1. (a) Determine the linear dependence (or independence) of set of the functions

$$-1, \sin^2 x, \cos^2 x$$

(b) Solve the differential equation :

$$\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+\left[x+\log x-x\sin y\right]dy=0$$

(c) Using index notation, verify that

$$\vec{a}.\left(\vec{b}\times\vec{c}\right)=\vec{c}.\left(\vec{a}\times\vec{b}\right)=\vec{b}.\left(\vec{c}\times\vec{a}\right)$$

(d) A certain student population, consisting of 70% from the government schools, selects 15 representatives to attend an international student meet. Find the mean representation of the students from government schools in the sample and calculate its standard deviation.

(e) Evaluate $\iint \vec{r}.\hat{n}\,dS$, where S is a closed surface.

(f) Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 4$ at the point $\left(1,\sqrt{2},-1\right)$.

(3×6=18)

2. (a) Solve : $\frac{dy}{dx}-y\tan x=-y^2\sec x$. (6)

(b) Solve the differential equation $\frac{d^2y}{dx^2}+\frac{g}{l}x=\frac{g}{l}L$ where g, l and L are constants subject to the conditions $x = a, \frac{dx}{dt} = 0$ at $t = 0$. (6)

(c) Evaluate $\vec{\nabla}.\left[r.\vec{\nabla}\left(\frac{1}{r^3}\right)\right]$ where $r=\sqrt{x^2+y^2+z^2}$. (6)

3. (a) Solve by the method of Undetermined coefficient

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=e^x+x. \quad (10)$$

(b) Verify $\vec{\nabla}\times\vec{B}=\vec{\nabla}\left(\vec{\nabla}.\vec{A}\right)-\vec{\nabla}^2\vec{A}$ where the vector $\vec{B}$ is defined as $\vec{B}=\vec{\nabla}\times\vec{A}$. (8)

4. (a) Solve the given differential equation using Variation of Parameters

$$\frac{d^2y}{dx^2}+y=x-\cot x \quad (8)$$

(b) Verify that a scalar product of vectors $\vec{A}$ and $\vec{B}$ is invariant under rotation. (6)

(c) Using Green' theorem, show that the area enclosed by the curve C is

$$\frac{1}{2}\int_C (xdy-ydx) \quad (4)$$

5. (a) Obtain the expression for the mean and variance of the poisson distribution. (8)

(b) Verify Stokes' theorem for the vector field (10) $\vec{F}=y^2\hat{i}-(x+z)\hat{j}+yz\hat{k}$ over the unit square bounded by $0\le x\le 1,\ 0\le y\le 1,\ 0\le z\le 1$.

6. (a) Show that the vector field

$$\vec{F}=2x\left(y^2+z^3\right)\hat{i}-2x^2y\hat{j}+3x^2z^2\hat{k}$$

is conservative. Find the corresponding scalar potential and compute the work done in moving the particle from (–1,2,1) to (2,3,4). (8)

(b) Find Talyor expansion of $f(x) = \ln(1 + v)$ near $x = 0$ and approximate $f(x = -0.1)$ by taking first four terms of the series. (4)

(c) Solve : $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$. (6)