1. Attempt **all** parts of this question.

(i) A body of mass 1 kg and initial velocity 10 ms^{-1} is sliding on a horizontal surface. If the coefficient of kinetic friction between the body and the surface is 0.5, then find the work done by friction when the body has traversed a distance of 5 m along the surface. (3)

(ii) Consider circular orbits in a central force potential $U(r) = -kr^{-n}$ where $k > 0$ and $0 < n < 2$. If the time period of circular orbit of radius R is T_1 and that of radius 2R is T_2, find the value of $\frac{T_2}{T_1}$. (3)

(iii) A nucleus initially at rest decays radioactively by emitting two particles, an electron with momentum 1.2×10^{-22} kg ms^{-1} and a neutrino at right angle to the electron with momentum 6.4×10^{-23} kg ms^{-1}. Find the direction and momentum of the recoiled nucleus. (3)

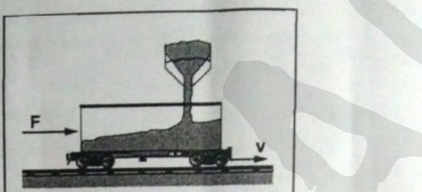
(iv) Three masses each of 2kg are situated at the vertices of an equilateral triangle whose sides measure 0.1 m each. Calculate the moment of inertia of the system and its radius of gyration with respect to an axis perpendicular to the plane determined by the triangle and passing through one of its vertices. (3)

(v) Calculate the speed of a 2 MeV electron, given that the rest mass of an electron is 0.5 MeV. (3)

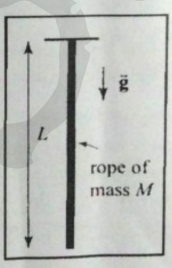
(vi) A bucket containing water is tied to one end of a rope and rotated about the other end in a vertical circle of radius 0.75 m. Find the minimum speed at the top to ensure that no water spills out. (3)

2. (i) Find the centre of mass of a right angled triangular sheet of mass M, base b, height h and small thickness t. (6)

(ii) Sand falls from a stationary hopper continuously on a freight car which is moving with uniform velocity v. The sand falls at the rate $\frac{dm}{dt}$. Find the force required to make the freight move with the same constant velocity v. Also prove that the power due to this force is twice the rate of increase of kinetic energy of the system. (6)

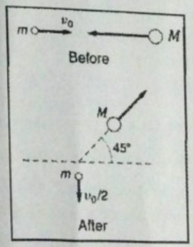


(iii) A rope of mass M and length L is suspended from a ceiling as shown in the figure. Find the tension in the rope as a function of the distance from the ceiling and hence show that the tension changes at a constant rate along the length of the rope. What is the tension in the rope at the upper end where the rope is fixed to the ceiling. (6)



3. (i) A particle of mass m moves in one dimension along the positive x axis. It is acted on by a constant force directed towards the origin with magnitude B and an inverse square law repulsive force with magnitude $\frac{A}{x^2}$. Find the potential energy function U(x) and the equilibrium position x_0 of the particle. What is the frequency of small oscillations about x_0? (6)

(ii) A particle of mass m and initial velocity v_0 collides elastically with a particle of unknown mass M coming from the opposite direction as shown in the diagram. After the collision, m has velocity $\frac{v_0}{2}$ at right angles to the incident direction, and M moves off in the direction shown in the figure. Find the ratio M/m. (6)



(iii) A particle slides back and forth on a frictionless track whose height as a function of horizontal position x is given by $y = bx^2$ where $b = 0.92$ m^{-1}. If the particle's maximum speed is 8.5 ms^{-1}, find the turning points of its motion. (6)

4. (i) Show that the total angular momentum of a system of particles about an axis of rotation is given by the relation $\vec{J} = \vec{J}_0 + \vec{J}_{c.m.}$, where $\vec{J}_0$ is the angular momentum of the centre of mass of the system about the given axis and $\vec{J}_{c.m.}$ is the angular momentum of the system about an axis parallel to the given axis passing through the centre of mass. (6)

(ii) A cylinder of radius R and mass M rolls without slipping down a plane inclined at angle θ. The coefficient of friction is μ. Determine the maximum value of $\theta = \theta_c$ for the cylinder to roll without slipping? What will be the acceleration of the cylinder as it rolls down the incline plane. (6)

(iii) A particle of mass 2 kg moves along a straight line given by the equation $y = \frac{x}{\sqrt{3}} + 3$, with a constant speed of 4ms^{-1}. Find the angular momentum of the particle about the origin. (6)

5. (i) Consider two inertial observers O and O′ moving with relative speed v with respect to each other. Their coordinate axes (Cartesian) are parallel to each other, with the x direction being the direction of relative motion such that observer O′ is observed to move with velocity v along the positive x direction of observer O. The observers observe the motion of a particle. Starting with Lorentz transformations, determine the relation between the velocity of the particle as measured by the two observers. (6)

(ii) What are space-like, time-like and light-like events? In a certain inertial frame, two firecrackers go off at the same instant of time, separated by distance *l*. Are these two events spacelike, time-like or light-like? Explain. (6)

(iii) A bug crawls towards the rim with a constant speed v_0 along the spoke of a wheel that is rotating with constant angular velocity ω about a vertical axis. Find all the apparent forces acting on the bug. Find how far the bug can crawl before it starts to slip, given that the coefficient of static friction between the bug and the spoke is μ_s. (6)

6. (i) A clock is observed to move in an inertial frame O with velocity v. Two events take place at the location of the clock, with the duration between these events measured to be Δτ by the clock. Between these two events, the clock moves past two identical clocks at rest in frame O. If the duration between these events is Δt as measured by these clocks, starting with Lorentz transformations, deduce a relationship between Δτ and Δt. (6)

(ii) Two spaceships approach each other, each moving with the same speed as measured by a stationary observer on the Earth. Their relative speed is 0.7c. Determine the velocities of each spaceship as measured by the stationary observer on Earth. (6)

(iii) A simple pendulum consisting of a mass m suspended by a string of negligible mass and length *l* is hanging from the ceiling of a tram car. The car is accelerating relative to the road with uniform acceleration of magnitude a. Analyzing the dynamics in the frame of reference of the car, derive an expression for the angle (relative to the vertical direction) at which the pendulum will be in equilibrium. What will be the time period of small oscillations of the pendulum about this equilibrium position? (6)

01/03/023