

1. (a) A salesperson has the following record of sales during three weeks of a particular month in respect of items P, Q and R. She earns sales commission at the rate of 2%, 2.5% and 1% respectively. The details of the sales and commission are given below :

Weeks	Sales in units			Total Commission ₹
	P	Q	R	
First	400	200	200	27,000
Second	500	400	700	45,000
Third	400	600	400	54,000

Using matrices, you are required to:

- (i) Determine the selling price per unit of items P, Q and R.
- (ii) Total commission received by the salesperson in the fourth week if sales are 670, 150 and 400 units for products P, Q and R respectively. (6)

OR

- (a) A manufacturer produces two products A and B that are processed on two machines I and II before completion. Machine I can process either 25 units of product A or 10 units of product B per hour. Machine II can process 40 of product B or 20 units of product A per hour.

Using matrices determine the following :

- (i) Monthly output of products A and B if machines I and II operate for 8 and 7 hours per day respectively in a 6-day working week with 4 weeks in a month.
- (ii) Per unit cost of production if the cost of operating per hour on two machines is ₹25,000 and ₹30,000 respectively.
- (iii) Total cost of production. (6)

(b) A hydro-electricity power plant in a small village generates electricity and water for irrigation which also serve as intermediate inputs in each other's production. To generate a unit of electricity, 0.40 units of electricity and 0.20 units of water are needed. Similarly, to produce a unit of water, 0.50 units of water and 0.25 units of electricity are required. 0.25 and 0.15 units of labour are required to produce a unit of electricity and a unit of water respectively and 0.15 and 0.10 units of capital are required to produce each unit of electricity and water respectively. The wage rate and interest rate are ₹500 per man-day and 10% respectively. The village needs 1800 kW (kilowatt) electricity and 4400 cubic feet of water for final consumption.

Using matrix algebra, you are required to :

- (i) Write the balancing equations.
- (ii) Test the Hawkins-Simon conditions of viability of the system.
- (iii) Construct the input-output transaction table for the gross output required to satisfy the above final demand.
- (iv) Determine the equilibrium prices.

(v) Compute the total value added. (12)

OR

(b) A manufacturing unit has three departments which produce output for inter-departmental consumption and for the final consumers. The inter-departmental flow of the products of three departments of the manufacturing unit is given as follows :

Producers	Inter-Department Consumption (in tons)			Final Demand (in tons)
	Department X	Department Y	Department Z	
Department X	100	50	0	50
Department Y	0	20	40	40
Department Z	40	70	80	60

Labour required by departments X, Y and Z is 60, 0 and 130 labour days respectively and the wage rate is ₹310 per labour day.

Using matrices, you are required to :

(i) Compute equilibrium prices for the three departments based on the above table.

(ii) Compute total value added.

(iii) If the domestic demand changes by 50, 0 and 20 tons respectively for departments

X, Y and Z respectively, what should be the change in gross output of each department to meet the demand?

- (iv) If the total labour available is 180 labour days, is the new gross output feasible?
(v) Find the change in total value added.

(12)

2. (a) An OTT service operator has a subscriber base of 2,000 subscribers, from each of whom he charges ₹300 per month. In order to increase his subscriber base, he proposes to decrease the rate of monthly subscriptions. His assessment is that for every decrease of ₹1 in the monthly subscription, ten additional subscribers would seek a subscription to the OTT service. You are required to :

- (i) Compute the decrease in service charge and the new number of subscribers that would enable him to maximize his total revenue.
(ii) Find the corresponding new revenue.

(9)

OR

2. (a) A factory follows an economic order quantity system for maintaining stocks of one of its component requirements. The annual demand is for 12,960 units, the cost of placing an order is ₹50, and the component cost is ₹100 per unit. The factory has imputed 10% as the inventory carrying rate.

You are required to determine the following :

- (i) Economic Order Quantity (EOQ) using calculus.
- (ii) Total cost of the EOQ.
- (iii) Optimal number of orders.
- (iv) Optimal interval for placing orders assuming that a year is equivalent to 360 days.
- (v) The discount rate that the supplier should offer to ensure that the purchase of 6480 units is equally attractive as the purchase of EOQ. (9)

- (b) A transport company purchases a pickup truck at a cost of ₹90,00,000. The company estimates that the average cost of capital and average operating cost is a function of x , i.e., the number of days

the truck is used. The salvage value of the truck (in rupees) is expressed by the function $(x) = 7800000 - 6000x$. The operating cost (OC) per day is given by the function, $OC(x) = 8000 + 0.3x$.

Using calculus determine :

- (i) The number of days the truck should be used before replacement if the objective is to minimize the sum of average capital.
- (ii) Average depreciation of the truck per day.
- (iii) Average operating cost of the truck per day. (9)

OR

- (b) A monopolist has the following demand and average cost functions :

$$p = 50 - \frac{x}{3} \text{ and } = 0.5x + 10 + \frac{400}{x}, \text{ where } p \text{ is price}$$

and x is quantity.

- (i) Find what is elasticity of demand when $p = 30$.

- (ii) Determine the level of output at which profit will be maximum.
- (iii) At the level of output computed in part (ii), prove that the elasticity of average cost is equal to the elasticity of total cost minus one. (9)

3. (a) Two firms A and B form a cartel for joint profit maximization. The cost function is $C_1 = 100 + 20x_1 + 2x_1^2$ and $C_2 = 48 + 36x_2 + 2x_2^2$ for firm A and B respectively, where x_1 and x_2 is the output of the firm A and B respectively. The market demand is $x = 50 - 0.5 p$, where $x = x_1 + x_2$. Find how many units of the product should be produced by each firm to maximise the joint profit. Compute the maximum joint profit and the price charged by the firms? (9)

OR

- (a) For the production function: $Q = \left[\frac{0.4}{K} + \frac{0.6}{L} \right]^{-1}$

where L is Labour, K is Capital and Q is production.

You are required to determine :

- (i) Nature of returns to scale.
- (ii) Marginal Rate of Technical Substitution.
- (iii) Elasticity of substitution.
- (iv) Whether the production function satisfies the Euler's theorem. (9)

(b) When the price of a smartphone averaged ₹4000, Sphone Co. Ltd. sold 20 units every week. When the price dropped to an average of ₹1000, 120 smartphones were sold every week. When the price was ₹4000, 200 smartphones were available per week for sale. When the price reached ₹1000, only 50 remained.

You are required to determine :

- (i) Demand and supply functions, assuming both are linear.
- (ii) Consumer's and producer's surplus. (9)

OR

(b) The marginal revenue for a commodity is $MR = 44 - 5x$. The marginal cost is $MC = 3x + 13$, and the cost of producing 75 units is ₹9,500.

Determine :

(i) Profit function.

(ii) Profit or loss from selling 100 units.

(iii) Profit-maximizing output.

(iv) Maximum profit.

(9)

4. Attempt any three :

(a) A piece of land leased with eucalyptus plantations shall start yielding timber and oil from the end of the sixth year onwards for 30 years in all and is expected to yield an annual income of ₹1,20,000. Find the cash price of the land lease if the money is worth 6% per annum.

(b) A person is contemplating installing a photocopying machine at a cost of ₹10,00,000 with a productive life span of 6 years during which the machine is to be maintained by the supplier free of cost. He knows that on average, he can photocopy 3,000

pages per day and charge 50 paise for each page from the customers. The toner cartridge needs replacement after 5,000 pages at a cost of ₹500. A loan can be obtained to buy this machine at 9% p.a. effective, find out whether the purchase of this machine is worthwhile or not, assuming that the machine will be used in a year for 360 days.

- (c) A loan of ₹50,000 due 10 years from now is instead to be paid off by three payments : ₹15,000 now, ₹10,000 in four years and a final payment of ₹10,000 at the end of n years. If the rate of interest is 5% compounded annually, find the value of n.
- (d) A machinery that was installed at a cost of ₹90,000 on a diminishing balance method, stood at ₹30,618 after the end of five years.
- (i) Compute the average rate of depreciation at which the machinery has been depreciated during the period of five years.
- (ii) The rate of depreciation applied in the first three years was 30%, 25% and 20% respectively. Find 'k' if depreciation at the rate of k % per year was applied in the last two years. (6×3)

5. (a) An agriculturist has a hydroponic farm with 1000 acres. He produces spinach, cucumbers, and tomato. Whatever he raises is fully sold in the market. He gets ₹10 for spinach per kg, ₹50 for cucumbers per kg and ₹40 for tomato per kg. The average yield is 5000 kg of spinach per acre, 700 kg of cucumbers per acre and 800 kg of tomato per acre. To produce 200 kg of spinach and cucumber each and to produce 100 kg of tomato, a sum of ₹250 is used for manure. The labour required for each acre to raise the crop is 100 man-days for spinach and 50 man-days each for cucumbers and tomatoes. A total of 2000 man-days of labour at a rate of ₹200 per man-day are available. Formulate this linear programming model to maximize the agriculturist's total profit. Do not solve. (6)

(b) A tablet manufacturing company makes three different kinds of smart tablets viz., 64 GB (X_1), 128 GB (X_2) and 256 GB (X_3). All the types of tablets can be profitably manufactured by the company, but the company's production is constrained by the capacity of three operations that process the units of hardware, i.e., touch screens (S_1), CPUs (S_2) and batteries (S_3) each month. The production manager will choose the

combination of tablets that maximises his profit based on the information available regarding the number of hardware components processed, monthly processing capacity and the profit per tablet in the following table:

Operation	Smart Tablets (units processed)			Monthly processing capacity (units)
	64 GB	128 GB	256 GB	
Touch screen Processing	12	7	9	630
CPU Processing	22	18	16	9504
Battery Processing	2	4	3	198
Profit per tablet	₹8,000	₹4,000	₹9,000	

By using the Simplex technique, the following solution was obtained.

C_j								
C_j	Basic Variable	X_1	X_2	X_3	S_1	S_2	S_3	Quantity
		1	-5/6	0	1/6	0	-1/2	6
		0	55/9	0	-17/9	1	1/3	8380
		0	17/9	1	-1/9	0	2/3	62
	Z_j							
	$C_j - Z_j$							

- (i) Complete the above table. Do you agree that this is the optimum production strategy? If not, improve the solution to get the best production strategy.

- (ii) How many units of each tablet should be produced to maximise profit and how much is the maximum profit?
- (iii) Are all three types of tablets produced? If not, why?
- (iv) Are all three hardware processing operations completely used? If not, how much of which processing operations are unused?
- (v) What are the shadow prices?
- (vi) Is there an alternative product mix which gives the same total profit? Give reasons.
- (vii) How many CPUs are processed to make all the tablets in the optimal solution?
- (viii) If 18 units of 128 GB Tablet are manufactured, what will be the new product mix and new profit? (12)

OR

A manufacturing company makes three products, each of which requires three operations as part of the manufacturing process. The company can sell all the products it can manufacture but its production capability is limited by the capacity of its operation centres. Additional data concerning the company are as follows :

Product	Manufacturing Requirements (Hours/Unit)			Cost (₹)	Selling Price (₹)
	Centre-I	Centre-II	Centre-III		
A	8	4	1	20	50
B	4	6	1	30	50
C	3	2	1	10	22
<i>Hours Available</i>	640	540	100		

- (i) Formulate the above problem as LPP and find the optimum product mix.
- (ii) What are shadow prices?
- (iii) If the production capacity is to be expanded then which operation should be given priority? Why?
- (iv) Which operation, if any, has unutilized capacity?
If so, how many hours?
- (v) Is the solution degenerate? Give reasons.

(18)