Physics Exam Answers

Question 1

(a) Forces on Freight Trucks

Given: Three trucks of mass m each pulled by force F with negligible friction.

Solution:

1. Total mass of system = $3m$
2. Acceleration of system (a) = $F/(3m)$
3. Force on first truck (closest to locomotive):

$$F_1 = ma = m(F/3m) = F/3 \text{ (to the right)}$$

4. Force on middle truck:

$$F_2 = 2ma - F_1 = 2m(F/3m) - F/3 = 2F/3 - F/3 = F/3 \text{ (to the right)}$$

5. Force on last truck:

$$F_3 = 3ma - F_1 - F_2 = F - F/3 - F/3 = F/3 \text{ (to the right)}$$

Answer: Each truck experiences a force of $F/3$ in the rightward direction.

(b) Moment of Inertia

Definition: Moment of inertia (I) of a body about an axis is the sum of the products of the mass of each particle and the square of its perpendicular distance from the axis.

Physical Significance:

- Rotational analogue of mass in linear motion
- Measures resistance to angular acceleration
- Depends on mass distribution relative to rotation axis
- Determines rotational kinetic energy ($K = \frac{1}{2}I\omega^2$)

(c) Work Done in Force Field

Given: Force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along path $(0,0,0)$ to $(2,1,3)$

Solution: Parametric equations of straight line: $x = 2t$, $y = t$, $z = 3t$ (t from 0 to 1)

Work done $W = \int \vec{F} \cdot d\vec{r} = \int (3x^2dx + (2xz - y)dy + zdz)$

$$\begin{aligned}
&= \int_0^1 [3(2t)^2(2dt) + (2(2t)(3t) - t)(dt) + 3t(3dt)] \\
&= \int_0^1 (24t^2 + 12t^2 - t + 9t)dt \\
&= \int_0^1 (36t^2 + 8t)dt \\
&= \left[12t^3 + 4t^2\right]_0^1 = 16 \text{ J}
\end{aligned}$$

Answer: 16 Joules

(d) Astronaut's Effective Weight

Given: $m = 70$ kg, $a = 5g$ upward

Solution: Effective weight $= m(g + a) = 70(9.81 + 5 \times 9.81) = 70 \times 6 \times 9.81 = 4120.2$ N

Answer: 4120.2 N ($\approx$420 kg equivalent)

(e) Loss in Mechanical Energy

Given: Mass M, initial speed V_0, spring constant k, $\mu = bx$

Solution: Initial energy $= \frac{1}{2}MV_0^2$ When comes to rest ($x = x_k$): Work against friction $= \int \mu Mgdx = \int bxMgdx = \frac{1}{2}bMgx_k^2$ Spring energy $= \frac{1}{2}kx_k^2$ By energy conservation: $\frac{1}{2}MV_0^2 = \frac{1}{2}kx_k^2 + \frac{1}{2}bMgx_k^2$ Loss = Initial − Final $= \frac{1}{2}MV_0^2 - \frac{1}{2}kx_k^2 = \frac{1}{2}bMgx_k^2$

Answer: Loss $= \frac{1}{2}bMgx_k^2$ where x_k is compression at rest

(f) Sphere vs Cylinder on Incline

Solution: Acceleration on incline: $a = \frac{g\sin\theta}{1+I/MR^2}$ For sphere: $I = \frac{2}{5}MR^2 \Rightarrow a_{\text{sphere}} = \frac{5}{7}g\sin\theta$ For cylinder: $I = \frac{1}{2}MR^2 \Rightarrow a_{\text{cylinder}} = \frac{2}{3}g\sin\theta$ Since $5/7 > 2/3$, sphere accelerates faster.

Answer: The sphere reaches the bottom first.

(g) Relative Speed of Spaceships

Given: Both moving apart at $0.99c$ in observer frame

Solution: Use relativistic velocity addition:

$$v = \frac{v_1 + v_2}{1 + v_1v_2/c^2} = \frac{0.99c + 0.99c}{1 + 0.99^2} \approx 0.99995c$$

Answer: Approximately $0.99995c$

Question 2

(a) Center of Mass Location

Answer: No, the center of mass doesn't necessarily lie within the body. Examples:

- A ring (COM at center where there's no material)
- A boomerang (COM often in empty space between arms)
- L-shaped object (COM may be outside main body)

(b) Bomb-shell Explosion

Proof:

1. Before explosion: Trajectory determined by initial velocity and gravity
2. During explosion: Internal forces act in pairs (Newton's 3rd law)
3. No external force (neglecting air resistance), so COM continues original path
4. Fragments' COM must follow same path as original shell

(c) Rocket Equation

Derivation: From momentum conservation:

$$MdV = udM \text{ (where } u = \text{exhaust velocity)}$$

$$\int dV = u \int \frac{dM}{M} \Rightarrow \Delta V = u \ln \left(\frac{M_0}{M_f} \right)$$

Thus $\frac{M_f}{M_0} = e^{-\Delta V/u}$

Question 3

(a) Fictitious Forces

Definition: Apparent forces that appear in non-inertial frames. Examples:

- Centrifugal force (in rotating frames)
- Coriolis force (for moving objects in rotating frames)
- Euler force (when rotation rate changes)

(b) Potential Energy Curve

Importance:

- Shows stable/unstable equilibrium points
- Predicts motion without solving equations
- Indicates allowed regions of motion

Definitions:

- Turning point: Where total energy = potential energy (kinetic energy zero)
- Equilibrium points: Where $dU/dx = 0$
  - Stable: $d^2U/dx^2 > 0$
  - Unstable: $d^2U/dx^2 < 0$
  - Neutral: $d^2U/dx^2 = 0$

(c) Kepler's Laws & Satellite Velocity

Kepler's Laws:

1. Planets move in elliptical orbits with Sun at one focus
2. Radius vector sweeps equal areas in equal times
3. $T^2 \propto a^3$ for all planets

Satellite Velocity Derivation: Centripetal force = Gravity:

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

Question 4

(a) Effective Gravity with Rotation

Solution:

$$g_{\text{eff}} = g - \Omega^2 R \cos^2 \lambda$$

(where λ = latitude, Ω = angular velocity)

(b) Angular Momentum Change

Given: $\vec{\tau} = (4, 4, 6)$, $\vec{L_0} = (2, 7, 8)$, $\Delta t = 6\text{s}$

Solution:

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t} \Rightarrow \Delta \vec{L} = \vec{\tau} \Delta t = (24, 24, 36)$$
$$\vec{L_f} = \vec{L_0} + \Delta \vec{L} = (26, 31, 44) \text{ kg m}^2/\text{s}$$

(c) Cube Moment of Inertia

Solution: For cube about central axis: $I = \frac{Ma^2}{6}$

Question 5

(a) Collision Problem

Given: m_1, $m_2 = 3m_1$, $v_{\text{cm}} = 3$ m/s

Solution:

1. CM speed remains 3 m/s (no external force)
2. Elastic collision: relative velocity reverses
3. Final velocities:

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$
$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

   Given $v_2 = 0$, $v_{\text{cm}} = \frac{m_1 v_1}{4m_1} = \frac{v_1}{4} \Rightarrow v_1 = 12$ m/s Thus $v_1' = -6$ m/s, $v_2' = 6$ m/s

(b) Conservative Force Proof

Given: $\vec{r} = a\cos(\omega t)\hat{i} + b\sin(\omega t)\hat{j}$

Solution:

1. Find acceleration: $\vec{a} = -\omega^2 \vec{r}$
2. Force $\vec{F} = m\vec{a} = -m\omega^2 \vec{r}$
3. $\nabla \times \vec{F} = 0 \Rightarrow$ force is conservative

(c) Angular Momentum of System

Concept: Total angular momentum $\vec{L} = \sum(\vec{r_i} \times \vec{p_i})$ In absence of external torque: $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L}$ is conserved

Question 6

(a) Relativistic Length Contraction

Given: $v = 0.6c$, $\theta = 30°$

Solution: Length contraction only in motion direction:

$$\begin{aligned}
L'_x &= L_x\sqrt{1-v^2/c^2} = (1\text{m}\cos 30^\circ)(0.8) = 0.693\text{m} \\
L'_y &= L_y = 1\text{m}\sin 30^\circ = 0.5\text{m} \\
\text{Total length } L' &= \sqrt{0.693^2+0.5^2} \approx 0.853\text{m} \\
\text{Orientation : } \tan\theta' &= \frac{L'_y}{L'_x} \Rightarrow \theta' \approx 35.8^\circ
\end{aligned}$$

(b) Speed Limit of Light

Reasons:

- Relativistic mass increases toward infinity as $v \to c$
- Would require infinite energy to reach c
- Causality violations would occur if $v > c$
- Supported by all experimental evidence

(c) Einstein's Velocity Addition

Formula:

$$w = \frac{u+v}{1+uv/c^2}$$

Given: $KE = 3m_0c^2$

Solution: Total energy $E = m_0c^2 + 3m_0c^2 = 4m_0c^2 = \gamma m_0c^2 \Rightarrow \gamma = 4$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \Rightarrow v = \frac{c\sqrt{15}}{4} \approx 0.968c$$