Homework 5 คำนวณต้นไม้

Decision Tree

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

ข้อมูล root

Class

$$\text{Info}(D) = -\sum_{i=1}^{M} p_i \log_2(p_i)$$

$$= I(9, 5)$$

$$= -\left(\frac{9}{14}\log_2\frac{9}{14}\right) + \left(-\frac{5}{14}\log_2\frac{5}{14}\right)$$

$$= -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14}$$

$$= -\frac{9}{14}(-0.637) - \frac{5}{14}(-1.485)$$

$$= 0.940 \#$$

Feature

$$\text{Info}_{age}(D) = \sum_{j=1}^{v} \left|\frac{D_j}{D}\right| \times \text{Info}(D_j)$$

<= 30 | 31...40 | > 40

$$= \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2)$$

$$= \frac{5}{14}\left[-\frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{3}{5}\log_2\left(\frac{3}{5}\right)\right] + \frac{4}{14}\left[-\frac{4}{4}\log_2\left(\frac{4}{4}\right) - \frac{0}{4}\log_2\left(\frac{0}{4}\right)\right] + \frac{5}{14}\left[-\frac{3}{5}\log_2\left(\frac{3}{5}\right) - \frac{2}{5}\log_2\left(\frac{2}{5}\right)\right]$$

$$= \frac{5}{14}(0.529 + 0.442) + \frac{4}{14}(0 + \text{หาค่าไม่ได้}) + \frac{5}{14}(0.442 + 0.529)$$

$$= \frac{5}{14}(0.971) + \text{หาค่าไม่ได้} + \frac{5}{14}(0.971)$$

$$= 0.347 + 0.347$$

$$= 0.694 \ \#$$

$$\text{Info}_{income}(D) = \sum_{j=1}^{v} \left|\frac{D_j}{D}\right| \times \text{Info}(D_j)$$

high | medium | low

$$= \frac{4}{14} I(2,2) + \frac{6}{14} I(4,2) + \frac{4}{14} I(3,1)$$

$$= \frac{4}{14}\left[-\frac{2}{4}\log_2\left(\frac{2}{4}\right) - \frac{2}{4}\log_2\left(\frac{2}{4}\right)\right] + \frac{6}{14}\left[-\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log_2\left(\frac{2}{6}\right)\right] + \frac{4}{14}\left[-\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right)\right]$$

$$= \frac{4}{14}(0.5 + 0.5) + \frac{6}{14}(0.390 + 0.528) + \frac{4}{14}(0.311 + 0.5)$$

$$= \frac{4}{14} + \frac{6}{7}(0.918) + \frac{4}{14}(0.811)$$

$$= 0.286 + 0.394 + 0.232$$

$$= 0.912 \ \#$$

$$\text{Info}_{student}(D) = \sum_{j=1}^{v} \left|\frac{D_j}{D}\right| \times \text{Info}(D_j)$$

No | Yes

$$= \frac{7}{14} I(3,4) + \frac{7}{14} I(6,1)$$

$$= \frac{7}{14}\left[-\frac{3}{7}\log_2\left(\frac{3}{7}\right) - \frac{4}{7}\log_2\left(\frac{4}{7}\right)\right] + \frac{7}{14}\left[-\frac{6}{7}\log_2\left(\frac{6}{7}\right) - \frac{1}{7}\log_2\left(\frac{1}{7}\right)\right]$$

$$= \frac{7}{14}(0.524 + 0.461) + \frac{7}{14}(0.191) + (0.401)$$

$$= \frac{7}{14}(0.985) + \frac{7}{14}(0.592)$$

$$= 0.493 + 0.296$$

$$= 0.789 \ \#$$

$Info_{credit}(D)$ = $\sum_{j=1}^{v} \left|\frac{D_j}{D}\right| \times Info(D_j)$

excellent Low

$= \frac{8}{14} I(6,2) + \frac{6}{14} I(3,3)$

$= \frac{8}{14}\left[-\frac{6}{8}\log_2\left(\frac{6}{8}\right) - \frac{2}{8}\log_2\left(\frac{2}{8}\right)\right] + \frac{6}{14}\left[-\frac{3}{6}\log_2\left(\frac{3}{6}\right) - \frac{3}{6}\log_2\left(\frac{3}{6}\right)\right]$

$= \frac{8}{14}(0.311 + 0.5) + \frac{6}{14}(0.5 + 0.5)$

$= \frac{8}{14}(0.8111) + \frac{6}{14}$

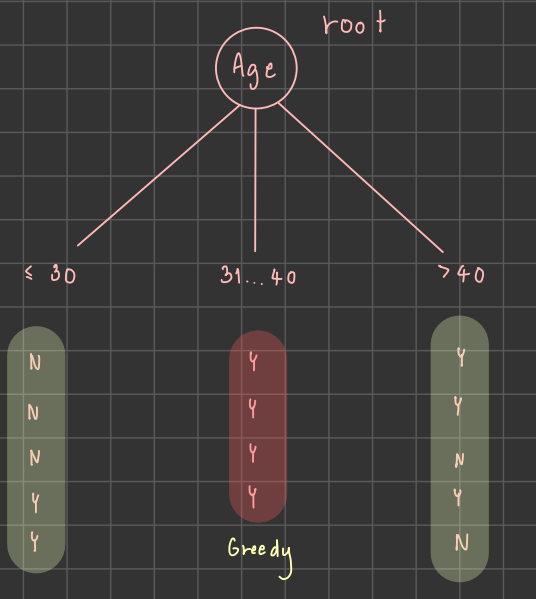
$= 0.464 + 0.429$

$= 0.893$ #

Gain

age	= Info(D) − $Info_{age}(D)$	= 0.940 − 0.649	=	0.291
income	= Info(D) − $Info_{income}(D)$	= 0.940 − 0.912	=	0.028
student	= Info(D) − $Info_{student}(D)$	= 0.940 − 0.789	=	0.151
credit_rating	= Info(D) − $Info_{credit}(D)$	= 0.940 − 0.893	=	0.047

∴ เลือก age เป็น root เพราะ มีค่าเยอะสุด ซึ่งเป็นทางเลือกที่ดีที่สุด #



" Feature 1 Age ≤ 30 "

Class

$$Info(D) = -\sum_{i=1}^{M} p_i \log_2(p_i)$$

Yes No

$$= I(2,3)$$

Yes No

$$= -\frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{3}{5}\log_2\left(\frac{3}{5}\right)$$

$$= 0.971 \ \#$$

Features

high medium low

$$Info_{income}(D) = \frac{2}{5}I(0,2) + \frac{2}{5}I(1,1) + \frac{1}{5}I(1,0)$$

$$= \frac{2}{5}\left[-\frac{0}{2}\log_2\left(\frac{0}{2}\right) - \frac{2}{2}\log_2\left(\frac{2}{2}\right)\right] + \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right] + \frac{1}{5}\left[-\frac{1}{1}\log_2\left(\frac{1}{1}\right) - 2\log_2(0)\right]$$

$$= 0.4 \ \#$$

Yes No

$$Info_{student}(D) = \frac{3}{5}I(0,3) + \frac{2}{5}I(2,0)$$

$$= \frac{3}{5}\left[-\frac{0}{3}\log_2\left(\frac{0}{3}\right) - \frac{3}{3}\log_2\left(\frac{3}{3}\right)\right] + \frac{2}{5}\left[-\frac{2}{2}\log_2\left(\frac{2}{2}\right) - \frac{0}{2}\log_2\left(\frac{0}{2}\right)\right]$$

$$= 0 \ \#$$

fair excellent

$$Info_{credit}(D) = \frac{3}{5}I(1,2) + \frac{2}{5}I(1,1)$$

$$= \frac{3}{5}\left[-\frac{1}{3}\log_2\left(\frac{1}{3}\right) - \frac{2}{3}\log_2\left(\frac{2}{3}\right)\right] + \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right]$$

$$= 0.551 + 0.4$$

$$= 0.951 \ \#$$

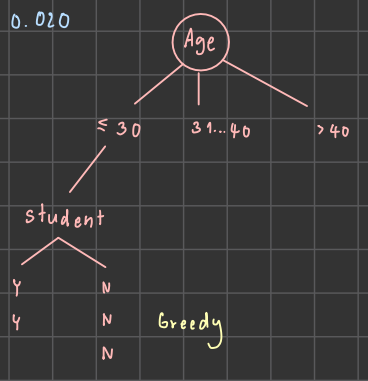
Gain

income = Info(D) − $Info_{income}(D)$ = 0.971 − = 2

student = Info(D) − $Info_{student}(D)$ = 0.971 − 0 = 0.971

credit_rating = Info(D) − $Info_{credit}(D)$ = 0.971 − 0.951 = 0.020

∴ เลือก student เพราะ มีค่าเยอะสุด ซึ่งเป็นทางเลือกที่ดีที่สุด #



" Feature 3 Age > 40 "

Class

$$\text{Info}(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

Yes No

$$= I(3, 2)$$

$$= -\frac{3}{5}\log_2\left(\frac{3}{5}\right) - \frac{2}{5}\log_2\left(\frac{2}{5}\right)$$

$$= 0.971 \#$$

Features

medium Low

$$\text{Info}_{income}(D) = \frac{3}{5} I(2,1) + \frac{2}{5} I(1,1)$$

$$= \frac{3}{5}\left[-\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right] + \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right]$$

$$= 0.551 + 0.4$$

$$= 0.951 \#$$

No Yes

$$\text{Info}_{student}(D) = \frac{2}{5} I(1,1) + \frac{3}{5} I(2,1)$$

$$= \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right] + \frac{3}{5}\left[-\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right]$$

$$= 0.4 + 0.551$$

$$= 0.951 \#$$

fair excellent

$$\text{Info}_{credit}(D) = \frac{3}{5} I(3,0) + \frac{2}{5} I(1,1)$$

$$= \frac{3}{5}\left[-\frac{3}{3}\log_2\left(\frac{3}{3}\right) - \frac{0}{3}\log_2\left(\frac{0}{3}\right)\right] + \frac{2}{5}\left[-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right]$$

$$= 0.4 \#$$

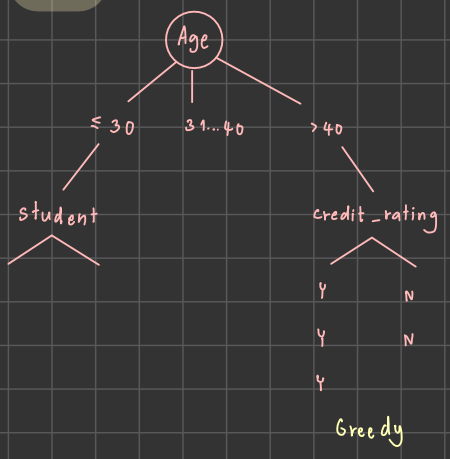
Gain

$$\text{income} = \text{Info}(D) - \text{Info}_{income}(D) = 0.971 - 0.951 = 0.2$$

$$\text{student} = \text{Info}(D) - \text{Info}_{student}(D) = 0.971 - 0.951 = 0.2$$

$$\text{credit\_rating} = \text{Info}(D) - \text{Info}_{credit}(D) = 0.971 - 0.4 = 0.571$$

∴ เลือก credit เพราะ มีค่าเยอะสุด ซึ่งเป็นทางเลือกที่ดีที่สุด #



" Decision Tree Induction "