Waves and Oscillations - BSc Physics Hons, DU

Waves and Oscillations

BSc Physics Hons, DU - Semester I: Exam-Focused Notes

This document provides a detailed overview of essential concepts in Waves and Oscillations, tailored for students of BSc Physics Honours.

1. Simple Harmonic Motion (SHM)

Simple Harmonic Motion is a special type of periodic motion where the restoring force is directly proportional to the displacement and acts in the opposite direction.

- Equation of SHM:
  - Displacement: $x(t) = A\cos(\omega t + \phi)$
    - A: Amplitude (maximum displacement)
    - ω: Angular frequency
    - ϕ: Phase constant (initial phase)
  - Velocity: $v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$
  - Acceleration: $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x$
    - This defining characteristic ($a \propto -x$) makes the motion SHM.
- Energy in SHM: The total mechanical energy in SHM remains constant if no damping forces are present.
  - Kinetic Energy (KE): $KE = \frac{1}{2}mv^2$
  - Potential Energy (PE): $PE = \frac{1}{2}kx^2$ (for a spring-mass system, where k is the spring constant).
  - Total Energy: Total Energy $= KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ (since $\omega^2 = k/m$, and $v^2 = \omega^2(A^2 - x^2)$).
    - Total energy is proportional to the square of the amplitude.
- Time Period (T): The time taken for one complete oscillation.
  - Formula: $T = 2\pi\sqrt{\frac{m}{k}}$ (for a spring-mass system)
  - Also, $T = \frac{2\pi}{\omega}$.

PYQ: Derive energy expressions in SHM.

2. Damped Oscillations

Damped oscillations occur when a dissipative force (like friction or air resistance) acts on an oscillating system, causing the amplitude of oscillations to gradually decrease over time.

- Equation of Damped SHM:
  - $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$
    - m: mass
    - b: damping constant (coefficient of the resistive force, $F_d = -bv$)
    - k: spring constant
- Solution Depends on Damping: The nature of the solution depends on the value of the damping constant b relative to m and k.
  - Damping Ratio (ζ): $\zeta = \frac{b}{2\sqrt{mk}}$
  - Types of Damping:
    - Underdamped ($\zeta < 1$ or $b^2 < 4mk$): The system oscillates with a decreasing amplitude.
    - Critically Damped ($\zeta = 1$ or $b^2 = 4mk$): The system returns to equilibrium as quickly as possible without oscillating.
    - Overdamped ($\zeta > 1$ or $b^2 > 4mk$): The system returns to equilibrium slowly without oscillating.

PYQ: State the conditions for over, under, and critical damping.

3. Forced Oscillations & Resonance

Forced oscillations occur when an oscillating system is subjected to an external periodic driving force. Resonance is a special condition where the amplitude of forced oscillations becomes maximum.

- Driven Equation:
  - $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$
    - F_0: Amplitude of the driving force
    - ω: Angular frequency of the driving force
- Amplitude Response ($A(\omega)$):
  - $A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+(b\omega/m)^2}}$
    - $\omega_0 = \sqrt{k/m}$ is the natural angular frequency of the undamped oscillator.
- Resonance:

- Definition: Resonance occurs when the driving frequency (ω) is equal to or very close to the natural frequency (ω_0) of the system.
- Phenomenon: At resonance, the amplitude of oscillations reaches a maximum.
- Importance: Resonance is crucial in applications (e.g., tuning radios, musical instruments, MRI) but can also be destructive (e.g., bridge collapse).

- Q-factor (Quality Factor): A measure of the "sharpness" of the resonance peak.
  - Formula: $Q = \frac{\omega_0}{\Delta\omega}$, where $\Delta\omega$ is the bandwidth.
  - Higher Q-factor: Implies less damping, a sharper resonance peak.
  - Lower Q-factor: Implies more damping, a broader resonance peak.

PYQ: Explain the phenomenon of resonance in forced oscillations.

4. Wave Motion

Wave motion is a disturbance that propagates through a medium or space, transferring energy without transferring matter.

- Definition: A wave is a propagating disturbance that carries energy and momentum.
- Types of Waves:
  - Transverse Waves: Particles oscillate perpendicular to the direction of wave propagation (e.g., waves on a string, light waves).
  - Longitudinal Waves: Particles oscillate parallel to the direction of wave propagation (e.g., sound waves).
  - Mechanical Waves: Require a material medium for propagation (e.g., sound waves, water waves).
  - Electromagnetic Waves: Do not require a medium and can travel through a vacuum (e.g., light, radio waves).
- Characteristics of Wave Motion:
  - Amplitude (A): The maximum displacement from equilibrium.
  - Wavelength (λ): The distance over which the wave's shape repeats.
  - Frequency (f): The number of oscillations per unit time (in Hz).
  - Period (T): The time for one complete oscillation. $T = 1/f$.
  - Wave Speed (v): $v = f\lambda$.
  - Wave Number (k): $k = \frac{2\pi}{\lambda}$.
  - Angular Frequency (ω): $\omega = 2\pi f = \frac{2\pi}{T}$.

5. Wave Equation

The wave equation is a partial differential equation that describes the propagation of a variety of waves.

- Derivation of 1D Wave Equation (for a transverse wave on a string):
  - Applying Newton's second law to a small segment of a stretched string under tension T, we arrive at:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

    * $y(x,t)$: Transverse displacement at position x and time t.
    * $v = \sqrt{\frac{T}{\mu}}$: Wave speed, with T (tension) and μ (linear mass density).
- General Solution:

$$y(x,t) = f(x - vt) + g(x + vt)$$

  - $f(x - vt)$: Wave traveling in the positive x-direction.
  - $g(x + vt)$: Wave traveling in the negative x-direction.
- Harmonic Wave (Sinusoidal Wave):
  - Equation: $y(x,t) = A\sin(kx - \omega t + \phi)$
    * A: Amplitude
    * k: Wave number
    * ω: Angular frequency
    * ϕ: Initial phase
  - Key Relations:
    * $v = \lambda f$
    * $\omega = 2\pi f$
    * $k = \frac{2\pi}{\lambda}$

PYQ: Derive the 1D wave equation and discuss its general solution.

6. Superposition & Interference

The principle of superposition is fundamental to understanding how waves combine, leading to phenomena like interference.

- Principle of Superposition:
  - When two or more waves overlap, the resultant displacement is the vector sum of the displacements due to individual waves.
  - Holds for linear waves where the medium's response is proportional to the disturbance.
- Interference:
  - Definition: The phenomenon when two or more waves overlap and combine to form a resultant wave.
  - Types of Interference:

        - Constructive Interference: Occurs when waves combine in phase (phase difference $\delta = 2n\pi$, where $n = 0, \pm1, \pm2, \ldots$). Path difference: $\Delta x = n\lambda$.
        - Destructive Interference: Occurs when waves combine out of phase (phase difference $\delta = (2n+1)\pi$). Path difference: $\Delta x = (n + \frac{1}{2})\lambda$.

PYQ: Explain the principle of superposition and conditions for constructive and destructive interference.

7. Beats & Stationary Waves

These are specific outcomes of wave superposition under certain conditions.

- Beats:
    - Definition: The periodic variation in amplitude produced by the superposition of two sound waves of slightly different frequencies.
    - Beat Frequency (f_{beat}): $f_{beat} = |f_1 - f_2|$
    - Applications: Used in tuning musical instruments and signal processing.
- Stationary Waves (Standing Waves):
    - Definition: Formed when two waves of the same amplitude and frequency traveling in opposite directions interfere. Energy is localized within the vibrating segments.
    - Equation: $y(x,t) = 2A\sin(kx)\cos(\omega t)$
    - Characteristics:
        - Nodes: Points where displacement is always zero ($\sin(kx) = 0$, i.e., $kx = n\pi$).
        - Antinodes: Points where displacement is maximum ($\sin(kx) = \pm1$, i.e., $kx = (n + \frac{1}{2})\pi$).
        - Distance between consecutive nodes (or antinodes): $\lambda/2$.
        - Distance between a node and an adjacent antinode: $\lambda/4$.
    - Formation: In stretched strings fixed at both ends, air columns in pipes, and across membranes.

PYQ: Derive the equation for beats or stationary waves and explain their characteristics.

8. Doppler Effect

The Doppler effect describes the change in observed frequency (and wavelength) of a wave when there is relative motion between the source and the observer.

- Definition: The apparent change in frequency and wavelength perceived by an observer moving relative to the source of the wave.
- Phenomenon:
    - When moving closer, observed frequency increases (wavelength decreases).
    - When moving away, observed frequency decreases (wavelength increases).

- Formula (General Case for Sound Waves):

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$$

  - f': Observed frequency
  - f: Actual frequency
  - v: Speed of sound in the medium
  - v_o: Speed of the observer
  - v_s: Speed of the source
  - Sign Conventions:
    * Use $+$ for v_o if observer moves towards the source.
    * Use $-$ for v_o if observer moves away from the source.
    * Use $-$ for v_s if source moves towards the observer.
    * Use $+$ for v_s if source moves away from the observer.
- Applications:
  - Radar Guns: Measure speed of vehicles.
  - Medical Imaging (Ultrasound): Measure blood flow.
  - Astronomy: Measure speed of stars and galaxies (redshift/blueshift).
  - Weather Forecasting: Doppler radar for wind speed and direction.

PYQ: Derive the Doppler shift formula for a moving source and observer.

9. PYQ Practice Topics (Frequent)

The following topics are frequently asked in examinations and require thorough preparation:

Topic	Type of Question
SHM	Differential equation & energy analysis
Damped Oscillations	Conditions for damping types
Forced Oscillations	Resonance explanation and conditions
Wave Equation	Derivation of 1D wave equation and its solution
Superposition & Interference	Conditions for constructive/destructive interference
Beats & Stationary Waves	Derivation of equations and characteristics
Doppler Effect	Derivation and applications