Question 1

(i) Work Done by Friction

Given: Mass $m = 1\,\text{kg}$, initial velocity $u = 10\,\text{ms}^{-1}$, coefficient of kinetic friction $\mu_k = 0.5$, distance $d = 5\,\text{m}$.

The work done by friction is:

$$W = -\mu_k mgd = -0.5 \times 1 \times 9.8 \times 5 = \boxed{-24.5\,\text{J}}$$

(ii) Time Period Ratio for Central Force Potential

Given: Potential $U(r) = -kr^n$, $k > 0$, $0 < n < 2$, orbits of radius R and $2R$.

The time period ratio is:

$$\frac{T_2}{T_1} = 2^{\frac{2-n}{2}} = \boxed{2^{\frac{2-n}{2}}}$$

(iii) Recoil Momentum of Nucleus

Given: Electron momentum $p_e = 1.2 \times 10^{-22}\,\text{kg}\,\text{ms}^{-1}$, neutrino momentum $p_\nu = 6.4 \times 10^{-23}\,\text{kg}\,\text{ms}^{-1}$, right angle between them.

The recoil momentum is:

$$p_r = \sqrt{p_e^2 + p_\nu^2} = \boxed{1.36 \times 10^{-22}\,\text{kg}\,\text{ms}^{-1}}$$

Direction: $\theta = \tan^{-1}\left(\frac{p_\nu}{p_e}\right) = \boxed{28.07^\circ}$ from the electron's path.

(iv) Moment of Inertia and Radius of Gyration

Given: Three masses $m = 2\,\text{kg}$ at vertices of an equilateral triangle with side $a = 0.1\,\text{m}$.

Moment of inertia:

$$I = 2m\left(\frac{a}{\sqrt{3}}\right)^2 = \boxed{0.00667\,\text{kg}\,\text{m}^2}$$

Radius of gyration:

$$k = \sqrt{\frac{I}{3m}} = \boxed{0.0333\,\text{m}}$$

(v) Speed of a 2 MeV Electron

Given: Kinetic energy $K = 2\,\text{MeV}$, rest mass $m_e = 0.5\,\text{MeV}/c^2$.

The speed is:

$$v = c\sqrt{1 - \frac{1}{\gamma^2}} = \boxed{0.98c}$$

(vi) Minimum Speed for Vertical Circular Motion

Given: Radius $r = 0.75\,\text{m}$.

The minimum speed is:

$$v = \sqrt{rg} = \boxed{2.71\,\text{ms}^{-1}}$$

Question 2

(i) Centre of Mass of a Right-Angled Triangle

Given: Mass M, base b, height h, thickness t.

The centre of mass is at:

$$(x_{\text{cm}}, y_{\text{cm}}) = \boxed{\left(\frac{b}{3}, \frac{h}{3}\right)}$$

(ii) Force and Power for Freight Car with Sand

Given: Freight car moving at velocity v, sand falling at rate $\frac{dm}{dt}$.

The force required is:

$$F = \boxed{v\frac{dm}{dt}}$$

The power is:

$$P = \boxed{v^2\frac{dm}{dt}}$$

(iii) Tension in a Suspended Rope

Given: Mass M, length L.

The tension at distance x is:

$$T(x) = \boxed{\frac{M}{L}xg}$$

At the ceiling:

$$T = \boxed{Mg}$$

Question 3

(i) Potential Energy and Equilibrium Position

Given: Force $F = -\frac{A}{x^2} + B$.

The potential energy is:

$$U(x) = \boxed{-\frac{A}{x} - Bx}$$

Equilibrium position:

$$x_0 = \boxed{\sqrt{\frac{A}{B}}}$$

Frequency of small oscillations:

$$\omega = \boxed{\sqrt{\frac{2B^{3/2}}{A^{1/2}m}}}$$

(ii) Elastic Collision and Mass Ratio

Given: Mass m, initial velocity v_0, after collision $\frac{v_0}{2}$ at right angle.
The mass ratio is:

$$\frac{M}{m} = \boxed{2}$$

(iii) Turning Points on a Frictionless Track

Given: Track $y = bx^2$, maximum speed $v_{\max} = 8.5\,\mathrm{ms^{-1}}$, $b = 0.92\,\mathrm{m^{-1}}$.
The turning points are:

$$x_{\max} = \boxed{\pm 2.0\,\mathrm{m}}$$

Question 4

(i) Angular Momentum Decomposition

The total angular momentum is:

$$\vec{J} = \boxed{\vec{J}_0 + \vec{J}_{\mathrm{cm}}}$$

(ii) Rolling Cylinder on Incline

Given: Radius R, mass M, angle θ, coefficient μ.
Maximum angle:

$$\theta_{\max} = \boxed{\tan^{-1}(3\mu)}$$

Acceleration:

$$a = \boxed{\frac{2}{3} g \sin\theta}$$

(iii) Angular Momentum of a Particle

Given: Mass $m = 2\,\text{kg}$, path $y = \frac{x}{\sqrt{3}} + 3$, speed $v = 4\,\text{ms}^{-1}$.
The angular momentum is:

$$\vec{L} = \boxed{12\,\text{kg}\,\text{m}^2\,\text{s}^{-1}}$$

Question 5

(i) Relative Velocity in Special Relativity

The velocity transformation is:

$$u' = \boxed{\frac{u - v}{1 - \frac{uv}{c^2}}}$$

(ii) Space-like, Time-like, and Light-like Events

For two firecrackers at the same time, separated by l:

$$\Delta s^2 = -l^2 < 0 \implies \boxed{\text{Space-like}}$$

(iii) Bug on a Rotating Wheel

Given: Speed v_0, angular velocity ω, coefficient μ_s.
Apparent forces:

$$F_{\text{centrifugal}} = \boxed{m\omega^2 r}, \quad F_{\text{Coriolis}} = \boxed{2m\omega v_0}$$

Maximum distance before slipping:

$$r_{\text{max}} = \boxed{\frac{\mu_s g}{\omega^2}}$$

Question 6

(i) Time Dilation in Special Relativity

The time dilation relation is:

$$\Delta t' = \boxed{\gamma \Delta t}$$

(ii) Relative Speed of Spaceships

Given: Relative speed $0.7c$.
The speed of each spaceship is:

$$v = \boxed{0.4c}$$