1. Attempt all questions. Each question carries equal marks. (3×5)

(a) Define simple harmonic motion (SHM). Prove that the principle of superposition holds in case of the Homogeneous Linear equations.

(b) Consider a mass $\boldsymbol{m}$ attached with two identical massless springs having spring constant $\boldsymbol{k}$, relaxed length $\boldsymbol{a_0}$ and equilibrium length $\boldsymbol{a}$ each as shown in Fig. 1. Show that the frequency of transverse oscillation (under slinky approximation) is:$\omega = \sqrt{\frac{2k}{m}}$.

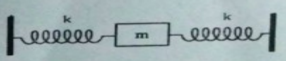

Fig. 1

(c) The equation of a damped harmonic oscillation is given by,

$$8\left(\frac{d^2y}{dt^2}\right) + 24\left(\frac{dy}{dt}\right) + 48\,y = 0$$

Find the frequency of the damped oscillations.

(d) A damped harmonic oscillator has the amplitude of 20 cm. It reduces to 2 cm after 100 oscillations each of the time period 4.6 s. Calculate its logarithmic damping constant. Compute the number of oscillations in which the amplitude drops by 50%.

(e) Distinguish between stationary and progressive waves.

2. (a) A uniform spring of length $\boldsymbol{l}$ has force constant $\boldsymbol{k}$. The spring is cut into two pieces of unstressed lengths $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$, where $(\boldsymbol{l_1}/\boldsymbol{l_2}) = (\boldsymbol{n_1}/\boldsymbol{n_2})$. Express the force constants k_1 and k_2 of the two pieces in terms of $\boldsymbol{k}$, $\boldsymbol{n_1}$ and $\boldsymbol{n_2}$. (5)

(b) Deduce an expression for the energy of a harmonic oscillator of mass $\boldsymbol{m}$, amplitude $\boldsymbol{a}$ and angular frequency $\boldsymbol{\omega}$. At what value of the displacement kinetic and potential energies become equal? (5)

(c) An alternating emf of peak-to-peak value of 40 V is applied across the series combination of an inductor of inductance 100 mH, capacitor of capacitance 1μF and resistance 100Ω. Determine maximum value of current drawn by circuit its bandwidth and quality factor. (5)

3. (a) Two collinear simple harmonic motions of nearly equal frequencies and different amplitude are superimposed on each other. Find out the resultant equation and explain the formation of beats. (10)

(b) A point is under the influence of two simultaneous simple harmonic motion is mutually perpendicular directions given by $x = a \cos \pi t$, $y = a \cos \pi t/2$. Find the trajectory of the resulting motion of that point. (5)

4. (a) Show that for forced damped harmonic oscillator in steady state, the average power is equals to the average power dissipated by the system. (10)

(b) An object of mass 0.1 kg is suspended from a spring of force constant 100 Nm1. The frictional force acting on the object is 5v Newton, where v is its velocity. If a harmonic force F = 2 cos 20t is applied on this object calculate the steady state amplitude of the forced oscillation. (5)

5. (a) The string of length 3a under tension T has two equal masses places at a and 2a as shown in fig.2.

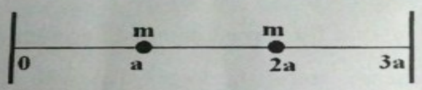

Fig. 2

Find out the normal mode of transverse and longitudinal vibrations using small- oscillation approximation. (10)

(b) Establish the classical wave equation $\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$ by considering a harmonic wave travelling in a medium with velocity v. (5)