Mechanics - BSc Physics Hons, DU

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Semester I Exam-Focused Notes

These notes provide a comprehensive and detailed overview of key concepts in Mechanics, essential for students of BSc Physics Honours at Delhi University.

1. Reference Frames & Newton's Laws

Understanding reference frames is fundamental to applying Newton's Laws of Motion.

1.1 Inertial and Non-Inertial Frames

- Inertial Frame: A reference frame in which Newton's laws of motion are valid without the introduction of fictitious forces. In such a frame, an object at rest remains at rest and an object in motion continues with constant velocity unless acted upon by a net external force. Examples include a frame moving at constant velocity or a frame at rest relative to distant stars.
- Non-Inertial Frame: A reference frame that is accelerating with respect to an inertial frame. In non-inertial frames, Newton's laws appear not to hold true, necessitating the introduction of pseudo forces (also known as fictitious forces or inertial forces) to explain observed accelerations. Common examples include:
  - Centrifugal Force: An apparent outward force experienced by an object moving in a circular path in a rotating frame of reference.
  - Coriolis Force: An apparent force that deflects moving objects sideways in a rotating frame of reference, significant in large-scale atmospheric and oceanic currents.

1.2 Newton's Laws of Motion

These three laws form the cornerstone of classical mechanics:

1. 1st Law (Law of Inertia): An object at rest stays at rest, and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced external force. This law defines inertia.
2. 2nd Law: The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. The direction of the acceleration

is in the direction of the net force. Mathematically, it is expressed as:

$$F = ma$$

This law quantifies the relationship between force, mass, and acceleration.

3. 3rd Law: For every action, there is an equal and opposite reaction. This means that when one object exerts a force on a second object, the second object simultaneously exerts a force equal in magnitude and opposite in direction on the first object.

PYQ: Define an inertial frame with an example.

2. Conservation Laws

Conservation laws are powerful principles in physics, stating that certain physical quantities remain constant in an isolated system.

2.1 Linear Momentum

- Conservation in Isolated Systems: In the absence of external forces, the total linear momentum of a system remains constant.

$$\sum p_i = \text{constant}$$

where p_i is the momentum of each particle in the system.

- Impulse-Momentum Theorem: This theorem relates the change in momentum of an object to the impulse applied to it. Impulse is the integral of force over time.

$$J = \int F dt = \Delta p$$

This is particularly useful in analyzing collisions.

2.2 Angular Momentum

Angular momentum is the rotational analogue of linear momentum.

- Definition: For a single particle, angular momentum **L** is the cross product of its position vector **r** and its linear momentum **p**:

$$L = \mathbf{r} \times \mathbf{p}$$

For a rigid body, $L = I\omega$, where I is the moment of inertia and ω is the angular velocity.

- Relation to Torque: The rate of change of angular momentum is equal to the net torque acting on the system:

$$\tau = \frac{d}{dt}L$$

In an isolated system where net external torque is zero, angular momentum is conserved.

2.3 Work-Energy Theorem

This theorem states that the net work done on an object is equal to the change in its kinetic energy.

- $$W = \Delta K.E. = K_f - K_i$$

  where K_f is the final kinetic energy and K_i is the initial kinetic energy. Kinetic energy is given by $K = \frac{1}{2}mv^2$.

PYQ: Collision problems, especially elastic/inelastic cases.

3. Central Force Motion

Central force motion describes the movement of an object under a force that is always directed towards or away from a fixed point (the center of force). Gravitational force is a classic example of a central force.

3.1 Kepler's Laws

These laws describe the motion of planets around the Sun, assuming the Sun is the center of force:

1. Law of Ellipses: All planets move in elliptical orbits with the Sun at one of the two foci.
2. Law of Equal Areas: A line segment joining a planet and the Sun sweeps out equal areas in equal intervals of time. This law is a direct consequence of the conservation of angular momentum in central force motion.
3. Law of Harmonies: The square of the orbital period (T) of a planet is directly proportional to the cube of the semi-major axis (r) of its orbit.

$$T^2 \propto r^3$$

3.2 Gravitational Force

Newton's law of universal gravitation describes the attractive force between any two objects with mass.

- $$F = G\frac{m_1 m_2}{r^2}$$

  where G is the gravitational constant, m_1 and m_2 are the masses of the two objects, and r is the distance between their centers. This force is a central force.

3.3 Effective Potential

For central force motion, it is often useful to introduce an effective potential energy function that combines the actual potential energy and a term related to angular momentum.

- $$V_{eff}(r) = \frac{L^2}{2mr^2} + V(r)$$

  Here, L is the angular momentum, m is the mass of the orbiting particle, r is the radial distance, and $V(r)$ is the actual potential energy (e.g., gravitational potential energy $V(r) = -\frac{GMm}{r}$). The first term, $\frac{L^2}{2mr^2}$, is known as the centrifugal potential. The effective potential helps in analyzing the radial motion of the particle.

PYQ: Derive the orbit equation under central force.

4. Rotational Dynamics

Rotational dynamics deals with the motion of rigid bodies, where an object rotates about an axis.

4.1 Moment of Inertia (MI)

The moment of inertia is a measure of an object's resistance to changes in its rotational motion (angular acceleration). It depends on the object's mass and how that mass is distributed relative to the axis of rotation.

- For a continuous mass distribution:

  $$I = \int r^2 dm$$

  where r is the perpendicular distance from the mass element dm to the axis of rotation.

- Common Moments of Inertia:
  - Rod (about center): $I = \frac{1}{12}ML^2$
  - Ring (about center): $I = MR^2$
  - Disk (about center): $I = \frac{1}{2}MR^2$
  - Sphere (about center): $I = \frac{2}{5}MR^2$

4.2 Torque and Angular Acceleration

Torque (τ) is the rotational equivalent of force, causing angular acceleration (α).

- $$\tau = I\alpha$$

  This is Newton's second law for rotational motion.

4.3 Parallel and Perpendicular Axis Theorems

4.3.1 Parallel Axis Theorem   This theorem relates the moment of inertia (I) about any axis to the moment of inertia about a parallel axis passing through the center of mass (I_{CM}).

- $$I = I_{CM} + Md^2$$

  where M is the total mass of the object and d is the perpendicular distance between the two parallel axes.

4.3.2 Perpendicular Axis Theorem This theorem applies only to planar objects (objects that lie entirely within a plane). It states that the moment of inertia about an axis perpendicular to the plane (I_z) is the sum of the moments of inertia about two perpendicular axes (I_x and I_y) lying in the plane and intersecting at the point where the perpendicular axis passes.

- For a planar object:
  $$I_z = I_x + I_y$$

PYQ: Use MI theorems to find rotation properties.

5. Rigid Body Motion

Rigid body motion combines translational and rotational motion.

5.1 Rolling Motion

Rolling motion is a common type of rigid body motion where an object rolls without slipping.

- For pure rolling without slipping: The velocity of the point of contact with the surface is instantaneously zero. This implies a direct relationship between translational and rotational velocities:
  $$v = \omega R$$
  where v is the translational velocity of the center of mass, ω is the angular velocity, and R is the radius of the rolling object.
- Total Kinetic Energy: The total kinetic energy of a rolling object is the sum of its translational and rotational kinetic energies:
  $$K_{total} = K_{trans} + K_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

5.2 Conditions for Rolling

- Pure Rolling: Occurs when there is no slipping between the rolling object and the surface. This means $v = \omega R$ and static friction is present.
- Slipping: Occurs when there is relative motion between the contact surfaces, meaning $v \neq \omega R$. Kinetic friction is typically present in this scenario.

5.3 Instantaneous Axis of Rotation

For a rigid body undergoing general planar motion, at any instant, there exists an axis about which the body appears to be purely rotating. This is called the instantaneous axis of rotation. Its position changes over time.

PYQ: Distinguish between pure and impure rolling.

6. Oscillations

Oscillations describe periodic motion, such as that of a pendulum or a mass on a spring.

6.1 Simple Harmonic Motion (SHM)

SHM is a specific type of periodic motion where the restoring force is directly proportional to the displacement from equilibrium and acts in the opposite direction.

- Key Relations:
  - Displacement: $x(t) = A\cos(\omega t + \phi)$
  - Acceleration: $a = -\omega^2 x$
  - Time Period: $T = 2\pi\sqrt{\frac{m}{k}}$
  - Angular Frequency: $\omega = \sqrt{\frac{k}{m}}$

  where A is amplitude, ϕ is phase constant, m is mass, and k is the spring constant.

6.2 Damped Oscillations

In real-world oscillations, energy is gradually lost due to resistive forces (like air resistance). This leads to damped oscillations, where the amplitude decreases over time.

- The differential equation for a damped harmonic oscillator is:

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0$$

  where 2β is the damping coefficient (related to the resistive force) and ω_0 is the natural angular frequency.

6.3 Forced Oscillations & Resonance

When an external periodic force acts on an oscillating system, it undergoes forced oscillations.

- Resonance: A phenomenon where the amplitude of oscillations becomes very large when the driving frequency of the external force approaches the natural frequency of the oscillating system.

PYQ: Solve the SHM differential equation.

7. Systems of Particles

This section deals with the mechanics of multiple interacting particles.

7.1 Centre of Mass (CM)

The center of mass is a unique point representing the average position of all the mass in a system. For translational motion, a system of particles behaves as if all its mass is concentrated at its center of mass.

- For a system of discrete particles:

$$R = \frac{\sum m_i r_i}{\sum m_i}$$

  where m_i is the mass of the i-th particle and r_i is its position vector.

7.2 Two-Body Problem

The two-body problem involves determining the motion of two interacting bodies. It can often be simplified by transforming it into an equivalent one-body problem.

- Reduced Mass: This simplification involves using a concept called reduced mass (μ). The motion of two bodies interacting via a central force can be reduced to the motion of a single particle with mass μ around a fixed center.

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

  where m_1 and m_2 are the masses of the two bodies.

PYQ: Find the CM of a 3-body system, and applications of reduced mass.

8. Constraints & Degrees of Freedom

These concepts are crucial in analytical mechanics (Lagrangian and Hamiltonian mechanics) and for describing the configuration space of a system.

8.1 Types of Constraints

Constraints are restrictions on the motion of a system.

- Holonomic Constraints: These can be expressed as algebraic equations relating the coordinates of the particles (and possibly time). They reduce the number of independent coordinates needed to describe the system. Examples include a bead moving on a fixed wire or a rigid rod.
- Non-holonomic Constraints: These cannot be expressed as simple algebraic equations and often involve inequalities or differential relations that cannot be integrated. Examples include a rolling wheel (no slipping condition) or a gas inside a container.
- Scleronomic Constraints: Constraints that are independent of time.
- Rheonomic Constraints: Constraints that explicitly depend on time.

8.2 Degrees of Freedom (DOF)

The degrees of freedom of a system refer to the minimum number of independent coordinates required to completely describe its configuration.

- For N particles with k holonomic constraints, the number of degrees of freedom is:

$$DOF = 3N - k$$

  where $3N$ represents the total number of coordinates if there were no constraints (3 coordinates for each particle).

PYQ: Classify systems by DOF.

9. PYQ Practice Topics (Frequent)

Regular practice with these topics will be highly beneficial for your exams.

Topic	Type of Question
SHM	Differential equation & graph analysis
Torque & MI	MI derivation using theorems
Central Force	Orbital equation and Kepler's laws
CM and 2-body problem	Definitions and applications
Newton's Laws	Applications in non-inertial frames
Rotational motion	Pure rolling vs slipping
Conservation Laws	Momentum & angular momentum problems
Oscillations	Damped and forced oscillations
Rigid Body Motion	Instantaneous axis of rotation
Constraints	Classification and DOF calculation

10. Important Formulas Summary

Kinematics

- Velocity: $v = \frac{dx}{dt}$
- Acceleration: $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Dynamics

- Newton's 2nd Law: $F = ma$
- Linear Momentum: $p = mv$
- Angular Momentum (for a particle): $L = \mathbf{r} \times \mathbf{p}$
- Work Done: $W = \int \mathbf{F} \cdot d\mathbf{r}$

Energy

- Kinetic Energy: $K = \frac{1}{2}mv^2$
- Gravitational Potential Energy: $U = mgh$ (near Earth's surface)

Rotation

- Angular Velocity: $\omega = \frac{d\theta}{dt}$
- Angular Acceleration: $\alpha = \frac{d\omega}{dt}$
- Rotational Dynamics (Newton's 2nd Law): $\tau = I\alpha$
- Angular Momentum (for rigid body): $L = I\omega$

Oscillations

- Angular Frequency (SHM): $\omega = \sqrt{\frac{k}{m}}$
- Time Period (SHM): $T = \frac{2\pi}{\omega}$
- Frequency (SHM): $f = \frac{1}{T}$

Central Force

- Gravitational Force: $F = \frac{GMm}{r^2}$
- Effective Potential: $V_{eff} = \frac{L^2}{2mr^2} + V(r)$
- Kepler's 3rd Law: $T^2 \propto r^3$

Remember: Practice numerical problems regularly.

Best of luck for your Mechanics exam!