Q1. (a) Reaction Time Deviations

i. Fifth Deviation

The sum of deviations from the mean must be zero:

$$0.3 + 0.9 + 1.0 + 1.3 + d_5 = 0$$

$$3.5 + d_5 = 0$$

$$d_5 = -3.5$$

ii. Sample Standard Deviation

Calculate variance:

$$s^2 = \frac{0.3^2 + 0.9^2 + 1.0^2 + 1.3^2 + (-3.5)^2}{5-1} = \frac{15.84}{4} = 3.96$$

Standard deviation:

$$s = \sqrt{3.96} \approx 1.99$$

When multiplied by 2: New variance $= 2^2 \times 3.96 = 15.84$

iii. Degrees of Freedom

Degrees of freedom = n-1 = 4 (not equal to n=5 because one parameter, the mean, is estimated from the data).

Q1. (b) Class Interval Data

i. Median Class

Cumulative relative frequencies:

- 0-5: 0.177
- 5-10: 0.343
- 10-15: 0.518 (first exceeding 0.5)

Median lies in 10-15 class.

ii. Density Calculation

Density = Relative frequency/Class width:

Class	Calculation	Density
0-5	0.177/5	0.0354
5-10	0.166/5	0.0332
10-15	0.175/5	0.0350
15-20	0.136/5	0.0272
20-30	0.194/10	0.0194
30-40	0.078/10	0.0078
40-60	0.044/20	0.0022
60-90	0.030/30	0.0010

iii. Proportion between 25-45

$$25 - 30 : (30 - 25)/10 \times 0.194 = 0.097$$
$$30 - 40 : 0.078$$
$$40 - 45 : (45 - 40)/20 \times 0.044 = 0.011$$
$$\text{Total} = 0.097 + 0.078 + 0.011 = 0.186$$

Q2. (a) College Attendance Probability

i. At Least One Comes

Given $P(\neg A_1 \cap \neg A_2 \cap \neg A_3) = 0.06$
$P(\text{at least one}) = 1 - 0.06 = 0.94$

ii. $P(A_1 \cup A_2)$

Since A_1 and A_2 are independent:

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1)P(A_2)$$

Given $P(A_2) = 4P(A_1)$ and $P(A_1 \cap A_2 \cap A_3) = 0.04$

$$0.25P(A_1)P(A_2) = 0.04 \Rightarrow P(A_1) = 0.2, P(A_2) = 0.8$$

Thus:

$$P(A_1 \cup A_2) = 0.2 + 0.8 - (0.2)(0.8) = 0.84$$

iii. $P(A_1 \cap A_3|A_2)$

$$P(A_1 \cap A_3|A_2) = P(A_1)P(A_3|A_1 \cap A_2) = 0.2 \times 0.25 = 0.05$$

Q2. (b) Bookstore Revenue

i. PMF of Y

Y = 12X - 6×3 + 2(3-X) = 10X - 12 Possible Y values: -12, -2, 8, 18 PMF:

Y	P(Y)
-12	0.1
-2	0.2
8	0.2
18	0.5

ii. E(X) and Var(X)

$$E(X) = 00.1 + 10.2 + 20.2 + 30.5 = 2.1$$
$$E(X^2) = 00.1 + 10.2 + 40.2 + 90.5 = 5.5$$
$$Var(X) = 5.5 - 2.1^2 = 1.09$$

iii. E(Y) and Var(Y)

$$E(Y) = 10E(X) - 12 = 9$$
$$Var(Y) = 100Var(X) = 109$$

iv. P(Y8)

$$P(Y = 8) + P(Y = 18) = 0.2 + 0.5 = 0.7$$

Q3. (a) Disease Testing

i. Tree Diagram

```
          Carrier (0.01)
         /       \
      Pos(0.9)   Neg(0.1)
     /     \     /      \
Pos(0.9) Neg(0.1) Pos(0.05) Neg(0.95)
```

ii. Same Result Probability

$$\begin{aligned}
&P(\text{both positive}|\text{carrier}) + P(\text{both negative}|\text{carrier}) \\
&+ P(\text{both positive}|\text{non-carrier}) + P(\text{both negative}|\text{non-carrier}) \\
&= 0.01(0.9^2 + 0.1^2) + 0.99(0.05^2 + 0.95^2) \\
&= 0.01 \times 0.82 + 0.99 \times 0.905 = 0.90415
\end{aligned}$$

iii. $P(\textbf{Carrier}|\textbf{both positive})$

$$\frac{0.9^2 \times 0.01}{0.01 \times 0.9^2 + 0.99 \times 0.05^2} \approx \frac{0.0081}{0.010575} \approx 0.766$$

Q3. (b) Random Variable CDF

Given:

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{(x-2)}{2}, & 2 \leq x \leq 4 \\ 1, & x \geq 4 \end{cases}$$

i. PDF

$$f(x) = \frac{dF}{dx} = \begin{cases} \frac{1}{2}, & 2 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

ii. $P(2/3 < X < 3)$

$$F(3) - F(2/3) = \frac{3-2}{2} - 0 = 0.5$$

iii. $P(X > 3.5)$

$$1 - F(3.5) = 1 - \frac{3.5-2}{2} = 0.25$$

iv. 60th Percentile

$$F(x) = 0.6 \Rightarrow \frac{x-2}{2} = 0.6 \Rightarrow x = 3.2$$

v. $P(X = 3)$

0 (since X is continuous)

Q4. (a) Weather Probabilities

i. Rainy → Rainy → Sunny → Rainy

$$0.8 \times 0.2 \times 0.6 = 0.096$$

ii. Rainy → Sunny → Sunny → Rainy

$$0.2 \times 0.4 \times 0.6 = 0.048$$

iii. No sunny days for 3 days

All rainy: $0.8^3 = 0.512$

iv. Rain two days later

Transition matrix P^2 where:

$$P = \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.76 & 0.24 \\ 0.72 & 0.28 \end{bmatrix}$$

From rainy: 0.76

Q4. (b) Project Completion

i. Expected Cost

$$E[\text{cost}] = \int_0^{15} 5(15-t)0.1e^{-0.1t}dt + \int_{15}^{\infty} 10(t-15)0.1e^{-0.1t}dt$$

After calculation 13.50

ii. $E(X)$ for Geometric

$$E(X) = \frac{1}{p}$$

Q5. (a) PC Lifetime

$$P(T > 4|T > 2) = P(T > 2) = e^{-0.25\times 2} \approx 0.6065$$

Q5. (b) Defective Components

i. $P(X \leq 3)$

Hypergeometric approximation 0.647

ii. $E(X)$ and $V(X)$

$$E(X) = 50 \times \frac{300}{10000} = 1.5$$
$$V(X) \approx 1.432$$

iii. Poisson Approximation

$= 1.5$, $P(X \leq 3) \approx 0.934$

Q5. (c) Candy Box Weights

i. Probability Calculations

$$P(\text{cordials} > 0.5) = P(X + Y < 0.5) \approx 0.1125$$
$$P(Y < 1/8|X = 3/4) \approx 0.3125$$

ii. Independence Check

X and Y are not independent since $f(x, y) \neq f_X(x)f_Y(y)$

Q6. (a) Grading Distribution

Grade	Probability
A $(Z > 1)$	15.87%
B $(0 < Z \leq 1)$	34.13%
C $(-1 \leq Z \leq 0)$	34.13%
D $(-2 \leq Z < -1)$	13.59%
F $(Z < -2)$	2.28%

Q6. (b) Typesetter Errors

i. Variances

$$\text{Var}(X_1) = 2.6, \text{Var}(X_2) = 3.8$$

ii. P(no errors)

$$0.6 \times e^{-2.6} + 0.4 \times e^{-3.8} \approx 0.0532$$

iii. P(Typesetter 2 — no errors)

$$\frac{0.022 \times 0.4}{0.0532} \approx 0.1654$$

Q6. (c) Coin Toss

i. Joint Distribution

A	B	P(A,B)
0	0	0.49
0	1	0.21
1	1	0.21
1	2	0.09

ii. Independence Check

Not independent since $P(A=1, B=1) \neq P(A=1)P(B=1)$

Q7. (a) Faulty Switches

Poisson approximation (= 3):

$$P(4 \leq X \leq 8) \approx 0.3489$$

Q7. (b) Steel Strength

i. 25th Percentile

$$x \approx 39.967 \text{ ksi}$$

ii. 90th Percentile

$$x \approx 48.769 \text{ ksi}$$

iii. 99% Interval

$$c \approx 11.592 \text{ ksi}$$

iv. $P($at most 3 $< 43)$

$$P(Y \leq 3) \approx 0.0904 \text{ for } Y \sim \text{Bin}(15, 0.5)$$

Q7. (c) Normal Variables

i. Corr(U,V)

$$\text{Corr}(U, V) = 0 \text{ (since X,Y independent)}$$

ii. Corr(X,W) and Corr(Y,W)

$$\mathrm{Corr}(X, W) = 0.6$$
$$\mathrm{Corr}(Y, W) = 0.8$$