

Mathematics Formula Sheet

1. Sets & Relations

- **Union:** $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- **Intersection:** $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- **Complement:** $A' = \{x \mid x \notin A\}$
- **De Morgans Laws:**
 - $(A \cup B)' = A' \cap B'$
 - $(A \cap B)' = A' \cup B'$
- **Cartesian Product:** $A \times B = \{(a, b) \mid a \in A, b \in B\}$

2. Functions

- **Domain & Range:** $f : A \rightarrow B$
- **Injective (1-1):** $f(x_1) = f(x_2) \implies x_1 = x_2$
- **Surjective (Onto):** $\forall y \in B, \exists x \in A \text{ s.t. } f(x) = y$
- **Bijjective:** Both injective and surjective.
- **Inverse Function:** $f^{-1}(y) = x \iff f(x) = y$ (if bijective).

3. Matrices & Determinants

- **Matrix Addition:** $(A + B)_{ij} = A_{ij} + B_{ij}$
- **Matrix Multiplication:** $(AB)_{ij} = \sum_k A_{ik} B_{kj}$
- **Transpose:** $(A^T)_{ij} = A_{ji}$
- **Determinant (2x2):**

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- **Determinant (3x3):**

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

- **Inverse (2x2):**

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad (\text{if } \det(A) \neq 0)$$

4. Complex Numbers

- **Standard Form:** $z = a + ib$
- **Modulus:** $|z| = \sqrt{a^2 + b^2}$
- **Conjugate:** $\bar{z} = a - ib$
- **Polar Form:** $z = r(\cos \theta + i \sin \theta)$
- **Eulers Formula:** $e^{i\theta} = \cos \theta + i \sin \theta$
- **Multiplication:** $|z_1 z_2| = |z_1| |z_2|$
- **De Moivres Theorem:**

$$(\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta)$$

5. Polynomials

- **Quadratic Formula:**

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Sum & Product of Roots:**

- Sum: $\alpha + \beta = -\frac{b}{a}$
- Product: $\alpha\beta = \frac{c}{a}$

- **Remainder Theorem:** If $P(x)$ is divided by $(x - a)$, remainder = $P(a)$.
- **Factor Theorem:** $(x - a)$ is a factor of $P(x)$ iff $P(a) = 0$.

6. Permutations & Combinations

- **Factorial:** $n! = n \times (n - 1)!$
- **Permutation:** ${}^n P_r = \frac{n!}{(n-r)!}$
- **Combination:** ${}^n C_r = \frac{n!}{r!(n-r)!}$

7. Binomial Theorem

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

where $\binom{n}{k} = {}^n C_k$.