BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
WORK INTEGRATED LEARNING PROGRAMMES

COURSE HANDOUT

Part A: Content Design

Course Title	Mathematical Foundations for Data Science
Course No(s)	DSECL ZC416
Credit Units	4
Course Author	G Venkiteswaran
Version No	3
Date	15.04.2021
Lead Instructor	Jyotsana Grover

Course Description

Vector and matrix algebra, systems of linear algebraic equations and their solutions; eigenvalues, eigenvectors and diagonalization of matrices; Calculus and optimization; Counting principles and combinatorics

Course Objectives

No	Objective- The course aims to
CO1	Introduce concepts in linear algebra and to use it as a platform to model physical problems.
CO2	Provide techniques for analytical and numerical solutions of linear equations
CO3	Utilize concepts of linear algebra and calculus in solving optimization problems.
CO4	Introduce combinatorics, induction and counting principles

Text Book(s)

No	Author(s), Title, Edition, Publishing House
T1	Erwin Kreyszig, Advanced Engineering Mathematics, Wiley India, 10th Edition, 2015 (earlier editions are also okay)
T2	Kenneth H. Rosen, Discrete Mathematics and its Applications, Tata McGraw Hill, 7th Ed., 2011.

Reference Book(s) & other resources

No	Author(s), Title, Edition, Publishing House
R1	K Hoffman and R Kunze, Linear Algebra, Pearson Education, 2nd Edition, 2005.
R2	Kolman, Busby, Ross and Rehman, Discrete Mathematical Structures for Computer Science, Pearson Education, 6th Edition, 2017
R3	Operations Research: An Introduction, Pearson Education, 10th Edition, 2017. (Earlier editions are also okay)

Content Structure

No	Title of the module	References
M1	1. Matrices, rank, determinants, solution of linear systems – analytical techniques 1.1. Solution of linear systems (A (m x n) x (n x 1) = b (m x 1); A has rank r.) – just a recapitulation 1.2. Solution using Gauss elimination with and without pivoting and operations count 1.3 LU decomposition methods 1.4 Iterative methods for linear systems	T1: Sec 7.3, 20.1, 20.2, 20.3
M2	2. Vector spaces and subspaces, basis and dimensions, Linear transformations and properties 2.1 Vector spaces, inner product spaces, properties, LI and LD, bases and dimensions, LT 2.2. LT and Rank-Nullity theorem • NS(A), RS(A) and CS(A) – illustration	T1: Sec 7.4, 7.9
M3	3. Eigenvalues, Eigenvectors and singular values 3.1. Eigenvalues and eigenvectors of special matrices and their properties 3.2. Eigenbases and diagonalization 3.3. Gerschgorin's Theorem 3.4 Power Method	T1: Sec 8.1,8.7, 20.7, 20.8

M4	4. Decomposition methods (Eigenvalue, decomposition, QR and SVD) • Gram-Schmidt Orthogonalization procedure • QR decomposition • SVD • Dimensionality reduction	T1: Sec 20.9 Class notes
M5	• Linear programming problems • Motivation – 3 problems • Application of LA to Simplex method	Class notes
M6	6. Calculus and Optimization (applications from probability theory to be used for exemplification in Calculus) 6.1 Continuous functions on closed intervals, differentiation (1d case) 6.2 Taylor series expansion 6.3 Maxima and minima 6.4 Integral properties (cdf and pdf, even and odd integrands, integration by parts and so on) for 1d 7. Calculus of several variables • Review limits, continuity and differentiability (graphically and algebraically) • Vector calculus and some of the identities • Maxima and minima (unconstrained) • Steepest gradient method • Lagrange multipliers (for more number of constraints)	Class notes

M7	8. Counting principles and combinatorics • Induction principle • Mathematical induction • Strong induction • Recursive definition and structural induction	T2: Sec 5.1-5.3

	• Counting Principles
	  • Basics of counting
	  • Pigeonhole principle
	  • Permutations and combinations
	  • Binomial coefficients and identities
		T2: Sec 6.1-6.4
		T2: Sec 8.1, 8.2, 8.4

	• Advanced counting
	  • Application of recurrence relations
	  • Solving linear recurrence relations
	  • Generating functions	

Learning Outcomes:

No	Learning Outcomes
LO1	Students will be able to effectively use matrix algebra tools to analyse and solve systems of linear equations.
LO2	Students will be able to use some numerical methods to solve linear systems of equations
LO3	Students would be able to use methods in linear algebra to solve linear programming problems and methods in calculus to solve non-linear optimization problems.
LO4	Students will be able to use the methods of counting principles and combinatorics

Part B: Contact Session Plan

Academic Term	First Semester 2021-2022
Course Title	Mathematical Foundations for Data Science
Course No	DSECL ZC416
Lead Instructor	Jyotsana Grover

Course Contents

Contact Hours (Week)	List of Topic Title	Text/Ref Book/external resource
1	Matrices, rank, determinants, solution of linear systems – analytical techniques • Solution of linear systems (A (m x n) x (n x 1) = b (m x 1); A has rank r.) – just a recapitulation • Solution using Gauss elimination with and without pivoting and operations count • LU decomposition methods	T1: Sec 7.3, 20.1, 20.2
2	Numerical solution for linear systems • LU decomposition methods (Continued) • Iterative methods for linear systems	T1: Sec 20.2, 20.3
3	Vector spaces and subspaces, basis and dimensions, Linear transformations and properties • Vector spaces, inner product spaces, properties, LI and LD, bases and dimensions, LT • LT and Rank-Nullity theorem • NS(A), RS(A) and CS(A) – illustration	T1: Sec 7.9

4	Eigenvalues and eigenvectors • Eigenvalues and eigenvectors of special matrices and their properties • Eigenbases and diagonalization • Gerschgorin's Theorem • Power Method	T1: Sec 8.1,8.7, 20.7, 20.8
5-6	Decomposition methods (Eigenvalue, decomposition, QR and SVD) • Gram-Schmidt Orthogonalization procedure • QR decomposition • SVD • Dimensionality reduction	T1: Sec 20.9 Class notes
7 -8	Application of linear algebra in optimization. Modelling linear programming problem and the basics of Simplex algorithm and sensitivity analysis. • Model a LPP in construction of buildings. • Model the currency arbitrage optimization problem. • Work out the graphical method of solution in the case of 2 variable case • Simplex method for simple cases • Outline how Gauss Jordan produces the inverse matrix.	Class notes
	Mid-Semester exam; all the topics covered above would be there for the test.	
9	Properties of functions • Continuous functions on closed intervals, differentiation (1d case) • Taylor series expansion • Maxima and minima • Integral properties (cdf and pdf, even and off integrands, integration by parts and so on) for 1d	Class notes
10	Calculus of several variables • Review limits, continuity and differentiability (graphically and algebraically) • Vector calculus and some of the identities • Maxima and minima (unconstrained) • Steepest gradient method • Lagrange multipliers (for more number of constraints)	Class notes

11-12	Induction principle • Recursive definition and structural induction • Mathematical induction • Strong induction	T2: Sec 5.1, 5.2, 5.3,
13-14	Counting Principles • Basics of counting • Pigeonhole principle • Permutations and combinations • Binomial coefficients and identities	T2: Sec 6.1, 6.2, 6.3,6.4
15-16	Advanced counting • Application of recurrence relations • Solving linear recurrence relations • Generating functions	T2: Sec 8.1, 8.2, 8.4

Evaluation Scheme

Legend: EC = Evaluation Component

No	Name	Type	Duration	Weight	Day, Date, Session, Time
1	Assignment 1	Online		10%	To be announced
2	Assignment 2	Online		10%	
3	Quiz 1	Online	*	5%	
4	Quiz 2	Online	*	5%	
5	Quiz 3	Online		5%	
6	Mid-Semester Exam	Open book	120 min	30%	
7	Comprehensive Exam	Open book	120 min	40%	

Important Information

Syllabus for Mid-Semester Test (Open Book): Topics in Weeks 1-8

Syllabus for Comprehensive Exam (Open Book): All topics (in sessions 1 to 16) given in plan of study

Evaluation Guidelines:

- EC-1 consists of two Assignments and two Quizzes (best two out of the three would be taken for grading). Announcements regarding the same will be made in a timely manner.
- For Closed Book tests: No books or reference material of any kind will be permitted. Laptops/Mobiles of any kind are not allowed. Exchange of any material is not allowed.
- For Open Book exams: Use of prescribed and reference text books, in original (not photocopies) is permitted. Class notes/slides as reference material in filed or bound form is permitted. However, loose sheets of paper will not be allowed. Use of calculators is permitted in all exams. Laptops/Mobiles of any kind are not allowed. Exchange of any material is not allowed.
- If a student is unable to appear for the Regular Test/Exam due to genuine exigencies, the student should follow the procedure to apply for the Make-Up Test/Exam. The genuineness of the reason for absence in the Regular Exam shall be assessed prior to giving permission to appear for the Make-up Exam.

  It shall be the responsibility of the individual student to be regular in maintaining the self-study schedule as given in the course handout, attend the lectures, and take all the prescribed evaluation components such as Assignment/Quiz, Mid-Semester Test and Comprehensive Exam according to the evaluation scheme provided in the handout.