Oscillations & Waves Formula Sheet

1. Simple Harmonic Motion (SHM)

Basic Equations

- **Displacement**:
$$x(t) = A\cos(\omega t + \phi)$$
- **Velocity**:
$$v(t) = -A\omega\sin(\omega t + \phi)$$
- **Acceleration**:
$$a(t) = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x(t)$$
- **Angular Frequency**:
$$\omega = \sqrt{\frac{k}{m}} \quad \text{(Spring-mass)}$$
$$\omega = \sqrt{\frac{g}{L}} \quad \text{(Simple pendulum)}$$

Energy in SHM

- **Potential Energy**:
$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$
- **Kinetic Energy**:
$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2\sin^2(\omega t + \phi)$$
- **Total Energy**:
$$E = U + K = \frac{1}{2}kA^2$$

2. Damped Oscillations

Equation of Motion

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

- **Damping Types**:
  - **Underdamped** ($b^2 < 4mk$):
$$x(t) = Ae^{-\gamma t}\cos(\omega_d t + \phi), \quad \omega_d = \sqrt{\omega_0^2 - \gamma^2}$$
  - **Critically damped** ($b^2 = 4mk$):
$$x(t) = (A + Bt)e^{-\gamma t}$$
  - **Overdamped** ($b^2 > 4mk$):
$$x(t) = Ae^{-\gamma_1 t} + Be^{-\gamma_2 t}$$
- **Damping Coefficient**:
$$\gamma = \frac{b}{2m}$$

3. Forced Oscillations & Resonance

Driven Harmonic Oscillator

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

- **Steady-State Solution**:

$$x(t) = A\cos(\omega t - \delta)$$

  - **Amplitude**:

$$A = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}}$$

  - **Phase Lag**:

$$\tan\delta = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$

- **Resonance Frequency**:

$$\omega_r = \sqrt{\omega_0^2 - 2\gamma^2}$$

4. Wave Motion

Wave Equation (1D)

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$$

- **General Solution**:

$$y(x,t) = f(x - vt) + g(x + vt)$$

Harmonic Waves

- **Displacement**:

$$y(x,t) = A\sin(kx - \omega t + \phi)$$

- **Wave Number (k)**:

$$k = \frac{2\pi}{\lambda}$$

- **Wave Speed**:

$$v = \frac{\omega}{k} = \lambda f$$

5. Standing Waves & Normal Modes

Fixed Ends (String)

- **Wavelengths**:

$$\lambda_n = \frac{2L}{n}, \quad n = 1, 2, 3, \ldots$$

- **Frequencies**:

$$f_n = \frac{nv}{2L}$$

Open/Closed Pipes

- **Open Pipe**:

$$f_n = \frac{nv}{2L}, \quad n = 1, 2, 3, \ldots$$

- **Closed Pipe**:

$$f_n = \frac{nv}{4L}, \quad n = 1, 3, 5, \ldots$$

6. Energy & Power in Waves

- **Energy Density**:

$$u = \frac{1}{2}\mu\omega^2 A^2$$

- **Power Transmitted**:

$$P = \frac{1}{2}\mu v \omega^2 A^2$$

where μ = mass per unit length.

7. Superposition & Interference

Principle of Superposition

$$y_{\text{total}}(x,t) = y_1(x,t) + y_2(x,t)$$

Interference

- **Constructive**:

$$\Delta\phi = 2n\pi \quad \text{or} \quad \Delta x = n\lambda$$

- **Destructive**:

$$\Delta\phi = (2n+1)\pi \quad \text{or} \quad \Delta x = \left(n + \frac{1}{2}\right)\lambda$$

8. Doppler Effect

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$$

- **Sign Convention**:
  - v_o: + if observer moves toward source
  - v_s: + if source moves away from observer