1. (a) A sample of 26 offshore oil workers took part in a simulated escape exercise, resulting in the following data on time (sec) to complete the escape. Construct a stem-and-leaf display and comment on any interesting features of the display.

P.T.O.

389	356	359	363	375	424	325	394	402	373
373	370	364	366	364	325	339	393	392	369
374	359	356	403	334	397				

(b) The following data gives the sample of total nitrogen loads (kg N/day) from a particular Chesapeake Bay location. Calculate the median, upper fourth (third quartile) and lower fourth (first quartile).

9.69	13.16	17.09	18.12	23.70	24.07	24.29	26.43	30.75	31.54

(c) Data was collected for the sale of homes for homes in a particular city. The following data gives the sale amounts of homes (in 1000's of \$) that were sold in the previous month.

590	815	575	608	350	1285	408	540	555	679

(i) Calculate the sample variance and standard deviation.

(ii) The evaluator realized that he had by mistake forgotten to multiply each observation by 5 while collecting data. What would be the resulting values of the sample variance and standard deviation after the correction? Answer without reperforming the calculations.

2. (a) A computer consulting firm presently has bids out on three projects. Let A_k = {awarded project k}, for k = 1,2,3, and suppose that $P(A_1) = .22$, $P(A_2) = .25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = .05$, $P(A_2 \cap A_3) = .07$, $P(A_1 \cap A_2 \cap A_3) = .01$. Compute the $P(A_1' \cap A_2')$ and $P(A_1 \cup A_2 \cup A_3)$.

(b) State Baye's Theorem. An individual has three different email accounts. 70%, of her messages come into account A1, whereas 20% come into account A2 and the remaining 10% into account A3. Of the messages into account A1, only 1% are spam, whereas the corresponding percentages for accounts A2 and A3 are 2% and 5%, respectively. A randomly selected mail is found to be a spam. What is the probability that it came in account A2?

(c) Components of a certain type are shipped to a supplier in batches of ten. Suppose that 50% of all such batches contain no defective components, 30% contain one defective component, and 20% contain two defective components. Two components from a batch are randomly selected and tested. What are the probabilities associated with 0,1, and 2 defective components being in the batch under the condition that one of the two tested components is defective?

3. (a) Starting at a fixed time, the gender of each new born child is observed at a certain hospital until a boy (B) is born. Let p = P(boy is born) = P(B) and assume that the successive births are independent. Let the random variable

X = the number of births upto and including that of the first boy

(i) Find the probability mass function (pmf) of X.

(ii) Determine the cumulative mass function (cmf) of X.

(b) Let the random variable be defined as :

X = 1 if a randomly selected vehicle passes an emission test and X = 0 otherwise

Assume that the probability mass function (pmf) of this variable is: p(1) = p and p(0) = 1 − p.

(i) Name the random variable.

(ii) Compute $E(X^2)$

(iii) Show that V(X) = p(1 − p)

(c) For any random variable X, let E(X) = 5 and E[X(X − 1)] = 27.5. Compute :

(i) $E(X^2)$

(ii) V(X)

(iii) V(2X + 3)

4. (a) The error involved in making a certain measurement is a continuous random variable with probability density function as follows

$$f(x) = \begin{cases} .09375(4 - x^2) & -2 \le x \le 2 \\ 0 & otherwise \end{cases}$$

(i) Obtain the cumulative density function F(x) of X

(ii) Compute P(−1 < X < 1)

(iii) Compute E[X]

(iv) Compute V[X]

(b) Suppose that 25% of all students at a large public University receive financial aid. Let X be the number of students in a random sample of size 50 who receive financial aid. Using normal approximations find the approximate probabilities that

(i) at most 10 students receive aid

(ii) between 5 and 15 (inclusive) of the selected students receive aid.

(c) Define exponential distribution. Find the mean and standard deviation of an exponentially distributed random variable X. Are they equal?

5. (a) If a publisher of non technical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is 0.005 and errors are independent from page to page, what is the probability that one of its 400 page novels will contain

(i) Exactly one page with errors?

(ii) At most three page with errors?

(b) An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let the random variable be defined as :

X = the number of months between successive payments with the cumulative density function(cdf) as follows :

$$F(x) = \begin{cases} 0 & x < 1 \\ .30 & 1 \le x < 3 \\ .40 & 3 \le x < 4 \\ .45 & 4 \le x < 6 \\ .60 & 6 \le x < 12 \\ 1 & 12 \le x \end{cases}$$

(i) Determine the probability mass function of X?

(ii) Using just the cdf, compute $P(3 \le X \le 6)$.

(c) The weight distribution of parcels sent in a certain manner is normal with mean value of 12 lb and standard deviation 3.5 lb. The parcel service wishes to establish a weight value c beyond which there will be a surcharge. What value of c is such that 99% of all parcels are under the surcharge weight?

6. (a) The Turbine Oil Oxidation Test (TOST) and Rotating Bomb Oxidation Test (RBOT) are two different procedures for evaluating the oxidation stability of steam turbine oils. The following table gives these observations on x = TOST time (in hours) and y = RBOT time (in minutes) for 10 oil specimens

X	4200	3600	3750	3675	4050	2770	4870	4500	3450	2700
y	370	340	375	310	350	200	400	375	285	225

(i) Calculate the value of the sample correlation coefficient. Based on this value, how would you describe the nature of relationship between the two variables?

(ii) If RBOT is also measured in hours, what happens to the value of r? Why?

(b) The following table gives the data on x = rainfull volume (m^3) and y = runoff volume (m^3) for a particular location.

x	5	12	14	17	23	30	40	47	55	67
y	4	10	13	15	15	25	27	46	38	46

(i) Determine the equation of the estimated regression line using the principle of least square.

(ii) Estimate the runoff volume when the rainfull volume is 70.

(c) The amount of a particular impurity in a batch of a certain chemical product is a random variable with mean 4.0 g and standard deviation 1.5 g. If $\bar{X}$ is the sample mean impurity for a random sample of 50 batches.

(i) Where is the sampling distribution of $\bar{X}$ centered?

(ii) What is the standard deviation of the $\bar{X}$ distribution?

(iii) What is the probability that the sample average amount of impurity $\bar{X}$ is between 3.5 and 3.8 g?