Homework 2

Tongji University 2022 Class Computer Science and Technology College Software Engineering Major Machine Intelligence Direction Computer Vision Course Assignment

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Homogeneous Coordinates of the Point at Infinity

First, convert the line equation into the homogeneous equation form in the projective plane:

$$X - 3Y + 4Z = 0$$

The point at infinity satisfies $Z = 0$, so substitute $Z = 0$ into the equation to get:

$$X - 3Y = 0 \quad \Rightarrow \quad X = 3Y$$

Therefore, the homogeneous coordinates of the point at infinity are:

$$[3Y, Y, 0]^T = [3, 1, 0]^T$$

In conclusion, the homogeneous coordinates of the point at infinity for the line $x - 3y + 4 = 0$ are:

$$[3, 1, 0]^T$$

Jacobian Matrix of Distortion Mapping

In the normalized retinal plane, assume p_n is an ideal projection point without considering distortion. If distortion is considered,$p_n = (x, y)^T$ is mapped to $p_d = (x_d, y_d)^T$, with the relationship represented by the following equations:

$$\begin{cases} x_d = x(1 + k_r r^2 + k_y r^4) + 2\rho_1 xy + \rho_2(r^2 + 2x^2) + xk_r r^6 \\ y_d = y(1 + k_r r^2 + k_y r^4) + 2\rho_2 xy + \rho_1(r^2 + 2y^2) + yk_r r^6 \end{cases}$$

where $r^2 = x^2 + y^2$。

To perform nonlinear optimization in the camera calibration process, we need to calculate the Jacobian matrix of p_d with respect to p_n

$$\frac{dp_d}{dp_n} = \begin{bmatrix} \frac{\partial x_d}{\partial x} & \frac{\partial x_d}{\partial y} \\ \frac{\partial y_d}{\partial x} & \frac{\partial y_d}{\partial y} \end{bmatrix}$$

After detailed derivation, the partial derivatives are as follows:

$$\frac{\partial x_d}{\partial x} = 1 + k_r r^2 + k_y r^4 + 2k_r x^2 + 4k_y r^2 x^2 + 2\rho_1 y + 6\rho_2 x + k_r r^6 + 6k_r x^2 r^4$$

$$\frac{\partial x_d}{\partial y} = 2k_r xy + 4k_y r^2 xy + 2\rho_1 x + 2\rho_2 y + 6k_r xyr^4$$

$$\frac{\partial y_d}{\partial x} = 2k_r xy + 4k_y r^2 xy + 2\rho_2 y + 2\rho_1 x + 6k_r xyr^4$$

$$\frac{\partial y_d}{\partial y} = 1 + k_r r^2 + k_y r^4 + 2k_r y^2 + 4k_y r^2 y^2 + 2\rho_2 x + 2\rho_1 y + k_r r^6 + 6k_r y^2 r^4$$

Therefore, the Jacobian matrix is:

$$\frac{dp_d}{dp_n} = \begin{bmatrix} 1 + k_r r^2 + k_y r^4 + 2k_r x^2 + 4k_y r^2 x^2 + 2\rho_1 y + 6\rho_2 x + k_r r^6 + 6k_r x^2 r^4 & 2k_r xy + 4k_y r^2 xy + 2\rho_1 x + 2\rho_2 y + 6k_r xyr^4 \\ 2k_r xy + 4k_y r^2 xy + 2\rho_2 y + 2\rho_1 x + 6k_r xyr^4 & 1 + k_r r^2 + k_y r^4 + 2k_r y^2 + 4k_y r^2 y^2 + 2\rho_2 x + 2\rho_1 y + k_r r^6

Jacobian Matrix of Rotation Matrix

1. Definition of Rotation Matrix

According to Rodrigues' formula, the rotation matrix R can be expressed as:

$$R = \beta I + \gamma n n^T + \alpha [n]_\times$$

where:

- $\beta = \cos\theta$
- $\gamma = 1 - \cos\theta$
- $\alpha = \sin\theta$
- I is the identity matrix
- $[n]_\times$ is the skew-symmetric matrix of n

2. Vectorization of Rotation Matrix

The vectorized form of the rotation matrix R is:

$$r = (r_{11}, r_{12}, r_{13}, r_{21}, r_{22}, r_{23}, r_{31}, r_{32}, r_{33})^T$$

3. Calculation of Jacobian Matrix

We need to calculate the Jacobian matrix $\frac{dr}{dd^T}$ of r with respect to d , where $d = \theta n$ and n is a unit vector.

First, calculate the partial derivatives of r_{ij} with respect to θ and n , then use the chain rule to find the partial derivatives of r_{ij} with respect to d.

3.1 Partial Derivatives of r_{ij} with respect to θ

For example:

$$\frac{\partial r_{11}}{\partial \theta} = -\sin\theta + (1 - \cos\theta) \cdot 2n_1^2$$

3.2 Partial Derivatives of r_{ij} with respect to n

For example:

$$\frac{\partial r_{11}}{\partial n_1} = 2(1 - \cos\theta) n_1$$

3.3 Partial Derivatives of θ and n with respect to d

Since $d = \theta n$ and n is a unit vector, therefore:

$$\theta = ||d||, \quad n = \frac{d}{||d||}$$

So:

$$\frac{\partial \theta}{\partial d_k} = \frac{d_k}{\theta}$$

$$\frac{\partial n_i}{\partial d_k} = \frac{\delta_{ik}\theta - n_k d_i}{\theta^2}$$

3.4 Combining Partial Derivatives

Using the chain rule, combine the above partial derivatives:

$$\frac{\partial r_{ij}}{\partial d_k} = \frac{\partial r_{ij}}{\partial \theta}\frac{\partial \theta}{\partial d_k} + \sum_{m=1}^{3} \frac{\partial r_{ij}}{\partial n_m}\frac{\partial n_m}{\partial d_k}$$

4. Conclusion

Through the above steps, we can calculate the partial derivatives of each r_{ij} with respect to d_k and finally construct the Jacobian matrix $\frac{dr}{dd^T}$. This process requires careful symbolic computation to ensure each step is accurate.

Bird's Eye View Generation

Environment: Windows 11

Platform: PyCharm Professional 2024.1.4

Python version: 3.12.4

Python libraries: numpy opencv-Python

Code location: ../Project1

Results are as follows:

相机标定参数

Reprojection Error

$ret = 1.3526290383110415$

Intrinsic Matrix

$$\text{mtx} = \begin{bmatrix} 1.06408820 \times 10^3 & 0.00000000 \times 10^0 & 6.97624043 \times 10^2 \\ 0.00000000 \times 10^0 & 1.05884544 \times 10^3 & 3.67820618 \times 10^2 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 1.00000000 \times 10^0 \end{bmatrix}$$

Distortion Coefficients

$$\text{dist} = \begin{bmatrix} 2.19183009 \times 10^{-1} & -9.71999184 \times 10^{-1} & 8.92226849 \times 10^{-4} & -7.72790370 \times 10^{-3} & 9.61389806 \times 10^{-1} \end{bmatrix}$$

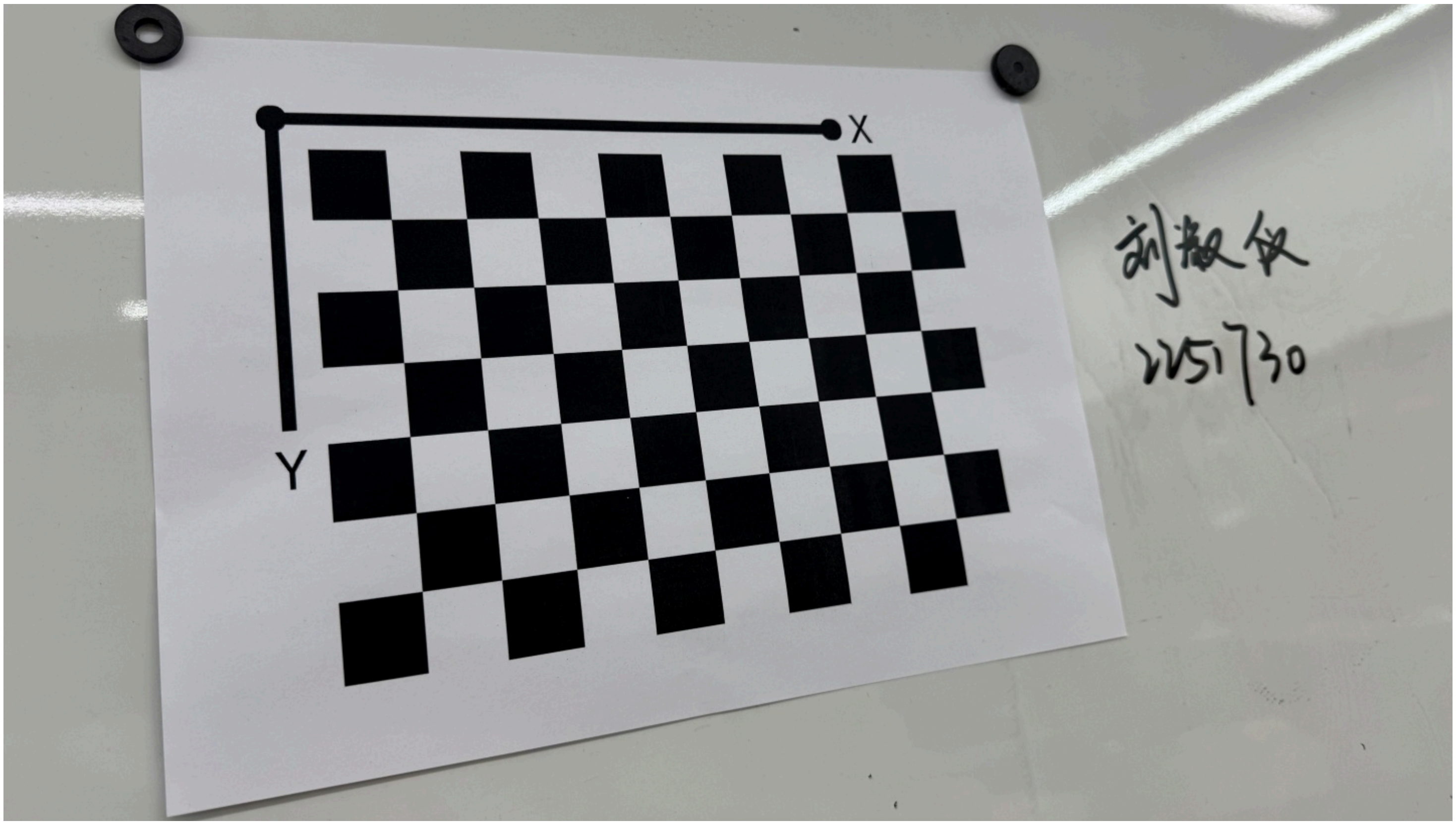
Rotation Vectors

$$\text{rvecs} = \left(\begin{bmatrix} -0.15060814 \\ 0.68259582 \\ -1.42548071 \end{bmatrix}, \begin{bmatrix} -0.62169292 \\ 0.02379807 \\ -1.54327886 \end{bmatrix}, \begin{bmatrix} -0.48797313 \\ 0.57757602 \\ -1.47210871 \end{bmatrix}, \begin{bmatrix} -0.00730448 \\ 0.19318482 \\ -1.58085337 \end{bmatrix}, \begin{bmatrix} 0.42666152 \\ 0.16677968 \\ -1.6343885 \end{bmatrix}, \begin{bmatrix} 0.42971655 \\ -0.20436555 \\ -1.63009947 \end{bmatrix}, \begin{bmatrix} 0.12150479 \\ -0.42919655 \\ -1.65266561 \end{bmatrix}\right.$$

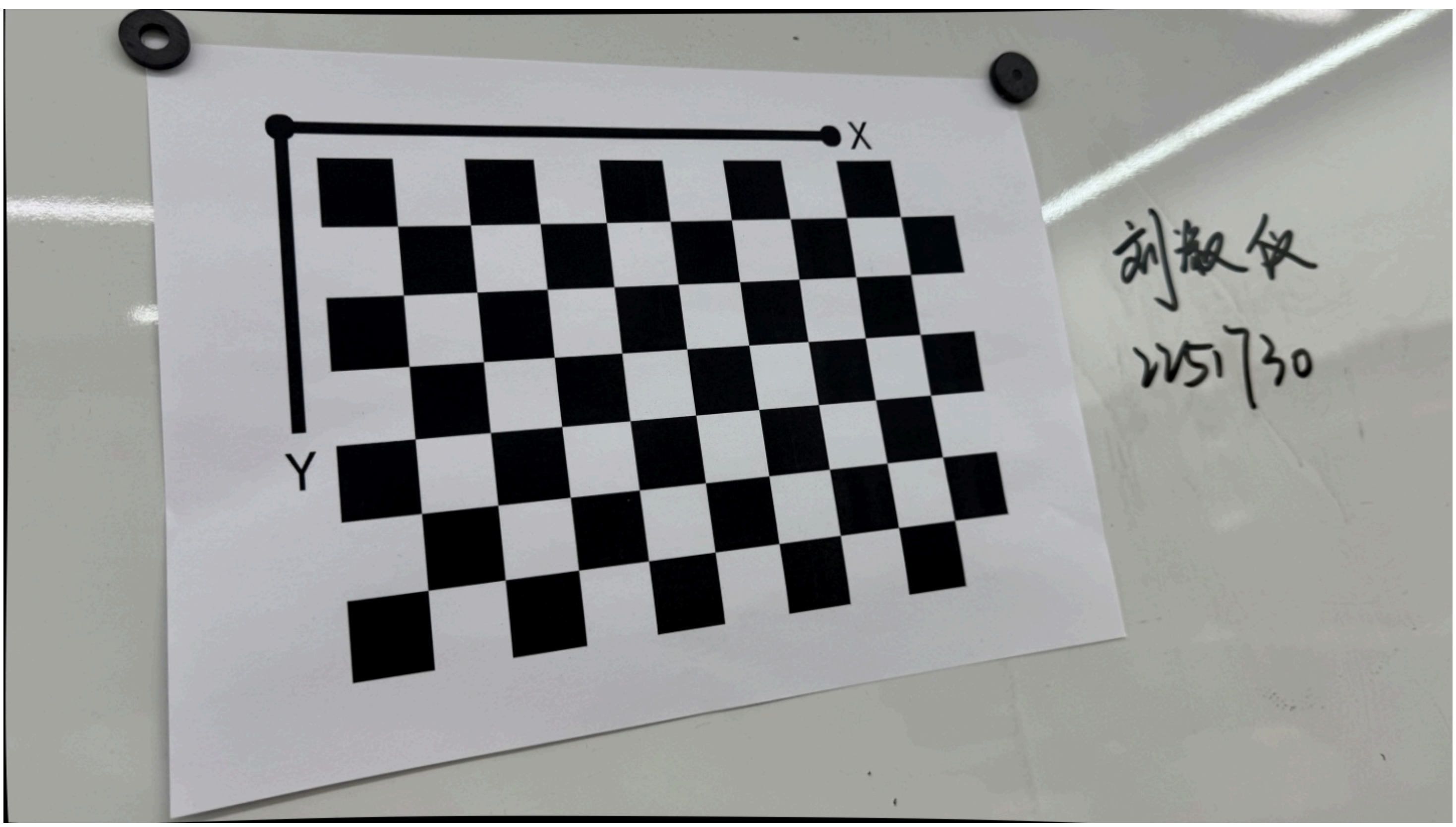
Translation Vectors

$$\text{tvecs} = \left(\begin{bmatrix} -0.16131816 \\ 0.04315679 \\ 0.4099573 \end{bmatrix}, \begin{bmatrix} -0.1625625 \\ 0.07084516 \\ 0.35582155 \end{bmatrix}, \begin{bmatrix} -0.17356461 \\ 0.05434441 \\ 0.38696302 \end{bmatrix}, \begin{bmatrix} -0.13625178 \\ 0.07758926 \\ 0.37540559 \end{bmatrix}, \begin{bmatrix} -0.08562375 \\ 0.08729008 \\ 0.34406331 \end{bmatrix}, \begin{bmatrix} -0.05401651 \\ 0.05972098 \\ 0.28404526 \end{bmatrix}, \begin{bmatrix} -0.07216266 \\ 0.05521112 \\ 0.27942144 \end{bmatrix}\right.$$

Original Image



Undistorted Image



Bird's Eye View