PHY Waves and Oscillations
Important Examination Questions

Question 1: Mathematical Analysis of Damped Harmonic Motion A damped harmonic oscillator has mass $m = 2\,\text{kg}$, damping constant $b = 8\,\text{N}\,\text{s}\,\text{m}^{-1}$, and spring constant $k = 32\,\text{N}\,\text{m}^{-1}$.

(a) Derive the equation of motion and determine whether the system is underdamped, critically damped, or overdamped. (5 marks)

(b) Solve for the displacement $x(t)$ if the initial conditions are $x(0) = 5\,\text{cm}$ and $\dot{x}(0) = 0$. (7 marks)

(c) Calculate the time for the amplitude to reduce to half its initial value. (3 marks)

Question 2: Superposition Principle and Beats Phenomenon

(a) Using trigonometric identities, show that the superposition of two waves $y_1 = A\cos(\omega_1 t)$ and $y_2 = A\cos(\omega_2 t)$ (where $\omega_1 \approx \omega_2$) results in beats. Derive the beat frequency. (6 marks)

(b) A tuning fork of frequency $256\,\text{Hz}$ produces 6 beats per second with another fork. What are the possible frequencies of the second fork? (4 marks)

(c) Explain how beats are used to tune musical instruments with two practical examples. (5 marks)

Question 3: Lissajous Figures and Phase Differences

(a) Derive the conditions under which two perpendicular SHMs $x = A\cos(\omega t)$ and $y = B\cos(\omega t + \phi)$ produce a straight-line Lissajous figure. (5 marks)

(b) Sketch the Lissajous figure for $\omega_x : \omega_y = 2 : 1$ with a phase difference of $\pi/4$. Label all critical points. (5 marks)

(c) Describe how Lissajous figures are used to measure unknown frequencies in laboratories, including the mathematical basis. (5 marks)

Question 4: Normal Modes in Coupled Oscillators Two masses m, connected by three identical springs (spring constant k), are fixed between walls.

(a) Derive the equations of motion and find the normal mode frequencies. (8 marks)

(b) Describe the symmetry of each mode and sketch the displacement patterns. (4 marks)

(c) Explain how energy distributes between the modes if the system is displaced symmetrically initially. (3 marks)

Question 5: Power Dissipation in Forced Oscillations A forced damped oscillator with $m = 0.5\,\text{kg}$, $b = 2\,\text{N}\,\text{s}\,\text{m}^{-1}$, and $k = 50\,\text{N}\,\text{m}^{-1}$ is driven by a force $F = 4\cos(12t)\,\text{N}$.

(a) Calculate the resonant frequency and amplitude at resonance. (5 marks)

(b) Prove that the average power input equals the power dissipated by damping at steady state. (7 marks)

(c) Plot the power vs. frequency curve and explain the significance of the quality factor Q. (3 marks)