Questions

Q1. Deviations & Sample Statistics

Given the deviations from the mean for four observations: $d_1 = 0.3$, $d_2 = 0.9$, $d_3 = 1.0$, $d_4 = 1.3$, and knowing that $\sum_{i=1}^{n} d_i = 0$:

(a) Calculate the fifth deviation d_5.

(b) Compute the sample variance and standard deviation.

(c) Determine the new variance if all observations are multiplied by 2.

Hint: Use $\sum d_i^2 = 15.84$ for part (b).

Q2. Probability & Conditional Probability

In a disease testing scenario:

- Carrier probability: $P(C) = 0.01$
- Test accuracy: $P(+|C) = 0.9$, $P(+|\neg C) = 0.05$

(a) Find the probability that two independent tests give the *same* result (both + or both −).

(b) Calculate $P(C \mid ++)$ (probability of being a carrier given both tests are positive).

Q3. Normal Distribution & Percentiles

Steel strength follows $N(\mu = 43, \sigma = 4.5)$:

(a) Find the 25th percentile of steel strength.

(b) What is the probability that at most 3 out of 15 randomly tested samples have strength < 43?

Q4. Quartiles & Outliers

For the datasets:

- **Group I:** 10, 10, 10, 15, 35, 75, 90, 95, 100, 175, 420, 490, 515, 515, 790
- **Group II:** 0, 5, 5, 15, 30, 45, 50, 50, 50, 60, 75, 110, 140, 240, 330

(a) Calculate Q_1, median, and Q_3 for both groups.

(b) Identify outliers using the 1.5×IQR rule.

Q5. Exponential Distribution

A PC's lifetime (years) follows an exponential distribution with $\lambda = 0.25$:

(a) Find $P(X > 6 \mid X > 2)$.

(b) Explain how the memoryless property applies here.

Bonus: Central Limit Theorem (CLT)

Weekly sales data has $\mu = 50$, $\sigma = 16$. For a 36-week period:

1. Find $P(1728 < \text{Total Sales} < 1872)$.
2. Why is CLT less reliable for $n = 12$?

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