1. (a) For each of the following utility functions, determine whether the underlying preferences are monotonic and convex.

(i) $u(x,y) = \min[(3x+2y),(2x+5y)]$

(ii) $u(x,y) = x^2 + y^2$

(iii) $u(x,y) = y/(100-x)$.

(iv) $u(x,y) = -\max[x,y]$

(b) Assume that due to the Russo-Ukraine war, the price of petrol (p) increases. The government decides to impose a specific tax (t) on the price of petrol. Due to protests by vehicle owners, the government simultaneously provides a lumpsum rebate based on an agent's post-tax consumption of petrol. Show that the agent is worse off due to the tax-rebate programme. Draw a suitable diagram.

(c) In a 2 commodity world, Rehmat's utility function $u(x,y) = \ln x + y$ (ln stands for 'natural log'). If (P_x, P_y, M) represents the prices-income vector, find the Engel curve for 'y'. Show that the price consumption curve is parallel to the 'x' axis, if the price of x falls. (8+6+6)

(ii) $u(x,y) = x^2 + y^2$

(iii) $u(x,y) = y/(100-x)$.

(iv) $u(x,y) = -\max[x,y]$

2. (a) Assume that the Delhi government wants to decrease the consumption of water (W); it creates an incentive to do so by providing water at low prices if its consumption is low. Zorawar consumes water (W) and sugar (S). His income is Rs.1000 and the price of sugar is Rs.5 per kg.

If water is priced at Rs.2 per unit for $W \leq 50$ and a quantity tax of Rs.2 per unit is imposed for $W > 50$, draw his budget constraint with 'W' on the horizontal axis. Find the slopes of both segments of the budget constraint and calculate the co-ordinates of both x and y intercepts and the kink.

(b) Chahat, a modern day entrepreneur, produces tomatoes (x) and mushrooms(y). She also consumes these vegetables, and her utility function is represented by $u(x,y) = \text{Min}[x,y]$. She produces 30 kg of 'x' and 10 kg of 'y', given that the market price vector is $(P_x,P_y) = (5,5)$. If P_y rises to 15 and P_x remains unchanged, calculate the substitution , income and the endowment income effects associated with this price change.

(c) A rational utility maximising agent , with strictly convex preferences (who buys and sells 'x' and 'y'), is a net supplier of 'x'. Assume that the price of 'x', a normal good, declines. If he remains a net seller, what is the impact on his consumption (of 'x') and utility? Draw a diagram and use the Slutsky equation.

(d) A risk loving agent has Rs.17 as her income and is contemplating a gamble that gives her Rs.25 with a probability of 0.5 and Rs.9 with a probability of 0.5. Let her utility function be $u = c^2$ where 'c' represents her income. What must be the relationship between the expected utility of the gamble and the utility from the expected value of the gamble? Calculate and draw a diagram.

(6+6+4+4)

3. (a) Ms. Anupriya Jolly has 'T' hours at her disposal and her well behaved utility function is defined over leisure (R) and a composite consumption good

(c). Assume that the wage rate is 'w' and her only source of income is her work. Further, let the price of the composite consumption good be 1.

Use the Slutsky Decomposition Equation (without drawing a diagram or proving the Slutsky Equation) to answer (ai) and (aii) :

(i) Is the labour supply schedule upward sloping if leisure is an inferior good?

(ii) Is the leisure demand schedule downward sloping when $u(R,c) = \min[R,c]$ and leisure is a normal good?

(b) Assume that Anupriya's preferences are convex (over 'R' and 'c') and her wage rate increases from w_0 to w_1. Draw a diagram (with 'R' on the horizontal axis) splitting the impact of the wage increase into its substitution and income effects. Hence, demonstrate that an increase in the overtime wage rate always results in an increase (non-decrease) in her labour supply.

(c) Let Anupriya's utility function be represented by $u(R,c) = R^{0.5} c^{0.5}$. She faces a wage rate of Rs.4/ hour and the price of 'c' remains at 1. Work is the only source of her income and her endowment of time is 80 hours:

(i) Find the optimal values of (R,c).

(ii) Assume that Anupriya is paid overtime wages at the rate of Rs.6/ hour (for the hours she puts in beyond her initial optimum labour supply at Rs 4/hour). Calculate her new optimum labour supply, using her overtime budget constraint. (4+6+10)

4. (a) Assume that the contingent consumption bundle of a risk averse economic agent is (c_1, c_2) in the 'bad' and 'good' states of the world respectively. Assume that the probability of the 'bad' state is π. To avoid this contingent consumption bundle, an insurance company offers him insurance at the rate of 'γ' per unit of insurance cover. Derive the

first order conditions for constrained expected utility maximization. Show that, under 'fair' insurance, the agent shall buy 'full' insurance cover, that is $c_1^* = c_2^*$.

[(c_1^*, c_2^*) denote the post insurance consumption bundle in the 'bad' and 'good' states respectively].

(b) Let unfair insurance be defined as positive expected profit for the insurance company. Show that this implies $\gamma > \pi$. Use the first order conditions for constrained expected utility maximization in this case to prove that the agent shall not buy full insurance, that is $c_1^* < c_2^*$ [Symbols have the same interpretation as in 4(a)].

(c) Zafar, a risk averse paddy farmer from Kerala, faces two states of the world: a 'bad' state (floods) with a probability of 0.5 and a 'good' state (normal monsoons) with a probability of 0.5. His contingent consumption bundle in the 2 states is (6400,10000). He buys insurance from 'Annapurna Insurance Company' at $\gamma = 0.5$, where γ is the insurance premium charged by the insurance company per unit (Rs1) of benefit sought. Let the insurance company's expected profits be zero and Zafar's utility function be $u = c^{1/2}$.

If Zafar maximises expected utility subject to his insurance budget constraint, compute his optimal consumption bundle. How much insurance cover does he buy? Why can it be termed 'full insurance'? (6+6+8)

5. (a) Two firms, Nike and Adidas, have technologies characterised by the following production functions:

$$y_1 = (L^{1/2} + K^{1/2}) \text{ and}$$

$$y_2 = (L^2 + K^2)$$

(i) Determine the economies of scale exhibited by the two production functions.

(ii) For the two production functions, find the associated long run cost functions by deriving the conditional input demand functions for labour and capital.

(iii) Derive the long run supply function of the firm 1, for the production function $y_1 = (L^{1/2} + K^{1/2})$.

(b) If the technology employed by a firm exhibits constant returns to scale or increasing returns to scale, show that the output supply functions do not exist. Draw suitable diagrams for both cases.

(1+6+4+4)

6. (a) Assume a short run production function $y = L^{1/2} K^{1/3}$ where the quantity of capital employed is fixed at K_0. Further the factor prices of labour and capital are specified as (w,v). 'AVC' denotes the average variable cost.

Derive the short run input demand functions, the short run cost function and the short run supply function (assume $p \geq$ Min AVC, where 'p' is the price of output).

(b) For the production function in 6(a), let $(w, v, K_0) = (1,2,64)$,

(i) Derive the short run supply function (assume $p \geq \text{Min AVC}$). Draw an appropriate diagram (for the parameter values in this question).

(ii) What is the total profit of the firm in the short run at $p = 2$? Should the firm continue to produce? Explain.

(c) Consider a profit maximising firm with a decreasing return to scale production function, $f(L, K)$, and input K fixed at K_0. What is the impact on the conditional factor demand for labour, L^*, and profit in each of the following situations:

(i) w, the price of labour, rises.

(ii) v, the price of capital, falls.

(iii) p, the price of the output, rises.

(7+5+3)

7. (a) (i) In a single input, single output production model, why are 'real world' production functions often thought to be convex for low levels of input and concave for high levels of input ?What does the production function imply about the cost curve? Draw a diagram.

(ii) In a two input production function, assume two efficient production techniques that produce the same level of output 'y'. The first technique uses (L,K) = (6,2), while the second technique uses (L,K) = (2,6). If the technology is strictly convex, show that any weighted average [where the weights lie in the open interval (0,1)] of these input bundles producing 'y' is not efficient.

(b) Consider the production function $y = L^{2/3} + (1/3)L$. (assume $L \geq 1$). Let the output price, P=9. Find the firm's input demand curve, $L^*(w)$.

(c) The inverse production with one input and 2 outputs is $L = y_1^2 + y_2^2 + y_1 y_2$. Assume that the price of the input labour (L) is w=1.

(i) Find the firm's total cost curve $C(y_1, y_2)$ and the marginal cost curves $MC_1(y_1)$ and $MC_2(y_2)$.

(ii) Find the firm's supply curves, $y_1^*(P_1, P_2)$ and $y_2^*(P_1, P_2)$ subject to the non-negative profit condition. [(3+3)+4+(3+2)]