Probability and Statistics

Department of Mathematics

Questions

Q1. Descriptive Statistics & Probability

(a) Given the oxidation-induction time data (in minutes):
87, 103, 105, 130, 132, 145, 160, 180, 195, 211

- Calculate the sample variance and standard deviation.
- Convert these measures to hours.

(6 marks)

(b) For events A and B:

- Prove $P(A \cap B') = P(A) - P(A \cap B)$ using axioms of probability.
- If $P(A) = 0.4$, $P(B) = 0.3$, and $P(A \cup B) = 0.6$, find $P(A \cap B)$.

(6 marks)

(c) Construct a stem-and-leaf plot for the data:
31, 35, 36, 36, 37, 38, 40, 40, 54, 55, 58, 62, 66, 66, 67, 68, 75
Identify two key features of the distribution. (4 marks)

(d) Find the median and quartiles for the dataset:
16, 20, 26, 30, 35, 39, 45, 46, 48, 50 (4 marks)

Q2. Probability Distributions

(a) A discrete random variable Y represents the trial number of the first success in independent Bernoulli trials with success probability $p = \frac{1}{3}$.

- Derive the PMF of Y.
- Sketch the line graph of the PMF for $Y = 1, 2, 3, 4$.

(6 marks)

(b) For a continuous random variable X with CDF:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{2}(x - \frac{x^3}{3}) & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

- Find $E(X)$ and $V(X)$.
- Calculate $P(0.2 \leq X \leq 0.8)$.

(8 marks)

(c) Prove that for any random variable X and constants a, b:

$$V(aX + b) = a^2 V(X)$$

(6 marks)

Q3. Bayesian Inference & Normal Distribution

(a) An aircraft emergency locator has:

- 70% detection rate ($P(D) = 0.7$).
- 60% of detected planes have locators ($P(L|D) = 0.6$).
- 10% of undetected planes have locators ($P(L|D') = 0.1$).

Find the probability that a plane with a locator was *not* detected ($P(D'|L)$). (6 marks)

(b) Reaction times X (in seconds) follow $N(1.25, 0.46^2)$. Calculate:

- $P(1.00 \leq X \leq 1.75)$.
- The time t such that $P(X \leq t) = 0.99$.

(6 marks)

(c) Water dispenser output $X \sim N(64, 0.78^2)$ oz. Find the cutoff c where only 0.5% of outputs exceed c. (4 marks)

(d) Visitors to a store make purchases with probabilities:

- Day visitors: 10%
- One-night visitors: 30%
- Two-night visitors: 20%

Given that a purchase was made, what is the probability it was by a day visitor? (4 marks)

Q4. Correlation & Regression

(a) For asparagus quality data ($n = 10$):

- Given $\sum x = 707$, $\sum y = 23.59$, $\sum xy = 1759.11$, $\sum x^2 = 54961$, $\sum y^2 = 56.3135$, compute the correlation coefficient r.
- Interpret the strength and direction of the relationship.

(6 marks)

(b) Cheese elongation data ($n = 7$) yields the regression line:

$$\hat{y} = 88.31 + 1.208x$$

- Predict elongation (%) when $x = 70$.
- If elongation is measured in decimals (e.g., 1.72 instead of 172%), how does this affect the regression slope?

(6 marks)

(c) State the *invariance property* of the correlation coefficient and verify it for part (a) if shear force is converted to pounds. (4 marks)

(d) For the dataset:

x	56	42	72	36	63	47	55	49	38	42
y	148	126	159	118	149	130	151	142	114	141

Calculate the least squares regression line. (4 marks)

Q5. Sampling Distributions & Binomial Models

(a) Piston ring diameters $X \sim N(12, 0.04^2)$ cm. For $n = 16$:

- Find $E(\bar{X})$ and $\sigma_{\bar{X}}$.
- Calculate $P(\bar{X} > 12.01)$.

(6 marks)

(b) Credit card purchases $X \sim \text{Binomial}(10, 0.75)$:

- Compute $E(X)$ and $V(X)$.
- Find $P(|X - \mu| \leq \sigma)$.

(8 marks)

(c) Prove that for $X \sim \text{Binomial}(n, p)$:

$$E(X) = np$$

(6 marks)