Physics Exam Solutions

Question 1

(i) Work done by friction

Given:

- Mass (m) = 1 kg
- Initial velocity (v_0) = 10 m/s
- Coefficient of kinetic friction (μ_k) = 0.5
- Distance traversed (d) = 5 m

Solution: Frictional force (f) = $\mu_k N = \mu_k mg = 0.5 \times 1 \times 9.8 = 4.9$ N
Work done by friction (W) = $-f \times d = -4.9 \times 5 = -24.5$ J
(Negative sign indicates work is done against the motion)

Answer: The work done by friction is $\boxed{-24.5 \text{ J}}$.

(ii) Time period ratio for central force potential

Given potential $U(r) = -kr^n$ with $0 < n < 2$.

Solution: For circular orbits in central potential, the time period $T \propto R^{(2-n)/2}$.
Thus, $\frac{T_2}{T_1} = \left(\frac{2R}{R}\right)^{(2-n)/2} = 2^{(2-n)/2}$.

Answer: $\boxed{2^{(2-n)/2}}$.

(iii) Recoil momentum of nucleus

Given:

- Electron momentum (p_e) = 1.2×10^{-22} kg·m/s (let's say in x-direction)
- Neutrino momentum (p_n) = 6.4×10^{-23} kg·m/s (at right angle, y-direction)

Solution: By conservation of momentum:

$$p_{Nx} = -p_e = -1.2 \times 10^{-22} \text{ kg·m/s}$$
$$p_{Ny} = -p_n = -6.4 \times 10^{-23} \text{ kg·m/s}$$

Magnitude: $p_N = \sqrt{(1.2 \times 10^{-22})^2 + (6.4 \times 10^{-23})^2} \approx 1.36 \times 10^{-22}$ kg·m/s
Direction: $\theta = \arctan(p_{Ny}/p_{Nx}) = \arctan(0.533) \approx 28°$ from negative x-axis.

Answer: The nucleus recoils with momentum $\boxed{1.36 \times 10^{-22} \text{ kg·m/s}}$ at $\boxed{28°}$ from the negative x-axis.

(iv) Moment of inertia and radius of gyration

Given:

- Three masses (m) = 2 kg each
- Equilateral triangle side (a) = 0.1 m
- Axis through one vertex perpendicular to plane

Solution: Moment of inertia (I) $= \Sigma mr^2 = 0 + 2 \times (0.1)^2 + 2 \times (0.1)^2 = 0.04$ kg·m^2
Total mass (M) = 6 kg
Radius of gyration (k): $I = Mk^2 \Rightarrow k = \sqrt{I/M} = \sqrt{0.04/6} \approx 0.0816$ m

Answer: Moment of inertia is $\boxed{0.04 \text{ kg·m}^2}$ and radius of gyration is $\boxed{0.0816 \text{ m}}$.

(v) Speed of relativistic electron

Given:

- Total energy (E) = 2 MeV
- Rest mass energy ($m_e c^2$) = 0.5 MeV

Solution: $\gamma = E/(m_e c^2) = 2/0.5 = 4$
$\gamma = 1/\sqrt{1 - v^2/c^2} \Rightarrow v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{15/16} \approx 0.968c$

Answer: The electron's speed is $\boxed{0.968c}$.

(vi) Minimum speed for vertical circular motion

Given:

- Radius (r) = 0.75 m

Solution: At the top: $mg = mv^2/r \Rightarrow v = \sqrt{gr} = \sqrt{9.8 \times 0.75} \approx 2.71$ m/s

Answer: The minimum speed is $\boxed{2.71 \text{ m/s}}$.

Question 2

(i) Center of mass of right triangular sheet

For a right triangle with base b and height h, the COM coordinates are: $\bar{x} = b/3$, $\bar{y} = h/3$ from the right angle.

Answer: The center of mass is at $\boxed{\left(\frac{b}{3}, \frac{h}{3}\right)}$ from the right angle.

(ii) Force on freight car with falling sand

Solution: Force required: $F = v(dm/dt)$
Power $(P) = Fv = v^2(dm/dt)$
Rate of KE increase: $dK/dt = d/dt(\frac{1}{2}mv^2) = \frac{1}{2}v^2(dm/dt)$
Thus, $P = 2(dK/dt)$.

Answer: The required force is $\boxed{v\frac{dm}{dt}}$ and the power is twice the rate of KE increase.

(iii) Tension in hanging rope

Solution: At distance y from bottom: $T(y) = (M/L)gy$
Tension changes at rate $dT/dy = (M/L)g = \text{constant}$.
At upper end $(y = L)$: $T = Mg$.

Answer: The tension varies as $\boxed{T(y) = \frac{Mgy}{L}}$ and is $\boxed{Mg}$ at the top.

Question 3

(i) Potential and oscillations

Potential $U(x) = Bx - A/x$
Equilibrium: $dU/dx = 0 \Rightarrow B + A/x_0^2 = 0 \Rightarrow x_0 = \sqrt{-A/B}$
Frequency: $\omega = \sqrt{k/m}$ where $k = d^2U/dx^2$ at x_0.

(ii) Elastic collision

Using momentum conservation in x and y directions: $M/m = 2\sqrt{3}$.

(iii) Turning points on track

Using energy conservation: $\frac{1}{2}mv_{\max}^2 = mgy \Rightarrow y_{\max} = v_{\max}^2/(2g)$
Then $x_{\max} = \pm\sqrt{y_{\max}/b}$.

Question 4

(i) Angular momentum decomposition

Total angular momentum $J = r_{\text{cm}} \times p_{\text{cm}} + \Sigma(r_i' \times p_i') = J_0 + J_{\text{cm}}$.

(ii) Rolling cylinder

Maximum θ: $\tan\theta = 3\mu$
Acceleration: $a = \frac{2}{3}g\sin\theta$.

(iii) Angular momentum

$L = r \times p = 2 \times 4 \times$ (distance from origin to line) $= 24$ kg·m^2/s.

Question 5

(i) Relative velocity

From Lorentz transformations: $u'_x = (u_x - v)/(1 - u_x v/c^2)$.

(ii) Spacelike events

If $\Delta x > c\Delta t$, spacelike. Here $\Delta t = 0 \Rightarrow$ spacelike.

(iii) Bug on rotating wheel

Apparent forces: Coriolis, centrifugal, Euler.
Slip condition: $\mu_s g = \omega^2 r + 2\omega v_0 \Rightarrow r_{\max} = (\mu_s g - 2\omega v_0)/\omega^2$.