Statistics Exam Solutions

Student Name

June 26, 2025

Q1. (a) Deviations and Sample Statistics

i. Fifth Observation Deviation

1. Given deviations from mean: $d_1 = 0.3$, $d_2 = 0.9$, $d_3 = 1.0$, $d_4 = 1.3$
2. Property of deviations: $\sum_{i=1}^{n} d_i = 0$
3. Calculation: $0.3 + 0.9 + 1.0 + 1.3 + d_5 = 0$
4. Solve: $3.5 + d_5 = 0 \Rightarrow d_5 = -3.5$

ii. Sample Standard Deviation and New Variance

1. Calculate sum of squared deviations:

$$\begin{aligned}\sum d_i^2 &= 0.3^2 + 0.9^2 + 1.0^2 + 1.3^2 + (-3.5)^2 \\ &= 0.09 + 0.81 + 1.0 + 1.69 + 12.25 = 15.84\end{aligned}$$

2. Sample variance formula:

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{15.84}{4} = 3.96$$

3. Standard deviation:

$$s = \sqrt{3.96} \approx 1.99$$

4. New variance when multiplied by 2:

$$\text{New Var} = 2^2 \times s^2 = 4 \times 3.96 = 15.84$$

iii. Degrees of Freedom

- Degrees of freedom $= n - 1 = 5 - 1 = 4$
- Reason: One parameter (mean) estimated from data reduces independent information by 1

Q1. (b) Frequency Distribution

i. Median Class

1. Cumulative relative frequencies:

$$\begin{aligned} 0-5 &: 0.177 \\ 5-10 &: 0.177+0.166=0.343 \\ 10-15 &: 0.343+0.175=0.518 \quad \text{(First exceeds 0.5)} \end{aligned}$$

2. Median lies in 10-15 class

ii. Density Calculation

Class	Width	Rel Freq	Density
0-5	5	0.177	$\frac{0.177}{5}=0.0354$
5-10	5	0.166	$\frac{0.166}{5}=0.0332$
10-15	5	0.175	$\frac{0.175}{5}=0.0350$
15-20	5	0.136	$\frac{0.136}{5}=0.0272$
20-30	10	0.194	$\frac{0.194}{10}=0.0194$
30-40	10	0.078	$\frac{0.078}{10}=0.0078$
40-60	20	0.044	$\frac{0.044}{20}=0.0022$
60-90	30	0.030	$\frac{0.030}{30}=0.0010$

iii. Proportion between 25 and 45

1. 20-30 class (25-30 portion): $\frac{5}{10}\times 0.194=0.097$
2. 30-40 class: 0.078
3. 40-60 class (40-45 portion): $\frac{5}{20}\times 0.044=0.011$
4. Total proportion: $0.097+0.078+0.011=0.186$

Q2. (a) Probability of Students Attending

i. P(at least one)

$$P(\text{at least one})=1-P(\text{none})=1-0.06=0.94$$

ii. P($A_1 \cup A_2$)

1. Given $P(A_2)=4P(A_1)$ and independent
2. $P(A_1\cap A_2)=P(A_1)P(A_2)=0.16$

3. Let $P(A_1) = p \Rightarrow 4p^2 = 0.16 \Rightarrow p = 0.2$
4. $P(A_1) = 0.2$, $P(A_2) = 0.8$
5. Union probability:
$$P(A_1 \cup A_2) = 0.2 + 0.8 - 0.16 = 0.84$$

Q3. (a) Disease Testing

ii. Both Tests Same Result

$$\begin{aligned} P(\text{same}) &= P(\text{carrier})[P(++) + P(--)] + P(\text{non-carrier})[P(++) + P(--)] \\ &= 0.01 \times (0.9^2 + 0.1^2) + 0.99 \times (0.05^2 + 0.95^2) \\ &= 0.01 \times 0.82 + 0.99 \times 0.905 \\ &= 0.0082 + 0.89595 \approx 0.904 \end{aligned}$$

iii. P(Carrier | Both Tests +)

$$\begin{aligned} P(\text{C}|++) &= \frac{P(++|\text{C})P(\text{C})}{P(++)} \\ &= \frac{0.9^2 \times 0.01}{0.9^2 \times 0.01 + 0.05^2 \times 0.99} \\ &= \frac{0.0081}{0.0081 + 0.002475} \approx 0.766 \end{aligned}$$

Q3. (b) Random Variable with Given CDF

i. PDF of X

$$f(x) = \frac{dF(x)}{dx} = \begin{cases} \frac{1}{2} & \text{for } 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

iv. 60th Percentile

$$\begin{aligned} F(x) &= 0.6 \\ \frac{x-2}{2} &= 0.6 \\ x - 2 &= 1.2 \\ x &= 3.2 \end{aligned}$$

Q5. (a) Exponential Lifetime of PC

$$\begin{aligned} P(X > 6 | X > 2) &= P(X > 4) \quad \text{(Memoryless property)} \\ &= e^{-\lambda t} = e^{-0.25 \times 4} = e^{-1} \approx 0.368 \end{aligned}$$

Q6. (a) Normal Distribution Grading

Standard normal probabilities:

$$\begin{aligned} \text{A} &: P(Z > 1) = 1 - \Phi(1) \approx 15.87\% \\ \text{B} &: P(0 < Z < 1) = \Phi(1) - \Phi(0) \approx 34.13\% \\ \text{C} &: P(-1 < Z < 0) = 34.13\% \\ \text{D} &: P(-2 < Z < -1) \approx 13.59\% \\ \text{F} &: P(Z < -2) \approx 2.28\% \end{aligned}$$

Q7. (a) Faulty Light Switches

Poisson approximation ($\lambda = np = 1500 \times 0.002 = 3$):

$$\begin{aligned} P(4 \leq X \leq 8) &= \sum_{k=4}^{8} \frac{e^{-3} 3^k}{k!} \\ &\approx 0.1680 + 0.1008 + 0.0504 + 0.0216 + 0.0081 \\ &\approx 0.3489 \end{aligned}$$

Q7. (b) Steel Strength

i. 25th Percentile

$$\begin{aligned} \Phi(z) = 0.25 &\Rightarrow z \approx -0.674 \\ x = \mu + z\sigma &= 43 + (-0.674)(4.5) \approx 39.967 \end{aligned}$$

iv. P(at most 3 $<$ 43)

$$\begin{aligned} Y &\sim \text{Binomial}(n = 15, p = 0.5) \\ P(Y \leq 3) &= \sum_{k=0}^{3} \binom{15}{k} 0.5^{15} \\ &\approx 0.00003 + 0.00046 + 0.00320 + 0.01389 \\ &\approx 0.01758 \end{aligned}$$