1. (a) Find the value of the Jacobian $J\frac{(u,v)}{(r,\theta)}$ where $u = x^2 - y^2$ and $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$.

(b) In what direction from the point (1,3,2) is the directional derivative of $\Phi = 2xz - y^2$, a maximum?

(c) Evaluate $\oiint \vec{r}.\hat{n}\, ds$ where S is a closed surface.

(d) Express $\vec{r}, \hat{r}$ and $\vec{a}.\hat{r}$ in index notation.

(e) Show that following equation is exact and solve it:

$$(y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)\, dy = 0$$

(f) Refer to the Probability distribution given below:

X:	−3	6	9
P(X=x)	1/6	1/2	1/3

Find E(X) and $E((2X+1)^2)$. (3×6)

2. (i) Show that following equation is inexact and then solve it :

$$(2x \log x - xy)\, dy + 2y\, dx = 0 \quad (6)$$

(ii) A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there is 50 milligrams of the material present and after two hours it is observed that the material has lost 10 percent of its original mass. With the help of the differential equation $dN/dt = kN$, N being the amount of material present at time t, find (a) an expression for the mass of the material remaining at any time t (b) the mass of the material after 4 hours. (6)

(iii) Solve by the method of Undetermined Coefficients :

$$d^2y/dx^2 + 2\, dy/dx + y = x - e^{-x} \quad (6)$$

3. (i) Solve the given differential equation :

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

given that $y = 0$ when $x = \pi/3$. (6)

(ii) Solve by the method of Variation of parameters:

$$d^2y/dx^2 + a^2 y = \tan ax \quad (6)$$

(iii) Solve the differential equation:

$$x^2 d^2y/dx^2 - 4x\, dy/dx + 6y = 42/x^4 \quad (6)$$

4. (i) Show that

$$\nabla^2 r^n = n(n+1)r^{n-2} \text{ where } r^2 = x^2 + y^2 + z^2 \quad (4)$$

(ii) Show that

$$\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$$

is irrotational. Find Φ such that $\vec{A} = \vec{\nabla}\Phi$ (6)

(iii) Prove $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2\vec{A}$. (8)

5. (i) Verify the divergence theorem for

$$\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$$

taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. (9)

(ii) Using Stake's theorem, evaluate

$$\int (x + 2y)dx + (x - z)dy + (y - z)dz$$

over the contour C, where C is the boundary of triangle bounded with vertices (2,0,0), (0,3,0) and (0,0,6) oriented in anticlockwise direction and S is the surface of the plane

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1 \quad (9)$$

6. (i) Show that scalar product of two vectors is invariant under rotation of axes. (4)

(ii) Find an expression for the mean and variance of the Binomial distribution. (8)

(iii) If $\vec{F} = 2y\hat{i} - z\hat{j} + x^2\hat{k}$.
and S is the surface $y^2 = 8x$ in the first octant bounded by the plane $y = 4$ and $z = 6$, evaluate

$$\iint \vec{F}.\hat{n}\, dS. \quad (6)$$