

1. (a) The total sales (S) in thousands of rupees of a firm selling two products X and Y is given by the relationship,  $S = a + bX + cY$ . Data for the first three months is given as under :

Month	Total Sales	X	Y
1	18	1	3
2	10	2	1
3	26	5	3

Using determinant method, determine the sales in the month when it sells 10 units of X and 20 units of Y.

**OR**

- (a) Mr. Y invested ₹50,000 divided into three investments. Part of the money is invested in a savings account with an annual return of 4%, partly in 9% annual yield bonds and the remainder in business. In 2019, when he lost 2% of the money that he invested in the business, his net income from all the three investments is ₹1,500. If he invested ₹10,000 more in the business than in the savings account, how much was invested in each. Use the matrix algebra. (6)

- (b) Suppose the inter-industry flow of the products of two industries is given as under :

Production Sector	Consumption Sector		Domestic Demand	Total Output
	X	Y		
X	20	90	90	200
Y	30	15	105	150
Labour (hours)	50	30		

- (i) Determine the technology matrix and test Simon-Hawkins conditions for the viability of the system.

- (ii) If the domestic demand changes to 144 and 72 units respectively, what should be the gross output of each sector in order to meet the demands?
- (iii) If the total labour available is 100 hours, is the solution feasible?
- (iv) Find the value added in each sector.
- (v) Find the equilibrium prices, if wage rate is ₹150. (12)

**OR**

(b) You are given the following transaction matrix for a two-sector economy :

Purchase Sector	Sales Sector		Final Demand	Gross Output
	I	II		
I	4	3	13	20
II	5	4	3	12
Primary Input	11	5		

- (i) Determine the technology matrix and test Simon-Hawkins conditions for the viability of the system.

- (ii) Rewrite the new transaction matrix when the final demand for the output of sector I increases to 23 units. (12)

2. (a) A publishing house purchases 9000 units of a particular item every year at a unit cost of ₹20. The ordering cost per order is ₹40 and the storage cost is 10% of the value of average inventory. Using calculus, find the optimal order quantity and the minimum total cost. Would you like to accept an offer by a supplier, who proposes to allow a discount of 10% on order size of 1000 units? (9)

**OR**

- (a) The demand function for a certain commodity is  $5q = 400 - 2p$  ( $0 \leq p \leq 200$ ).

- (i) Determine where the demand is elastic, inelastic and of unit elasticity with respect to price.
- (ii) Use the result of part (i) to determine the intervals of increase and decrease of the revenue function and the price at which revenue is maximized. (9)

- (b) A cable service operator has a subscriber base of 1000 households, from each of whom he charges ₹500 per month. In order to increase his subscriber base, he proposes to decrease the rate of monthly subscriptions. His assessment is that for every decrease of ₹50, 200 additional households would seek connection of cable service. You are required to find out the decrease in service charge that enables him to maximize his total revenue.

(9)

OR

- (b) The cost of fuel in running an engine is proportional to the square of the speed in kms per hour, and is ₹48 per hour when the speed is 16 kms. Other costs amount to ₹300 per hour. Find the most economical speed.

(9)

3. (a) Assume that a monopolist sells his product in two separate markets and the demand functions are:  $p_1 = 12 - x_1$ ,  $p_2 = 20 - 3x_2$ . The total cost function for his product is given by:  $C = 3 + 2x$ , where  $x = x_1 + x_2$ . Determine the prices that the monopolist will charge and the quantities he will sell in each of the markets to maximize his profit.

(9)



**OR**

(a) A production function is given by  $Q = AL^{2/7}K^{2/7}$ , where L is labour and K is capital.

(i) What is nature of returns to scale?

(ii) What is nature of returns to inputs?

(iii) What is the total reward of labour and capital if each factor is paid a price equal to its marginal product? Show that total product is not exhausted. (9)

(b) The elasticity of demand of a commodity with

respect to price is calculated to be  $\frac{p}{(p-1)(p-2)}$ ,

where p is price. Find the demand function if it is known that the quantity demanded is 4 units at

$p = 3$ . (9)

**OR**

(b) The demand and supply laws for a commodity are  $p_d = 56 - x^2$  and  $p_s = 8 + (x^2/3)$ . Find the consumer surplus and produce surplus at equilibrium price.

(9)

4. Attempt **any three** questions. Each question carries 6 marks.

(a) A debt of ₹6000 due 8 years from now and ₹6000 due 12 years from now is to be repaid by a payment of ₹3000 in 3 years and a payment of ₹3000 in 5 years and a final payment at the end of 6 years if the rate of interest is 9% compounded annually, how much is the final payment.

(b) Machine A costs ₹8,000 and has a useful life of 7 years. Machine B costs ₹6,000 and has a useful life of 5 years. Suppose Machine A generates an annual labour savings of ₹2,000 while Machine B generates an annual labour savings of ₹1,800. Assuming that the time value of money of the corporation is 10% per annum, which machine is preferable?

(c) Mr. Harshdeep makes 8 annual payments of ₹1,00,000 each, the first being made at the end of 5 years. Find the amount of the annuity if the money is worth 7% compounded annually.

(d) A machine depreciates at the rate of 8% of its value at the beginning of a year. The machine was purchased for ₹1,00,000 and the scrap value realized when sold was ₹43,440. Find the number of years the machine was used.

5. Given below are the objective function, the constraints and the final simplex tableau for a linear programming product-mix problem :

$$\text{Max. } Z = 15x_1 + 12x_2 + 11x_3$$

Subject to Constraints:

$$3x_1 + 4x_2 + 5x_3 \leq 63 \text{ (hours, Dept. A)}$$

$$4x_1 + x_2 + 4x_3 \leq 64 \text{ (hours, Dept. B)}$$

$$x_1 + 4x_2 + 2x_3 \leq 40 \text{ (hours, Dept. C)}$$

$$x_1, x_2, x_3 \geq 0$$



**Final Simplex Table :**

$C_j$	Product Mix	15	12	11	0	0	0	Quantity
		$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	
		0	1	$8/13$	$4/13$	$-3/13$	0	$60/13$
		1	0	$11/13$	$-1/13$	$4/13$	0	$193/13$
		0	0	$-17/13$	$-15/13$	$8/13$	1	$87/13$
	$Z_j$							
	$C_j - Z_j$							

- (i) Complete the table and test whether the solution is optimal or not.
- (ii) Write the optimal product mix and the profit contribution shown by the above solution.
- (iii) Is the solution feasible? Give reason.
- (iv) Does the problem have any alternative solution? If so, show one such solution.
- (v) Indicate the shadow prices of three departments.
- (vi) If the company wishes to expand the production capacity, which of the three departments should be given priority.

(vii) If the company produces 13 units of  $x_3$ , how many units of  $x_1$  and  $x_2$  shall have to be reduced, if any?

(viii) If a customer is prepared to pay higher prices for product  $x_3$ , how much should the price be increased so that the company's profit remains unchanged.

(ix) Indicate whether the solution given in the table is degenerate. If yes, which variable is degenerated? (18)

**OR**

(a) A 24-hour supermarket has the following minimal requirement for sales personnel :

Period	1	2	3	4	5	6
Time of day (24 hr clock)	04-08	08-12	12-16	16-20	20-00	00-04
Minimum no. required	8	18	16	25	8	4

Period 1 follows immediately after period 6. Sales personnel works for 8 consecutive hours, starting at the beginning of one of the six periods. To determine a daily employee worksheet which satisfies the requirement with the least number of personnel, formulate the problem as a linear programming problem. (6)

- (b) The final product of a firm has a requirement that it must weigh exactly 5 kg. The two raw materials used in the manufacture of this product are A with a cost of ₹5 per kg and B with a cost of ₹8 per kg. At least 2 kgs of B and not more than 4 kg of A must be used. How much of each type of raw material should be used for each unit of the final product if the cost is to be minimized? Use simplex method. (12)