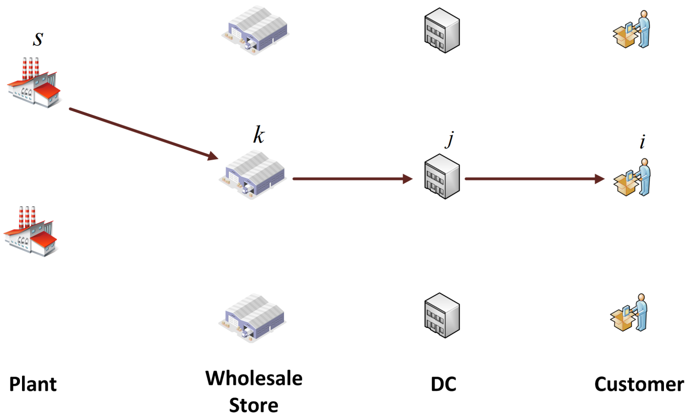
Multi-Stage End-to-End Supply Chain Network Design to Minimize the Cost and Maximize the Service Level

Supply Chain Network Model



Supply Chain Network Model (contd.)

Network designer's objectives:

- **Minimize the total supply-chain cost**. Total supply-chain cost includes:
  - Operating costs of different network entities such as Wholesale stores and DCs
  - Transportation cost of shipping the products from Plants $\rightarrow$ Wholesale stores $\rightarrow$ DCs $\rightarrow$ Customers
  - Inventory cost at warehouses
- **Maximize the service-level** i.e. maximize the satisfiability of customer demand within a specified interval of time

Supply Chain Network Model (contd.)

Factors:

- Which of the network entities (e.g. Wholesale stores and DCs) to be used?
- Which DC will serve to which customers because in general a customer is uniquely assigned to a DC
- Quantity of each product to be transported from Plant $\rightarrow$ Wholesale store
- Quantity of each product to be transported from Wholesale store $\rightarrow$ DC
- Quantity of each product to be transported from DC $\rightarrow$ Customer

Supply Chain Network Model (contd.)

Necessary data:

- Demand of the customers for each product
- Maximum supply level for each Plant
- Maximum capacity of Wholesale stores and DCs
- Transportation cost per unit amount of product from one location to other
- Cost of holding unit amount of each product at warehouses
- Maximum allowable delivery time to ship the products from DCs to Customers

Mathematical Notations

Indices:

- I : Set of customers indexed by i
- J : Set of DCs indexed by j
- K : Set of wholesale stores indexed by k
- S : Set of manufacturing plants indexed by s
- L : Set of products indexed by l

Model Variables:

- b_{lsk} : Quantity of product l to be shipped from plant s to wholesale store k
- f_{lkj} : Quantity of product l to be shipped from wholesale store k to DC j
- q_{lji} : Quantity of product l to be shipped from DC j to customer i

Mathematical Notations (contd.)

Model Variables (contd.):

$$z_j = \begin{cases} 1, & \text{if DC } j \text{ is open} \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ji} = \begin{cases} 1, & \text{if DC } j \text{ serves the product to the customer } i \\ 0, & \text{otherwise} \end{cases}$$

$$p_k = \begin{cases} 1, & \text{if warehouse } k \text{ is open} \\ 0, & \text{otherwise} \end{cases}$$

Mathematical Notation (contd.)

Model parameters:

- D_k : Capacity of wholesale store k
- W_j : Capacity of DC j
- sup_{ls} : Supply limit of plant s for product l
- d_{li} : Demand for product l at customer i
- v_j : Fixed cost for operating DC j
- g_k : Fixed cost for wholesale store k
- h_{lj} : Holding cost for product l at DC j
- c^{pw}_{lsk} : Unit transportation cost for product l from plant s to wholesaler k

Mathematical Notation (contd.)

Model parameters (contd.):

- c^{wd}_{lkj} : Unit transportation cost for product l from wholesaler k to Dc j
- c^{dc}_{lji} : Unit transportation cost for product l from DC j to customer i
- τ : Maximum allowable delivery time (hours) from DCs to customers to customers
- $x_{ji}(\tau)$: Binary values indicating whether customer i can be reached from DC j in τ hours
- η_{li} : Ratio of ordered amount to the demand amount given at customer i for product l

Optimization Problem Formulation

Objective function to be minimized:

- f_1 : Total cost of supply-chain network.

$$f_1 = \underbrace{\sum_k g_k p_k}_{\text{Cost of operating wholesale stores}} + \underbrace{\sum_j v_j z_j}_{\text{Cost of operating DCs}} + \underbrace{\sum_l \sum_s \sum_k c^{pw}_{lsk} b_{lsk}}_{\text{Costs of transportation of products from plants to wholesalers}}$$

$$+ \underbrace{\sum_l \sum_k \sum_j c^{wd}_{lkj} f_{lkj}}_{\text{Tansportation cost of products from wholesalers to DCs}} + \underbrace{\sum_l \sum_j \sum_i \eta_{li} c^{dc}_{lji} q_{lji}}_{\text{Tansportation cost of products from DCs to customers}} + \underbrace{\sum_l \sum_j \sum_i (1 - \eta_{li}) h_{lj} q_{lji}}_{\text{Inventory holding cost for unsold products}}$$

Optimization Problem Formulation (contd.)

Objective function to be maximized:

- f_2 : The fraction of total customer demand that can be delivered within the stipulated access time τ.

$$f_2 = \frac{\sum_l \sum_j \sum_i \eta_{li} q_{lji} x_{ji}(\tau)}{\sum_l \sum_i \eta_{li} d_{li}}$$

Optimization Problem Formulation (contd.)

Constraints:

- Unique assignment of a DC to a customer

$$\sum_{j} y_{ji} \leq 1, \forall i \in I$$

- Outflow from DC $\leq$ Capacity constraint for DC

$$\sum_{l} \sum_{i} q_{lji} \leq W_j z_j, \forall j \in J$$

- Inward flow into a DC $\leq$ Capacity constraint for DC

$$\sum_{l} \sum_{k} f_{lkj} \leq W_j z_j, \forall j \in J$$

Optimization Problem Formulation (contd.)

Constraints (contd.):

- Satisfaction of customer demand for the product

$$q_{lji} = d_{li} y_{ji}, \ \forall l \in L, i \in I, j \in J$$

- Ensure that $y_{ji} = 0$ when $z_j = 0$

$$\sum_{i} y_{ji} \leq z_j |I|, \ \forall j \in J$$

- Inward flow into a wholesale store = Outflow from the wholesale store

$$\sum_{k} f_{lkj} = \sum_{i} q_{lji}, \ \forall l \in L, j \in J$$

- Factory supply restriction

$$\sum_{k} b_{lsk} \leq sup_{ls}, \ \forall l \in L, s \in S$$

Optimization Problem Formulation (contd.)

Constraints (contd.):

- Number of DCs that can be opened

$$\sum_{j} z_j \leq |J|$$

- Inward flow into a wholesale store $\leq$ Capacity of wholesale store

$$\sum_{l} \sum_{s} b_{lsk} \leq D_k p_k, \quad \forall k \in K$$

- Wholesale store outflow $\leq$ Inward flow of the wholesale store

$$\sum_{j} f_{lkj} \leq \sum_{s} b_{lsk}, \forall l \in L, k \in K$$

Optimization Problem Formulation (contd.)

Constraints (contd.):

- Wholesale store outflow $\leq$ Wholesale store capacity

$$\sum_{l}\sum_{j} f_{lkj} \leq D_k p_k, \; \forall k \in K$$

- Outflow from a wholesale store $\leq$ Inward flow into the wholesale store

$$\sum_{j} f_{lkj} \leq \sum_{s} b_{lsk}, \; \forall l \in L, k \in K$$

- Number of plants that are opened

$$\sum_{k} p_k \leq |K|$$

Optimization Problem Formulation (contd.)

Constraints (contd.):

- Binary restriction on the decision variables z_j, p_k, y_{ij}

$$z_j \in \{0,1\},\ \forall j \in J$$

$$p_k \in \{0,1\},\ \forall k \in K$$

$$y_{ij} \in \{0,1\},\ \forall i \in I, j \in J$$

- Non-negativity restriction on decision variables $b_{lsk}, f_{lkj}, q_{lij}$

$$b_{lsk} \geq 0,\ \forall l \in L, s \in S, k \in K$$

$$f_{lkj} \geq 0,\ \forall l \in L, j \in J, k \in K$$

$$q_{lji} \geq 0,\ \forall l \in L, i \in I, j \in J$$

ϵ-Constraint Method for Multi-objective Optimization

- Keep just one of the objectives and restrict the other objective to be greater than or equal to a user-specified value (ϵ).

$$
\begin{aligned}
&\mathit{min} && f_1 \\
&s.t. && f_2 \geq \epsilon \\
& && \text{and other constraints.}
\end{aligned}
$$

- For example, $\epsilon = 0.9$ means 90% of the customer demand should be satisfied within time τ while minimizing the total cost of the end-to-end supply chain.

Results (contd.)

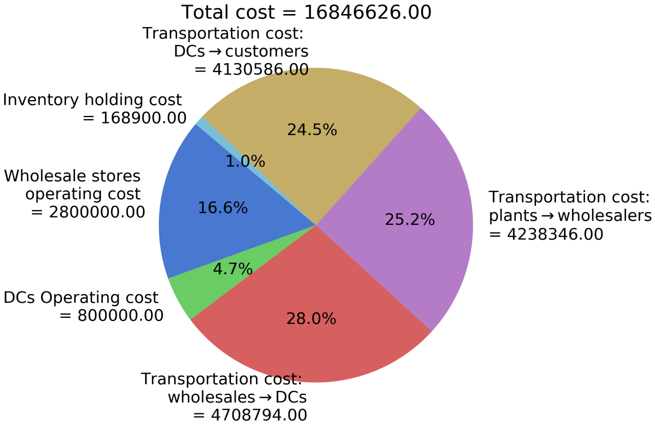


Figure: Total supply chain cost to provide 90% service level in 24 hours time.

The End