1. (a) For the following function

$$f(x) = \ln(4 - x^2)$$

(i) Find the Domain

(ii) Find the Asymptotes

(b) Given $g(2) = -4$ and $g'(x) = \sqrt{(x^2+5)}$ for all x.

use linear approximation to estimate $g(2.05)$.

(5+5)

2. (a) Find the limit

$$\lim_{x\to\infty}\left[x \ln\left(1-\frac{2}{3x}\right)\right]$$

Is the function continuous everywhere?

(b) Let $f(x)=\begin{cases} x^3-1 & \text{for } x<2 \\ x^2+3 & \text{for } x\geq 2\end{cases}$

3. (a) Check the convergence of the following:

(i) $\sum_{k=0}^{\infty} b\left(1+\frac{p}{100}\right)^{-k} \quad P>0$

(ii) $\left\{(-1)^{n-1}\left(\frac{1}{n}\right)\right\}$

(b) Solve the following Inequality

$$\frac{\frac{1}{y}-1}{\frac{1}{y}+1}\geq 1$$

(5+5)

4. (a) Let $A = (x: x \in R, |x| < 1\}$ and $B = \{x: x \in R, |x - 1| \geq 1\}$ where R is real numbers, If $A \cup B = R - D$, then find set D.

(b) Draw the graph of $f(x) = \ln|x - 2|$ from the graph of $f(x) = \ln|x|$

5. (a) (i) If x is restricted by the condition $0 < x < 2$, Find the range that y can take, given

$$y = (x - 1)^2$$

(ii) Show that $f(x) = 20x - e^{-4x}$ has exactly one real root.

(b) (i) Show that if F^{-1} exists then it is unique.

(ii) Discuss the continuity of $|x| + |x - 1|$ in the interval $[-1,2]$

(5+5)

6. (a) Solve for x

$$[\ln(x + e)]^3 - [\ln(x + e)^2]^2 = \ln(x + e) - 4$$

(b) Suppose we know that $f(x)$ is continuous and differentiable on the interval $[-3,4]$, that $f(-3) = 7$ and that $f'(x) \leq -17$. What is the largest possible value for $f(4)$?

(5+5)

7. (a) Does f have a local maxima/minima? Is it global? Is f differentiable at 0. Identify the cusp

$$f(x)=x^{\frac{2}{3}}(2x+5)$$

(b) Show that $Ax = e^x$ has 2 solutions when $e < A < \infty$.

(5+5)

8. (a) Find Elasticity of y with respect to x when f is given by

$$x^3y^3 + 3x^3 = 2$$

(b) For what values of the numbers α and β does the function

$$f(x) = \alpha x e^{-\beta x}$$

have the maximum value $f(2) = 1$.

(5+5)

9. (a) Let α and β be positive constants. Find

$$\lim_{x\to 0^+}\frac{1-\left(1+x^{\alpha}\right)^{-\beta}}{x}$$

(b) It is estimated that t years from now the population of a certain town will be

$$F(t) = 40 - \frac{8}{t+2} \text{ million.}$$

Use differentials to estimate the amount by which population will increase during the next 6 months.

(5+5)

10. (a) Show that the tangent to the curve $y = x^3$ at any point meets the curve again at point z where the slope is four times the slope at (c, c^3).

(b) If f and g are continuous functions in [a, b] such that $f(a) > g(a)$ and $f(b) < g(b)$. Prove the existence of c within (a, b) such that $f(c) = g(c)$.

(5+5)

11. (a) Find the third degree taylor formula for $\ln(1 + x)$ at 0. Use this to approximate $\ln(1.1)$. Estimate the upper bound to the error $R_3(x)$.

(b) Show that the function $f(x) = x|x|$ has an inflection point at (0,0) but $f''(0)$ does not exist. Draw the graph.

(5+5)

12. For the following function (10)

$$f(x)=\frac{3}{x^4-x^2+1}$$

Determine :

(a) The intervals for which the function is increasing/ decreasing.

(b) Find the points of local maxima and minima.

(c) Find the global maxima and minima.

13. (a) The population of a country grows according to the following function of time, t;

$$P(t) = \frac{a}{b+e^{-at}}$$

(i) Find dP/dt when t=0.

(ii) Find the proportional rate of growth of the population.

(iii) Show that the population has a limiting value and find its value.

(iv) At what time is the population rising most rapidly.

(b) The initial value of a rare diamond is given as

$$F(t)=25000(1.75)^{4\sqrt{t}}$$

If the rate of interest is compounded continuously and is 7%, how long should the painting be held?

(5+5)