Causality Constraints on Massive Gravity (1610.02033)

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Causality Constraints on Massive Gravity
https://arxiv.org/abs/1610.02033

Definitions

The Minkowski null frame and its inverse:

$$\eta = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

```
iη = Inverse@η;  iη // MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The inverse metric g:

$$g = \begin{pmatrix} F & -1/2 & 0 \\ -1/2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

A vielbien of g:

$$E = \begin{pmatrix} -2^{-1/2} F & 2^{-1/2} & 0 \\ 2^{-1/2} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$g == E^{\mathsf{T}}.\eta.E$$

```
True
```

The inverse metric g^{-1}:

$$ig = \begin{pmatrix} 0 & -2 & 0 \\ -2 & -4F & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

```
ig == Inverse@g
```

```
True
```

A vielbien of g^{-1}:

$$itE = \begin{pmatrix} 0 & 2^{1/2} & 0 \\ 2^{1/2} & 2^{1/2} F & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$ig == itE^{\mathsf{T}}.i\eta.itE \bigwedge itE == \text{Inverse@Transpose@E}$$

```
True
```

The particular field F for the sandwich pp-wave solution:

$$A[u\_, a\_, \lambda\_] := \begin{cases} \text{Exp}\left[-a \frac{\lambda^2 u^2}{\left(u^2-\lambda^2\right)^2}\right] & -\lambda < u < \lambda \\ 0 & \text{True} \end{cases}$$

$$\text{Fg}[u\_, r\_, a\_, \lambda\_, m\_] := A[u, a, \lambda]\ \text{BesselK}[0, m\, r]$$

The null cone of g:

$$\text{nc\$g} = \left(\{\{\mathbb{T}u, \mathbb{T}v, \mathbb{T}r\}\}.g.\{\{\mathbb{T}u, \mathbb{T}v, \mathbb{T}r\}\}^{\mathsf{T}}\right)[\![1, 1]\!]\ //\ \text{Simplify}$$

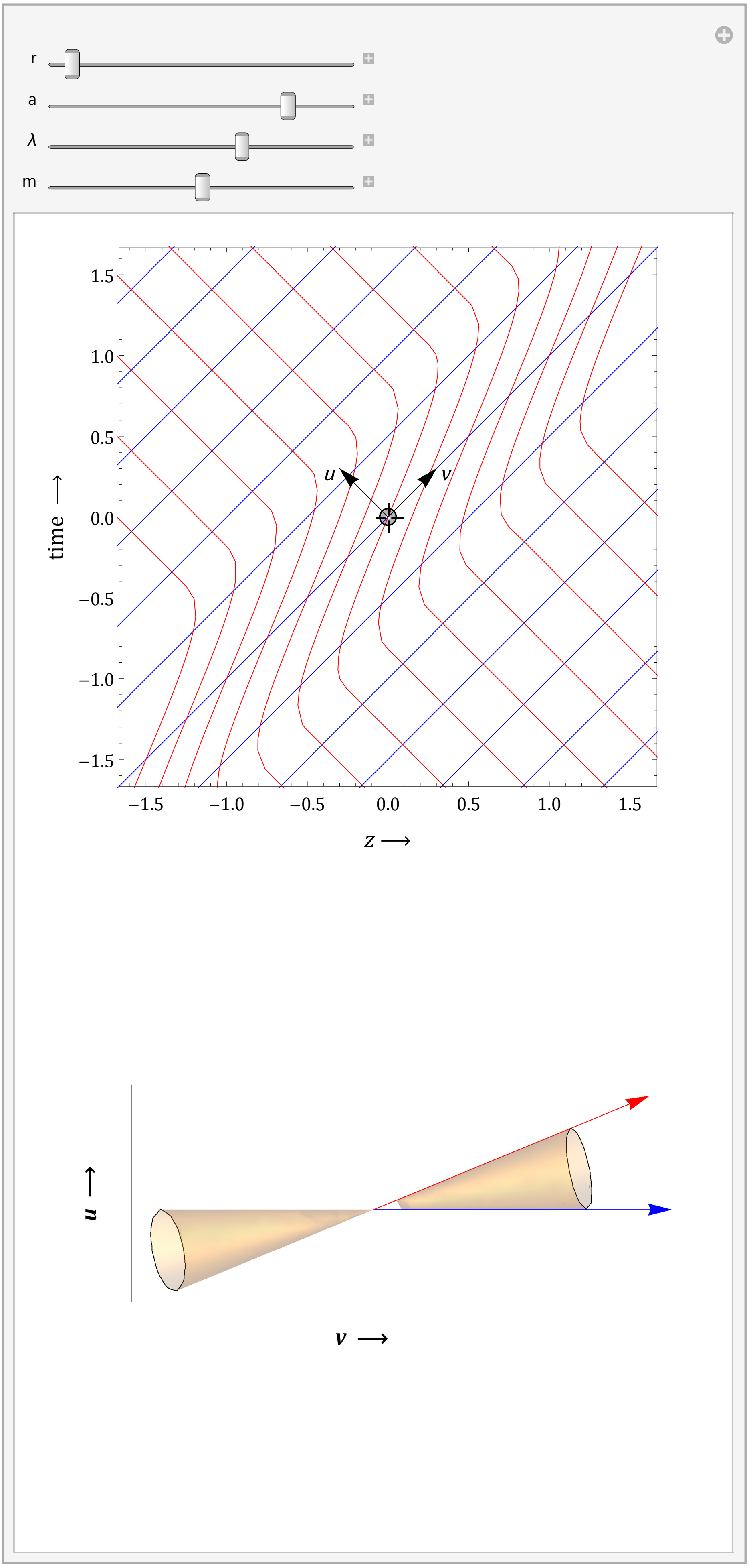
$$\mathbb{T}r^2 + \mathbb{T}u\,(F\,\mathbb{T}u - \mathbb{T}v)$$

```
nc$3D[u_, v_, r_, a_, λ_, m_, scale1_: 1, scale2_: 1] :=
 Block[
  {tvec$g, F},
  F = Fg[N@u, N@r, a , λ, m];
  tvec$g = Normalize[ Normalize@itE[[1]] + Normalize@itE[[2]] ];
  Translate[Scale[
    Join[
     Normal@ContourPlot3D[
        nc$g,
        {𝕋v, -1, 1},
        {𝕋r, -1, 1},
        {𝕋u, -1, 1},
        PlotRange → {{-3, 3}, {-3, 3}, {-3, 3}},
        Contours → {0}, ContourStyle → Opacity[0.2],
        Mesh → None, Boxed → False, Axes → False,
        RegionFunction → Function [ {vv, rr, uu}, Abs[ {uu, vv, rr}.tvec$g ] ≤ 0.7]
       ][[1]],
     {
      Blue, Arrow@{{0, 0, 0}, Normalize@itE[[1, {2, 3, 1}]]},
      Red, Arrow@{{0, 0, 0}, Normalize@itE[[2, {2, 3, 1}]]}
     }
    ], scale1
   ], {v, r, u} * scale2]
 ]
```

```
If[False, plot = Table[
   nc$3D[u, 0, r, 0.93, 1, 1, 0.07],
   {u, Range[-1.2, 1.2, 0.2]},
   {r, Range[0.2, 1.2, 0.2]}
  ];
 Graphics3D[plot, Axes → True, AxesLabel → {"r", "v", "u"},
  PlotRange → All, ViewPoint → {-8, -3, 0}, ImageSize → 500]
]
```

Null cone field and null geodesics

```
Manipulate[
 Block[{F},
  F = Fg[N@u, N@r, N@a, N@λ, N@m];
  Row@{
    Show[
     StreamPlot[
      {#⟦2⟧ - #⟦1⟧, #⟦1⟧ + #⟦2⟧} / 2 &@ (itE⟦1, {1, 2}⟧ /. {u → t - z, v → t + z}),
      {z, -3, 3},
      {t, -3, 3},
      StreamStyle → {Blue, "Line"},
      StreamPoints → {Table[{(-u + 0)/2, (u + 0)/2}, {u, -3, 3, 0.5}], 0.1, 10},
      ImageSize → {400, 400}
     ],
     StreamPlot[
      {#⟦2⟧ - #⟦1⟧, #⟦1⟧ + #⟦2⟧} / 2 &@ (itE⟦2, {1, 2}⟧ /. {u → t - z, v → t + z}),
      {z, -3, 3},
      {t, -3, 3},
      StreamStyle → {Red, "Line"},
      StreamPoints → {Table[{(-0 + v)/2, (0 + v)/2}, {v, -3, 3, 0.5}], 0.1, 10}
     ],
     Graphics[{
       Arrow[{zt, zt + 0.3 {1, 1}}], Text[Style[v, 15], zt + 0.4 {0.9, 0.7}],
       Arrow[{zt, zt + 0.3 {-1, 1}}], Text[Style[u, 15], zt + 0.4 {-0.9, 0.7}]
      }],
     PlotRange → {{-1.5, 1.5}, {-1.5, 1.5}},
     BaseStyle → {FontFamily → "Cambria", 12},
     FrameLabel → {Style[Row@{z, "  ⟶"}, 15], Style["time   ⟶", 15]}
    ],
    Spacer[20],
    Graphics3D[nc$3D[zt⟦2⟧ - zt⟦1⟧, zt⟦2⟧ + zt⟦1⟧, r, a, λ, m, 1],
     Boxed → False, Axes → {True, False, True},
     AxesLabel → {Row@{v, "   ⟶"}, None, Rotate[Row@{u, "   ⟶"}, 90 Degree]},
     Ticks → None, PlotRange → All,
     BaseStyle → {FontFamily → "Cambria", Bold, 15},
     ImageSize → {400, 400}, ViewPoint → Front]
   }
 ],
 {{r, 0.1}, 0.01, 3},
 {{a, 0.93}, -1.5, 1.5},
 {{λ, 1}, 0.1, 1.5},
 {{m, 1}, 0.5, 1.5},
 {{zt, {0, 0}}, Locator},
 SaveDefinitions → True
]
```


r
a
λ
m
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
time ⟶
u
v
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
z ⟶
u ⟶
v ⟶

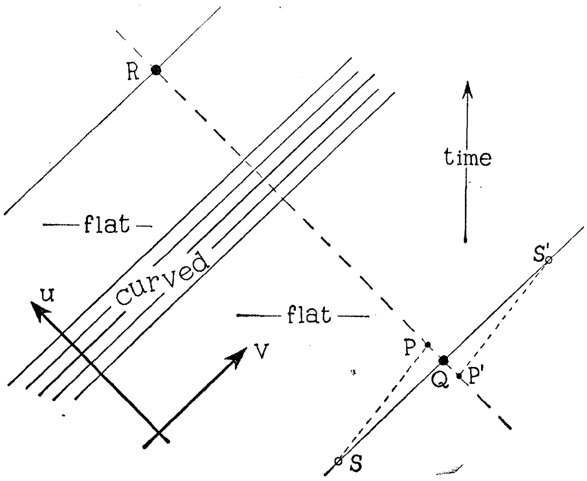


FIG. 1. The sandwich wave $\mathcal{W}_4$ and the related configuration of points mentioned in the test. The diagram essentially represents the two-dimensional section of $\mathcal{W}_4$ by $x_i = 0$.

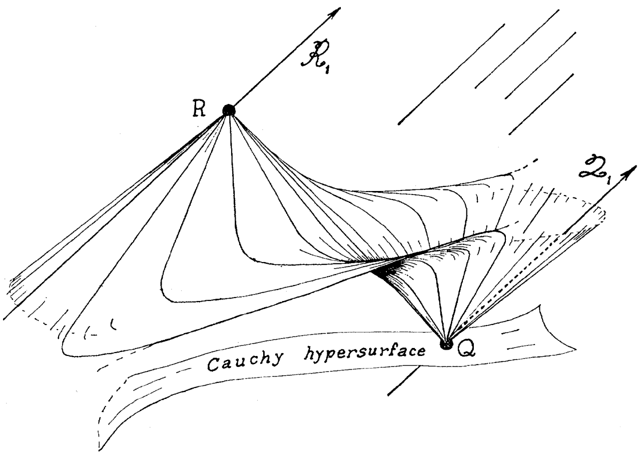


FIG. 2. The purely electromagnetic plane-wave space–times have exact analogs in two space and one time dimension. A null cone can be focused again to a second vertex. The situation is depicted above. A connected spacelike surface through Q can never meet the null line $\mathcal{R}_1$ (if the surface has no boundary).