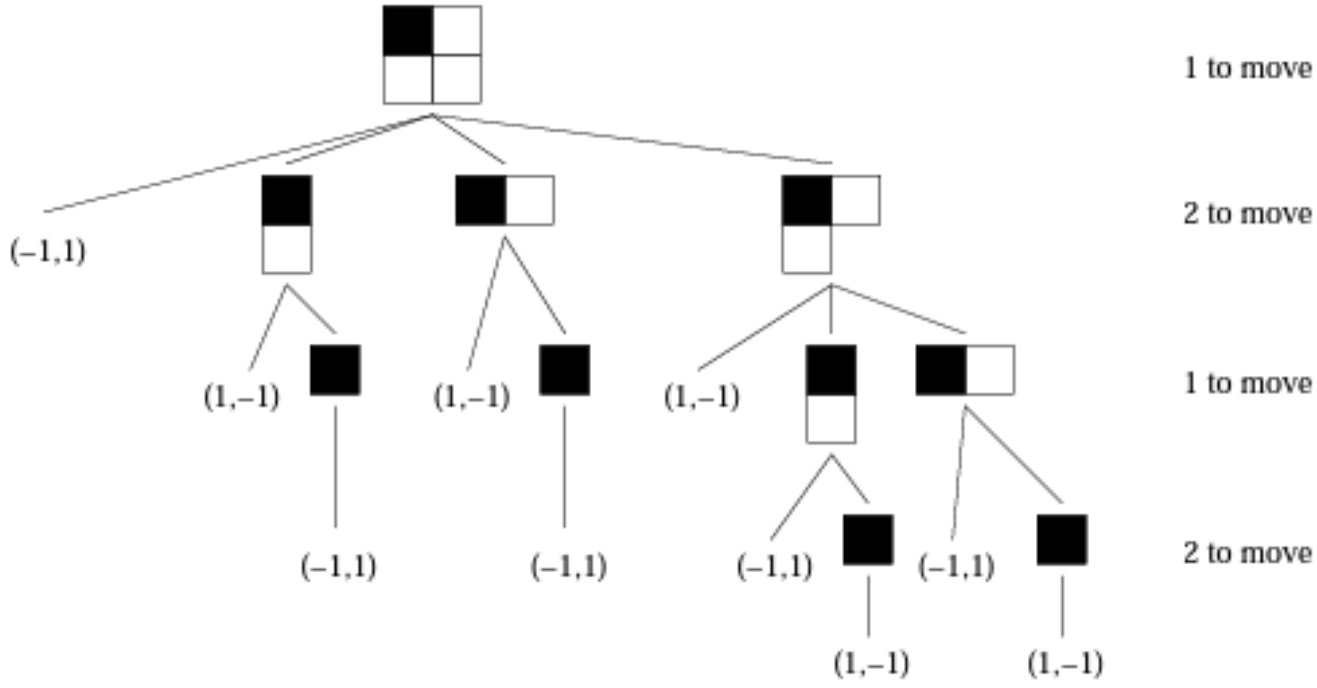


Figure: The game tree for 2×2 chomp (Example 1.2 of the notes).

Which side should you choose to play?

Points to notice:

1. One play of the game equals one path through the game tree.
2. Certain positions appear more than once in the tree. Why?
3. Is there a way for one of the players to win? Which one?

Example: Nim-N

Start with N matches (some integer N).

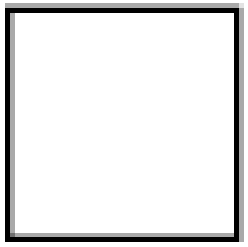
1. Players take turns.
2. At its turn, a player can take 1, 2, or 3 matches.
3. Whoever takes the last match *loses*

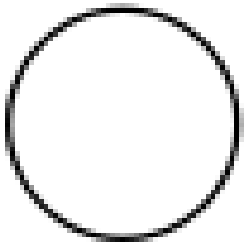
Exercise: Draw the game tree for $N = 3$, NIM-3.

Can either player guarantee a win?

Visualizer or board

Note: I use shapes to represent different players.

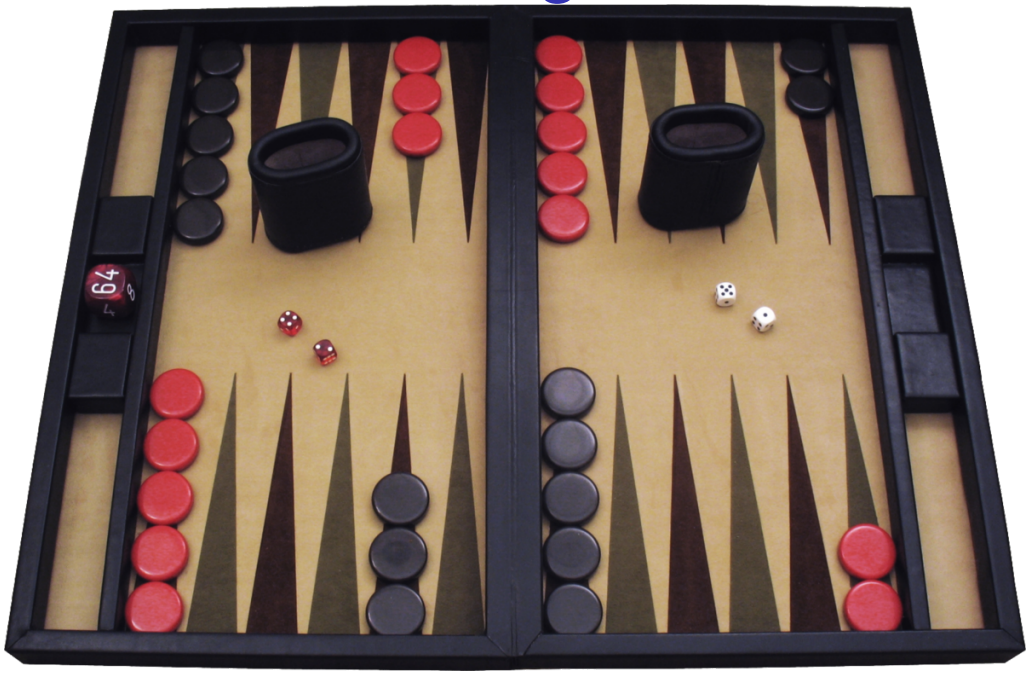
Player 1: 

Player 2: 

Questions: Consider NIM-N with N=37.

1. Can either player guarantee a win?
2. Can you compute the size of the game tree?

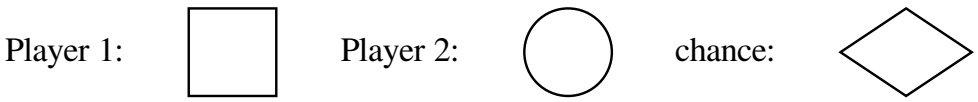
What about these games



Handling chance

Elements of chance: rolls of dice, dealing of cards, uncertain outcomes (real-life games).

A new player: called *Chance* or *Nature*,



- Each action from Nature has an associated probability;
- Nature gets no payoff.

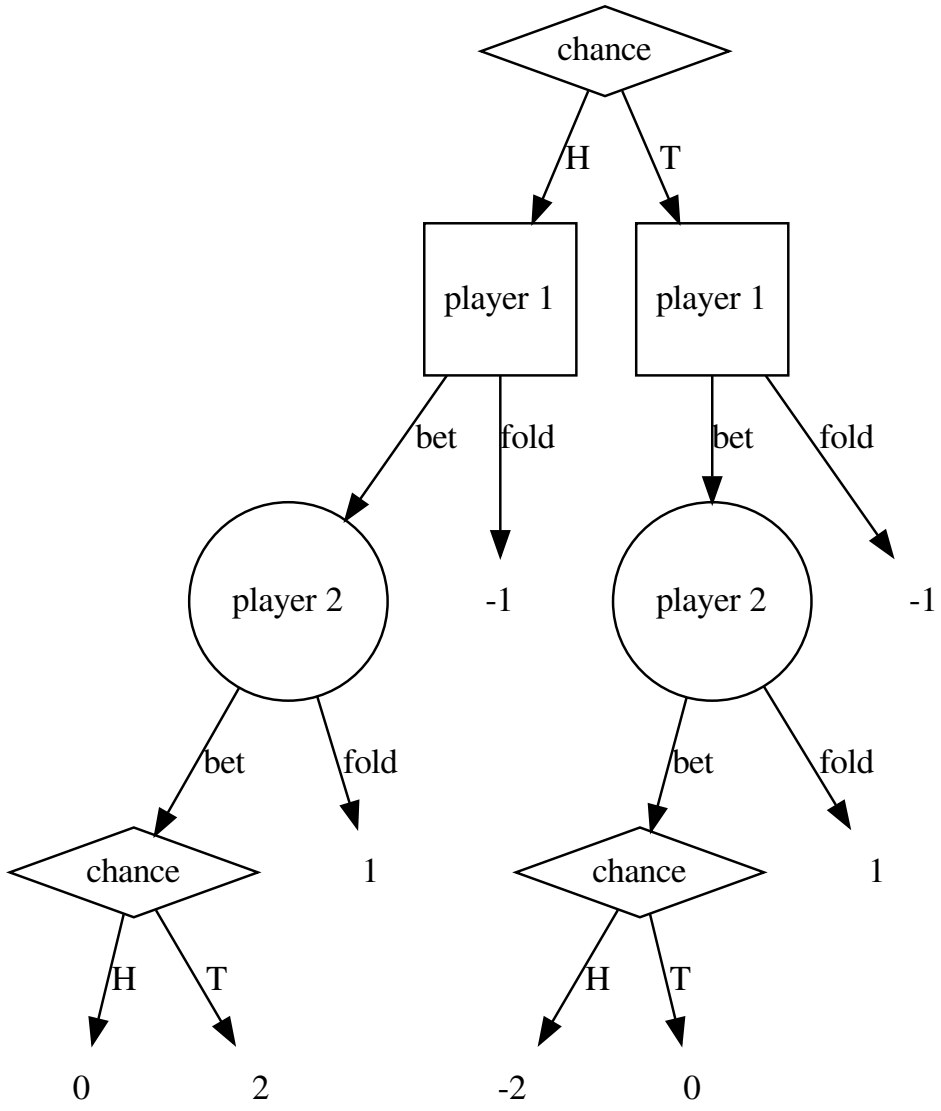
A (stupid) game with coins

1. Each player puts 1 unit in the pot.
2. Player 1 flips a coin which both players see.
3. Player 1 can bet 1 unit or fold (losing 1 unit which Player 2 gains).
4. Player 2 flips a coin which both players see.
5. Player 2 can bet 1 unit or fold (losing 1 unit which Player 1 gains).

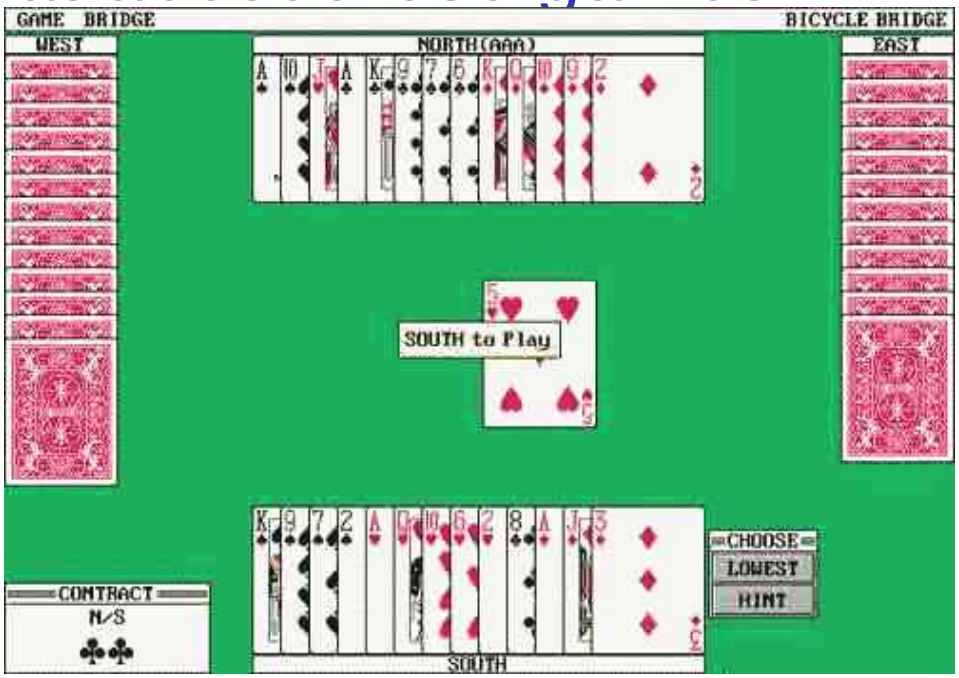
Heads beats tails; heads vs heads and tails vs tails are both draws.

The game tree

What is the game tree?



What about these games?



Games with incomplete (hidden) information

Handling Imperfect Information

In many games, players do not know what other players know. Examples:

- Simultaneous play games
- Card games (bridge, poker, etc.
- Auctions.

Difficulty: Player does not know where it is in the game tree.

Information set: The set of all nodes consistent with the information which the player has.

Information sets

Information set: The set of nodes a player could be at given the information it has.

Action taken: must be the same for all nodes in the information set.

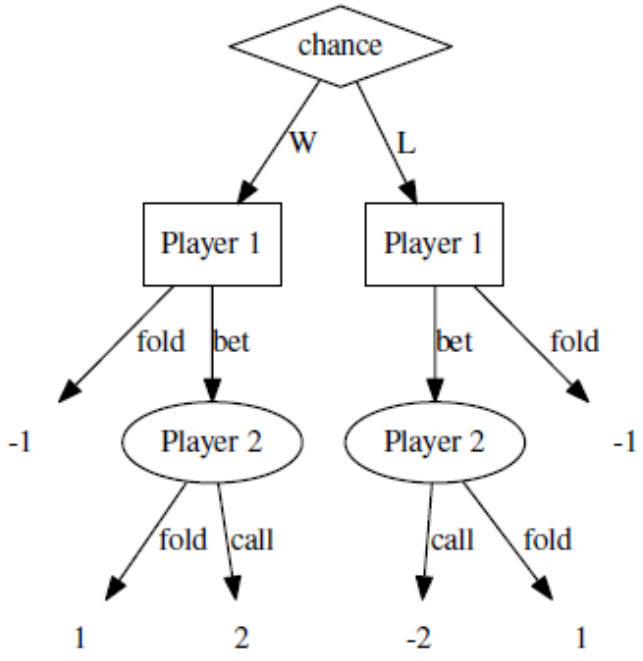
2-Player, 2-card tiny poker

Equipment: A deck of cards consisting of two types of cards: W and L.

1. Each player puts £1 into the pot.
2. Player 1 draws a card from the deck, but does not show it to player 2.
3. Player 1 can then bet or fold. If she folds, player 2 gets the pot. If she bets, she puts another £1 into the pot.
4. If player 1 bet, player 2 can call or fold. If he folds, player 1 gets the pot. To call, player 2 must put an additional £1 in the pot.
5. If player 2 calls, there is a showdown, and player 1 must show her card.
6. A W card means player 1 gets the pot; an L card means player 2 gets the pot.

Draw the game tree.

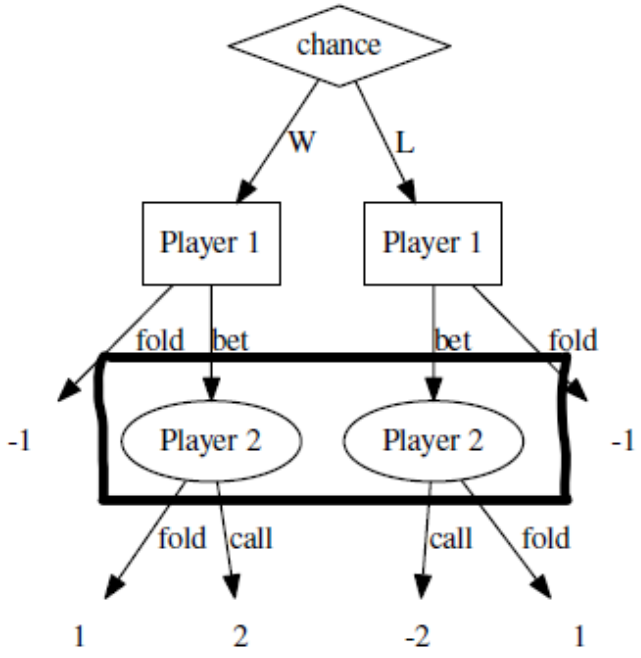
The game tree for Tiny Poker



What are the information sets?

What are the information sets?

The game tree for Tiny Poker



What are the information sets?

The information set

- Which side would you want to play, and why?
- Can you see a winning strategy?

Some Definitions

(See Definition 2 on page 15 of the notes.)

2-player game: A game with 2 players, not counting Nature.

Zero-sum game: A game with the property that the sum of the payoffs for all players is 0 at each leaf node (terminal node) in the game tree.

A game of perfect information: All players know which node of the game tree they are in. (All information sets are of size 1.) Otherwise, it is a *game of imperfect information*.

A game without chance: A game in which no node of the tree is controlled by nature. Otherwise, it is a *game with chance*.

What kind of game is Tiny Poker?

Tiny Poker is a 2-player, zero-sum game with chance and imperfect information.

Summary

1. Define a game as a game tree with nodes associated with particular players at a particular point in the game, and links as allowed actions at that point.
2. Games of chance can be handled by introducing an additional player call Nature (or Chance).
3. Games of imperfect information can be handled by the introduction of *information sets*
4. For two-player games with no chance and perfect information (Chomp, NimN), we could find a winning strategy for one of the players. For a game with imperfect information (Tiny Poker), we could not.