Question 1: Work and Energy

(a) A block of mass 1 kg moving at 10 m/s slides on a rough surface ($\mu_k = 0.5$) for 5 m.

i. Calculate the work done by friction.

ii. Explain the significance of the negative sign in your answer.

(b) A force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ acts on a particle moving along the path from $(0, 0, 0)$ to $(2, 1, 3)$. Compute the work done.

Question 2: Rotational Dynamics

(a) Derive the moment of inertia for three 2 kg masses placed at the vertices of an equilateral triangle (side 0.1 m) about an axis perpendicular to the plane through one vertex. Also find the radius of gyration.

(b) Explain why a sphere rolls down an inclined plane faster than a cylinder, using their moments of inertia.

Question 3: Central Forces and Orbits

(a) For a central potential $U(r) = -kr^n$ $(0 < n < 2)$, show that the time period ratio for two circular orbits of radii R and $2R$ is $2^{(2-n)/2}$.

(b) State Kepler's laws and derive the orbital velocity of a satellite.

Question 4: Conservation Laws

(a) An electron ($p_e = 1.2 \times 10^{-22}\,\text{kg}\cdot\text{m/s}$) and a neutrino ($p_\nu = 6.4 \times 10^{-23}\,\text{kg}\cdot\text{m/s}$) are emitted at right angles in a nuclear decay. Calculate the recoil momentum and direction of the nucleus.

(b) Prove that the center of mass of an exploding projectile follows the original parabolic trajectory (neglect air resistance).

Question 5: Special Relativity

(a) An electron's total energy is 2 MeV, and its rest mass energy is 0.5 MeV. Find its speed.

(b) Two spaceships move apart at $0.99c$ each in a lab frame. Using relativistic velocity addition, show that their relative speed is $\approx 0.99995c$.