Bimetric computations in xAct 3

```
(*********************************************************************)
(* Bimetric computations in xAct. Part III.                          *)
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(*********************************************************************)

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  USA.
*)
```

Generate TEX Report

First, display the configuration and the geometry of spacetimes:

```
tex$log={
"\\allowdisplaybreaks[1]","",
"\\section*{\\center Bimetric BH solutions}","",
"\\noindent\\centerline{Report generated: " <> DateString[{
 "Year","-","Month","-","Day"," ",
 "Hour",":","Minute",":","Second"}] <> " (MBK)}",
"\\vspace{1mm}","",
"\\noindent This file was generated automatically by the notebook: "<>
"\\texttt{"<>FileNameSplit[NotebookFileName[]][[-1]]<>"}"
};

texDumpAssumptions[ If[inOldVars,altFields,theFields] ];
```

```
log/@{
"\\subsubsection*{Transformations}",
"Old to new variables:",
"\\begin{align} &",
StringRiffle[texNice/@oldToNewVarsRelation,", \\quad "]//StringReplace[#,"="→" := "]&,
"\\end{align}",
"New to old variables:",
"\\begin{align} &",
StringRiffle[texNice/@newToOldVarsRelation,", \\quad "]//StringReplace[#,"="→" := "]&,
"\\end{align}",""
};
texDumpMetric [ "g",g■ ];
texDumpMetric [ "f",f■ ];
texDumpInteractionTerm [];
texDumpGeometryOf[ "g",g⌘, ChristoffelCDPD𝓑, RicciScalarCD, KretschmannCD, EinsteinCD ];
texDumpGeometryOf[ "f",f⌘, ChristoffelCDfPD𝓑, RicciScalarCDf, KretschmannCDf, EinsteinCDf ];
```

Display the full set of equations of motion:

```
log/@Flatten[{
"\\section*{The Full Set of EoM}",
"EoM for $g$:","\\begin{align}",
Riffle[Table[
{"& 0 = ", gEoM[[i]]//Expand//texNice//myTexBreak},
{i,Length@gEoM}
],"\\\\"],
"\\end{align}",
"EoM for $f$:","\\begin{align}",
Riffle[Table[
{"& 0 = ", fEoM[[i]]//Expand//texNice//myTexBreak},
{i,Length@fEoM}
],"\\\\"],
"\\end{align}",
"Bianchi constraint for $g$:","\\begin{align}",
Riffle[Table[
{"& 0 = ", gBianchi[[i]]//Expand//texNice//myTexBreak},
{i,Length@gBianchi}
],"\\\\"],
"\\end{align}",
"Bianchi constraint for $f$:","\\begin{align}",
Riffle[Table[
{"& 0 = ", fBianchi[[i]]//Expand//texNice//myTexBreak},
{i,Length@fBianchi}
],"\\\\"],
"\\end{align}"
}];
```

Display the reduced set of equations of motion:

```
log/@Flatten[{
 "\\section*{The Reduced Set of EoM}",
 "Combined EoM:","\\begin{align}",
 Riffle[texDisplayExpandedForm[coeffSet$full],"\\\\"],
 "\\end{align}"
}];
```

Report how to find the normal form:

```
If[inNewVars,log /@{"",
 "\\subsection*{Normal form of EoM}",
 "Procedure how to find the normal form of EoM (in new variables):",
 "\\begin{itemize}",
 "\\item Solve $F^{\\prime}$ from $G_g{}^1{}_1$.",
 "\\item Solve $b^{\\prime}$ from $G_g{}^0{}_0$.",
 "\\item Solve $\\sigma^{\\prime}$ from $G_f{}^0{}_0$.",
 "\\item Solve $\\tau^{\\prime}$ from $G_f{}^1{}_1$ as a function of $R^{\\prime}$.",
 "\\item Solve $\\tau^{\\prime}$ from Bianchi constraint as a function of $R^{\\prime}$.",
 "\\item Solve algebraically $\\tau$ from $G_f{}^1{}_1$ and Bianchi so that $R^{\\prime}$ drops ou
 "\\item Calulate $\\tau^{\\prime}$ from $\\tau$.",
 "\\item Find (reversed) $R^{\\prime}$ from the calculated $\\tau^{\\prime}$.",
 "\\item Verify validity of $F^{\\prime}$, $b^{\\prime}$, $\\sigma^{\\prime}$, $\\tau^{\\prime}$ a
 "\\end{itemize}"
}];
If[inOldVars,log /@{"",
 "\\subsection*{Normal form of EoM}","Procedure how to find the normal form of EoM (in old variabl
 "\\begin{itemize}",
 "\\item Solve $a$ from $G_f{}^1{}_1$ and Bianchi",
 "\\item Solve $Q^{\\prime}$ from $G_g {}^1{}_ 1$",
 "\\item Calculating $a^{\\prime}$ from $a$",
 "\\item Solve $N^{\\prime}$, $Y^{\\prime}$ and $U^{\\prime}$ from $G_g{}^0{}_0$, $G_f{}^0{}_0$ an
 "\\item Simplify $N^{\\prime}$, $Y^{\\prime}$ and $U^{\\prime}$",
 "\\end{itemize}"
}];
```

Display the simplified set of equations of motion:

```
log/@Flatten[{
 "\\section*{An Example of EoM}",
 "Combined EoM using parameters:",
 "$m=1$, $\\kappa=1$, $c=1$, $\\beta_2=0$, $\\beta_3=0$",
 "\\begin{align}",
 Riffle[texDisplayExpandedForm[coeffSet],"\\\\"],
 "\\end{align}"
}];
```

Display the normal form

```
texSplitLong[expr_]:=If[Head@expr=!=Times,
expr//texNice,
Riffle[Table[
 If[Head@expr[[i]]=!=Power,
 texNice[expr[[i]]//Expand,"{\\color{blue}\\big(}","{\\color{blue}\\big)}"],
 texNice[expr[[i]][[1]]//Expand,"{\\color{red}\\big(}","{\\color{red}\\big)}"]<>
 texNice[expr[[i]][[2]],"^{\\color{red}","}"]
 ],
 {i,Length@expr}]," \\cdot " ]/.List→StringJoin
]//myTexBreak

If[inNewVars∧coordSys==1,log/@{
"\\subsubsection*{The normal form}",
"\\begin{align}",
texNice[τ,"","  &= "],
τ/.sol$τ45//texSplitLong,
"\\\\",
texNice[F′,"","  &= "],
F′/.sol$F/.sol$τ45//Expand//Simplify//texSplitLong,
"\\\\",
texNice[b′,"","  &= "],
b′/.sol$b/.sol$τ45//Expand//Simplify//texSplitLong,
"\\\\",
texNice[σ′,"","  &= "],
σ′/.sol$σ/.sol$τ45//Expand//Simplify//texSplitLong,
"\\\\",
texNice[R′,"","  &= "],
R′/.sol$R//texSplitLong,
"\\end{align}"
}];

If[inOldVars∧coordSys==1,log/@{
"\\subsubsection*{The normal form}",
"\\begin{align}",
texNice[a/Q,"","  &= "],
a/Q/.sol$a//texSplitLong,
"\\\\",
texNice[Q′/Q,"","  &= "],
Q′/Q/.sol$Q//Expand//Simplify//texSplitLong,
"\\\\",
texNice[Y′,"","  &= "],
N′/.sol$NYU//Expand//Simplify//texSplitLong,
"\\\\",
texNice[N′,"","  &= "],
Y′/.sol$NYU//Expand//Simplify//texSplitLong,
"\\\\",
texNice[U′,"","  &= "],
U′/.sol$NYU//texSplitLong,
"\\end{align}"
}];
```

Some notes about coordinate transformations.

```
Quiet@DefTensor[G[a,-b],M]
```

```
If[coordSys==1,Block[{coordSys=2},
log/@Flatten[{
"\\section*{The EoM in the off-diagonal form}",
"Assume the chart transition map $J$:",
"\\begin{align}",
"& J^{\\mu}{}_{\\underline{\\alpha}} := \\frac{\\partial x^\\mu}{\\partial x^{\\underline{\\alpha
J■ // redefineFields // Simplify // texNiceMatrix,
"\\!,\\quad (J^{-1})^{\\underline{\\alpha}}{}_{\\mu} := \\frac{\\partial x^{\\underline{\\alpha}}
iJ■ // redefineFields // Simplify // texNiceMatrix,
"\\!,\\end{align}",
"In the chart $\\{\\underline{v},\\underline{r},\\underline{\\theta},\\underline{\\phi}\\}$, we h
"\\begin{align}",
"g_{\\alpha\\beta} &= ",
g■ // redefineFields // Simplify // texNiceMatrix,
"\\!,\\\\",
"f_{\\alpha\\beta} &= ",
f■ // redefineFields // Simplify // texNiceMatrix,
"\\!,\\\\",
"S^{\\alpha}{}_{\\beta} &= ",
iJ■.S■.J■ // redefineFields // Simplify // texNiceMatrix,
"\\!.",
"\\end{align}",
"If the equations of motion $G$ are in the diagonal form,",
"\\begin{align}",
"G &= ",
DiagonalMatrix@{G[{0,ℬ},{0,-ℬ}],G[{1,ℬ},{1,-ℬ}],G[{2,ℬ},{2,-ℬ}],G[{3,ℬ},{3,-ℬ}]} // redefineFiel
"= 0,",
"\\end{align}",
"then after the coordinate transformation, we have:",
"\\begin{align}",
"J^{-1}\,G\,J &= ",
iJ■.DiagonalMatrix@{G[{0,ℬ},{0,-ℬ}],G[{1,ℬ},{1,-ℬ}],G[{2,ℬ},{2,-ℬ}],G[{3,ℬ},{3,-ℬ}]}.J■ // redef
"= 0.",
"\\end{align}",
"That is, we can consider EoM only in the bi-diagonal form without loss of generality."
}]
]];
```

Finally, save the report to a 'tex' file and compile it to get a 'pdf'.

```
saveToTexFile[ NotebookDirectory[]<>"bh-report-"<>If[inOldVars,"O","N"]<>ToString@coordSys ][tex$
```