

1. (a) Test scores out of 800 of a competitive exam of two group of 15 students each are given below :

Group I	10	10	10	15	35	75	90	95	100	175	420	490	515	515	790
Group II	0	5	5	15	30	45	50	50	50	60	75	110	140	240	330

- (i) Calculate the median, first quartile, third quartile for each group of students.  
(ii) Are there any extreme or mild outliers in both groups of data?

- (b) Twenty cars of distinct colours park in the same parking lot every day. Ten of these cars are Indian-made, while the other ten are foreign-made. The parking lot has exactly twenty spaces all in a row. So, the cars park side by side. However, the drivers have varying schedules so, the position any car might take on a certain day is random.

- (i) In how many different ways can the cars line up?  
(ii) What is the probability that on a given day, the cars will park in such a way that they alternate (no two Indian-made cars are adjacent and no two foreign-made cars are adjacent)? Give an explanation for your answer. (1+4)

2. (a) Find 27% trimmed mean of 2, 4, 6, 7, 11, 21, 81, 90, 105, 121. What would be the trimming percentage if one observation is removed from each end of the dataset? (3+2)

- (b) Tags are attached to the left and right hind legs of a cow in a pasture. Let  $A_1$  be the event that the left leg tag is lost and  $A_2$  the event that the right leg tag is lost. Suppose these two events are independent and  $P(A_1) = P(A_2) = 0.3$ .  
(i) Find the probability that at least one leg tag is lost.  
(ii) Find the probability that exactly one tag is lost, given that at least one tag is lost. (2+3)

3. (a) Ganesh asks his neighbour to water a flowering plant while he is on vacation. Without water it will die with probability 0.8; with water it will die with probability 0.15. He is 90 percent certain that his neighbour will remember to water the plant.  
(i) What is the probability that the plant will be alive when Ganesh returns?  
(ii) If it is dead, what is the probability that his neighbour forgot to water it? (3+2)

- (b) A sample of automobiles was selected, and each was subjected to a 5-mph crash test. Denoting a car with no visible damage by S (for success) and a car with such damage by F, results were as follows :

S S F S S S F F S S

What is the value of the sample proportion of successes? Suppose it is decided to include 15 more cars in the experiment. How many of these would have to be successes so that sample proportion of successes is 0.80 for the entire sample of 25 cars? (2+3)

- (i) Airlines sometimes overbook flights. Suppose that for a plane with 50 seats, 55 passengers have tickets. Define the random variable Y as the number of ticketed passengers who actually show up for the flight. The probability mass function of Y appears in the following table.

Y	45	46	47	48	49	50	51	52	53	54	55
P(Y)	0.05	0.1	0.12	0.14	0.25	0.17	0.06	0.05	0.03	0.02	0.01

- (i) What is the probability that the flight will accommodate all ticketed passengers who show up?  
(ii) What is the probability that not all ticketed passengers who show up can be accommodated?  
(iii) If you are the first person on the standby list (which means you will be the first one to get on the plane if there are any seats available after all ticketed passengers have been accommodated), what is the probability that you will be able to take the flight? What is this probability if you are the third person on the standby list? (1+2+3)

- (b) Let X have the PMF as below :

x	1	2	3	4
P(x)	0.4	0.3	0.1	0.2

In a win-win game, the player will win a monetary prize, but has to decide between the fixed price of Rs. 1000/E(X) and the random price of Rs. 1000/X, where the random variable X has the PMF given above. Which choice would you recommend the player to make? (4)

5. A random variable Z, which represents the weight (in grams) of an article, has density function,

$$f(z) = \begin{cases} (z-8) & \text{for } 8 \leq z \leq 9 \\ (10-z) & \text{for } 9 < z \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Calculate the mean of the random variable Z.  
(ii) The manufacturer sells the article for a fixed price of Rs. 2. She guarantees to refund the purchase money to any customer who finds the weight of his/her article to be less than 8.25 grams. Her cost of production is related to the weight of the article by the relation  $z/15 + 0.35$ . Find the expected profit per article. (4+6)

6. (a) In a batch of 10 of a product three are known to be defective. A simple random sample of size  $n = 3$  is drawn from a batch of these product items. Find the PMF and the CDF of the random variable X = number of defective items in the sample. (5)  
(b) Find an expression for median and 90<sup>th</sup> percentile of the random variable X if

$$f(x) = \begin{cases} \frac{1}{B-A} & A \leq x \leq B \\ 0 & \text{otherwise} \end{cases}$$

7. (a) Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. That is, they arrive at 7, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that he waits at least 12 minutes for a bus. (5)  
(b) Let number of claims handled daily by an insurance company is Poisson random variable. If the average number of claims handled daily by an insurance company is 5,  
(i) What proportion of days have less than 3 claims?  
(ii) What is the probability that there will be 4 claims in exactly 3 of the next 5 days?

Assume that the number of claims on different days is independent. (5)

- (c) The components of a 6-component system are to be randomly chosen from a bin of 20 used components. The resulting system will be functional if at least 4 of its 6 components are in working condition. If 15 of the 20 components in the bin are in working condition, what is the probability that the resulting system will be functional? What is the expected number of components working? (5)

- (a) Suppose that a person taking Vitamin C supplements contracts an average of three colds per year and that this average increases to five colds per year for persons not taking Vitamin C supplements. Suppose further that the number of colds a person contracts in a year is a Poisson random variable.

- (i) Find the probability of no more than two colds  
• for a person taking supplements  
• person not taking supplements.

- (ii) Suppose 70% of the population takes Vitamin C supplements. Find the probability that a randomly selected person will have no more than two colds in a given year. (6)

- (b) In a certain country, 30% of the adult male population smoke regularly. In a random sample of 750 adults, what is the approximate probability that

- (i) fewer than 200 are smokers?  
(ii) 240 or more are smokers?

Which distribution will you use and why? (5)

- (c) The lifespan of a car battery averages six years. Suppose the battery lifespan follows an exponential distribution.

- (i) Find the probability that a randomly selected car battery will last more than four years.

- (ii) Suppose a three-year-old battery is still going strong. Find the probability the battery will last an additional five years. (4)

- (a) Suppose 70% of all purchases in a certain store are made with credit card. Let X denote the number of credit card uses in the next 10 purchases. Find

- (i) Expected value and Variance of X.

- (ii)  $P(5 \leq X \leq 8)$ .

- (b) Suppose the monthly death rate due to a certain disease in a certain county is 1 per 100,000 people. Give an approximation to the probability that in a city of 500,000 in this county there will be no more than six deaths due to disease in the next month. (5)

- (c) The mean and standard deviation of SAT math exam scores were 527 and 120, respectively, in 2022. Assume that the scores are normally distributed and answer the following questions.

- (i) Find the 30th percentile.

- (ii) If your score is 700, what percentage of the students got higher scores than you in 2022? (5)

10. Let X and Y denote the number of four-wheeler and two-wheeler carried on a trip. Suppose the joint distribution of X & Y is given in the table :

$x \backslash y \rightarrow$	2	4	6
1	1/5	0	1/5
2	0	1/5	0
3	1/5	0	1/5

- (i) Are X & Y independent? Explain.

- (ii) Find the conditional distribution of Y given X=3. (2,3)

11. Suppose that X & Y are two Continuous Random variables, given Joint PDF

$$f(x,y) = \begin{cases} c(1-x)(2-y) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Compute c.

- (ii) Find the marginal PDFs of X and Y. (2,3)

12. If X & Y are two independent random variables where  $P_X(1)=1/2$ ,  $P_X(2)=1/2$ ,  $P_Y(1)=5/12$ ,  $P_Y(2)=1/3$ ,  $P_Y(3)=1/4$  then

- (i) Display the joint PMF of (X,Y) in a joint probability table.

- (ii) Find  $P(x+y \leq 3)$ . (3,2)

13. The number of units serviced in a week at a certain service facility is a random variable having mean 50 and variance 16, Find an approximation to the probability that the total number of units to be serviced at the facility over the next 36 weeks is between 1728 and 1872. (5)

14. The average salary of Eco Hons graduated students is Rs. 53,600 with a SD of Rs. 3200

- (i) What is the probability that the average salary of 36 students is between Rs. 52,000 and Rs. 55,000?

- (ii) If the sample size has been reduced to 12 students, can we still calculate the required probability in part (i) using the same method? Explain. (3,2)