1. Maximize Profits : $\pi = 64x - 2x^2 + 96y - 4y^2 - 13$

subject to the production constraint: $x + y \le 36$

लाभ अधिकतमकरण: $\pi = 64x - 2x^2 + 96y - 4y^2 - 13$

उत्पादन बाधा: $x + y \le 36$ के अधीन होते हैं।

2. Suppose that an economy's investment flow every year is $I(t) = 10t^{\frac{1}{2}}$. Let $K(t)$ represent the current stock of capital at time t. If $K(0) = K_0$ and there is

no depreciation, find the level of capital stock five years from now. What happens to the capital stock after five years if investment flow is the same as before and capital stock depreciates every year by 500?

3. (a) Show that $u_1 = e^{2t}$ and $u_2 = te^{2t}$ both solve the $\ddot{x} - 4\dot{x} + 4x = 0$. What is the general solution?

(b) Find the general solutions of the following differential equation and determine if it is stable or not :

$$\dot{y} + \frac{1}{2}y = \frac{3}{4}$$

4. Two firms share the market for a product. Firm 1's output is x; firm 2's output is y. The two reaction functions of the firms are

$$x_{t+1} + \beta y_t = b; \qquad \beta \neq 1$$

$$y_{t+1} + \alpha x_t = b; \qquad \alpha \neq 1$$

Derive and solve the second-order difference equation for x implied by the above model. Also, discuss the conditions under which the steady state is stable.

5. Solve the consumer demand problem

$\max x^a + y^a$ subject to $px + qy = m$ where $a \in (0,1)$ and $x > 0$, $y > 0$

Also, check the second order sufficient conditions.

6. A central planner controls an economy with two sectors, producing outputs y_1 and y_2. Prices of the goods are $p_1 = 1$ and $p_2 = 2$ respectively. The planner wishes to maximise the national output at these given prices. Labour is the only input and it is available in a fixed total amount $L_0 = 1000$. The production functions in the two sectors are

$$y_1 = 100L_1^{\frac{1}{2}} \quad \text{and} \quad y_2 = 50L_2^{\frac{1}{2}}$$

(a) Calculate the optimal labour allocations, outputs and the shadow wage rate.

(b) Write the value function and Find $\frac{\partial Y^*}{\partial p_1}$, $\frac{\partial Y^*}{\partial p_2}$ and, $\frac{\partial Y^*}{\partial L_0}$ using the envelope theorem.

7. A sports student is trying to decide on the lowest-cost diet. Following is the data for two types of eatables (sprouts and Banana) and three types of nutrients (Calcium, protein and Vitamin)

	Nutritional Content (mg/ unit)			Cost (Rs Per Unit)
	Calcium	Protein	Vitamin	
Sprouts	5	4	2	6
Banana	7	2	1	3.5

The requirement per day of calcium, proteins and vitamins is 8, 15 and 3 respectively. The problem is to find how much of each eatable to consume per day to get the required amount per day of each nutrient at a minimal cost.

(a) Write the linear programming problem and its dual and solve them.

(b) If the sports coach increases the calcium requirement from 8 to 9 and reduces the protein requirement from 15 to 14, then what will be the change in the cost?

8. Suppose that a fish population grows according to the function, $g(y) = 2y\left(1 - \frac{y}{2}\right)$ where y is the stock of fish. If the fish population is harvested by the fishing industry at a constant rate of ¾, write down the equation for the rate of change in the stock of fish. Also, draw the phase diagram to show the change in the stock of the population as a function of the stock of fish and discuss the nature of the steady states.

9. An electric company is setting up a power plant and it has to plan its capacity. The demand for power in period 1 and period 2 are q_1 and q_2 respectively. Assume that a regulatory authority fixes the corresponding prices for period 1 and period 2 to be Rs. 1 per unit and Rs. 3 per unit respectively. The total operating cost over the two periods is $q_1^2 + q_2^2$. The cost of maintaining output capacity k cost is k^2 which is paid only once and is used in both periods.

(a) Write the producer's total profit function with all constraints.

10. Consider the LP problem

$$x + y \leq 3$$

$$2x + y - z \leq 1$$

$$\max\ 3x + 2y \text{ subject to } \begin{cases} x + 2y - z \leq 1 \\ x \geq 0 \\ y \geq 0 \\ z \geq 0 \end{cases}$$

(a) Assuming z to be a fixed number, solve the problem for $z = 0$ and $z = 4$.

(b) Solve the problem for any fixed value of $z \in [0, \infty)$. The maximal value of the criterion function $3x + 2y$ becomes a function of z. Find this function and maximise it.