

Formal Analysis of Real-World Security Protocols

*Lecture 3: Attacker Model and
Trace Properties*



Exercises

- We have finished grading the first exercise sheet
- You will receive feedback through CMS later today
- Questions about the grading? Send an email to alexander.dax@cispa.de
 - Include your **team number** and question
 - e.g., “I am member of Team #1. I do not understand the feedback we got for Exercise sheet 1, Ex. 1b. Could you please clarify what our mistake was?”
- The next exercise sheet will be published today at 16:00
 - Feel free to ask questions on the **forum** if something is unclear!



Model components

What **components** do we need to model protocols?

- | | | |
|---|---|-----------|
| 1. All possible sent and received messages | } | Lecture 1 |
| 2. All possible protocol behaviors | } | Lecture 2 |
| 3. The attacker | } | Lecture 3 |
| 4. Security properties that we want to verify | | |



This lecture

Actions and Action Traces

Protocol Model

Attacker Model

Trace Properties

Actions and Action Traces



Action facts

- **Actions**, like regular facts, are built from predicates applied to terms
- They model actions taken by agents during protocol execution and steps taken during protocol initialization

```
// Send message
[ Fr(~m) ] --[ Send(~m) ]-> [ Out(~m) ]

// Receive message
[ In(m) ] --[ Receive(m) ]-> [ ]
```

- Actions are analogous to labels in labelled transition systems and can be used for **property specification**



Executions

An execution is a sequence of steps that a system can take, starting from an initial state and following a set of rules. Each step transforms the current state into a new state by applying a specific rule.

- Let R be a set of rules constructed over a given signature, and let S be a state of the system, i.e., a multiset of facts
- An **execution** of R with respect to an equational theory E is an alternating sequence of states and ground rule instances:

$$[S_0, l_1 \multimap [a_1] \rightarrow r_1, S_1, l_2 \multimap [a_2] \rightarrow r_2, \dots, S_{k-1}, l_k \multimap [a_k] \rightarrow r_k, S_k]$$

such that the following three conditions hold:

1. $S_0 = []$,
 2. $\forall i \in \{1 \dots k\}, (S_{i-1}, (l_i \multimap [a_i] \rightarrow r_i), S_i) \in \text{steps}(R)$, and
 3. $\forall i, j \in \{1 \dots k\}, r_i = [] \multimap [] \rightarrow [Fr(n)]$ and $r_j = [] \multimap [] \rightarrow [Fr(n)]: i = j$.
- We denote the set of executions of a set of rules R by $\text{execs}(R)$

- For each execution, we define the corresponding trace as the sequence $[set(a_1), set(a_2), \dots, set(a_k)]$ and denote the set of all traces of a set of rules R by $traces(R)$
- Consider the following protocol:

```
[      ] --[ Init() ]-> [ A('1') ] // Create A('1')
[ A(x) ] --[ Step(x) ]-> [ B(x)   ] // Convert A(x) to B(x)
```

- One possible execution:

```
[      ] --[ Init() ]->
[ A('1') ] --[ Init() ]->
[ A('1'), A('1') ] --[ Step('1') ]->
[ A('1'), B('1') ]
```

- Corresponding trace:

```
[ Init(), Init(), Step('1') ]
```


Example

Condition 1: Initial State

What it says: The system starts from an empty state.

What it means: At the very beginning (time S), there are no facts in the system. It's like starting with a blank slate.

Condition 2: Valid Transitions

What it says: Each step in the sequence must:

Start from the current state (S).

Apply a rule (a) from the set of rules (R).

End in a new valid state (S).

What it means:

The system can only move forward if a rule allows it.

Every state transition must make sense according to the rules in R.

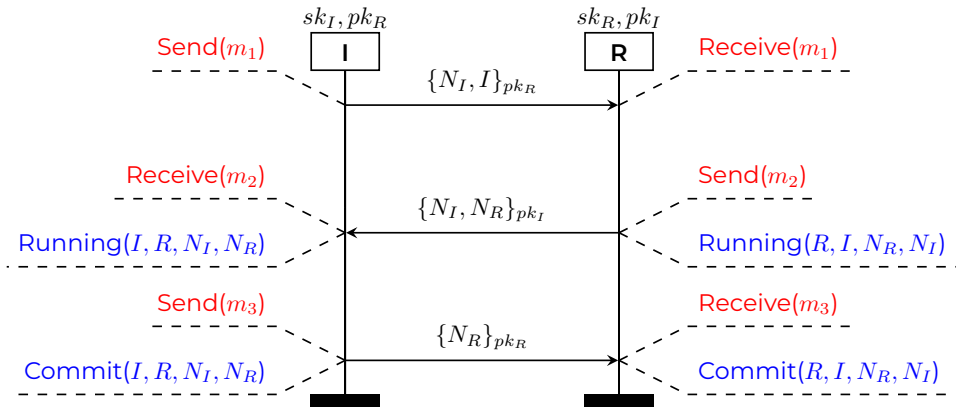
Condition 3: each rule application is unique and doesn't produce conflicting outputs:

If $i=j$ it's the same rule, so it should produce consistent results.

If $i \neq j$ they are different rules, and their outputs should be independent.

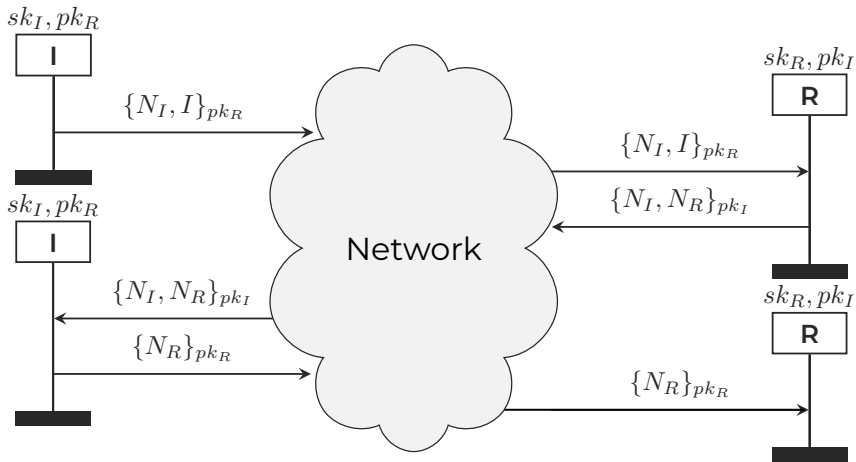


Needham-Schroeder Public-Key protocol (NSPK)



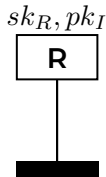
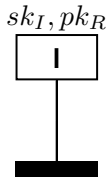


Protocol model





Initialization



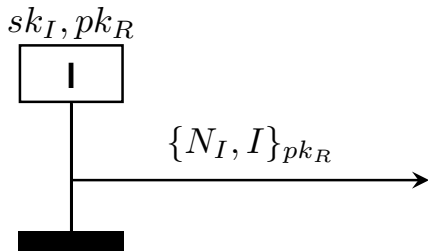
```
builtins: asymmetric-encryption

/* Public key infrastructure */
rule register_pk:
  let
    public_key = pk(~secret_key)
  in
    [ Fr(~secret_key) ]
    -->
    [ !Sk($ID, ~secret_key)
      , !Pk($ID, public_key)
      , Out(public_key) ]

/* Reveal secret key */
rule reveal_sk:
  [ !Sk(ID, secret_key) ]
  --[ Reveal(ID) ]->
  [ Out(secret_key) ]
```



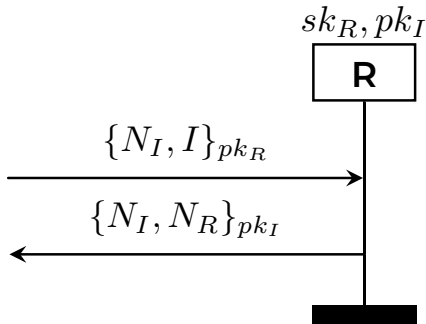
Initiator (1/2)



```
/* Generate a fresh nonce nI and
   send an encrypted message
   to R. */
rule initiator_1:
  let
    m1 = aenc{'1', ~nI, $I}pkR
  in
    [ Fr(~nI)
      , !Pk($R, pkR) ]
  --[ Send(m1) ]->
    [ Out(m1)
      , St_I_1($I, $R, ~nI) ]
```



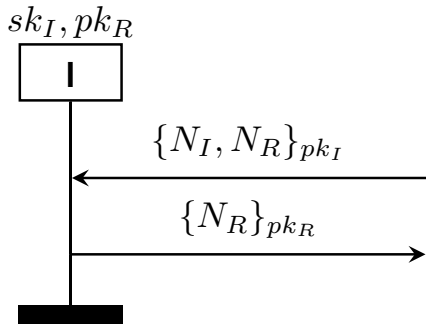
Responder (1/2)



```
/* Receive an encrypted message
   from I and decrypt it.
   Derive a fresh nonce nR and
   reply to I. */
rule responder_1:
  let
    m1 = aenc{'1', nI, I}pk(skR)
    m2 = aenc{'2', nI, ~nR}pkI
  in
  [ In(m1)
    , !Sk(R, skR)
    , !Pk(I, pkI)
    , Fr(~nR) ]
-- [ Receive(nI, m1)
    , Send(m2)
    , Running(R, I, ~nR, nI) ]->
  [ Out(m2)
    , St_R_1(R, I, nI, ~nR) ]
```



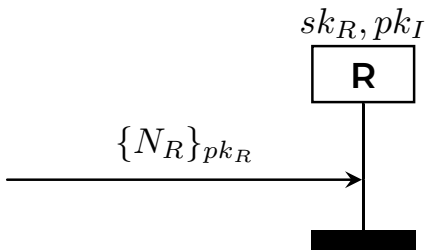
Initiator (2/2)



```
/* Receive an encrypted message
   from R and decrypt it.
   Respond to R. */
rule initiator_2:
  let
    m2 = aenc{'2', nI, nR}pk(skI)
    m3 = aenc{'3', nR}pkR
  in
    [ In(m2)
      , St_I_1(I, R, nI)
      , !Sk(I, skI)
      , !Pk(R, pkR) ]
  -- [ Receive(nR, m2)
      , Running(I, R, nI, nR)
      , Commit(I, R, nI, nR) ] ->
    [ Out(m3) ]
```



Responder (2/2)



```
/* Receive a message from I. */  
rule responder_2:  
  let  
    m3 = aenc{'3', nR}pk(skR)  
  in  
    [ In(m3)  
      , St_R_1(R, I, nI, nR)  
      , !Sk(R, skR) ]  
  --[ Commit(R, I, nR, nI) ]->  
    [ ]
```


Protocol Model



Protocol model in Tamarin

- **Term algebra**

- $\Sigma_{DH} = \{enc(-, -), dec(-, -), h(-), \langle -, - \rangle, fst(-), snd(-), \hat{-}, -^{-1}, - \times -, 1\}$

- **Equational theory**

- $E_{DH} = \{dec(enc(m, k), k) =_E m, x \times (y \times z) =_E (x \times y) \times z, \dots\}$

- **Facts**

- $F(t_1, \dots, t_n)$

- **Transition system**

- State: multiset of facts

- Rules: $l \multimap [a] \rightarrow r$

- **Special facts and rules**

- Facts: $In(), Out(), K()$

- Special fresh rule: $[] \multimap [] \rightarrow [Fr(x)]$



- **Transition relation**

$S \dashv [a] \rightarrow_R ((S \setminus^\# I) \cup^\# r)$, where

- $I \dashv [a] \rightarrow r$ is a ground instance of a rule in R , and
- $I \subseteq^\# S$ wrt the equational theory

- **Executions**

- $\text{execs}(R) = \{ [] \dashv [a_1] \rightarrow \dots \dashv [a_n] \rightarrow S_n \mid \forall n. Fr(n) \text{ appears only once on the right-hand side of the rule} \}$

- **Traces**

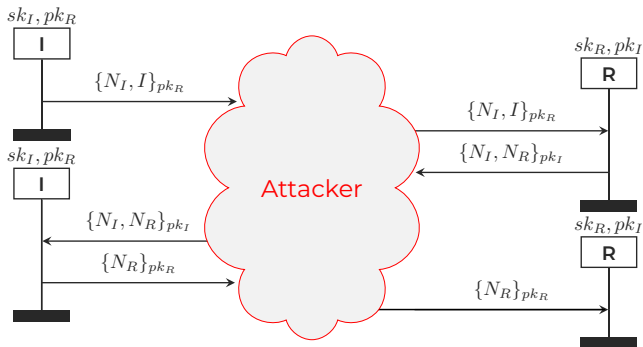
- $\text{traces}(R) = \{ [a_1, \dots, a_n] \mid [] \dashv [a_1] \rightarrow \dots \dashv [a_n] \rightarrow S_n \in \text{execs}(R) \}$

Attacker Model



Attacker model

Recall the **protocol execution model** from earlier:





The Dolev-Yao model

- All messages are sent to the attacker who can either **drop, modify,** or **forward** them
- The attacker sees all the messages and maintains a knowledge set of all the information sent over public channels
- When the attacker learns a cryptographic key, it can perform cryptographic operations, such as **encryption, decryption,** and **signing**, to add new messages to its knowledge set
- The attacker can also deconstruct messages into their components and create new messages from the parts it knows
- However, it cannot forge or read cryptographically protected messages **without knowing the corresponding keys**



Some potential attacks

Man-in-the-middle: c impersonates a to b

Replay: reuse previous messages

Reflection: send message back to its sender

Oracle: use normal protocol responses to gain information

Binding: use messages in an unintended context

Type flaw: substitute message fields



Attacker knowledge and interaction

- A persistent fact **K(m)** denotes that m is known to the adversary
- A linear fact **Out(m)** denotes that the protocol has sent the message m , which can be received by the adversary
- A linear fact **In(m)** denotes that the protocol can receive the message m , which might have been sent by the attacker
- The semantics of these three fact symbols is given by the following set of **message deduction rules**



Message deduction rules

$\left\{ \frac{\text{Out}(x)}{K(x)} \right\}$	// Receive message from the protocol $[\text{Out}(x)] \rightarrow [K(x)]$
$\left\{ \frac{K(x)}{\text{In}(x)} [K(x)] \right\}$	// Send message to the protocol $[K(x)] \rightarrow [\text{In}(x)]$
$\left\{ \frac{}{K(x : \text{pub})} \right\}$	// Learn public value $[] \rightarrow [K(\$x)]$
$\left\{ \frac{\text{Fr}(x : \text{fresh})}{K(x : \text{fresh})} \right\}$	// Generate fresh value $[\text{Fr}(\sim x)] \rightarrow [K(\sim x)]$
$\left\{ \frac{K(x_1) \dots K(x_k)}{K(f(x_1 \dots x_k))} \right\}$	// Apply functions to known messages $[K(x_1) \dots K(x_k)] \rightarrow [K(f(x_1 \dots x_k))]$

Trace Properties



Trace properties

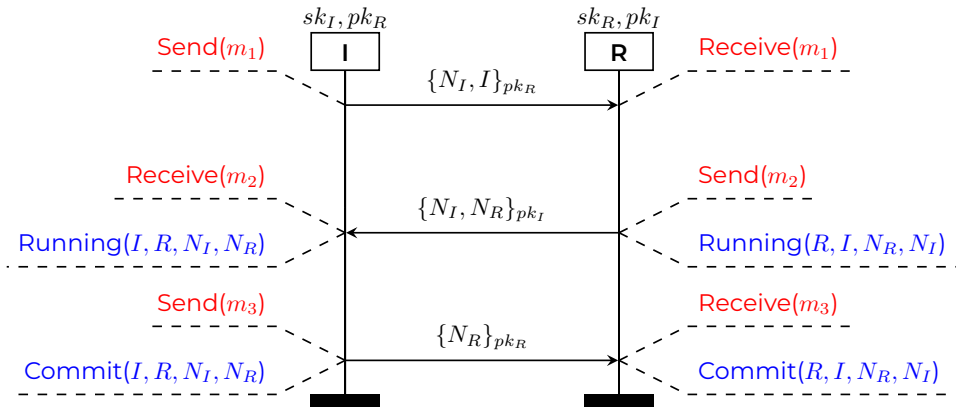
- A trace property specifies a set of traces representing a set of desired protocol behaviors
- If the protocol state machine includes behaviors that are not included in the specified property, then we have a violation
→ **This constitutes an attack on the protocol!**
- In Tamarin, trace properties are specified as formulas in **first-order logic**, built from actions and quantifying over message terms and timepoints
- Timepoints are used to order actions; they enable the specification of properties that depend on the events' relative ordering

All	Universal quantification (\forall)
Ex	Existential quantification (\exists)
\Rightarrow	Implication
&	Conjunction
	Disjunction
not	Negation (\neg)
$f@i$	An action f at a timepoint $\#i$
$\#i < \#j$	Timepoint $\#i$ occurring before $\#j$
$\#i = \#j$	Timepoint equality
$x = y$	Message variable equality
$\text{Pred}(t_1, \dots, t_n)$	The predicate Pred applied to the terms t_1 to t_n

Example



Needham-Schroeder Public-Key protocol (NSPK)





Lemma 1: Executability

To rule out (some) modeling mistakes, we use **reachability lemmas** to make sure that it is **possible** to reach the end of the protocol model. Our goal is to find a completed protocol trace where the steps are the expected ones taken by honest agents without adversary interference.

```
/* Executability */  
lemma trace: exists-trace  
  " Ex A B nA nB #i #j .  
    Commit(A, B, nA, nB)@i  
    & Commit(B, A, nB, nA)@j  
    & not(Ex #r. Reveal(A)@r)  
    & not(Ex #r. Reveal(B)@r)  
  "
```



Lemma 2: Injective agreement

Whenever somebody commits to running a session and the adversary did not reveal the long-term key of the participants, there is somebody running a session with the same parameters and there is no other commit on the same parameters.

```
/* Injective agreement */
lemma injective_agreement:
  " All A B nA nB #i .
    Commit(A, B, nA, nB)@i
    ==> (Ex #j. Running(B, A, nB, nA)@j & j < i
        & not(Ex A2 B2 #i2 .
            Commit(A2, B2, nA, nB)@i2 & not(#i = #i2)))
    | (Ex #r. Reveal(A)@r)
    | (Ex #r. Reveal(B)@r)
  "
```


Summary



Next lecture

- We now know how to model..
 - ..protocol behavior as **multiset rewriting rules**
 - ..protocol properties as **first-order logic formulas**
- Together, these two languages allow us to **model protocols, specify security properties, and analyze them in the presence of a Dolev-Yao attacker**
- In the next lecture, we will talk about how Tamarin uses this model to find attacks



Reading material

Recommended reading:

[Bas+24, Ch. 4.2.2, 5, 6–6.4], [Mei13, Ch. 7.3], [Sch+12]

- [Bas+24] D. Basin, C. Cremers, J. Dreier, and R. Sasse. **Modeling and Analyzing Security Protocols with Tamarin: A Comprehensive Guide**. Draft v0.5. Sept. 2024.
- [Mei13] S. Meier. **Advancing Automated Security Protocol Verification**. PhD thesis. ETH Zurich, 2013.
- [Sch+12] B. Schmidt, S. Meier, C. Cremers, and D. Basin. **Automated Analysis of Diffie-Hellman Protocols and Advanced Security Properties**. In: 2012 IEEE 25th Computer Security Foundations Symposium. 2012.