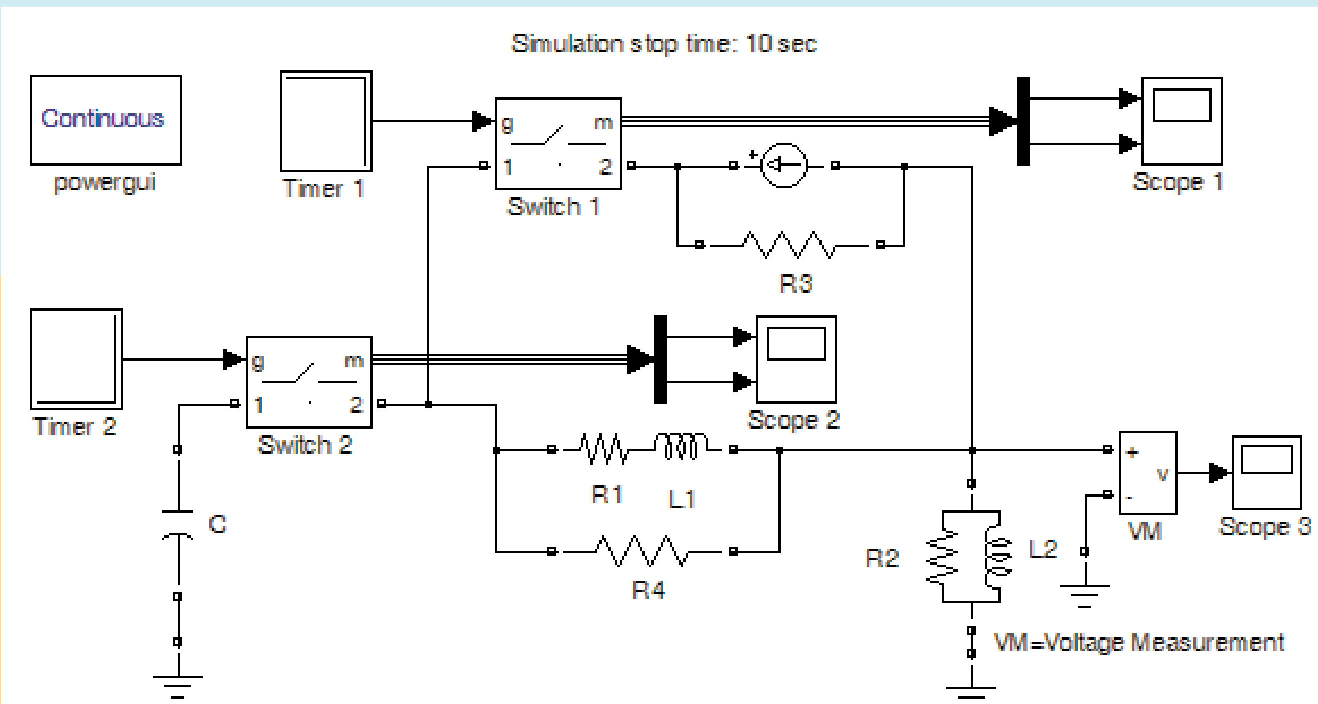
Circuit Analysis II

with MATLAB® Computing and Simulink®/SimPowerSystems® Modeling

Steven T. Karris



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Circuit Analysis II

with MATLAB® Computing and Simulink®/SimPowerSystems Modeling

Students and working professionals will find Circuit Analysis II with MATLAB® Computing and Simulink®/SimPowerSystems® Modeling to be a concise and easy-to-learn text. It provides complete, clear, and detailed explanations of the traditional second semester circuit analysis, and these are illustrated with numerous practical examples.

This text includes the following chapters and appendices:

• Second Order Circuits • Resonance • Elementary Signals • The Laplace Transformation • The Inverse Laplace Transformation • Circuit Analysis with Laplace Transforms • State Variables and State Equations • Frequency Response and Bode Plots • Self and Mutual Inductances - Transformers • One- and Two-Port Networks • Balanced Three-Phase Systems • Unbalanced Three-Phase Systems • Introduction to MATLAB® • Introduction to Simulink® • Introduction to SimPowerSystems® • Review of Complex Numbers • Matrices and Determinants • Scaling • Matrices and Determinants • Per Unit System • Review of Differential Equations

Each chapter and each appendix contains numerous practical applications supplemented with detailed instructions for using MATLAB, Simulink, and SimPowerSystems to obtain quick and accurate results.

Steven T. Karris is the founder and president of Orchard Publications, has undergraduate and graduate degrees in electrical engineering, and is a registered professional engineer in California and Florida. He has more than 35 years of professional engineering experience and more than 30 years of teaching experience as an adjunct professor, most recently at UC Berkeley, California. His area of interest is in The MathWorks, Inc. ™ products and the publication of MATLAB® and Simulink® based texts.

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Preface

This text is written for use in a second course in circuit analysis. It encompasses a spectrum of subjects ranging from the most abstract to the most practical, and the material can be covered in one semester or two quarters.The reader of this book should have the traditional undergraduate knowledge of an introductory circuit analysis material such as *Circuit Analysis I with MATLAB® Computing and Simulink® / SimPowerSystems® Modeling, ISBN 978-1-934404-17-1*. Another prerequisite would be a basic knowledge of differential equations, and in most cases, engineering students at this level have taken all required mathematics courses. Appendix H serves as a review of differential equations with emphasis on engineering related topics and it is recommended for readers who may need a review of this subject.

There are several textbooks on the subject that have been used for years. The material of this book is not new, and this author claims no originality of its content. This book was written to fit the needs of the average student. Moreover, it is not restricted to computer oriented circuit analysis. While it is true that there is a great demand for electrical and computer engineers, especially in the internet field, the demand also exists for power engineers to work in electric utility companies, and facility engineers to work in the industrial areas.

Chapter 1 is an introduction to second order circuits and it is essentially a sequel to first order circuits discussed in the last chapter of *Circuit Analysis I with MATLAB® Computing and Simulink® / SimPowerSystems® Modeling, ISBN 978-1-934404-17-1*. Chapter 2 is devoted to resonance, and Chapter 3 presents practical methods of expressing signals in terms of the elementary functions, i.e., unit step, unit ramp, and unit impulse functions. Accordingly, any signal can be represented in the complex frequency domain using the Laplace transformation.

Chapters 4 and 5 are introductions to the unilateral Laplace transform and Inverse Laplace transform respectively, while Chapter 6 presents several examples of analyzing electric circuits using Laplace transformation methods. Chapter 7 is an introduction to state space and state equations. Chapter 8 begins with the frequency response concept and Bode magnitude and frequency plots. Chapter 9 is devoted to transformers with an introduction to self and mutual inductances. Chapter 10 is an introduction to one- and two-terminal devices and presents several practical examples. Chapters 11 and 12 are introductions to three-phase circuits.

It is not necessary that the reader has previous knowledge of MATLAB® . The material of this text can be learned without MATLAB. However, this author highly recommends that the reader studies this material in conjunction with the inexpensive MATLAB Student Version package that is available at most college and university bookstores. Appendix A of this text provides a practical introduction to MATLAB, Appendix B is an introduction to Simulink, and Appendix C introduces SimPowerSystems. The pages where MATLAB scripts, Simulink / SimPowerSystems models appear are indicated in the Table of Contents.

The author highly recommends that the reader studies this material in conjunction with the inexpensive Student Versions of The MathWorks™ Inc., the developers of these outstanding products, available from:

The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA, 01760
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Appendix D is a review of complex numbers, Appendix E is an introduction to matrices, Appendix F discusses scaling methods, Appendix G introduces the per unit system used extensively in power systems and in SimPwerSystems examples and demos. As stated above, Appendix H is a review of differential equations. Appendix I provides instructions for constructing semilog templates to be used with Bode plots.

In addition to numerous examples, this text contains several exercises at the end of each chapter. Detailed solutions of all exercises are provided at the end of each chapter. The rationale is to encourage the reader to solve all exercises and check his effort for correct solutions and appropriate steps in obtaining the correct solution. And since this text was written to serve as a self-study or supplementary textbook, it provides the reader with a resource to test his knowledge.

The author is indebted to several readers who have brought some errors to our attention. Additional feedback with other errors, advice, and comments will be most welcomed and greatly appreciated.

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Chapter 1

Second Order Circuits

This chapter discusses the natural, forced and total responses in circuits that contain resistors, inductors and capacitors. These circuits are characterized by linear second–order differential equations whose solutions consist of the natural and the forced responses. We will consider both DC (constant) and AC (sinusoidal) excitations.

1.1 Response of a Second Order Circuit

A circuit that contains n energy storage devices (inductors and capacitors) is said to be an *nth–order circuit*, and the differential equation describing the circuit is an *nth–order differential equation*. For example, if a circuit contains an inductor and a capacitor, or two capacitors or two inductors, along with other devices such as resistors, it is said to be a second–order circuit and the differential equation that describes it will be a second order differential equation. It is possible, however, to describe a circuit having two energy storage devices with a set of two first–order differential equations, a circuit which has three energy storage devices with a set of three first–order differential equations and so on. These are called *state equations* and are discussed in Chapter 7.

As we know from previous studies,* the response is found from the differential equation describing the circuit, and its solution is obtained as follows:

1. We write the differential or integrodifferential (nodal or mesh) equation describing the circuit. We differentiate, if necessary, to eliminate the integral.
2. We obtain the forced (steady–state) response. Since the excitation in our work here will be either a constant (DC) or sinusoidal (AC) in nature, we expect the forced response to have the same form as the excitation. We evaluate the constants of the forced response by substitution of the assumed forced response into the differential equation and equate terms of the left side with the right side. The form of the forced response (particular solution), is described in Appendix H.
3. We obtain the general form of the natural response by setting the right side of the differential equation equal to zero, in other words, solve the homogeneous differential equation using the characteristic equation.
4. We add the forced and natural responses to form the complete response.
5. Using the initial conditions, we evaluate the constants from the complete response.

* *The natural and forced responses for first–order circuits are discussed in Circuit Analysis I with MATLAB® Computing and Simulink® / SimPowerSystems® Modeling, ISBN 978–1–934404–17–1.*

```
Solution was entered as y0 =
115*exp(-200*t)-110*exp(-300*t)
1st derivative of solution is y1 =
-23000*exp(-200*t)+33000*exp(-300*t)
2nd derivative of solution is y2 =
4600000*exp(-200*t)-9900000*exp(-300*t)
Differential equation is satisfied since y = y2+y1+y0 = 0
1st initial condition is satisfied since at t = 0, i0 = 5
2nd initial condition is also satisfied since vC+vL+vR=15 and vC0
= 2.5000
```

We denote the first term as $i_1(t) = 115e^{-200t}$, the second term as $i_2(t) = 110e^{-300t}$, and the total current $i(t)$ as the difference of these two terms. The response is shown in Figure 1.6.

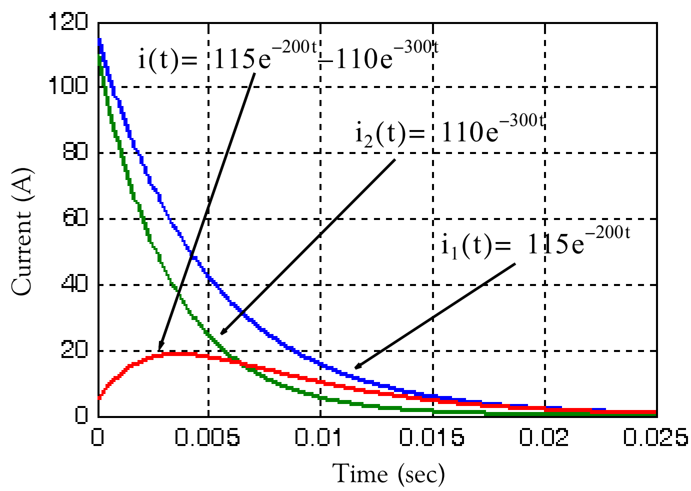

Figure 1.6. Plot for $i(t)$ *of Example 1.1*

In the above example, differentiation eliminated (set equal to zero) the right side of the differential equation and thus the total response was just the natural response. A different approach however, may not set the right side equal to zero, and therefore the total response will contain both the natural and forced components. To illustrate, we will use the following approach.

The capacitor voltage, for all time *t*, may be expressed as $v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt$ and as before, the circuit can be represented by the integrodifferential equation

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt = 15u_0(t) \tag{1.18}$$

and since

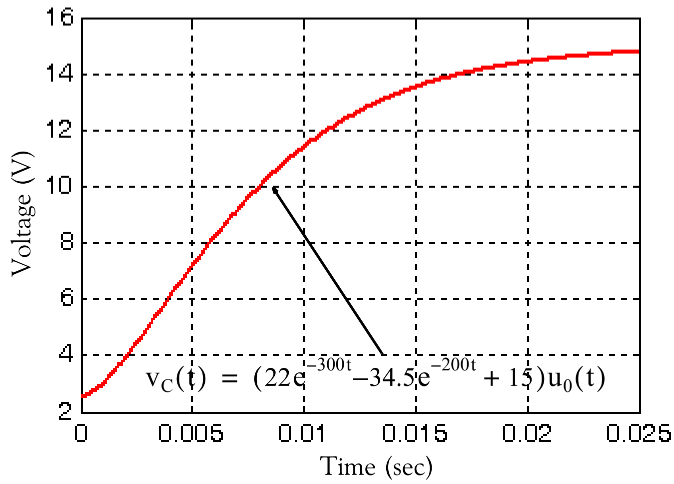

Figure 1.7. Plot for $v_C(t)$ of Example 1.1

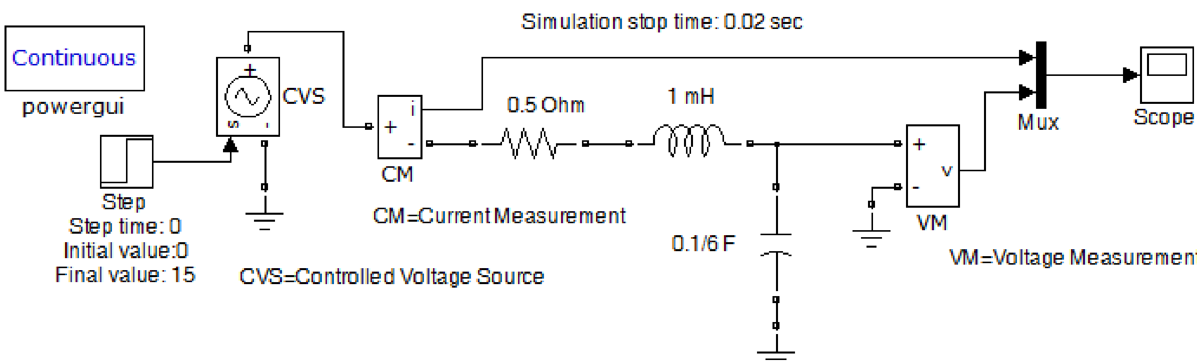

Figure 1.8. Simulink/SimPowerSystems model for the circuit in Figure 1.5

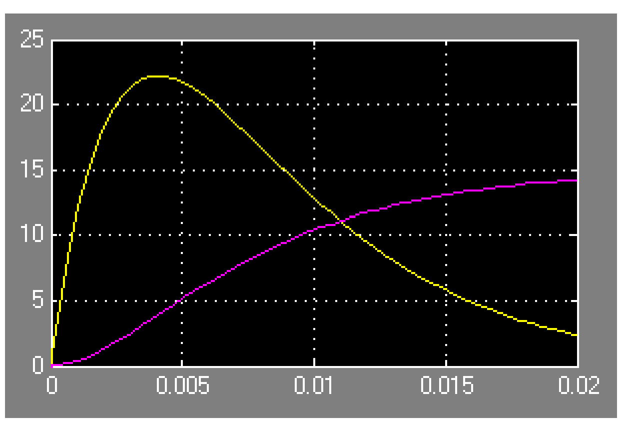

Figure 1.9. Waveforms produced by the Simulink/SimPowerSystems model in Figure 1.8

Chapter 2

Resonance

This chapter defines series and parallel resonance. The quality factor Q is then defined in terms of the series and parallel resonant frequencies. The half–power frequencies and bandwidth are also defined in terms of the resonant frequency.

2.1 Series Resonance

Consider phasor series RLC circuit of Figure 2.1.

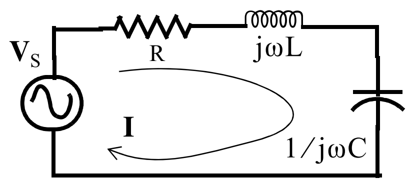

Figure 2.1. Series RLC phasor circuit

The impedance Z is

$$\text{Impedance} = Z = \frac{\text{Phasor Voltage}}{\text{Phasor Current}} = \frac{\mathbf{V_S}}{\mathbf{I}} = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{2.1}$$

or

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \angle \tan^{-1}(\omega L - 1/\omega C)/R \tag{2.2}$$

Therefore, the magnitude and phase angle of the impedance are:

$$|Z| = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \tag{2.3}$$

and

$$\theta_Z = \tan^{-1}(\omega L - 1/\omega C)/R \tag{2.4}$$

The components of $|Z|$ are shown on the plot in Figure 2.2.

The frequency at which the capacitive reactance $X_C = 1/\omega C$ and the inductive reactance $X_L = \omega L$ are equal is called the *resonant frequency*. The resonant frequency is denoted as ω_0 or f_0 and these can be expressed in terms of the inductance L and capacitance C by equating the reactances, that is,

$$\omega_0 L = \frac{1}{\omega_0 C}$$

The quality factor Q is also a measure of *frequency selectivity*. Thus, we say that a circuit with a high Q has a high selectivity, whereas a low Q circuit has low selectivity. The high frequency selectivity is more desirable in parallel circuits as we will see in the next section.

We will see later that

$$\boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\text{Resonant Frequency}}{\text{Bandwidth}}} \tag{2.11}$$

Figure 2.5 shows the relative response versus ω for $Q = 25, 50$, and 100 where we observe that highest Q provides the best frequency selectivity, i.e., higher rejection of signal components outside the bandwidth $BW = \omega_2 - \omega_1$ which is the difference in the $3\ dB$ frequencies. The curves were created with the MATLAB script below.

```
w=450:1:550; x1=1./(1+25.^2*(w./500–500./w).^2); plot(w,x1);...
x2=1./(1+50.^2*(w./500–500./w).^2); plot(w,x2);...
x3=1./(1+100.^2*(w./500–500./w).^2); plot(w,x3);...
plot(w,x1,w,x2,w,x3); grid
```

We also observe from (2.9) that selectivity depends on R and this dependence is shown on the plot of Figure 2.6.

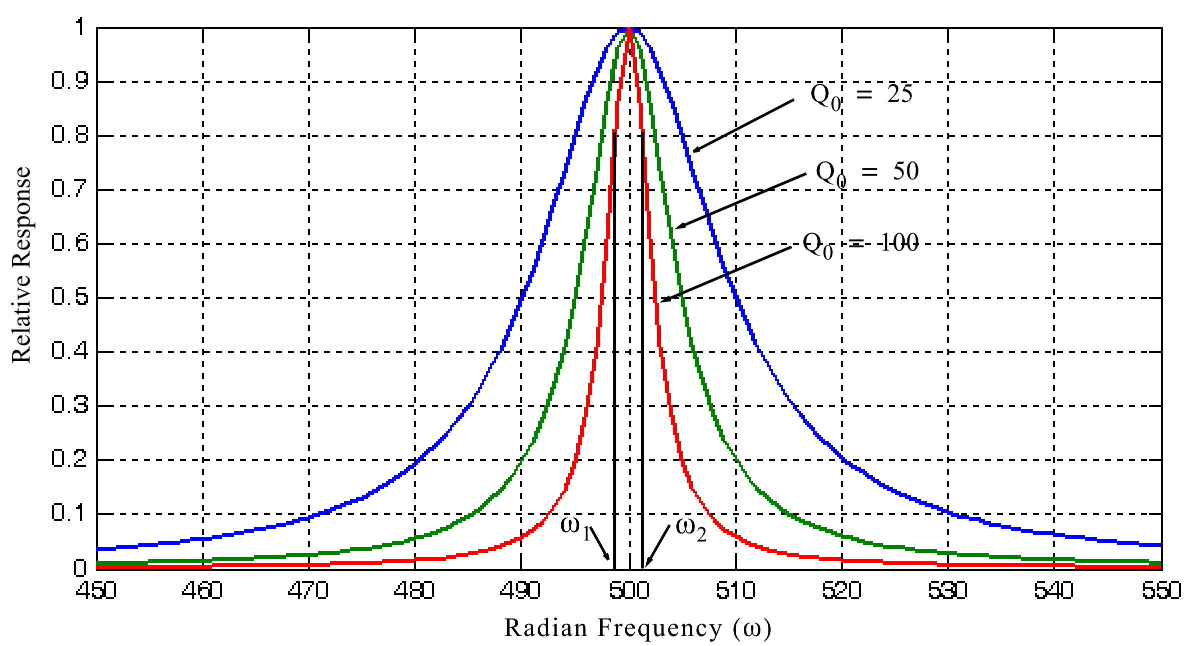

Figure 2.5. Selectivity curves with $Q = 25, 50$, and 100

The curves in Figure 2.6 were created with the MATLAB script below.

```
w=0:10:6000; R1=0.5; R2=1; L=10^(–3); C=10^(–4); Y1=1./sqrt(R1.^2+(w.*L–1./(w.*C)).^2);...
Y2=1./sqrt(R2.^2+(w.*L–1./(w.*C)).^2); plot(w,Y1,w,Y2)
```

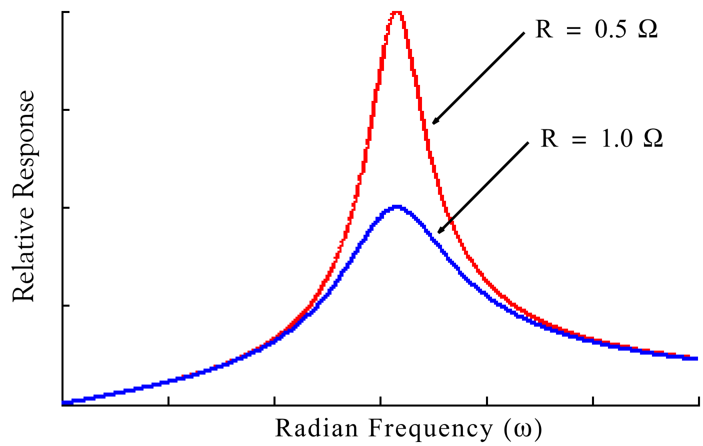

Figure 2.6. Selectivity curves with different values of R

If we keep one reactive device, say L, constant while varying C, the relative response "shifts" as shown in Figure 2.7, but the general shape does not change.

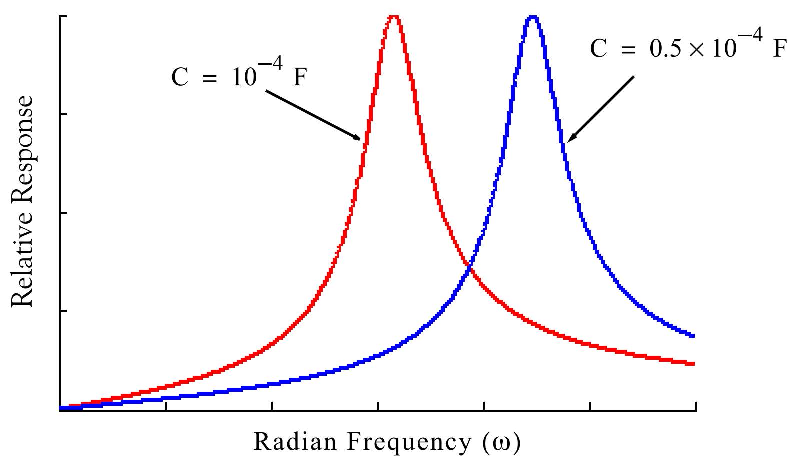

Figure 2.7. Relative response with constant L and variable C

The curves in Figure 2.7 were created with the MATLAB script below.

```
w=0:10:6000; R=0.5; L=10^(–3); C1=10^(–4); C2=0.5*10^(–4);...
Y1=1./sqrt(R.^2+(w.*L–1./(w.*C1)).^2);...
Y2=1./sqrt(R.^2+(w.*L–1./(w.*C2)).^2); plot(w,Y1,w,Y2)
```

2.3 Parallel Resonance

Parallel resonance (*antiresonance*) applies to parallel circuits such as that shown in Figure 2.8. The admittance Y for this circuit is given by

$$\text{Admittance} = Y = \frac{\text{Phasor Current}}{\text{Phasor Voltage}} = \frac{\mathbf{I}_S}{\mathbf{V}} = G + j\omega C + \frac{1}{j\omega L} = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

Chapter 3

Elementary Signals

This chapter begins with a discussion of elementary signals that may be applied to electric networks. The unit step, unit ramp, and delta functions are then introduced. The sampling and sifting properties of the delta function are defined and derived. Several examples for expressing a variety of waveforms in terms of these elementary signals are provided.

3.1 Signals Described in Math Form

Consider the network of Figure 3.1 where the switch is closed at time $t = 0$.

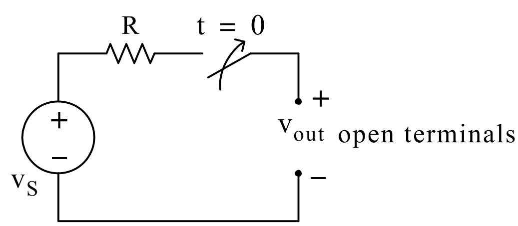

Figure 3.1. A switched network with open terminals

We wish to describe v_{out} in a math form for the time interval $-\infty < t < +\infty$. To do this, it is convenient to divide the time interval into two parts, $-\infty < t < 0$, and $0 < t < \infty$.

For the time interval $-\infty < t < 0$, the switch is open and therefore, the output voltage v_{out} is zero. In other words,

$$v_{out} = 0 \quad \text{for} \quad -\infty < t < 0 \tag{3.1}$$

For the time interval $0 < t < \infty$, the switch is closed. Then, the input voltage v_S appears at the output, i.e.,

$$v_{out} = v_S \quad \text{for} \quad 0 < t < \infty \tag{3.2}$$

Combining (3.1) and (3.2) into a single relationship, we obtain

$$\boxed{v_{out} = \begin{cases} 0 & -\infty < t < 0 \\ v_S & 0 < t < \infty \end{cases}} \tag{3.3}$$

We can express (3.3) by the waveform shown in Figure 3.2.

The waveform of Figure 3.2 is an example of a discontinuous function. A function is said to be *discontinuous* if it exhibits points of discontinuity, that is, the function jumps from one value to another without taking on any intermediate values.

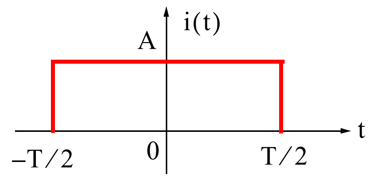

Figure 3.11. Symmetric rectangular pulse for Example 3.3

$$i(t) = Au_0\left(t+\frac{T}{2}\right) - Au_0\left(t-\frac{T}{2}\right) = A\left[u_0\left(t+\frac{T}{2}\right) - u_0\left(t-\frac{T}{2}\right)\right] \tag{3.14}$$

Example 3.4

Express the symmetric triangular waveform of Figure 3.12 as a sum of unit step functions.

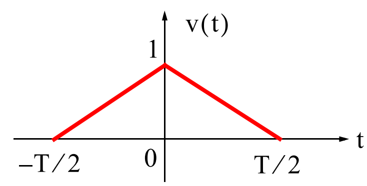

Figure 3.12. Symmetric triangular waveform for Example 3.4

Solution:

We first derive the equations for the linear segments ① and ② shown in Figure 3.13.

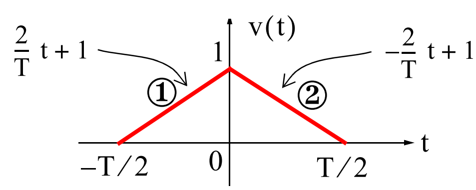

Figure 3.13. Equations for the linear segments in Figure 3.12

For line segment ①,

$$v_1(t) = \left(\frac{2}{T}t+1\right)\left[u_0\left(t+\frac{T}{2}\right) - u_0(t)\right] \tag{3.15}$$

and for line segment ②,

$$v_2(t) = \left(-\frac{2}{T}t+1\right)\left[u_0(t) - u_0\left(t-\frac{T}{2}\right)\right] \tag{3.16}$$

Combining (3.15) and (3.16), we obtain

$$v(t) = v_1(t) + v_2(t) = \left(\frac{2}{T}t+1\right)\left[u_0\left(t+\frac{T}{2}\right) - u_0(t)\right] + \left(-\frac{2}{T}t+1\right)\left[u_0(t) - u_0\left(t-\frac{T}{2}\right)\right] \tag{3.17}$$

Chapter 4

The Laplace Transformation

This chapter begins with an introduction to the Laplace transformation, definitions, and properties of the Laplace transformation. The initial value and final value theorems are also discussed and proved. It continues with the derivation of the Laplace transform of common functions of time, and concludes with the derivation of the Laplace transforms of common waveforms.

4.1 Definition of the Laplace Transformation

The *two–sided* or *bilateral* Laplace Transform pair is defined as

$$\boxed{\mathcal{L}\{f(t)\} = F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt} \tag{4.1}$$

$$\boxed{\mathcal{L}^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{\sigma - j\omega}^{\sigma + j\omega} F(s)e^{st}ds} \tag{4.2}$$

where $\mathcal{L}\{f(t)\}$ denotes the Laplace transform of the time function $f(t)$, $\mathcal{L}^{-1}\{F(s)\}$ denotes the Inverse Laplace transform, and s is a complex variable whose real part is σ, and imaginary part ω, that is, $s = \sigma + j\omega$.

In most problems, we are concerned with values of time t greater than some reference time, say $t = t_0 = 0$, and since the initial conditions are generally known, the two–sided Laplace transform pair of (4.1) and (4.2) simplifies to the *unilateral* or *one–sided Laplace transform* defined as

$$\boxed{\mathcal{L}\{f(t)\} = F(s) = \int_{t_0}^{\infty} f(t)e^{-st}dt = \int_{0}^{\infty} f(t)e^{-st}dt} \tag{4.3}$$

$$\boxed{\mathcal{L}^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{\sigma - j\omega}^{\sigma + j\omega} F(s)e^{st}ds} \tag{4.4}$$

The Laplace Transform of (4.3) has meaning only if the integral converges (reaches a limit), that is, if

$$\left|\int_{0}^{\infty} f(t)e^{-st}dt\right| < \infty \tag{4.5}$$

$$\boxed{tf(t) \Leftrightarrow -\frac{d}{ds}F(s)} \tag{4.21}$$

Proof:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st}dt$$

Differentiating with respect to s and applying *Leibnitz's rule** *for differentiation under the integral,* we obtain

$$\frac{d}{ds}F(s) = \frac{d}{ds}\int_0^{\infty} f(t)e^{-st}dt = \int_0^{\infty} \frac{\partial}{\partial s}e^{-st}f(t)dt = \int_0^{\infty} -te^{-st}f(t)dt = -\int_0^{\infty} [tf(t)]e^{-st}dt = -\mathcal{L}[tf(t)]$$

In general,

$$\boxed{t^n f(t) \Leftrightarrow (-1)^n \frac{d^n}{ds^n}F(s)} \tag{4.22}$$

The proof for $n \ge 2$ follows by taking the second and higher–order derivatives of $F(s)$ with respect to s.

4.2.7 Integration in Time Domain Property

This property states that *integration in time domain* corresponds to $F(s)$ divided by s plus the initial value of $f(t)$ at $t = 0^-$, also divided by s. That is,

$$\boxed{\int_{-\infty}^{t} f(\tau)d\tau \Leftrightarrow \frac{F(s)}{s} + \frac{f(0^-)}{s}} \tag{4.23}$$

Proof:

We begin by expressing the integral on the left side of (4.23) as two integrals, that is,

$$\int_{-\infty}^{t} f(\tau)d\tau = \int_{-\infty}^{0} f(\tau)d\tau + \int_0^{t} f(\tau)d\tau \tag{4.24}$$

The first integral on the right side of (4.24), represents a constant value since neither the upper, nor the lower limits of integration are functions of time, and this constant is an initial condition denoted as $f(0^-)$. We will find the Laplace transform of this constant, the transform of the sec-

* *This rule states that if a function of a parameter* α *is defined by the equation* $F(\alpha) = \int_a^b f(x, \alpha)dx$ *where f is some known function of integration x and the parameter* α*, a and b are constants independent of x and* α*, and the partial derivative* $\partial f / \partial \alpha$ *exists and it is continuous, then* $\frac{dF}{d\alpha} = \int_a^b \frac{\partial(x, \alpha)}{\partial(\alpha)}dx$.

Chapter 5

The Inverse Laplace Transformation

This chapter is a continuation to the Laplace transformation topic of the previous chapter and presents several methods of finding the Inverse Laplace Transformation. The partial fraction expansion method is explained thoroughly and it is illustrated with several examples.

5.1 The Inverse Laplace Transform Integral

The Inverse Laplace Transform Integral was stated in the previous chapter; it is repeated here for convenience.

$$\boxed{\mathcal{L}^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{\sigma - j\omega}^{\sigma + j\omega} F(s)e^{st}ds} \tag{5.1}$$

This integral is difficult to evaluate because it requires contour integration using complex variables theory. Fortunately, for most engineering problems we can refer to Tables of Properties, and Common Laplace transform pairs to lookup the Inverse Laplace transform.

5.2 Partial Fraction Expansion

Quite often the Laplace transform expressions are not in recognizable form, but in most cases appear in a rational form of s, that is,

$$F(s) = \frac{N(s)}{D(s)} \tag{5.2}$$

where $N(s)$ and $D(s)$ are polynomials, and thus (5.2) can be expressed as

$$F(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0} \tag{5.3}$$

The coefficients a_k and b_k are real numbers for $k = 1, 2, \ldots, n$, and if the highest power m of $N(s)$ is less than the highest power n of $D(s)$, i.e., $m < n$, $F(s)$ is said to be expressed as a *proper rational function*. If $m \ge n$, $F(s)$ is an *improper rational function.*

In a proper rational function, the roots of $N(s)$ in (5.3) are found by setting $N(s) = 0$; these are called the *zeros* of $F(s)$. The roots of $D(s)$, found by setting $D(s) = 0$, are called the *poles* of $F(s)$. We assume that $F(s)$ in (5.3) is a proper rational function. Then, it is customary and very convenient to make the coefficient of s^n unity; thus, we rewrite $F(s)$ as

Solution:

Using (5.6), we obtain

$$F_1(s) = \frac{3s+2}{s^2+3s+2} = \frac{3s+2}{(s+1)(s+2)} = \frac{r_1}{(s+1)} + \frac{r_2}{(s+2)} \tag{5.9}$$

The residues are

$$r_1 = \lim_{s \to -1}(s+1)F(s) = \left.\frac{3s+2}{(s+2)}\right|_{s=-1} = -1 \tag{5.10}$$

and

$$r_2 = \lim_{s \to -2}(s+2)F(s) = \left.\frac{3s+2}{(s+1)}\right|_{s=-2} = 4 \tag{5.11}$$

Therefore, we express (5.9) as

$$F_1(s) = \frac{3s+2}{s^2+3s+2} = \frac{-1}{(s+1)} + \frac{4}{(s+2)} \tag{5.12}$$

and from Table 4.2, Chapter 4, Page 4–22, we find that

$$e^{-at}u_0(t) \Leftrightarrow \frac{1}{s+a} \tag{5.13}$$

Therefore,

$$F_1(s) = \frac{-1}{(s+1)} + \frac{4}{(s+2)} \Leftrightarrow (-e^{-t} + 4e^{-2t})\,u_0(t) = f_1(t) \tag{5.14}$$

The residues and poles of a rational function of polynomials such as (5.8), can be found easily using the MATLAB **residue(a,b)** function. For this example, we use the script

```
Ns = [3, 2]; Ds = [1, 3, 2]; [r, p, k] = residue(Ns, Ds)
```

and MATLAB returns the values

```
r =
     4
    -1
p =
    -2
    -1
k =
     []
```

For the MATLAB script above, we defined Ns and Ds as two vectors that contain the numerator and denominator coefficients of $F(s)$. When this script is executed, MATLAB displays the r and p vectors that represent the residues and poles respectively. The first value of the vector r is associated with the first value of the vector p, the second value of r is associated with the second

Chapter 6

Circuit Analysis with Laplace Transforms

This chapter presents applications of the Laplace transform. Several examples are presented to illustrate how the Laplace transformation is applied to circuit analysis. Complex impedance, complex admittance, and transfer functions are also defined.

6.1 Circuit Transformation from Time to Complex Frequency

In this section we will show the voltage–current relationships for the three elementary circuit networks, i.e., resistive, inductive, and capacitive in the time and complex frequency domains. They are described in Subsections 6.1.1 through 6.1.3 below.

6.1.1 Resistive Network Transformation

The time and complex frequency domains for purely resistive networks are shown in Figure 6.1.

Figure 6.1. Resistive network in time domain and complex frequency domain

6.1.2 Inductive Network Transformation

The time and complex frequency domains for purely inductive networks are shown in Figure 6.2.

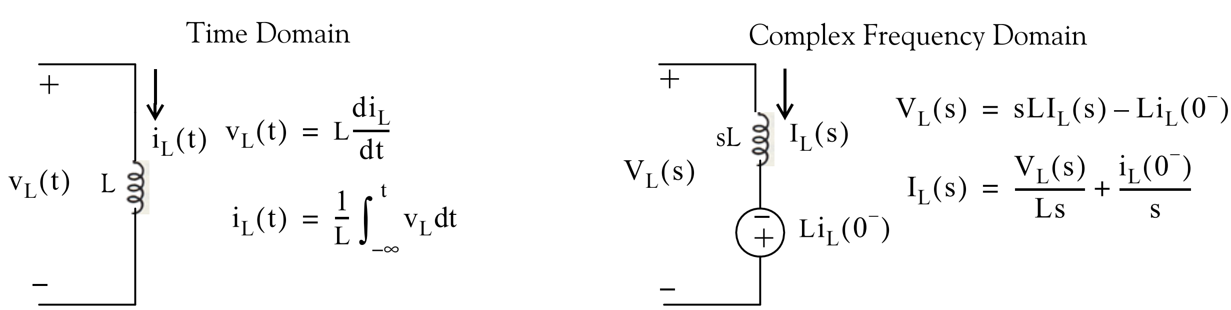

Figure 6.2. Inductive network in time domain and complex frequency domain

From (6.13), we observe that as $t \to \infty$, $v_{out}(t) \to 0$. This is to be expected because $v_{out}(t)$ is the voltage across the inductor as we can see from the circuit of Figure 6.9. The MATLAB script below will plot the relation (6.13) above.

```
t=0:0.01:10;...
Vout=1.36.*exp(–6.57.*t)+0.64.*exp(–0.715.*t).*cos(0.316.*t)–1.84.*exp(–0.715.*t).*sin(0.316.*t);...
plot(t,Vout); grid
```

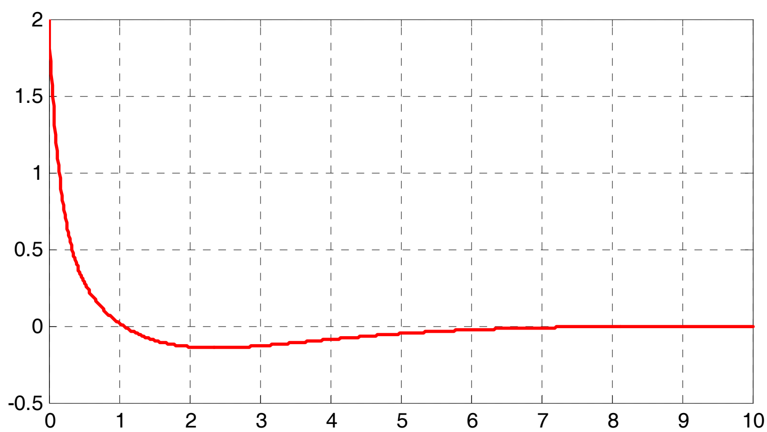

Figure 6.10. Plot of $v_{out}(t)$ for the circuit of Example 6.3

Figure 6.11 shows the Simulink/SimPower Systems model for the circuit in Figure 6.8.

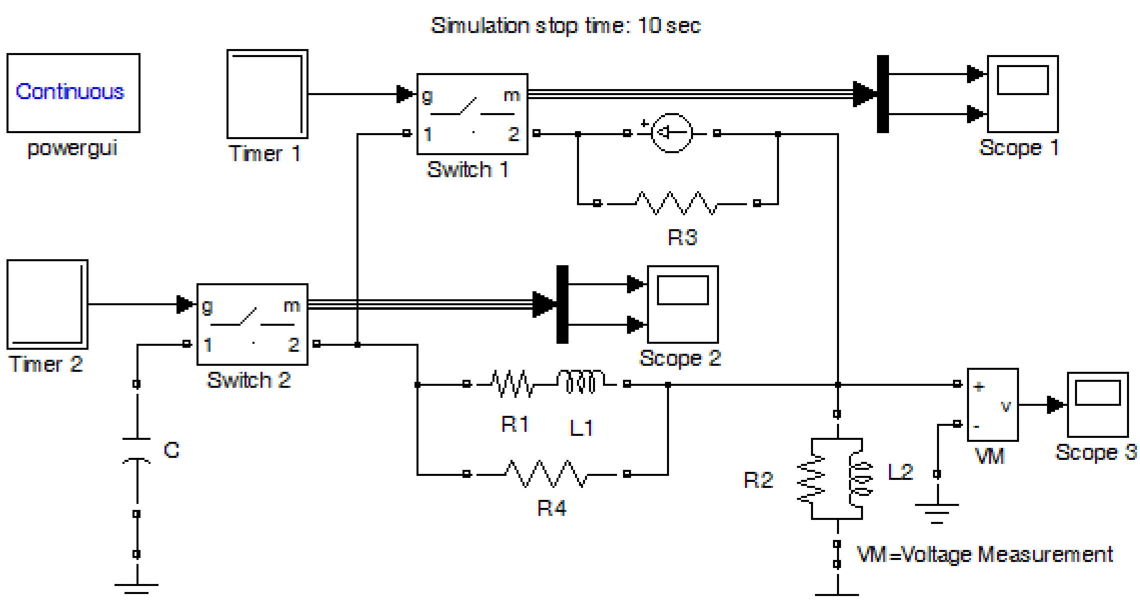

Figure 6.11. The Simulink/SimPowerSystems model for the circuit in Figure 6.8

Chapter 7

State Variables and State Equations

This chapter is an introduction to state variables and state equations as they apply in circuit analysis. The state transition matrix is defined, and the state–space to transfer function equivalence is presented. Several examples are presented to illustrate their application.

7.1 Expressing Differential Equations in State Equation Form

As we know, when we apply Kirchoff's Current Law (KCL) or Kirchoff's Voltage Law (KVL) in networks that contain energy–storing devices, we obtain integro–differential equations. Also, when a network contains just one such device (capacitor or inductor), it is said to be a *first–order circuit*. If it contains two such devices, it is said to be *second–order circuit*, and so on. Thus, a first order linear, time–invariant circuit can be described by a differential equation of the form

$$a_1\frac{dy}{dt} + a_0\,y(t) = x(t) \tag{7.1}$$

A second order circuit can be described by a second–order differential equation of the same form as (7.1) where the highest order is a second derivative.

An *nth–order* differential equation can be resolved to n first–order simultaneous differential equations with a set of auxiliary variables called *state variables*. The resulting first–order differential equations are called *state–space equations*, or simply *state equations*. These equations can be obtained either from the nth–order differential equation, or directly from the network, provided that the state variables are chosen appropriately. The state variable method offers the advantage that it can also be used with non–linear and time–varying devices. However, our discussion will be limited to linear, time–invariant circuits.

State equations can also be solved with numerical methods such as Taylor series and Runge–Kutta methods, but these will not be discussed in this text*. The state variable method is best illustrated with several examples presented in this chapter.

Example 7.1

A series RLC circuit with excitation

$$v_S(t) = e^{j\omega t} \tag{7.2}$$

* *These are discussed in Numerical Analysis using MATLAB and Excel, ISBN 978–1–934404–03–4.*

To prove that (7.34) is also satisfied, we differentiate the assumed solution

$$x(t) = e^{A(t-t_0)}x_0 + e^{At}\int_{t_0}^{t} e^{-A\tau}bu(\tau)d\tau$$

with respect to t and we use (7.40), that is,

$$\frac{d}{dt}e^{At} = Ae^{At}$$

Then,

$$\dot{x}(t) = Ae^{A(t-t_0)}x_0 + Ae^{At}\int_{t_0}^{t} e^{-A\tau}bu(\tau)d\tau + e^{At}e^{-At}bu(t)$$

or

$$\dot{x}(t) = A\left[e^{A(t-t_0)}x_0 + e^{At}\int_{t_0}^{t} e^{-A\tau}bu(\tau)d\tau\right] + e^{At}e^{-At}bu(t) \qquad (7.42)$$

We recognize the bracketed terms in (7.42) as $x(t)$, and the last term as $bu(t)$. Thus, the expression (7.42) reduces to

$$\dot{x}(t) = Ax + bu$$

In summary, if A is an $n \times n$ matrix whose elements are constants, $n \geq 2$, and b is a column vector with *n* elements, the solution of

$$\dot{x}(t) = Ax + bu \qquad (7.43)$$

with initial condition

$$x_0 = x(t_0) \qquad (7.44)$$

is

$$\boxed{x(t) = e^{A(t-t_0)}x_0 + e^{At}\int_{t_0}^{t} e^{-A\tau}bu(\tau)d\tau} \qquad (7.45)$$

Therefore, the solution of second or higher order circuits using the state variable method, entails the computation of the state transition matrix e^{At}, and integration of (7.45).

7.4 Computation of the State Transition Matrix e^{At}

Let A be an $n \times n$ matrix, and I be the $n \times n$ identity matrix. By definition, the *eigenvalues* λ_i, $i = 1, 2, \ldots, n$ of A are the roots of the nth order polynomial

$$\boxed{\det[A - \lambda I] = 0} \qquad (7.46)$$

We recall that expansion of a determinant produces a polynomial. The roots of the polynomial of (7.46) can be real (unequal or equal), or complex numbers.

Chapter 8

Frequency Response and Bode Plots

This chapter discusses frequency response in terms of both amplitude and phase. This topic will enable us to determine which frequencies are dominant and which frequencies are virtually suppressed. The design of electric filters is based on the study of the frequency response. We will also discuss the Bode method of linear system analysis using two separate plots; one for the magnitude of the transfer function, and the other for the phase, both versus frequency. These plots reveal valuable information about the frequency response behavior.

Note: Throughout this text, the common (base 10) logarithm of a number x will be denoted as $\log(x)$ while its natural (base e) logarithm will be denoted as $\ln(x)$. However, we should remember that in MATLAB the $\log(x)$ function displays the natural logarithm, and the common (base 10) logarithm is defined as $\log10(x)$.

8.1 Decibel Defined

The ratio of any two values of the same quantity (power, voltage or current) can be expressed in *decibels* (dB). For instance, we say that an amplifier has $10\ dB$ power gain or a transmission line has a power loss of $7\ dB$ (or gain $-7\ dB$). If the gain (or loss) is $0\ dB$, the output is equal to the input. We should remember that a negative voltage or current gain A_V or A_I indicates that there is a $180°$ phase difference between the input and the output waveforms. For instance, if an amplifier has a gain of -100 (dimensionless number), it means that the output is $180°$ out-of-phase with the input. For this reason we use absolute values of power, voltage and current when these are expressed in dB terms to avoid misinterpretation of gain or loss.

By definition,

$$dB = 10\log\left|\frac{P_{out}}{P_{in}}\right| \tag{8.1}$$

Therefore,

$10\ dB$ represents a power ratio of 10

$10n\ dB$ represents a power ratio of 10^n

$20\ dB$ represents a power ratio of 100

$30\ dB$ represents a power ratio of $1,000$

$60\ dB$ represents a power ratio of $1,000,000$

Also,

$$u \equiv \omega / a \tag{8.17}$$

and

$$\phi(u) = \tan^{-1} u \tag{8.18}$$

Then,

$$(a + j\omega)^n = a^n(1 + ju)^n = a^n\left(\sqrt{1 + u^2}\angle\tan^{-1}u\right)^n = a^n(1 + u^2)^{n/2}e^{jn\phi(u)} \tag{8.19}$$

Figure 8.13 shows plots of the magnitude of (8.16) for $a = 10$, $n = 1$, $n = 2$, and $n = 3$.

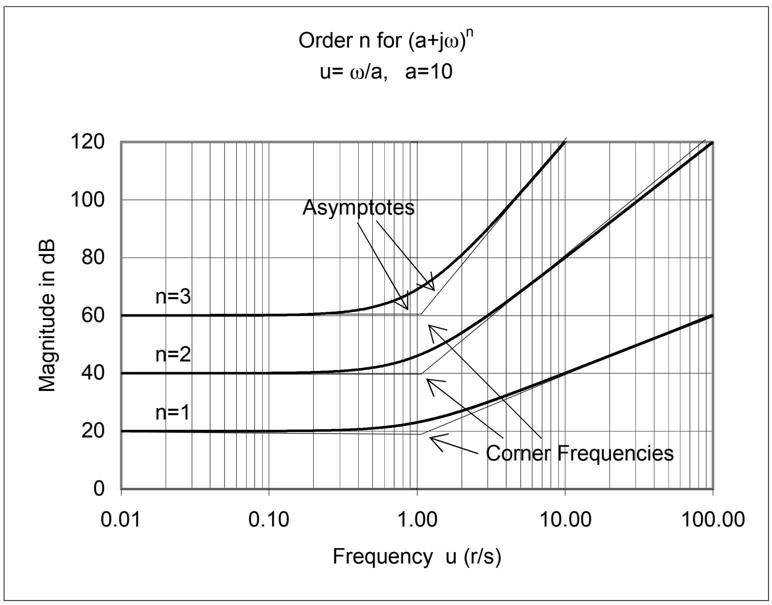

Figure 8.13. Magnitude for zeros of Order n for $(a + j\omega)^n$

As shown in Figure 8.13, a quick sketch can be obtained by drawing the straight line asymptotes given by $10\log 1 = 0$ and $10n\log u^2$ for $u \ll 1$ and $u \gg 1$ respectively.

The phase angle of (8.19) is $n\phi(u)$. Then, with (8.18) and letting

$$n\phi(u) = \theta(u) = n\tan^{-1}u \tag{8.20}$$

we obtain

$$\lim_{u \to 0} \theta(u) = \lim_{u \to 0} n\tan^{-1}u = 0 \tag{8.21}$$

and

$$\lim_{u \to \infty} \theta(u) = \lim_{u \to \infty} n\tan^{-1}u = \frac{n\pi}{2} \tag{8.22}$$

At the corner frequency $u = a$ we obtain $u = 1$ and with (8.20)

Chapter 9

Self and Mutual Inductances – Transformers

This chapter begins with the interactions between electric circuits and changing magnetic fields. It defines self and mutual inductances, flux linkages, induced voltages, the dot convention, Lenz's law, and magnetic coupling. It concludes with a detailed discussion on transformers.

9.1 Self–Inductance

About 1830, Joseph Henry, while working at the university which is now known as Princeton, found that electric current flowing in a circuit has a property analogous to mechanical momentum which is a measure of the motion of a body and it is equal to the product of its mass and velocity, i.e., Mv. In electric circuits this property is sometimes referred to as the *electrokinetic momentum* and it is equal to the product of Li where i is the current analogous to velocity and the *self–inductance* L is analogous to the mass M. About the same time, Michael Faraday visualized this property in a magnetic field in space around a current carrying conductor. This electrokinetic momentum is denoted by the symbol λ, that is,

$$\boxed{\lambda = Li} \tag{9.1}$$

Newton's second law states that the force necessary to change the velocity of a body with mass M is equal to the rate of change of the momentum, i.e.,

$$\mathbf{F} = \frac{d}{dt}(Mv) = M\frac{dv}{dt} = M\mathbf{a} \tag{9.2}$$

where $\mathbf{a}$ is the acceleration. The analogous electrical relation says that the voltage v necessary to produce a change of current in an inductive circuit is equal to the rate of change of electrokinetic momentum, i.e,

$$\boxed{v = \frac{d}{dt}(Li) = L\frac{di}{dt}} \tag{9.3}$$

9.2 The Nature of Inductance

Inductance is associated with the magnetic field which is always present when there is an electric current. Thus when current flows in an electric circuit, the conductors (wires) connecting the devices in the circuit are surrounded by a magnetic field. Figure 9.1 shows a simple loop of wire

In Figure 9.47, the value of the applied voltage V_S is set at its rated value*, and the voltmeter, ammeter, and wattmeter readings, denoted as V_{OC}, I_{OC}, and P_{OC} respectively, are measured and recorded. Then,

$$|Z_P| = \frac{V_{OC}}{I_{OC}} = \sqrt{R_P^2 + X_P^2} \tag{9.110}$$

from which

$$|X_P| = \sqrt{|Z_P|^2 - R_P^2} \tag{9.111}$$

The magnitude of the admittance Y_P in the excitation branch consisting of the parallel connection of R_C and X_M is found from

$$|Y_P| = \frac{V_{OC}}{I_{OC}} = \frac{I_{OC}}{V_{OC}} = \sqrt{G_C^2 + B_M^2} \tag{9.112}$$

where $G_C = 1/R_C$ and $B_M = 1/X_M$, and the phase angle θ_{OC} is found using the relation

$$\cos\theta_{OC} = \frac{P_{OC}}{V_{OC} \cdot I_{OC}} \tag{9.113}$$

from which

$$\theta_{OC} = \arccos\frac{P_{OC}}{V_{OC} \cdot I_{OC}} \tag{9.114}$$

Then,

$$\begin{aligned} G_C &= |Y_P|\cos\theta_{OC} \\ B_M &= |Y_P|\sin\theta_{OC} \end{aligned} \tag{9.115}$$

II. Short–Circuit Test

The short–circuit test is used to determine the magnitude of the series impedances referred to the primary side of the transformer denoted as Z_{SC} For this test, the secondary is shorted, and an ammeter, a voltmeter, and a wattmeter are connected as shown in Figure 9.48.

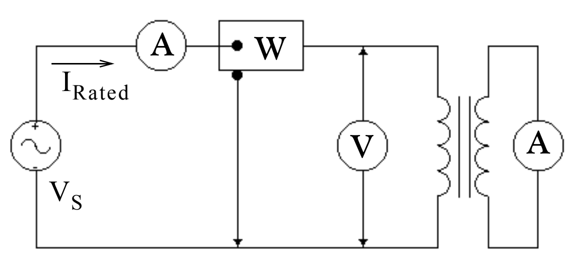

Figure 9.48. Configuration for transformer short–circuit test

* *It is important to use rated values so that the impedances and admittances will not have different values at different voltages.*

Chapter 10

One– and Two–Port Networks

This chapter begins with the general principles of one and two–port networks. The z, y, h, and g parameters are defined. Several examples are presented to illustrate their use. It concludes with a discussion on reciprocal and symmetrical networks.

10.1 Introduction and Definitions

Generally, a network has two pairs of terminals; one pair is denoted as the *input terminals*, and the other as the *output terminals*. Such networks are very useful in the design of electronic systems, transmission and distribution systems, automatic control systems, communications systems, and others where electric energy or a signal enters the input terminals, it is modified by the network, and it exits through the output terminals.

A *port* is a pair of terminals in a network at which electric energy or a signal may enter or leave the network. A network that has only one pair a terminals is called a *one–port network*. In an one–port network, the current that enters one terminal must exit the network through the other terminal. Thus, in Figure 10.1, $i_{in} = i_{out}$.

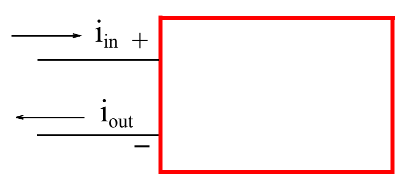

Figure 10.1. One–port network

Figures 10.2 and 10.3 show two examples of practical one–port networks.

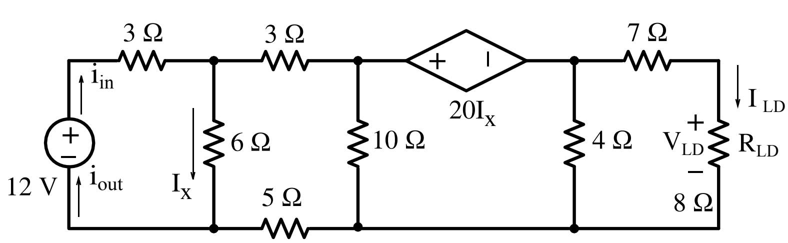

Figure 10.2. An example of an one–port network

and the voltage across that resistor is

$$\frac{5}{8} \times 15 = 75/8 \text{ V}$$

Therefore, the open circuit output impedance z_{22} is

$$z_{22} = \frac{v_1}{i_2} = \frac{75/8}{1} = 75/8\ \Omega \tag{10.91}$$

10.4.3 The h Parameters

A two–port network can also be described by the set of equations

$$v_1 = h_{11}i_1 + h_{12}v_2 \tag{10.92}$$

$$i_2 = h_{21}i_1 + h_{22}v_2 \tag{10.93}$$

as shown in Figure 10.34.

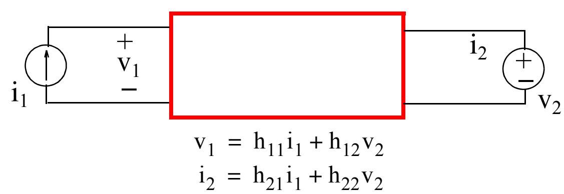

Figure 10.34. The h parameters for $i_1 \neq 0$ *and* $v_2 \neq 0$

The h parameters represent an impedance, a voltage gain, a current gain, and an admittance. For this reason they are called *hybrid* (different) parameters.

Let us assume that $v_2 = 0$ as shown in Figure 10.35.

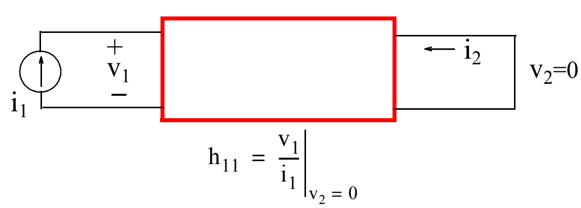

Figure 10.35. Network for the definition of the h_{11} *parameter*

Then, (10.92) reduces to

$$v_1 = h_{11}\, i_1 \tag{10.94}$$

or

$$h_{11} = \frac{v_1}{i_1} \tag{10.95}$$

Chapter 11

Balanced Three–Phase Systems

This chapter is an introduction to three–phase power systems. The advantages of three–phase system operation are listed and computations of three phase systems are illustrated by several examples.

11.1 Advantages of Three–Phase Systems

The circuits and networks we have discussed thus far are known as *single–phase* systems and can be either DC or AC. We recall that AC is preferable to DC because voltage levels can be changed by transformers. This allows more economical transmission and distribution. The flow of power in a three–phase system is constant rather than pulsating. Three–phase motors and generators start and run more smoothly since they have constant torque. They are also more economical.

11.2 Three–Phase Connections

Figure 11.1 shows three single AC series circuits where, for simplicity, we have assumed that the internal impedance of the voltage sources and the wiring have been combined with the load impedance. We also have assumed that the voltage sources are $120°$ out–of–phase, the load impedances are the same, and thus the currents I_a, I_b, and I_c have the same magnitude but are $120°$ out–of–phase with each other as shown in Figure 11.2.

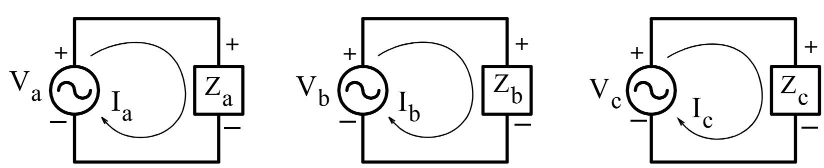

Figure 11.1. Three circuits with $120°$ out–of–phase voltage sources

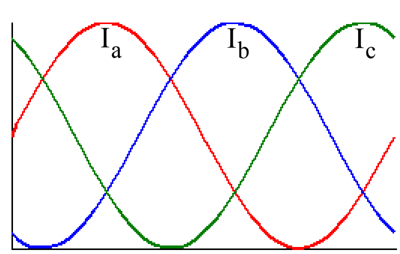

Figure 11.2. Waveforms for three $120°$ out–phase currents

$$I_{ab} = I_a + I_{ca}$$

or

$$I_a = I_{ab} - I_{ca} \tag{11.11}$$

The line currents I_b and I_c are derived similarly, and the phase–to–line current relationship in a Δ–connected load is shown in the phasor diagram of Figure 11.17.

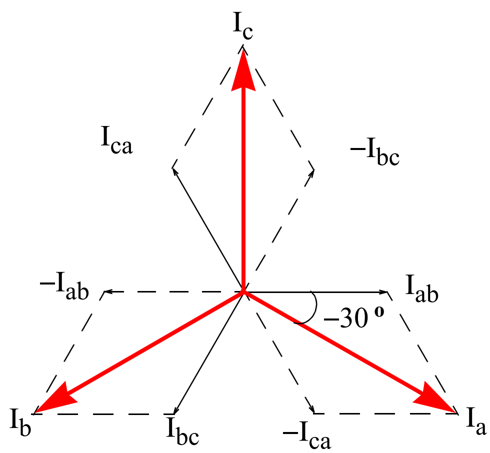

Figure 11.17. Phasor diagram for line and phase currents in Δ–connected load

From geometry and the law of sines we find that a balanced three–phase, positive phase sequence Δ–connected load, the line and phase currents are related as

$$\boxed{\begin{array}{c} I_a = \sqrt{3}I_{ab}\angle{-30°} \\ \Delta-\text{connected load} \end{array}} \tag{11.12}$$

The other two line currents can be easily obtained from the phasor diagram of Figure 11.17.

11.5 Equivalent Y and Δ Loads

In this section, we will establish the equivalence between the Y and Δ combinations shown in Figure 11.18.

Chapter 12

Unbalanced Three-Phase Systems

This chapter is an introduction to unbalanced three–phase power systems. It presents several practical examples of analysis applied to unbalanced three–phase systems and a number of observations are made based on the numerical examples. The method of symmetrical components is introduced and a phase sequence indicator serves as an illustration of a Y–connection with floating neutral.

12.1 Unbalanced Loads

Three–phase systems deliver power in enormous amounts to single–phase loads such as lamps, heaters, air–conditioners, and small motors. It is the responsibility of the power systems engineer to distribute these loads equally among the three–phases to maintain the demand for power fairly balanced at all times. While good balance can be achieved on large power systems, individual loads on smaller systems are generally unbalanced and must be analyzed as unbalanced three–phase systems.

Fortunately, many problems involving unbalanced loads can be handled as single–phase problems even though the computations can be three times as long as illustrated by the example below.

Example 12.1

In the three–phase system in Figure 12.1, the load consisting of electric heaters, draw currents as follows:

$$I_a = 150\text{ A} \qquad I_b = 100\text{ A} \qquad I_c = 50\text{ A}$$

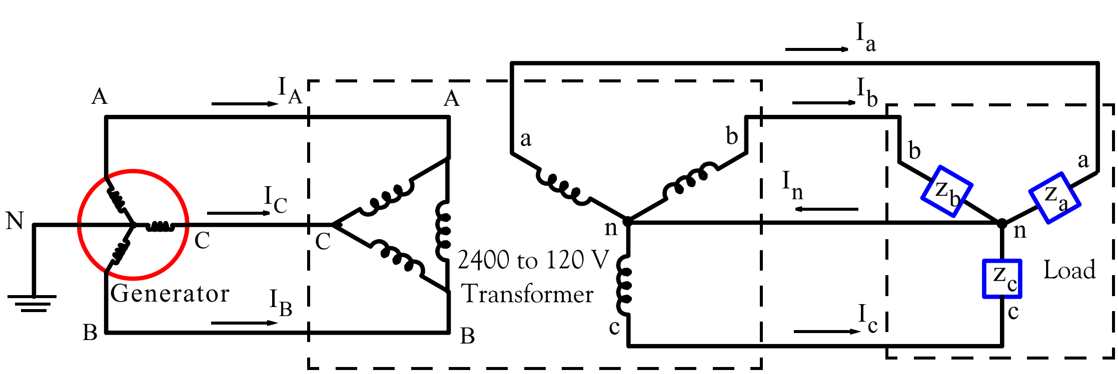

Figure 12.1. Three–phase system for Example 12.1

sequence voltages or currents are determined independently and the actual unbalanced voltages or currents are found by adding these three−phase sequences. Thus the solution of a difficult problem involving unbalanced voltages or currents is simplified to the solution of three easy problems involving only balanced voltages or currents.

Example 12.5

Show that the three unbalance current phasors in Figure 12.12(a) are the sum of the three balanced currents shown in Figure 12.12(b).

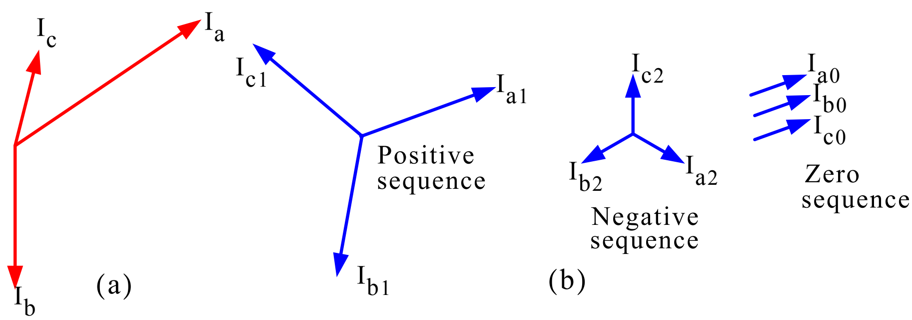

Figure 12.12. (a) Unbalanced currents and (b) their symmetrical components.

In symmetrical components, a symmetrical set of vectors as shown in Figure 12.12(b) above, are equal in length, and equally spaced in angle. The symmetrical sets of three vectors such as those shown in Figure 12.12(b) are related by equation (12.12) below.

$$I_{an} = I_{bn}\angle n\cdot 120° = I_{cn}\angle 2n\cdot 120° \qquad (12.12)$$

For the positive−sequence we set $n = 1$, and thus

$$I_{a1} = I_{b1}\angle 120° = I_{c1}\angle 240° \qquad (12.13)$$

In other words, for the positive−phase sequence set the order is $a-b-c-a-b-c-\dots$ as shown in Figure 12.13 below.

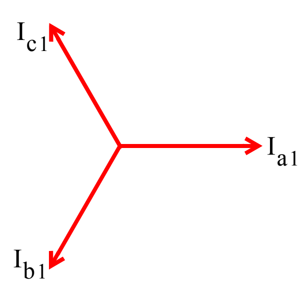

Figure 12.13. Positive sequence phasor diagram

For the negative−sequence we set $n = 2$, and thus

Appendix A

Introduction to MATLAB®

This appendix serves as an introduction to the basic MATLAB commands and functions, procedures for naming and saving the user generated files, comment lines, access to MATLAB's Editor / Debugger, finding the roots of a polynomial, and making plots. Several examples are provided with detailed explanations.

A.1 MATLAB® and Simulink®

MATLAB and Simulink are products of The MathWorks,™ Inc. These are two outstanding software packages for scientific and engineering computations and are used in educational institutions and in industries including automotive, aerospace, electronics, telecommunications, and environmental applications. MATLAB enables us to solve many advanced numerical problems rapidly and efficiently.

A.2 Command Window

To distinguish the screen displays from the user commands, important terms, and MATLAB functions, we will use the following conventions:

Click: Click the left button of the mouse
`Courier Font`: Screen displays
Helvetica Font: User inputs at MATLAB's command window prompt >> or `EDU>>`*
Helvetica Bold: MATLAB functions

Times Bold Italic: Important terms and facts, notes and file names
When we first start MATLAB, we see various help topics and other information. Initially, we are interested in the *command screen* which can be selected from the Window drop menu. When the command screen, we see the prompt >> or `EDU>>`. This prompt is displayed also after execution of a command; MATLAB now waits for a new command from the user. It is highly recommended that we use the *Editor/Debugger* to write our program, save it, and return to the command screen to execute the program as explained below.

To use the Editor/Debugger:

1. From the *File* menu on the toolbar, we choose *New* and click on *M–File*. This takes us to the *Editor Window* where we can type our *script* (list of statements) for a new file, or open a previously saved file. We must save our program with a file name which starts with a letter.

* EDU>> *is the MATLAB prompt in the Student Version*

Appendix B

Introduction to Simulink®

This appendix is a brief introduction to Simulink. This author feels that we can best introduce Simulink with a few examples. Some familiarity with MATLAB is essential in understanding Simulink, and for this purpose, Appendix A is included as an introduction to MATLAB.

B.1 Simulink and its Relation to MATLAB

The MATLAB® and Simulink® environments are integrated into one entity, and thus we can analyze, simulate, and revise our models in either environment at any point. We invoke Simulink from within MATLAB. We will introduce Simulink with a few illustrated examples.

Example B.1

For the circuit of Figure B.1, the initial conditions are $i_L(0^-) = 0$, and $v_c(0^-) = 0.5\ V$. We will compute $v_c(t)$.

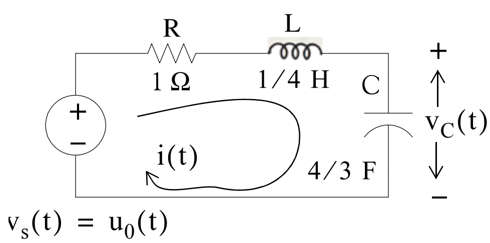

Figure B.1. Circuit for Example B.1

For this example,

$$i = i_L = i_C = C\frac{dv_C}{dt} \tag{B.1}$$

and by Kirchoff's voltage law (KVL),

$$Ri_L + L\frac{di_L}{dt} + v_C = u_0(t) \tag{B.2}$$

Substitution of (B.1) into (B.2) yields

Appendix C

Introduction to SimPowerSystems®

This appendix is a brief introduction to **SimPowerSystems®** blockset that operates in the **Simulink®** environment. An introduction to **Simulink** is presented in Appendix B. For additional help with Simulink, please refer to the Simulink documentation.

C.1 Simulation of Electric Circuits with SimPowerSystems

As stated in Appendix B, the MATLAB® and Simulink® environments are integrated into one entity, and thus we can analyze, simulate, and revise our models in either environment at any point. We can invoke **Simulink** from within MATLAB or by typing simulink at the MATLAB command prompt, and we can invoke **SimPowerSystems** from within Simulink or by typing powerlib at the MATLAB command prompt. We will introduce **SimPowerSystems** with two illustrated examples, a DC electric circuit, and an AC electric circuit

Example C.1

For the simple resistive circuit in Figure C.1, $v_S = 12v$, $R_1 = 7\Omega$, and $R_2 = 5\Omega$. From the voltage division expression, $v_{R2} = R_2 \times v_S / (R_1 + R_2) = 5 \times 12/12 = 5v$ and from Ohm's law, $i = v_S / (R_1 + R_2) = 1A$.

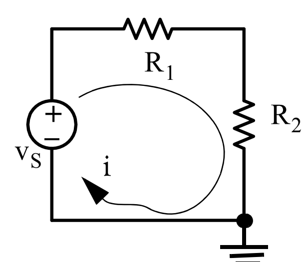

Figure C.1. Circuit for Example C.1

To model the circuit in Figure C.1, we enter the following command at the MATLAB prompt.

```
powerlib
```

and upon execution of this command, the powerlib window shown in Figure C.2 is displayed.

From the **File** menu in Figure C.2, we open a new window and we name it Sim_Fig_C3 as shown in Figure C.3.

Appendix D

Review of Complex Numbers

This appendix is a review of the algebra of complex numbers. The basic operations are defined and illustrated by several examples. Applications using Euler's identities are presented, and the exponential and polar forms are discussed and illustrated with examples.

D.1 Definition of a Complex Number

In the language of mathematics, the square root of minus one is denoted as i, that is, $i = \sqrt{-1}$. In the electrical engineering field, we denote i as j to avoid confusion with current i. Essentially, j is an operator that produces a 90–degree counterclockwise rotation to any vector to which it is applied as a multiplying factor. Thus, if it is given that a vector A has the direction along the right side of the x–axis as shown in Figure D.1, multiplication of this vector by the operator j will result in a new vector jA whose magnitude remains the same, but it has been rotated counterclockwise by $90°$.

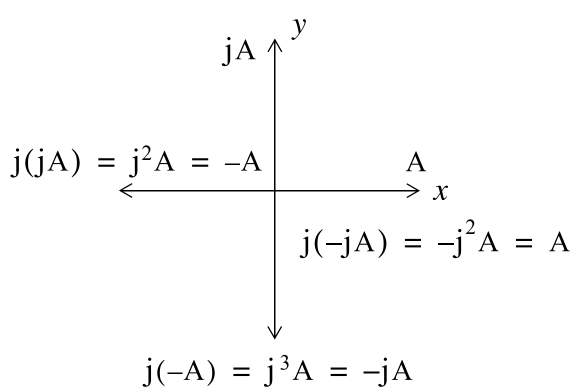

Figure D.1. The j operator

Also, another multiplication of the new vector jA by j will produce another $90°$ counterclockwise direction. In this case, the vector A has rotated $180°$ and its new value now is $-A$. When this vector is rotated by another $90°$ for a total of $270°$, its value becomes $j(-A) = -jA$. A fourth $90°$ rotation returns the vector to its original position, and thus its value is again A. Therefore, we conclude that $j^2 = -1$, $j^3 = -j$, and $j^4 = 1$.

Appendix E

Matrices and Determinants

This appendix is an introduction to matrices and matrix operations. Determinants, Cramer's rule, and Gauss's elimination method are reviewed. Some definitions and examples are not applicable to the material presented in this text, but are included for subject continuity, and academic interest. They are discussed in detail in matrix theory textbooks. These are denoted with a dagger (†) and may be skipped.

E.1 Matrix Definition

A *matrix* is a rectangular array of numbers such as those shown below.

$$\begin{bmatrix} 2 & 3 & 7 \\ 1 & -1 & 5 \end{bmatrix} \qquad \text{or} \qquad \begin{bmatrix} 1 & 3 & 1 \\ -2 & 1 & -5 \\ 4 & -7 & 6 \end{bmatrix}$$

In general form, a matrix A is denoted as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \tag{E.1}$$

The numbers a_{ij} are the *elements* of the matrix where the index i indicates the row, and j indicates the column in which each element is positioned. For instance, a_{43} indicates the element positioned in the fourth row and third column.

A matrix of m rows and n columns is said to be of $m \times n$ *order matrix*.

If $m = n$, the matrix is said to be a *square matrix of order* m (or n). Thus, if a matrix has five rows and five columns, it is said to be a square matrix of order 5.

Appendix F

Scaling

This chapter discusses magnitude and frequency scaling procedures that allow us to transform circuits that contain passive devices with unrealistic values to equivalent circuits with realistic values.

F.1 Magnitude Scaling

Magnitude scaling is the process by which the impedance of a two terminal network is changed by a factor k_m which is a real positive number greater or smaller than unity.

If we increase the input impedance by a factor k_m, we must increase the impedance of each device of the network by the same factor. Thus, if a network consists of R, L, and C devices and we wish to scale this network by this factor, the magnitude scaling process entails the following transformations where the subscript *m* denotes magnitude scaling.

$$\begin{aligned} R_m &\rightarrow k_m R \\ L_m &\rightarrow k_m L \\ C_m &\rightarrow \frac{C}{k_m} \end{aligned} \tag{F.1}$$

These transformations are consistent with the time–domain to frequency domain transformations

$$\begin{aligned} R &\rightarrow R \\ L &\rightarrow j\omega L \\ C &\rightarrow \frac{1}{j\omega C} \end{aligned} \tag{F.2}$$

and the t–domain to s–domain transformations

$$\begin{aligned} R &\rightarrow R \\ L &\rightarrow sL \\ C &\rightarrow \frac{1}{sC} \end{aligned} \tag{F.3}$$

F.2 Frequency Scaling

Frequency scaling is the process in which we change the values of the network devices so that at the new frequency the impedance of each device has the same value as at the original frequency.

Appendix G

Per Unit System

This chapter introduces the per unit system. This system allows us to work with normalized power, voltage current, impedance, and admittance values known as per unit (pu) values. The relationship between units in a per-unit system depends on whether the system is single–phase or three–phase. Three-phase systems are discussed in Chapters 11 and 12.

G.1 Per Unit Defined

By definition,

$$\text{Per Unit Value} = \frac{\text{Actual Value}}{\text{Base Value}} \tag{G.1}$$

A per unit (pu) system defines per unit values for volt–ampere (VA) power, voltage, current, impedance, and admittance, and of these only two of these are independent. It is customary to choose VA (or KVA) power and nominal voltage as the independent base values, and others are specified as multiples of selected base values.

For single-phase systems, the pu values are based on rated VA (or KVA) rated power and on the nominal voltage of the equipment, e.g., single–phase transformer, single–phase motor.

Example G.1

A single–phase transformer is rated 10 KVA and the nominal voltage on the primary winding is 480 V RMS. Compute its pu impedance.

Solution:

$$\begin{aligned}\text{Base Current (amperes)} &= \frac{\text{Base KVA}}{\text{Base Volts}} = \frac{10000 \text{ VA}}{480 \text{ V}} = 20.83 \text{ A RMS} \\ \text{Base Impedance (Ohms)} &= \frac{\text{Base Volts}}{\text{Base Current}} = \frac{480 \text{ V}}{20.83 \text{ A}} = 23.04\ \Omega\end{aligned} \tag{G.2}$$

and assuming that the actual primary winding voltage, current, and impedance are 436 Volts RMS, 15 A RMS, and $5\ \Omega$, respectively, the per unit values are computed as follows:

Appendix H

Review of Differential Equations

This appendix is a review of ordinary differential equations. Some definitions, topics, and examples are not applicable to introductory circuit analysis but are included for continuity of the subject, and for reference to more advance topics in electrical engineering such as state variables. These are denoted with an asterisk and may be skipped.

H.1 Simple Differential Equations

In this section we present two simple examples to show the importance of differential equations in engineering applications.

Example H.1

A $1\ F$ capacitor is being charged by a constant current I. Find the voltage v_C across this capacitor as a function of time given that the voltage at some reference time $t = 0$ is V_0.

Solution:

It is given that the current, as a function of time, is constant, that is,

$$i_C(t) = I = \text{constant} \tag{H.1}$$

We know that the current and voltage in a capacitor are related by

$$i_C(t) = C\frac{dv_C}{dt} \tag{H.2}$$

and for our example, $C = 1$. Then, by substitution of (H.2) into (H.1) we obtain

$$\frac{dv_C}{dt} = I$$

By separation of the variables,

$$dv_C = Idt \tag{H.3}$$

and by integrating both sides of (H.3) we obtain

$$v_C(t) = It + k \tag{H.4}$$

where k represents the constants of integration of both sides.

Appendix I

Constructing Semilog Paper with Excel® and with MATLAB®

This appendix contains instructions for constructing semilog plots with the Microsoft Excel spreadsheet. Semilog, short for semilogarithmic, paper is graph paper having one logarithmic and one linear scale. It is used in many scientific and engineering applications including frequency response illustrations and Bode Plots.

I.1 Instructions for Constructing Semilog Paper with Excel

Figure I.1 shows the Excel spreadsheet workspace and identifies the different parts of the Excel window when we first start Excel.

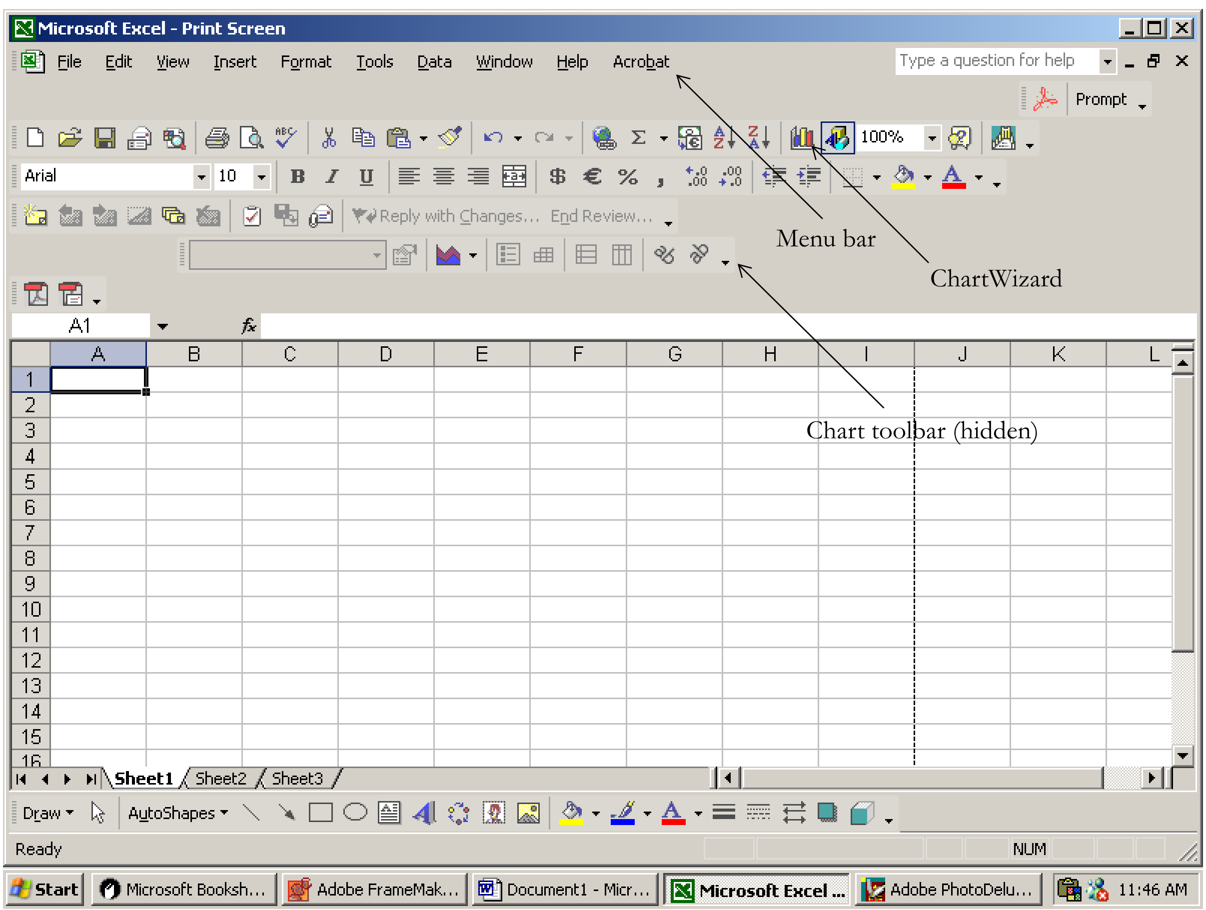

Figure I.1. The Excel Spreadsheet Workspace

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