

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-IV

DISCIPLINE SPECIFIC CORE COURSE – 10: SEQUENCES AND SERIES OF FUNCTIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Sequences and Series of Functions	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus DSC-8: Riemann Integration

Learning Objectives: The objective of the course is to introduce:

- The sequences and series of real-valued functions as a generalization to the sequences and series of real numbers.
- The situations under which the process of convergence of a sequence and series of real-valued functions may commute with the processes of calculus while taking differentiation, or integration.
- An important class of series functions (i.e., power series), and the elementary functions-exponential, logarithmic and trigonometric.

Learning Outcomes: This course will enable the students to:

- Learn about Cauchy criterion for uniform convergence and Weierstrass M -test for uniform convergence of series of real-valued functions.
- Know about the constraints for the inter-changeability of differentiation, and integration with infinite sum of a series of functions.
- Handle the convergence of power series and properties of the limit function, including differentiation and integration of power series.
- Appreciate utility of polynomials in the space of continuous functions.

SYLLABUS OF DSC-10

UNIT – I: Sequences of Functions

(18 hours)

Pointwise and uniform convergence of sequence of functions, The uniform norm, Cauchy criterion for uniform convergence, Continuity of the limit function of a sequence of functions, Interchange of the limit and derivative, and the interchange of the limit and integral of a sequence of functions, Bounded convergence theorem.

UNIT – II: Series of Functions

(12 hours)

Pointwise and uniform convergence of series of functions, Theorems on the continuity, differentiability and integrability of the sum function of a series of functions, Cauchy criterion and the Weierstrass M -test for uniform convergence.

UNIT – III: Power Series

(15 hours)

Definition of a power series, Radius of convergence, Absolute convergence (Cauchy-Hadamard theorem), Differentiation and integration of power series, Abel's theorem, Weierstrass's approximation theorem; The exponential, logarithmic and trigonometric functions: Definitions and their basic properties.

Essential Readings

1. Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). Wiley India Edition. Indian Reprint.
2. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

Suggestive Readings

- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
- Denlinger, Charles G. (2011). Elements of Real Analysis. Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

DISCIPLINE SPECIFIC CORE COURSE – 11: MULTIVARIATE CALCULUS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Multivariate Calculus	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus DSC-8: Riemann Integration

Learning Objectives: The primary objective of this course is to introduce:

- The extension of the studies of single variable differential and integral calculus to functions of two or more independent variables.
- The geometry and visualisation of curves and surfaces in two dimensions (plane) and three dimensions (space).
- The techniques of integration to functions of two and three independent variables.
- The applications of multivariate calculus tools to physics, economics, optimization etc.

Learning Outcomes: This course will enable the students to:

- Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double, and triple integral formulations.
- Familiarize with Green's, Stokes' and Gauss divergence theorems, and learn applications.

SYLLABUS OF DSC-11

UNIT – I: Calculus of Functions of Several Variables (18 hours)

Basic concepts, Limits and continuity, Partial derivatives, Tangent planes, Total differential, Differentiability, Chain rules, Directional derivatives and the gradient, Extrema of functions of two variables, Method of Lagrange multipliers with one constraint.

UNIT – II: Double and Triple Integrals (15 hours)

Double integration over rectangular and nonrectangular regions, Double integrals in polar coordinates, Triple integrals over a parallelepiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

UNIT – III: Green's, Stokes' and Gauss Divergence Theorem (12 hours)

Vector field, Divergence and curl, Line integrals and applications to mass and work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

Essential Reading

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. Pearson Education. Indian Reprint.

Suggestive Reading

- Marsden, J. E., Tromba, A., & Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). Indian Reprint.

DISCIPLINE SPECIFIC CORE COURSE – 12: NUMERICAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Numerical Analysis	4	3	0	1	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus

Learning Objectives: The main objective of this course is to introduce:

- Various computational techniques to find approximate value for possible root(s) of algebraic and non-algebraic equations.
- Methods to solve system of linear equations and ordinary differential equations.
- The use of computer algebra system (CAS) by which the numerical problems can be solved both numerically and analytically, and to enhance the problem-solving skills.

Learning Outcomes: This course will enable the students to:

- Learn some numerical methods to find the zeroes of nonlinear functions of a single variable, up to a certain given level of precision.
- Learn Gauss–Jacobi, Gauss–Seidel methods to solve system of linear equations.
- Get aware of using interpolation techniques, for example in finding values of a tabulated function at points which are not part of the table.
- Learn finding numerical solutions of difference equations which are obtained converting differential equations using techniques from calculus.

SYLLABUS OF DSC-12

UNIT – I: Methods for Solving Algebraic and Transcendental Equations (12 hours)

Rate and order of convergence; Bisection method, Method of false position, Fixed point iteration method, Newton's method, and Secant method, their order of convergence and convergence analysis.

UNIT – II: Techniques to Solve Linear Systems and Interpolation (15 hours)

LU decomposition and its applications; Iterative methods: Gauss–Jacobi, Gauss–Seidel methods; Lagrange and Newton interpolation, Piecewise linear interpolation.

UNIT – III: Numerical Differentiation and Integration (18 hours)

First and higher order approximation for the first derivative, Approximation for the second derivative; Numerical integration by closed Newton–Cotes formulae: Trapezoidal rule, Simpson's rule and its error analysis; Euler's method to solve ODE's, Modified Euler method, Runge–Kutta Method (fourth-order).

Essential Reading

1. Bradie, Brian. (2006). A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third impression 2011.

Suggestive Readings

- Gerald, Curtis F., & Wheatley, Patrick O. (2007). Applied Numerical Analysis (7th ed.). Pearson Education. India.
- Jain, M. K., Iyengar, S. R. K., & Jain, R. K. (2012). Numerical Methods for Scientific and Engineering Computation. (6th ed.). New Age International Publisher, India, 2016.

Note: Non programmable scientific calculator may be allowed in the University examination.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab: Use of computer algebra system (CAS) software: Python/SageMath/Mathematica/MATLAB/Maple/Maxima/Scilab etc., for developing the following numerical programs:

1. Bisection method.

2. Newton-Raphson method.
3. Secant method.
4. LU decomposition method.
5. Gauss–Jacobi method.
6. Gauss–Seidel method.
7. Lagrange interpolation.
8. Newton interpolation.
9. Trapezoidal rule.
10. Simpson's rule.
11. Euler's method.
12. Runge–Kutta Method (fourth-order).

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

B.Sc. (Hons) Mathematics, Semester-IV, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(i): BIOMATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Biomathematics	4	3	1	0	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to:

- Develop and analyse the models of the biological phenomenon with emphasis on population growth and predator-prey models.
- Interpret first-order autonomous systems of nonlinear differential equations using the Poincaré phase plane.
- Apply the basic concepts of probability to understand molecular evolution and genetics.

Learning Outcomes: This course will enable the students to:

- To learn and appreciate study of long-term behavior arising naturally in study of mathematical models and their impact on society at large.
- To understand spread of epidemic technically through various models and impact of recurrence phenomena.
- Learn what properties like Chaos and bifurcation means through various examples and their impact in Bio-Sciences.

SYLLABUS OF DSE-2(i)

UNIT – I: Mathematical Modeling for Biological Processes (15 hours)

Formulation a model through data, A continuous population growth model, Long-term behavior and equilibrium states, The Verhulst model for discrete population growth, Administration of drugs, Differential equation of chemical process and predator-prey model (Function response: Types I, II and III).

UNIT – II: Epidemic Model: Formulation and Analysis (15 hours)

Introduction to infectious disease, The SIS, SIR and SEIR models of the spread of an epidemic, Analyzing equilibrium states, Phase plane analysis, Stability of equilibrium points, Classifying the equilibrium state; Local stability, Limit cycles, Poincaré-Bendixson theorem.

UNIT – III: Bifurcation, Chaos and Modeling Molecular Evolution (15 hours)

Bifurcation, Bifurcation of a limit cycle, Discrete bifurcation and period-doubling, Chaos,

Stability of limit cycles, Introduction of the Poincaré plane; Modeling molecular evolution: Matrix models of base substitutions for DNA sequences, Jukes-Cantor and Kimura models, Phylogenetic distances.

Essential Readings

1. Robeva, Raina S., et al. (2008). An Invitation to Biomathematics. Academic press.
2. Jones, D. S., Plank, M. J., & Sleeman, B. D. (2009). Differential Equations and Mathematical Biology (2nd ed.). CRC Press, Taylor & Francis Group.
3. Allman, Elizabeth S., & Rhodes, John A. (2004). Mathematical Models in Biology: An Introduction. Cambridge University Press.

Suggestive Readings

- Linda J. S. Allen (2007). An Introduction to Mathematical Biology. Pearson Education.
- Murray, J. D. (2002). Mathematical Biology: An Introduction (3rd ed.). Springer.
- Shonkwiler, Ronald W., & Herod, James. (2009). Mathematical Biology: An Introduction with Maple and MATLAB (2nd ed.). Springer.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(ii): MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Modeling	4	3	0	1	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: Primary objective of this course is to introduce:

- Mathematical modeling as the representation of a system by a set of mathematical relations or equations.
- Mathematical epidemiological models susceptible-infectious-recovered (SIR) and its variant SEIR (S-Exposed-IR) for the spread of diseases.
- Monte Carlo simulation techniques, and simplex method for solving linear programming problems.

Learning Outcomes: This course will enable the students to:

- Understand the methodology of solving SIR models for disease spread.
- Learn significance of dieting model that provides important insights and guides to a biomedical issue that is of interest to the general public.
- Understand nonlinear systems and phenomena with stability analysis ranges from phase plane analysis to ecological and mechanical systems.

- Use Monte Carlo simulation technique to approximate area under a given curve, and volume under a given surface.

SYLLABUS OF DSE-2(ii)

UNIT – I: Mathematical Epidemiological and Dieting Models (15 hours)

Modeling concepts and examples, Scaling of variables, and approximations of functions; SIR and SEIR models for disease spread: Methodology, Standard and solvable SIR models, Basic reproduction number; Dieting model with analysis and approximate solutions.

UNIT – II: Modeling with Nonlinear Systems and Phenomena (15 hours)

Stability and the phase plane, Almost linear systems; Ecological models: Predators and competitors, Critical points, Oscillating populations, Survival of single species, Peaceful coexistence of two species, Interaction of logistic populations, Wildlife conservation preserve; Nonlinear mechanical systems: Hard and soft spring oscillations, Damped nonlinear vibrations.

UNIT – III: Simulation and Optimization Modeling (15 hours)

Monte Carlo simulating deterministic, and probabilistic behavior, Generating random numbers; Linear programming model: Geometric and algebraic solutions, Simplex method and its tableau format, Sensitivity analysis.

Essential Readings

1. Mickens, Ronald E. (2022). Mathematical Modelling with Differential Equations. CRC Press, Taylor & Francis Group.
2. Edwards, C. Henry, Penney, David E., & Calvis, David T. (2023). Differential Equations and Boundary Value Problems: Computing and Modeling (6th ed.). Pearson.
3. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). Brooks/Cole, Cengage Learning India Pvt. Ltd.

Suggestive Readings

- Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modeling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press. Taylor & Francis Group.
- Ross, Shepley L. (2014). Differential Equations (3rd ed.). Wiley India Pvt. Ltd.
- Simmons, George F. (2017). Differential Equations with Applications and Historical Notes (3rd ed.). CRC Press. Taylor & Francis Group.

Practical (30 hours)- Practical work to be performed in Computer Lab: Modeling of the following problems using: R/Python/SageMath/Mathematica/MATLAB/Maxima/Scilab etc.

1. a) Simulation of SIR model and its variants using some initial parameter values, and finding basic reproduction number for analysis.
b) Analysis of the dieting process, which includes both body-mass loss and gain.
2. Nonlinear Systems and Phenomena.
a) Plot phase plane portraits and solutions of first-order equations.
b) Obtain interesting and complicated phase portraits for almost linear systems.

- c) Discuss large wildlife conservation preserve model and obtain (i) The period of oscillation of the rabbit and fox populations, (ii) The maximum and minimum numbers of rabbits and foxes.
 - d) Discuss the Rayleigh and van der Pol models.
3. (i) Random number generation and then use it for the following:
 - a) Simulate area under a given curve.
 - b) Simulate volume under a given surface.
- (ii) [2] Chapter 7 (Projects 7.4 and 7.5).

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(iii): MECHANICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mechanics	4	3	1	0	Class XII pass with Mathematics	DSC-5: Calculus DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to:

- Starting Newtonian laws, learning various technical notions which explains various states of motion under given forces.
- Deals with the kinematics and kinetics of the rectilinear and planar motions of a particle including constrained oscillatory motions of particles, projectiles, and planetary orbits.
- Understand hydrostatic pressure and thrust on plane surfaces.

Learning Outcomes: This course will enable the students to:

- Understand necessary conditions for the equilibrium of particles acted upon by various forces and learn the principle of virtual work for a system of coplanar forces.
- Apply the concepts of center of gravity, laws of static and kinetic friction.
- Learn that a particle moving under a central force describes a plane curve and know the Kepler's laws of the planetary motions.
- Evaluate the hydrostatic pressure at any given depth in a heavy homogeneous liquid at rest under gravity.

SYLLABUS OF DSE-2(iii)

UNIT – I: Statics

(15 hours)

Fundamental laws of Newtonian mechanics, Law of parallelogram of forces, Equilibrium of a particle, Lamy's theorem, Equilibrium of a system of particles, External and internal forces, Couples, Reduction of a plane force system, Work, Principle of virtual work, Potential energy and conservative field, Mass centers, Centers of gravity, Friction.

UNIT – II: Dynamics**(18 hours)**

Kinematics of a particle, Motion of a particle, Motion of a system, Principle of linear momentum, Motion of mass center, Principle of angular momentum, Motion relative to mass center, Principle of energy, D'Alembert's principle; Moving frames of reference, Frames of reference with uniform translational velocity, Frames of reference with constant angular velocity; Applications in plane dynamics- Motion of a projectile, Harmonic oscillators, General motion under central forces, Planetary orbits.

UNIT – III: Hydrostatics**(12 hours)**

Shearing stress, Pressure, Perfect fluid, Pressure at a point in a fluid, Transmissibility of liquid pressure, Compression, Specific gravity, Pressure of heavy fluid- Pressure at all points in a horizontal plane, Surface of equal density; Thrust on plane surfaces.

Essential Readings

1. Synge, J. L., & Griffith, B. A. (2017). Principles of Mechanics (3rd ed.). McGraw-Hill Education. Indian Reprint.
2. Ramsey, A. S. (2017). Hydrostatics. Cambridge University Press. Indian Reprint.

Suggestive Readings

- Roberts, A. P. (2003). Statics and Dynamics with Background Mathematics. Cambridge University Press.
- Ramsey, A. S. (1985). Statics (2nd ed.). Cambridge University Press.