1. (a) Different students at World's Greatest University (WGU) have different preferences about economics. Draw the Indifference curves associated with each of the following statements. Measure "economics books" along horizontal axis and "books about other subjects" along vertical axis. Draw arrows indicating the direction in which utility is increasing.

   (i) "I care only about the total amount of knowledge I acquire. It is the same whether that is economics knowledge or knowledge of any other kind. That is, all books on all subjects are perfect substitutes for me."

(ii) "I hate the ABC textbook and all other economics books. On the other hand, I love everything else in the WGU curriculum."

(iii) "I really like books about economics because I want to understand the economic world. Books about other subjects make no difference to me."

(iv) "I like all my courses and the liberal education that WGU offers. That is, I prefer to read books on a variety of different subjects, rather than to read lots on one subject and little on the others."

(b) (i) Explain the axioms of consumer's preferences.

(ii) Show that the following CES function :

$$\alpha \frac{x^{\delta}}{\delta} + \beta \frac{y^{\delta}}{\delta}$$

(a) is homothetic.

(b) Show that if x = y, marginal rate of substitution (MRS) for this function depends only on the relative sizes of α and β. (8+7)

2. (a) There are 2 goods in the world, pumpkins and apple cider. Pumpkins are \$2 each. Cider is \$7 per gallon for the first 2 gallons. After the second gallon, price of cider drops to \$4 per gallon.

(b) Olivia gets an allowance of \$50 this week, but it will have to last for two weeks, as Mom pays her every other week. Let be her consumption this week (measured in units of stuff) and let be her consumption next week (measured in the same units). The price of one unit of stuff this week is \$1. Next week the price will be higher because of inflation. Assume the inflation rate is 1% per week or 0.01 per week. Therefore, the price of one unit of stuff next week will be $\$1(1+\pi) = \$1.01$. Olivia can borrow or save at the local bank, whether she is borrowing or saving, the interest rate is 1% per week or 0.01 when expressed as decimal. Olivia's utility function is, $u(c_1, c_2) = \ln c_1 + \ln c_2$.

(i) Write down Olivia's budget constraint first in abstract (with M, π, i) and then with given values incorporated.

(ii) Find her optimal consumption bundle (c_1^*, c_2^*).

(iii) Assume the inflation rate rises to 10% and the interest rate drops to zero, find her new optimal consumption bundle.

(9+6)

3. (a) Each day Paul who is in the third grade, eats lunch at school. He likes only Twinkies (t) and Soda (s) and these provide him a utility of,

$$u(t,s) = \sqrt{ts}$$

(i) If Twinkies cost \$0.10 each and soda cost \$0.25 per cup. How should Paul spend \$1 his mother gives him to maximize his utility.

(ii) If the school tries to discourage Twinkie consumption by increasing the price to \$0.40 by how much will Paul's mother have to increase his lunch allowance to provide him with the same level of utility he received in part a).

(b) (i) "Tangency is only a necessary condition but not a sufficient condition for a maximum". Comment.

(ii) Show that lump sum tax makes the consumer better off compared to quantity tax which costs the government the same amount.

4. (a) Suppose that Ram took part in a lottery that had a chance to increase, decrease or have no effect on his level of income. With probability 0.4, his income remains at its original level \$400, with probability 0.4 his income increases to \$800 and with probability 0.2 his income decreases to \$200. His utility function is,

$$u(I) = 125 + 3I$$

where 'I' denotes his level of income.

(i) Show that Ram's risk preferences are risk neutral.

(ii) Calculate Expected Utility and the utility equivalent of the Expected Value of Ram's income.

(iii) Suppose now that Ram had the option to either accept this lottery or walk away with initial \$400. Should he accept the lottery, why or why not?

(b) (i) Explain the methods of managing risk.

(ii) Describe Risk Aversion using Indifference curves. (9+6)

5. (a) Constant Returns to Scale CES Production function is given as :

$$q=\left(k^{\rho}+l^{\rho}\right)^{\frac{\gamma}{\rho}}$$

(i) Show that $MP_k=\left(\frac{q}{k}\right)^{1-\rho}$ and $MP_l=\left(\frac{q}{l}\right)^{1-\rho}$.

(ii) Show that $RTS = \left(\frac{k}{l}\right)^{1-\rho}$. Use this to show that elasticity of substitution, $\sigma = \frac{1}{1-\rho}$.

(iii) Prove that $\frac{q}{l} = \left(\frac{\partial q}{\partial l}\right)^{\sigma}$.

(b) Explain the properties of Profit function.

(9+6)

6. (a) Acme Heavy equipment school teaches students how to drive construction machinery.

Number of students that the school can educate per week is given by $q = 10 \min(k, l)^{\gamma}$ where is 'k' is the number of backhoes the firm rents per week, 'l' is the number of instructors hired each week and 'γ' is a parameter indicating the returns to scale in this production function.

(i) Supposing $\gamma = 0.5$, calculate firm's total cost function and profit function.

(ii) If $v = 1000$, $w = 500$, $P = 600$, how many students will Acme serve and what are its profits.

(iii) If price students are willing to pay rises to $P = 900$, how much will profit change?

(b) Hard red winter wheat is planted in the fall in order to be harvested in the spring. Suppose that wheat production uses acres of land A and labour L in its production as follows :

$$q = \alpha A + l^{\beta},$$

where 'q' is in thousands of bushels. Calculate the total cost function for wheat. (9+6)

7. (a) Suppose the production function for widgets is given by

$$q = kl - 0.8k^2 - 0.2l^2$$

Where 'q' represents the annual quantity of widgets produced, 'k' represents annual capital input and 'l' represents annual labour input.

(i) Suppose k = 10, graph the total and average productivity curves of labour. At what level of labour input does this average productivity reach a maximum? How many widgets are produced at that point?

(ii) Again, assuming k = 10, graph the MP_l curve. At what level of labor input does $MP_l = 0$.

(iii) Does widget production exhibit constant, increasing or decreasing returns to scale?

(b) Use Shepherd's Lemma to compute the (constant output) demand functions for inputs L and K for the following cost functions :

(i) $C = qw^{\frac{2}{3}}v^{\frac{1}{3}}$

(ii) $C = q\left(v + 2\sqrt{vw} + w\right)$ (9+6)