# Questions

**Q1. Implicit Differentiation & Tangent Lines**

Consider the curve defined by:

$$(x^2 + y^2)^3 = 8x^2y^2$$

(a) Find the equation of the tangent line at the point $(-1, 1)$.

(b) Explain why implicit differentiation is necessary here.

(c) What would be the slope at $(1, -1)$? Interpret this result economically as a marginal rate of substitution.

**(5+3+2 marks)**

**Q2. Taylor Series Approximation**

Let $f(x) = \sqrt{x}$.

(a) Construct the second-order Taylor polynomial for $f$ centered at $x = 4$.

(b) Use this polynomial to approximate $\sqrt{4.2}$.

(c) Estimate the maximum error for $x \in [4, 4.2]$. Why is this useful in economic forecasting?

**(4+3+3 marks)**

**Q3. Elasticity & Optimization**

Given the implicit demand function:

$$x^3y^3 + 3x^3 = 2$$

(a) Find the elasticity of $y$ with respect to $x$ ($E_{y,x}$).

(b) Calculate $E_{y,x}$ when $y = 1$. Interpret this result in terms of price sensitivity.

(c) What does $E_{y,x} = -1 - \frac{3}{y^3}$ imply about the relationship between $x$ and $y$?

**(5+3+2 marks)**

**Q4. Continuity in Economic Functions**

A firm's cost function is:

$$f(x) = \begin{cases} x^3 - 1 & \text{if } x < 2 \\ x^2 + 3 & \text{if } x \geq 2 \end{cases}$$

(a) Prove whether $f(x)$ is continuous at $x = 2$ using limits.

(b) If discontinuous, suggest an economic reason (e.g., fixed cost jump at production level 2).

(c) Sketch the graph and label any critical points.

**(4+3+3 marks)**

**Q5. Population Growth Model**

A population grows according to:

$$P(t) = \frac{a}{b + e^{-at}}$$

(a) Find the initial growth rate $\left.\frac{dP}{dt}\right|_{t=0}$.

(b) Determine when growth is most rapid.

(c) What is the limiting population as $t \to \infty$? Relate this to carrying capacity in resource economics.

**(4+4+2 marks)**

# Bonus Question (Optional)

Solve the inequality:

$$\frac{3p - 4}{p + 2} > 4 - p$$

Interpret the solution in terms of price elasticity thresholds for a product.
**(5 marks)**