MATLAB 的使用

第一章　MATLAB 使用環境

- 視窗介面
  1. 命令指令視窗 command window
     i. pwd, ls, cd, …
     ii. who, whos, …
     iii. edit, workspace, demo, …
  2. M-file editor：編輯檔案，如 Edit vrml.m。
  3. Workspace：觀看變數和變數值，使用 workspace 以呼叫視窗。
  4. Command history：觀看指令記錄。
  5. demo：觀看各種 demo。

- 線上說明
  1. help：命令指令視窗的說明，如 help sin、help min、help diff…。
  2. helpwin：說明輔助視窗，如 helpwin sin、helpwin min、helpwin diff…。

- 程式語言
  1. 變數：matlab 提供變數資料型態的種類，如 char、double、struct⋯，且在 matlab 中，不需事先宣告變數資料型態。
  2. 運算子：
     i. 數學運算子：.’(轉置矩陣)、.^(矩陣的次方運算)、’(共軛複數) 、.*(方陣次方)、./(右除法)、.\(左除法)、+、_、*、/…。
     ii. 關係運算子：<、<=、>、>=、==、~=。
     iii. 邏輯運算子：&、|、~。
  3. 判斷(if、else、elseif)
     i. if (a>=0)
        a=a;
        else a=a*(-1);
        ……………………………………………………………………………………
  4. 迴圈(for、while)
     i. for　index = start : increment : end
        end
     ……………………………………………………………………………………
  5. 函數(function)
  6. 函式庫(如 sin(o)、sin(pi/2)、sin(pi)…)
  7. 檔案的輸出與輸入(fopen、fclose)

- 數學應用
  1.解線性代數
  2.2D & 3D 繪圖
  3.數值分析
  4.統計方法
  5.微分 & 積分

  ……………………………………………………………………………………

- demo
  1. 在命令視窗中使用 demo，以呼叫出視窗。
  2. MATLAB >> Matrices >> Basic matrix operation >> RUN。
  3. MATLAB >> Graphics >> 2-D Plots >> RUN。
  4. MATLAB >> Graphics >> 3-D Plots >> RUN。
  5. Toolboxes >> Mapping >> Animated Satellite Orbits >> RUN。
  6. Toolboxes >> Mapping >> Map Projections Comparison >> RUN。

  ……………………………………………………………………………………

- Toolbox
- 變數與矩陣

<table>
<tr>
<td>

```
>> A=[3.5]
>> B=[1.5,3.1]
>> C=[-1,0,0;1,1,0;1,-1,0;0,0,2]
C =
    -1     0     0
     1     1     0
     1    -1     0
     0     0     2
>> X= C (:,1)
X =
    -1
     1
     1
     0
>> Y=C(:,2:3)
Y =
     0     0
     1     0
    -1     0
     0     2
```

</td>
<td>

```
>> A=[1 2 3; 4 5 6 ; 7 8 9]
A =
     1     2     3
     4     5     6
     7     8     9
>> B=inv(A)
>> S=1-1/2+1/3-1/4+1/5-1/6+1/7-1/8…
    +1/9+1/10
>> format short
>> format short g
>> format long
>> format long e
>> format long g
>> format bank
>> format rat
>> format hex
```

</td>
</tr>
</table>

● 實例演練一 (chap1_1.m)

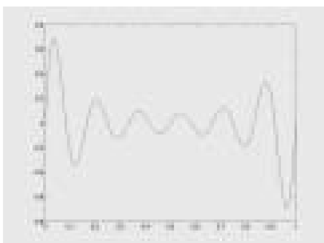
```
% File: ../ch1/ch1_1.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Introduce the Input prompt

a=[1 2 3]
c=[4;5;6]
a*c
dot(a,c)
A=c*a
b=a.^2
a.*b
exp(a)
log(ans)
sqrt(a)
format long
sqrt(a)
format
2^(-24)
sum(b),mean(c)
pi
y=tan(pi/6)
B=[-3 0 1; 2 5 -7; -1 4 8]
x=B\c
norm(B*x-c)
e=eig(B)
[V,D]=eig(B);
v=1:6;
w=2:3:10,y=1:-0.25:0;
C=[A,[8;9;10]],D=[B;a]
C(2,3)
C(2:3,1:2)
I3=eye(3,3),Y=zeros(3,5),Z=ones(2)
rand('state',20),randn('state',20)
F=rand(3),G=randn(1,5)
who
workspace
```

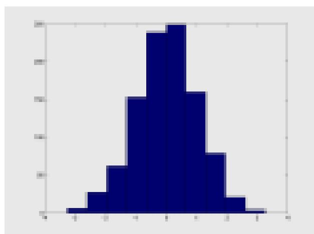
```
a =
     1     2     3
c =
     4
     5
     6
ans =
    32
ans =
    32
A =
     4     8    12
     5    10    15
     6    12    18
b =
     1     4     9
ans =
     1     8    27
ans =
    2.7183    7.3891   20.0855
ans =
     1     2     3
ans =
    1.0000    1.4142    1.7321
ans =
   1.00000000000000
1.41421356237310
1.73205080756888
ans =
  5.9605e-008
ans =
    14
ans =
     5
ans =
    3.1416
y =
    0.5774
B =
    -3     0     1
     2     5    -7
    -1     4     8
x =
   -1.3717
    1.3874
   -0.1152
```

```
ans =
  8.8818e-016
e =
  -2.8601
   6.4300 + 5.0434i
   6.4300 - 5.0434i
C =
     4     8    12     8
     5    10    15     9
     6    12    18    10
D =
    -3     0     1
     2     5    -7
    -1     4     8
     1     2     3

I3 =
     1     0     0
     0     1     0
     0     0     1
Y =
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
Z =
     1     1
     1     1
F =
    0.7062    0.3586    0.8468
    0.5260    0.8488    0.3270
    0.2157    0.0426    0.5541
G =
    1.4051         1.1780         -1.1142
0.2474     -0.8169
Your variables are:
A     C     F     I3     Y     a     b
e     w     y

B     D     G     V     Z     ans     c

v     x
```

● 實例演練二 (chap1_2.m)

```
% File: ../ch1/ch1_2.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Indroduce the loop and plot

g=2;
for k=1:10,g=1+1/g;end
g

t=0:0.005:1,z=exp(10*t.*(t-1)).*sin(12*pi*t);
figure(1);plot(t,z);
figure(2);hist(randn(1000,1));
```



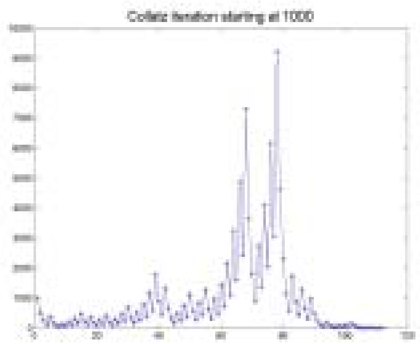


- 實例演練三 (chap1_3.m)

```
% File: ../ch1/ch1_3.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Random Fibonacci sequence

%Set random number state
rand('state',100)
%Number of iterations
m=1000;

%Inital Condition
x=[1 2];
%Main loop
for n=2:m-1
    x(n+1)=x(n)+sign(rand-0.5)*x(n-1);
end

semilogy(1:m,abs(x))
c=1.13198824
hold on
semilogy(1:m,c.^(1:m))
title('Random Fibonacci sequence')
hold off
```

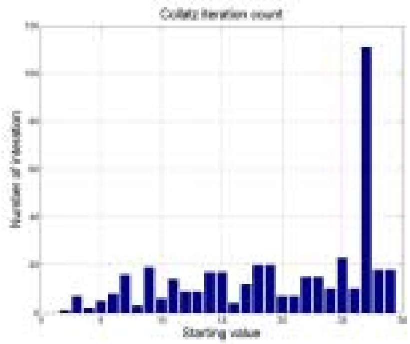
- 實例演練四 (chap1_4.m)

```
%File: ../ch1/ch1_4.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Collatz iteration

n=input('Enter an interger bigger than 2: ');
narray=n;

count=1;
while n ~= 1
    if rem(n,2) == 1 %Remainder modulo 2
        n=3*n+1;
   else
        n=n/2;
   end
   count=count+1;
   narray(count)=n; %Store the current iterate
end

%Plot with * marker and solid line style
plot(narray,'*-')
title(['Collatz iteration starting at '
int2str(narray(1))],'FontSize',16)
```



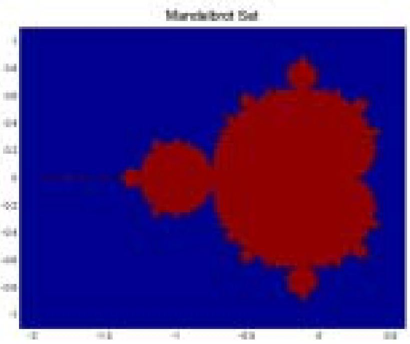
- 實例演練五 (chap1_5.m)

```
%File: ../ch1/ch1_5.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Collatz interation bar graphy

%Use starting value 1,2,...,N
N=29;
%Preallocate array
niter=zeros(N,1);

for i=1:N
	count=0;
	n=i;
	while n~= 1
		if rem(n,2) == 1
			n=3*n+1;
		else
			n=n/2;
		end
		count=count+1;
	end
	niter(i)=count;
end

%Bar graphy
bar(niter)
%Add horizontal and vertical grid lines
grid
title('Collatz iteration count','FontSize',16)
%Label of x and y label
xlabel('Starting value','FontSize',16)
ylabel('Number of interation','FontSize',16)
```



- 實例演練六 (chap1_6.m)

```
%File: ../ch1/ch1_6.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Mandelbrot set

h=waitbar(0,'Computing...');
x=linspace(-2.1,0.6,301);
y=linspace(-1.1,1.1,301);
[X,Y]=meshgrid(x,y);
C=complex(X,Y);
Z_max=1e6;it_max=50;
Z=C;
for k=1:it_max
	Z=Z.^2+C;
	waitbar(k/it_max)
end
close(h)
contourf(x,y,abs(Z)<Z_max,1)
title('Mandelbrot Set','FontSize',16)
```



第二章　常用函式

- 矩陣產生函式

函式	功能
zero	產生零矩陣(zeros array)
ones	產生全部元素均為一的矩陣(ones array)
eyes	產生單位矩陣(identity matrix)
repmat	產生元素區塊重複矩陣
rand	產生均勻分佈亂數
randn	產生正規分佈亂數
linspace	產生線性空間
logspace	產生對數空間
meshgrid	產生給 3-D plots 的 X 和 Y 陣列

- 基本陣列資料函式

函式	功能
size	矩陣大小
length	向量長度
ndims	陣列的階數
disp	顯示陣列或文字
isempty	檢測空矩陣
isequal	檢測單位矩陣
isnumeric	檢測數值矩陣
islogical	檢測邏輯矩陣
logical	轉換數值矩陣為邏輯矩陣

● 矩陣運算操作函式

函式	功能
reshape	更改大小
diag	對角矩陣或陣列
tril	取出矩陣下三角部份
triu	取出矩陣上三角部份
fliplr	將矩陣左右對調
flipud	將矩陣上下對調
flipdim	將矩陣沿特定方向對調
rot90	將矩陣旋轉 90 度
find	找出非零元素的指標
end	最後指標
sub2ind	Linear index from multiple subscripts.
ind2sub	Multiple subscripts from linear index.

● 特殊值與特殊矩陣

函式	功能
ans	Most recent answer.
eps	Floating point relative accuracy.
realmax	Largest positive floating point number.
realmin	Smallest positive floating point number.
pi	3.1415926535897...., PI = 4*atan(1) = imag(log(-1)) = 3.1415926535897....
i,j	Imaginary unit.
inf	Returns the IEEE arithmetic representation for positive infinity.
nan	The IEEE arithmetic representation for Not-a-Number.
isnan	True for Not-a-Number.
isinf	True for infinite elements.
isfinite	True for finite elements.
flops	Obsolete floating point operation count.
clock	Current date and time as date vector.
date	Current date as date string.

- 一般數學函式

函式	功能
abs(x)	The absolute value of the elements of X
sqrt(x)	The square root of the elements of X.
round(x)	Rounds the elements of X to the nearest integers.
fix(x)	Rounds the elements of X to the nearest integers towards zero.
floor(x)	Rounds the elements of X to the nearest integers towards minus infinity.
ceil(x)	Rounds the elements of X to the nearest integers towards infinity.
sign(x)	Signum function.
rem(x)	Remainder after division.
exp(x)	The exponential of the elements of X, e to the X.
log(x)	The natural logarithm of the elements of X.
log10(x)	The base 10 logarithm of the elements of X.

- 三角函式

函式	功能
sin(x)	The sine of the elements of X.
cos(x)	The cosine of the elements of X.
tan(x)	The tangent of the elements of X.
asin(x)	Inverse sine.
acos(x)	Inverse cosine.
atan(x)	Inverse tangent.
atan2(y,x)	Four quadrant inverse tangent.

- hyperbolic 函式

函式	功能
asinh(x)	The inverse hyperbolic sine of the elements of X.
cosh(x)	The hyperbolic cosine of the elements of X.
tanh(x)	The hyperbolic tangent of the elements of X.
asinh(x)	The inverse hyperbolic sine of the elements of X.
acosh(x)	The inverse hyperbolic cosine of the elements of X.
atanh(x)	The inverse hyperbolic tangent of the elements of X.

- 複數函式

函式	功能
conj(x)	The complex conjugate of X.
real(x)	The real part of X.
imag(x)	Displays matrix C as an image.
abs(x)	The absolute value of the elements of X.
angle(x)	Returns the phase angles, in radians, of a matrix with complex elements.
polar(theta,r)	Makes a plot using polar coordinates of the angle THETA, in radians, versus the radius RHO.

- 多項式函式

函式	功能
polyval(a,x)	Evaluate polynomial.
conv(a,b)	Convolution and polynomial multiplication.
[q,r]=deconv(n,d)	Deconvolution and polynomial division.
roots(a)	Find polynomial roots.
poly(r)	Convert roots to polynomial.

- 2D 繪圖函式

函式	功能
plot(x,y)	Plots vector Y versus vector X.
semilogx(x,y)	Semi-log scale plot.
semilogy(s,y)	Semi-log scale plot.
loglog(x,y)	Log-log scale plot.
Axis,axis(v)	Control axis scaling and appearance.
subplot	Create axes in tiled positions.
pause	Wait for user response.
title	Graph title.
xlabel, ylabel	Adds text beside the X-axis and Y-axis on the current axis.
grid	Adds grid lines to the current axes.
meshgrid	X and Y arrays for 3-D plots.

- 3D 繪圖函式

函式	功能
mesh(x_pts,y_pts,z)	3-D mesh surface.
surf(x_pts,y_pts,z)	3-D colored surface.
contour(x,y,z)	X and Y specify the (x,y) coordinates of the surface as for SURF.
contour(x,y,z,v)	Draw LENGTH(V) contour lines at the values specified in vector V.
meshc(x_pts,y_pts,z)	Combination mesh/contour plot.

- 數值分析函式

函式	功能
max(x)	The largest element in X
[y,k]=max(x)	Returns the indices of the maximum values in vector I.
max(x,y)	Returns an array the same size as X and Y with the largest elements taken from X or Y.
min(x)	The smallest element in X.
[y,k]=min(x)	Returns the indices of the minimum values in vector I.
min(x,y)	Returns an array the same size as X and Y with the smallest elements taken from X or Y.
sum(x)	The sum of the elements of X.
prod(x)	The product of the elements of X.
cumsum(x)	A vector containing the cumulative sum of the elements of X.
cumprod(x)	A vector containing the cumulative product of the elements of X.
Mean(x)	The mean value of the elements in X.
mediam(x)	The median value of the elements in X.
sort(x)	Sorts the elements of X in ascending order.
std(x)	Returns the standard deviation.
hist(x)	Bins the elements of Y into 10 equally spaced containers
hist(x,n)	Returns the number of elements in each container.

- 邏輯函式

函式	功能
any(x)	True if any element of a vector is nonzero.
all(x)	True if all elements of a vector are nonzero.
find(x)	Find indices of nonzero elements.
isnan(x)	True for Not-a-Number.
isempty(x)	True for empty matrix.

- 亂數函式

函式	功能
rand(n)	N-by-N matrix with random entries,
rand(m,n)	M-by-N matrices with random entries.
rand(‘seed’,n)	Cause the MATLAB 5 generator to be used.
rand(‘seed’)	Returns the current seed of the MATLAB 4 uniform generator.
randn(n)	An N-by-N matrix with random entries
randn(m,n)	M-by-N matrices with random entries.

- 輸入與輸出函式

函式	功能
fprintf	Write formatted data to file.
save	Save workspace variables to disk.
load	Load workspace variables from disk.
input	Prompt for user input.
disp	Displays the array, without printing the array name.

- 數值表現格式(format)

MATLAB command	display
format short	Default, ex:/ 15.2345
format long	14 decimals, ex:/ 15.23453333333333
format bank	2 decimals, ex:/ 15.23
format short e	4 decimals, ex:/ 1.5235e+01
format long e	15 decimals, ex:/ 1.523453333333333e+01
format +	+,-,blank, ex:/ +

- 實例演練一 (chap2_1.m)

% File: ../ch2/ch2_1.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Interaction and Script Files
 	
(1+sqrt(5))/2
2^(-53)
x=sin(22)
y=2*x+exp(-3)/(1+cos(.1));
x=2,y=cos(.3),z=3*x*y
 	
exmark=[12 0 5 28 97 3 56]
exsort=sort(exmark)
exmean=mean(exmark)
exmed=median(exmark)
exstd=std(exmark)
 	
A=rand(3)
inv(A)
ans*A
format short e
ans
 	
%help sqrt
%lookfor elliptic
 	
x=1+1/2+1/3+1/4+1/5+1/6+...
    1/7+1/8+1/9+1/10
 	
w=(-1)^0.25
exp(i*pi)
 	
save filename A x
 	
A=eye(2);disp(A);
disp('Result:'),disp(1/7)
	ans =
	  1.6180e+000
	ans =
	  1.1102e-016
	x =
	 -8.8513e-003
	x =
	     2
	y =
	  9.5534e-001
	z =
	  5.7320e+000
	exmark =
	    12 0 5 28 97 3 56
	exsort =
	     0 3 5 12 28 56 97
	exmean =
	  2.8714e+001
	exmed =
	    12
	exstd =
	  3.5906e+001
	A =
	  5.6361e-001 6.7911e-001 2.6952e-001
	  5.1677e-001 4.8714e-001 8.0017e-002
	  2.5689e-001 3.7027e-001 3.2226e-001
	ans =
	 -1.3388e+001 1.2515e+001 8.0896e+000
	  1.5345e+001 -1.1815e+001 -9.9007e+000
	 -6.9595e+000 3.5983e+000 8.0303e+000
	ans =
	  1.0000e+000 0 4.4409e-016
	  1.7764e-015 1.0000e+000 -4.4409e-016
	  0 0 1.0000e+000
	ans =
	  1.0000e+000 0 4.4409e-016
	  1.7764e-015 1.0000e+000 -4.4409e-016
	  0 0 1.0000e+000
	x =
	  2.9290e+000
	w =
	  7.0711e-001 +7.0711e-001i
	ans =
	 -1.0000e+000 +1.2246e-016i
	     1 0
	     0 1
	Result:
	  1.4286e-001

- 實例演練二 (chap2_2.m)

```
% File: ../ch2/ch2_2.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Distinctive Feature and Arithmetic of MATLAB

%Distinctive Feature
clear;
x(3)=0
x(6)=0
x=[3 4]
norm(x)
norm(x,1)
m=max(x)
[m,k]=max(x)
A=size(5,3)
s=size(A)
[m,n]=size(A)

%Arithmetic
computer
isieee
eps
realmax
-2*realmax
1.1*realmax
0/0
inf-inf
inf-inf
NaN-NaN
realmin
realmin*eps
realmin*eps/2
2^10/10
2+3*4
-2-3*4
1+2/3*4
1+2/(3*4)
```

```
x =
      0      0      0
x =
      0      0      0      0      0      0
x =
      3      4
ans =
      5
ans =
      7
m =
      4
m =
      4
k =
      2
A =
      1
s =
      1      1
m =
      1
n =
      1
ans =
PCWIN
Warning: ISIEEE is obsolete.   MATLAB only runs on IEEE machines.
> In C:\MATLABR12\toolbox\matlab\general\isieee.m at line 9
  In D:\書籍資料\Matlab\ch2\ch2_2.m at line 20
ans =
      1
ans =
  2.2204e-016
ans =
  1.7977e+308
ans =
  -Inf
ans =
   Inf
Warning: Divide by zero.
> In D:\書籍資料\Matlab\ch2\ch2_2.m at line 25
ans =
   NaN
ans =
   NaN
ans =
   NaN
ans =
   NaN
ans =
  2.2251e-308
ans =
  4.9407e-324
ans =
      0
ans =
  1.0240e+002
ans =
    14
ans =
   -14
ans =
  3.6667e+000
ans =
  1.1667e+000
```

第三章　M-File 程式設計

- 資料型態：Matlab 提供有 char、double、sparse、unit8、cell、struct 等六種型態，每一種都是陣列模式。

型態類別	範例	說明
char	char 'Luoson-1'	字元陣列。每一個字元用 16 個位元表示。
double	[1.5 2;0.01 0]	倍精準數值陣列。一般運算子、函式、陣列函式都支援。
sparse	speye(4)	倍精準數植稀疏陣列。僅適用於二維陣列，只儲存非零元素與其向量。運算時須配合特殊的運算函式，例如：splu、spchol。
unit8	unit(magic(3))	無符號的八位元整數陣列。表 0~255 的整數。目前無運算元可用，一般使用於資料儲存或配合影像處理工具箱使用。
cell	{'Luoson' 100 eye(2)}	密室陣列。可以包含其它不同類別的資料陣列，適用於大型資料庫使用。
struct	Cat.name='mimi' Cat.age=1.2 Cat.color='yellow'	結構陣列。與密室陣列一樣，可以將不同的資料型態包在同一個變數名稱下，但結構陣列另外含有欄位名稱。
UserObject		使用者定義的資料型態。

- 運算子：Matlab 運算子型態有數學(ex:/ +, -, *, / …)、關係(ex:/ > , < …)、邏輯運算子(and, or…)三種，處理順序為數學運算子>關係運算子>邏輯運算子。

數學運算子

運算子符號	功能	範例
.'	轉置矩陣 A^T	A=[10 1 2]; A.'=[10; 1; 2]
.^	矩陣的次方運算	A.^3=[1000 1 4]; A.^0.5=[3.162 1.000 1.414]
'	共軛複數	Z=4+5i; Z'=4-5i
^	方陣次方	M=[1 2; 3 4]; M^2=[7 10; 15 22]
.*	乘法運算	10 .* 100=1000
./	右除法	100 ./ 10=10
.\	左除法	100 .\ 10=0.1000
*	矩陣乘法運算	A=[10.5 2.5]; B=[3.5; -5]; A*B=24.2500
/	矩陣右除法運算	A=[10.5 2.5]; A/2=[5.2500 1.2500]
\	矩陣左除法運算	B=[3.5;-5]; 2\B=[1.7500 -2.500]
+	加法運算	M=[10 20 30]; N=[70 60 50];M+N=[80 80 80]
-	減法運算	M-N=[-60 -40 -20]

關係運算子

運算子符號	功能	範例 (A=[1 2 -1 -5]; B=[0 2 3 1])
<	小於	A<B; ans: [0 0 1 1]// A<1; ans:[0 0 1 1]
<=	小於或等於	A<=B; ans: [0 1 1 1]// A<=1; ans: [1 0 1 1]
>	大於	A>B; ans: [1 0 0 0]// A>1; ans: [0 1 0 0]
>=	大於或等於	A>=B; ans: [1 1 0 0]// A>=1; ans: [1 1 0 0]
==	相等	A==B; ans: [0 1 0 0]// A==1; ans: [1 0 0 0]
~=	不相等	A~=B; ans: [1 0 1 1]// A~=1; ans: [0 1 1 1]

邏輯運算子

運算子符號	功能	範例 (C=[5 -4 0 0.5]; D=[0 1 0 9])
&	And	C&D; ans: [0 1 0 1]// C&1; ans: [1 1 0 1]
|	Or	C|D; ans: [1 1 0 1]// C|1; ans: [1 1 1 1]
~	Not	~C; ans: [0 0 1 0]// ~1; ans: 0

邏輯函式

(J=[1 0 1 0]; K=[0 0 1 1]; L=[0 1 NaN -inf]; M=[0 0 9 inf])

函式名稱	功能	範例
xor(X,Y)	執行 XY 運算	xor(J,K); ans: [1 0 0 1]// xor(J,1); ans: [0 1 0 1]
all(X)	檢查是否 X 全為有 True	all(J); ans: 0// all(J|1); ans: 1
isfinite(X)	檢查是否 X 全為限大值	isfinite(L); ans: [1 1 0 0] isfinite(L./M); ans:[0 0 0 0]
isnan(X)	檢查是否 X 全為 NaN	isnan(L); ans: [0 0 1 0] isnan(L./M); ans: [1 0 1 1]
isinf(X)	檢查是否 X 全為無限大值	isinf(L); ans: [0 0 0 1];// isinf(L./M); ans: [0 1 0 0]

- 流程控制(迴圈、判斷)

關鍵字	功能
if, else, elseif	根據邏輯條件，來執行一群運算。
switch, case, otherwise	根據條件值，來選擇執行項目。
while	根據邏輯條件，來決定迴圈的執行次數。
for	執行一固定次數的迴圈。

if, else, else, 與 switch 語法

<table>
<tr>
<td>if　邏輯表示式子
運算指令
end</td>
<td rowspan="2">if 邏輯條件一
運算指令一
elseif　邏輯條件二
運算指令二
elseif　邏輯條件三
運算指令三
……………………..
else
運算指令四
end</td>
<td rowspan="2">switch 評估描述子 (整數或字串)
case 數值(或字串)條件一
運算指令一
case　數值(或字串)條件二
運算指令二
…………………………..
otherwise
運算指令 N
end</td>
</tr>
<tr>
<td>if 邏輯表示式子
運算指令一
else
運算指令二
end</td>
</tr>
</table>

while, 與 for 語法

運算指令一 white (終止條件) 運算指令二 end	for (索引數值=起始數:遞增數:終止數) 運算指令 end

- 函式：如第二章所述

- 檔案輸入與輸出

- **實例演練一 (chap3_1.m)**

```
% File: ../ch3/ch3_1.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Operations

clear;
A=[1 2; 3 4];
B=2*ones(2);
A==B
A>2
isequal(A,B)
```

```
x=[-1 1 1];y=[1 2 -3];
x>0 & y<0
x>0 | y>0
xor(x>0,y>0)
any(x>0)
all(x>0)
%x | y & z
%(x | y) & z
%x | (y & z)
x=[-3 1 0 -inf 0];
f=find(x)
x(find(isfinite(x)))
x(find(x<0))=0
```

```
A=[4 2 16;12 4 3],B=[12 3 1;10 -1 7]
f=find(A<B)
A(f)=0
clear;
y=[1 2 0 -3 0]
i1=logical(y)
i2=(y ~= 0)
i3=[1 1 0 1 0]
whos
y(i1)
y(i2)
isequal(i2,i3)
%y(i3)
```

```
ans =
     0     1
     0     0
ans =
     0     0
     1     1
ans =
     0
ans =
     0     0     1
ans =
     1     1     1
ans =
     1     0     1
ans =
     1
ans =
     0
f =
     1     2     4
ans =
    -3     1     0     0
x =
     0     1     0     0     0
A =
     4     2    16
    12     4     3
```

```
B =
    12     3     1
    10    -1     7
f =
     1
     3
     6
A =
     0     0    16
    12     4     0
y =
     1     2     0    -3     0
i1 =
     1     2     0    -3     0
i2 =
     1     1     0     1     0
i3 =
     1     1     0     1     0
  Name      Size           Bytes  Class
  i1        1x5               40  double array (logical)
  i2        1x5               40  double array (logical)
  i3        1x5               40  double array
  y         1x5               40  double array
Grand total is 20 elements using 160 bytes
ans =
     1     2    -3
ans =
     1     2    -3
ans =
     1
```

- **實例演練二 (chap3_2.m)**

```
% File: ../ch3/ch3_2.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Flow Control: if, elseif
x=[8], y=[6],
if x>y
	temp=y;
	y=x;
	x=temp;
end
if x>0
	x=sqrt(x)
end
```

```
e=exp(1);
if 2^e > e^2
	disp('2^e is bigger')
else
	disp('e^2 is bigger')
end
if isnan(x)
	disp('Not a Number')
elseif isinf(x)
	disp('Plus or minus infinity')
else
	disp('A"regular"float point number')
end
```

```
x =
     8
y =
     6
x =

2.44948974278318
e^2 is bigger
A'regular'float point number
```

- 實例演練三 (chap3_3.m)

```
% File: ../ch3/ch3_3.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Flow Control: for, while, switch

s=0;
for i=1:25,s=s+1/i,end,
s
for x=[pi/6 pi/4 pi/3],disp([x,sin(x)]),end

n=5;A=eye(n);
for j=2:n
	for i=i:j-1
		A(i,j)=i/j;
		A(j,i)=i/j;
	end
end

x=1;
while x>0,xmin=x;x=x/2;end
xmin

x=1;
while 1
	xmin=x;
	x=x/2;
	if x== 0,break,end
end
xmin

for i=1:10
	if i<5, continue,end
	disp(i)
end

p=2;
switch p
case 1
	y=sum(abs(x));
case 2
	y=sort(x'*x);
case inf
	y=max(abs(x));
otherwise
	error('P must be 1, 2 or inf.');
end

x=input('Enter a real number:');
switch x
case {inf,-inf}
	disp('Plus or minus infinity')
case 0
	disp('Zero')
otherwise
	disp('Nonzero and finite')
end
```

```
s =
       1
s =
    1.50000000000000
s =
    1.83333333333333
s =
    2.08333333333333
s =
    2.28333333333333
s =
    2.45000000000000
s =
    2.59285714285714
s =
    2.71785714285714
s =
    2.82896825396825
s =
    2.92896825396825
s =
    3.01987734487734
s =
    3.10321067821068
s =
    3.18013375513376
s =
    3.25156232656233
s =
    3.31822899322899
s =
    3.38072899322899
s =
    3.43955252264076
s =
    3.49510807819631
s =
    3.54773965714368
s =
    3.59773965714368
s =
    3.64535870476273
s =
    3.69081325021727
s =
    3.73429151108684
s =
    3.77595817775351
s =
    3.81595817775351
```

```
s =
    3.81595817775351
    0.52359877559830    0.50000000000000
    0.78539816339745    0.70710678118655
    1.04719755119660    0.86602540378444
xmin =
     4.940656458412465e-324
xmin =
     4.940656458412465e-324
     5
     6
     7
     8
     9
    10

Enter a real number:5
Nonzero and finite
>>
```

- 實例演練四 (chap3_4.m)

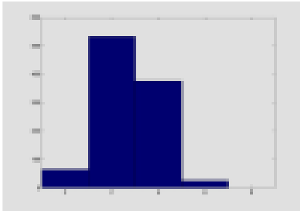
```
function y=maxentry(A)
%MAXENTRY File: ../ch3/maxentry.m
%              Author: Ming-Kai Chen, 20 Oct 2002
%              Prupose: function y=maxentry(A)
% =====>    Largest absolute value of matrix entries.
%              MAXENTRY(A) is the maximum of the absolute values
%              of the entries of A.

y=max(max(abs(A)));
```

```
function [f,fprime]=flogist(x,a)
%MAXENTRY File: ../ch3/flogist.m
%              Author: Ming-Kai Chen, 20 Oct 2002
%              Prupose: [f,fprime]=flogist(x,a)
% =====>    Logistic function and its derivative.
%              [F,FPRIME]=FLOGIST(X,A) evaluates the logistic
%              function F(X)=X.*(1-A*X) and its derivative FPRIME
%              at the matrix argument X, where A is a scalar parameter

f=x.*(1-a*x);
fprime=1-2*a*x;
```

```
function [x,iter]=sqrtn(a,tol)
%MAXENTRY File: ../ch3/sqrtn.m
%              Author: Ming-Kai Chen, 20 Oct 2002
%              Prupose: [x,iter]=sqrtn(a,tol)
% =====>    Square root of a scalar by Newton's method.
%              X=SQRTN(A,TOL) computes the square root of the scalar
%              A by Newtom's method (also know as Heron's method).
%              A is assumed to be >=0.
%              TOL is convergence tolerance (default FSP).
%              [X,ITER]=SQRTN(A,TOL) returns also the number of
%              interations ITER for convergence.

if nargin < 2,tol=eps;end
x=a;
iter=0;
xdiff=inf;
fprintf('k           x_k              rel. change\n')
while xdiff>tol
    iter=iter+1;
    xold=x;
    x=(x+a/x)/2;
    xdiff=abs(x-xold)/abs(x);
    fprintf('%2.0f: %20.16e %9.2e\n',iter,x,xdiff)
    if iter > 50
        error('Not converged after 50 interation.')
    end
end
```

```
function [x_sort,x_mean,x_med,x_std]=marks2(x)
%MAXENTRY File: ../ch3/marks2.m
%              Author: Ming-Kai Chen, 20 Oct 2002
%              Prupose: [x_sort,x_mean,x_med,x_std]=marks2(x)
% =====>    Statistical analysis of marks vector.
%              Given a vector of marks X,
%              [X_SORT,X_MEAN,X_MED,X_STD]=MARKS2(X) computes a
%              sorted marks list and the mean, medium and standard deviation
%              of the marks.
x_sort=sort(x);
if nargout > 1, x_mean=mean(x); end
if nargout > 2, x_med=median(x); end
if nargout > 3, x_std=std(x); end
```

```
% File: ../ch3/ch3_4.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Script M-files and Function M-files

clear;
%counts number of real eigenvalus of random matrix
A=randn(8);sum(abs(imag(eig(A)))<.0001)

%Empirical distribution of number of real eigenvalues
k=1000;
wheel=zeros(k,1);
for i=1:k
    A=randn(8);
    wheel(i)=sum(abs(imag(eig(A)))<.0001);
end
hist(wheel,[0 2 4 6 8]);

%maxentry function
maxentry(1:10)
maxentry(magic(4))

%flogist function
f=flogist(2,1)
[f,fprime]=flogist(2,.1)
flogist(1:4,2)

%squtn function
[x,iter]=sqrtn(2)
x=sqrtn(2,1e-4)

%marks2 function
exmark=[12 0 5 28 87 3 56]
x_sort=marks2(exmark)
[x_sort,x_mean,x_med]=marks2(exmark)

%Command/Function Duality
comfun('ab','cd','ef')
comfun ab cd ef

format long, format('long')
disp('Hello'),disp Hello
diary mydiary,diary('mydiary')
warning off, warning('off')

sqrt 2
```



```
ans =
     4
ans =
    10
ans =
    16
f =
    -2
f =

1.60000000000000
fprime =
   0.60000000000000
ans =
    -1    -6   -15   -28
k        x_k            rel. change
  1: 1.5000000000000000e+000 3.33e-001
  2: 1.4166666666666665e+000 5.88e-002
  3: 1.4142156862745097e+000 1.73e-003
  4: 1.4142135623746899e+000 1.50e-006
  5: 1.4142135623730949e+000 1.13e-012
  6: 1.4142135623730949e+000 0.00e+000
x =
   1.41421356237309
iter =
     6
k        x_k            rel. change
  1: 1.5000000000000000e+000 3.33e-001
  2: 1.4166666666666665e+000 5.88e-002
  3: 1.4142156862745097e+000 1.73e-003
  4: 1.4142135623746899e+000 1.50e-006
x =
   1.41421356237469
exmark =
    12     0     5    28    87     3    56
x_sort =
     0     3     5    12    28    56    87
x_sort =
     0     3     5    12    28    56    87
x_mean =
  27.28571428571429
x_med =
    12
ab
cd
ef
ab
cd
ef
Hello
Hello
ans =
   7.07106781186548
```

- 實例演練五 (chap3_5.m)

```
% File: ../ch3/ch3_5.m
% Author: Ming-Kai Chen, 20 Oct 2002
% Prupose: Input and Output

%Input
x=input('Starting point: ')
mytitle=input('Title fo plot: ','s')
axis([0 1 0 1])
[x,y]=ginput(4);
P=[x';y'];

%Output to the Screen
disp('Here is a 3-by-3 magic square')
disp(magic(3))
fprintf('%6.3f\n',pi)
fprintf('%6.3f\n',pi*10)
fprintf('%12.3e\n',pi)
fprintf('%5.2f\n%5.2f\n',exp(1),-exp(1))
fprintf('%5.0f\n%5.2f\n',9,103)
fprintf('%-5.0f\n%-5.0f\n',9,103)

m=5;iter=11;U=orth(randn(m))+1e-10;
fprintf('iter=%2.0f\n',iter)
fprintf('norm(U''*U-I)=%11.4e\n',norm(U'*U-eye(m)))
fprintf('%g %g\n',exp(1),exp(20))

A=[30 40 50 60 70]
fprintf('%g miles/hour=%g kilometers/hour\n',[A;8*A/5])

n=16;
erroe_msg=sprintf('Must supply a %d-by-%d matrix',n,n)

i=3;
title_str=['Result of experiment' int2str(i)]

%File Input and Output
A=[30 40 60 70];
fid=fopen('myoutput','w');
fprintf(fid,'%g miles/hour=%g kilometers/hour\n',[A;8*A/5]);
fclose(fid)

fid=fopen('myoutput','r');
X=fscanf(fid,'%g miles/hour=%g kilometers/hour')
fclose(fid);

fid=fopen('myoutput','r');
X=reshape(X,2,4)'
X=fscanf(fid,'%g miles/hour=%g kilometers/hour',[2 inf]);
X=X'
```

```
Starting point: 0.5
x =
    0.50000000000000
Title fo plot: experiment2
mytitle =
experiment2
Here is a 3-by-3 magic square
     8     1     6
     3     5     7
     4     9     2
 3.142
31.416
   3.142e+000
 2.72
-2.72
    9
103.00
9
103
iter=11
norm(U'*U-I)=6.0346e-010
2.71828 4.85165e+008
A =
    30    40    50    60    70
30 miles/hour=48 kilometers/hour
40 miles/hour=64 kilometers/hour
50 miles/hour=80 kilometers/hour
60 miles/hour=96 kilometers/hour
70 miles/hour=112 kilometers/hour
erroe_msg =
Must supply a 16-by-16 matrix
title_str =
Result of experiment3
ans =
     0
X =
    30
    48
    40
    64
    60
    96
    70
   112
X =
    30    48
    40    64
    60    96
    70   112
X =
    30    48
    40    64
    60    96
    70   112
```

第四章　矩陣與線性代數

● 向量

一向量 $\vec{A}=(a_x\hat{i},a_y\hat{j},a_z\hat{k})$，其中 $\hat{i}=\begin{bmatrix}1\\0\\0\end{bmatrix}$、$\hat{j}=\begin{bmatrix}0\\1\\0\end{bmatrix}$、$\hat{k}=\begin{bmatrix}0\\0\\1\end{bmatrix}$，則 $\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$、$\hat{i}\cdot\hat{j}=\hat{i}\cdot\hat{k}=\hat{j}\cdot\hat{k}=0$。

因：$\hat{i}\cdot\hat{i}=\hat{i}^T\cdot\hat{i}=\begin{bmatrix}1 & 0 & 0\end{bmatrix}_{1\times3}\cdot\begin{bmatrix}1\\0\\0\end{bmatrix}_{3\times1}=1$

$\hat{i}\cdot\hat{j}=\hat{i}^T\cdot\hat{j}=\begin{bmatrix}1 & 0 & 0\end{bmatrix}_{1\times3}\cdot\begin{bmatrix}0\\1\\0\end{bmatrix}_{3\times1}=0$

● 向量加法與減法

若 $\vec{A}=(a_x,a_y,a_z),\vec{B}=(b_x,b_y,b_z)$，則 $\vec{C}=\vec{A}+\vec{B}=(a_x+b_x,a_y+b_y,a_z+b_z)$、$\vec{D}=\vec{A}-\vec{B}=(a_x-b_x,a_y-b_y,a_z-b_z)$。

>> A=fix(10*rand(2,3)) A = 4 7 0 8 4 8 >> B=fix(10*rand(2,3)) B = 4 7 7 6 9 1	>> C=A+B C = 8 14 7 14 13 9 >> D=A-B D = 0 0 -7 2 -5 7

- 向量範數

向量 X 的二階範數(2-norm)為$\|X\|=\left(\sum x_i^2\right)^{1/2}\Rightarrow\sqrt{x_1^2+x_2^2}\,or\sqrt{x_1^2+x_2^2+\ldots+x_n^2}$，向量 X 的 P 階範數(p-norm)為$\|X\|=\left(\sum x_i^p\right)^{1/p}\Rightarrow\sqrt{x_1^p+x_2^p}\,or\sqrt{x_1^p+x_2^p+\ldots+x_n^p}$，其中 P>1、P=1,2,3…∞，以 norm(variable,P)函數求取。

>> A=[3 4] A = 3 4 >> norm(A,2) ans = 5 >> B=[9 12] B = 9 12 >> norm(B,2) ans = 15	>> C=[1 2 3] C = 1 2 3 >> norm(C,2) ans = 3.7417 >> norm(C,3) ans = 3.3019

- 向量積

長度大小相等的行向量或列向量相乘，有向量內積(dot・)和向量外積(cross X)。若

$A=[a_i]\rightarrow A_{1\times3}$ ， $B=[b_j]\rightarrow B_{1\times3}$ ， $C=A_{1\times3}*B_{1\times3}{}'=A_{1\times3}*B_{1\times3}{}^T\Rightarrow A_{1\times3}*BB_{3\times1}=C_{1\times1}$ 、

$D=B_{1\times3}{}'*A_{1\times3}=B_{1\times3}{}^T*A_{1\times3}\Rightarrow BB_{3\times1}*A_{1\times3}=C_{3\times3}$。

>> A=fix(10*rand(1,3)) A = 4 9 9 >> B=fix(10*rand(1,3)) B = 4 8 0 >> BB=B' BB = 4 8 0	>> A*BB ans = 88 >> BB*A ans = 16 36 36 32 72 72 0 0 0

- 特徵值與特徵向量

若存在一向量 v 與一常數 λ，使得一方陣 a 滿足 $Av = \lambda v$，稱 λ 為特徵值，v 為特徵向量。

A=[-2 2 -3;2 1 -6;-1 -2 0],[U,S,V]=svd(A) A = -2 2 -3 2 1 -6 -1 -2 0 U = 0.4450 -0.8944 0.0448 0.8899 0.4472 0.0895 -0.1001 0.0000 0.9950	S = 7.0321 0 0 0 3.0000 0 0 0 2.1331 V = 0.1408 0.8944 -0.4245 0.2816 -0.4472 -0.8489 -0.9492 0.0000 -0.3148

- 單位矩陣

>> A=eye(3) A = 1 0 0 0 1 0 0 0 1	>> B=eye(3,2) B = 1 0 0 1 0 0

- 零矩陣

>> zeros(3) ans = 0 0 0 0 0 0 0 0 0	>> zeros(3,2) ans = 0 0 0 0 0 0

- 轉置矩陣

將 i 行 j 列元素變到 j 行 i 列，若 $A = [a_{ij}]$則 $A^T = [a_{ji}]$。

>> A=[5,4,6] A = 5 4 6 >> E=A’ E = 5 4 6	>> B=[1,2,3;4,5,6] B = 1 2 3 4 5 6 >> E=B’ E = 1 4 2 5 3 6

- 矩陣加法與減法

大小相等的矩陣方可進行加減動作，若 $A = \lfloor a_{ij} \rfloor$、$B = \lfloor b_{ij} \rfloor$ 則 $C = A + B \Rightarrow \lfloor c_{ij} \rfloor = \lfloor a_{ij} + b_{ij} \rfloor$、$D = A - B \Rightarrow \lfloor d_{ij} \rfloor = \lfloor a_{ij} - b_{ij} \rfloor$。

>> A=fix(10*rand(3,3)) A = 9 4 4 2 8 0 6 7 8 >> B=fix(10*rand(3,3)) B = 4 9 4 6 7 9 7 1 9	>> C=A+B C = 13 13 8 8 15 9 13 8 17 >> D=A-B D = 5 -5 0 -4 1 -9 -1 6 -1

- 矩陣乘法

A 的行數等於 **B** 的列數，若 $A = \lfloor a_{ij} \rfloor$、$B = \lfloor b_{ij} \rfloor$、$C = \lfloor c_{ij} \rfloor$，且存在 $C = AB$ 關係，則 $\lfloor c_{ij} \rfloor = \left[\sum_k a_{ik} b_{kj} \right]$。若 $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \end{bmatrix}$、$B = \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 2 & 1 \end{bmatrix}$，

$$則 A_{2\times3} B_{3\times2} = C_{2\times2} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 1+0+4 & 0+0+2 \\ 0+0+2 & 0+9+1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 10 \end{bmatrix}$$

$$B_{3\times2} A_{2\times3} = D_{3\times3} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 & 2+0 \\ 0+0 & 0+9 & 0+3 \\ 2+0 & 0+3 & 4+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 9 & 3 \\ 2 & 3 & 5 \end{bmatrix}。$$

>> A=[1 0 2;0 3 1] A = 1 0 2 0 3 1 >> B=[1 0;0 3;2 1] B = 1 0 0 3 2 1	>> A*B ans = 5 2 2 10 >> B*A ans = 1 0 2 0 9 3 2 3 5

- 行列式

- 反矩陣

　若 $AX = I$，則 $X = A^{-1}$，以 inv(variable)函數求值。

>> A=[1 2; 3 4] A = 1　2 3　4 >> inv(A) ans = -2.0000　1.0000 1.5000　-0.5000 >> det(A) ans = -2	>> B=[1 2 3;2 1 3; 3 2 1] B = 1　2　3 2　1　3 3　2　1 >> inv(B) ans = -0.4167　0.3333　0.2500 0.5833　-0.6667　0.2500 0.0833　0.3333　-0.2500 >> det(B) ans = 12

- 矩陣除法

　若 $AX = B$，則以 $X = A\backslash B$ 或 $X = A^{-1}B$ 求之。

若 $\begin{aligned} -3x+0y+z&=4 \\ 2x+5y-7z&=5 \\ -x+4y+8z&=6 \end{aligned}$，令 $A=\begin{bmatrix} -3 & 0 & 1 \\ 2 & 5 & -7 \\ -1 & 4 & 8 \end{bmatrix}$、$X=\begin{bmatrix} x \\ y \\ x \end{bmatrix}$、$B=\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

利用 $X = A\backslash B$ 或 $X = A^{-1}B$ 求取 X，得 $X=\begin{bmatrix} -1.3717 \\ 1.3874 \\ -0.1152 \end{bmatrix}$。

>> A=[-3 0 1;2 5 -7;-1 4 8] A = -3　0　1 2　5　-7 -1　4　8 >> B=[4;5;6] B = 4 5 6	>> X=A\B X = -1.3717 1.3874 -0.1152 >> X=inv(A)*B X = -1.3717 1.3874 -0.1152

- 矩陣冪次方項與指數

$A\^P$ 表矩陣 **A** 自乘 P 次，$A\^(-P)$ 表矩陣 **A** 之反矩陣 **A**$^{-1}$ 自乘 P 次(**A** 非異矩陣)，如 a^(1/2)，亦可以 sqrtm(A)求之。另有.^表示矩陣內同一位置元素自乘，如 $A.\^(1/2)$，亦可以 sqrt(A)求之，

>> A=[1 2;4 9] A = 1 2 4 9 >> A^2 ans = 9 20 40 89 >> A^(1/2) ans = 0.5774 0.5774 1.1547 2.8868 >> sqrtm(A) ans = 0.5774 0.5774 1.1547 2.8868	>> A.^2 ans = 1 4 16 81 >> A.^(1/2) ans = 1.0000 1.4142 2.0000 3.0000 >> sqrt(A) ans = 1.0000 1.4142 2.0000 3.0000

- 矩陣 Y 的 P 階範數

$\|Y\|_P = \frac{\max}{x}\|Y_x\|_P / \|X\|_P$，以 norm(Y,P)求之。

- 特徵值與特徵矩陣

- 解線性方程式

  若 N 為資料量個數(number of data)，M 為未知的控制變數個數(number of model parameter)。

  (1)方陣系統(even-determined)：N=M，存在維一的一組解，若 $AX = B$，則以 $X = A\backslash B$ 或 $X = A^{-1}B$ 求之。

  (2)過限定系統(over-determined)：N>M，以 min error(least square)求之。下面以 t=[0 0.3 0.8 1.1 1.6 2.3]，y=[0.82 0.72 0.63 0.60 0.55 0.50]為例，認為數值有 $y(t) \sim c_1 + c_2 e^{-t}$ 的衰減指數趨勢，過程如下，求得趨勢方程式為 $y(t) \sim 0.4760 + 0.3803e^{-t}$。

  (3)限定不足系統(under-determined)：N<M，以 min length(simplify model)求之。在 MATLAB 中會找出一組構成通解的基底，而方程式的特解則由 QR 法來決定。

```
>> A=fix(15*rand(3,3))
A =
    14     7     6
     3    13     0
     9    11    12
>> B=fix(15*rand(3,1))
B =
     6
     9
    11
>> X=A\B
X =
   -0.0588
    0.7059
    0.3137
>> X=inv(A)*B
X =
   -0.0588
    0.7059
    0.3137
>> C=A*X
C =
    6.0000
    9.0000
   11.0000
```

```
>> t=[0 0.3 0.8 1.1 1.6 2.3]'
t =
         0
    0.3000
    0.8000
    1.1000
    1.6000
    2.3000
>> y=[0.82 0.72 0.63 0.60 0.55 0.50]'
y =
    0.8200
    0.7200
    0.6300
    0.6000
    0.5500
    0.5000
    0.4760
    0.3413
>> E= [ ones( size(t) ) exp(-t) ]
E =
    1.0000    1.0000
    1.0000    0.7408
    1.0000    0.4493
    1.0000    0.3329
    1.0000    0.2019
    1.0000    0.1003
```

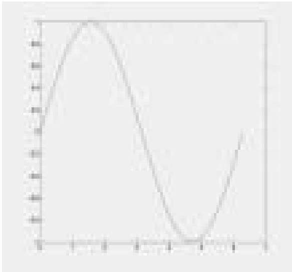
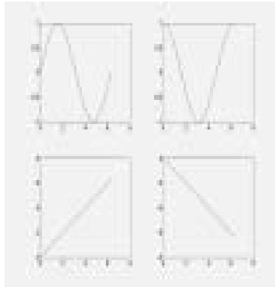
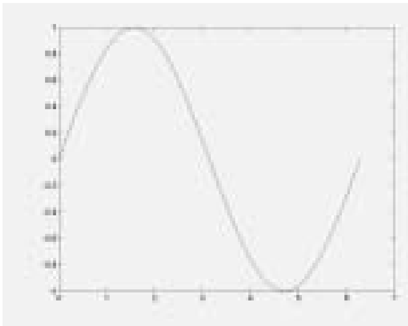
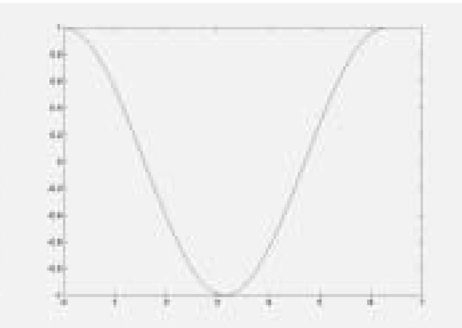
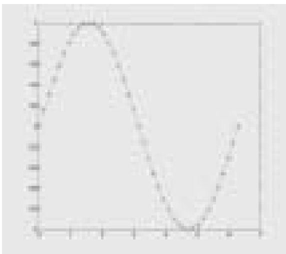
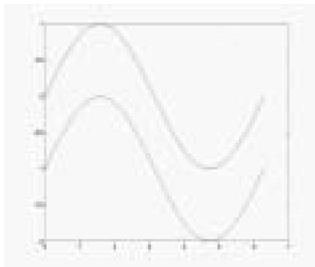
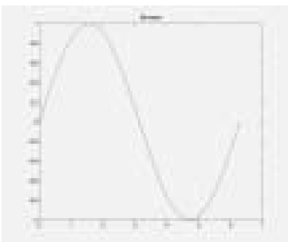
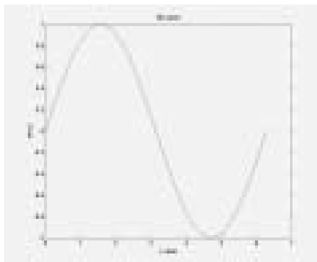
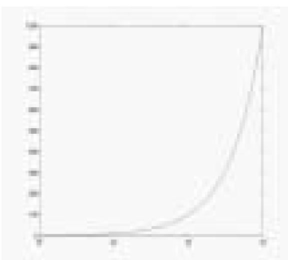
```
>> c=E\y
c =
    0.4760
    0.3413
>> A=fix(15*rand(3,4))
A =
    13     6     6     5
    11    14    13    12
     2    13     0     0
>> B=fix(15*rand(3,1))
B =
     2
     3
     2
>> P=A\B
P =
    0.0859
    0.1406
    0.0067
         0
```

- LU，QR，Cholesky 分解

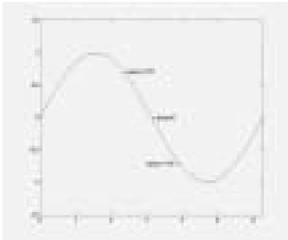
- 奇異值分解

第五章　2D & 3D 繪圖

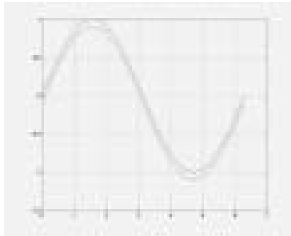
- 2D 繪圖

<table>
<tr><td>clear;
x=0:pi/20:2*pi;
plot(x,sin(x))
</td><td>clear;
x=0:pi/20:2*pi;
subplot(2,2,1);plot(x,sin(x));
subplot(2,2,2);plot(x,cos(x));
subplot(2,2,3);plot(x,x);
subplot(2,2,4);plot(x,-x);
</td></tr>
<tr><td colspan="2">clear;
x=0:pi/20:2*pi;
figure(1);plot(x,sin(x));
figure(2);plot(x,cos(x));
</td></tr>
<tr><td>clear;
x=0:pi/20:2*pi;
plot(x,sin(x),'r+:')
</td><td>clear
x=0:pi/20:2*pi;
plot(x,sin(x),'r',x,sin(x)-1,'b');
</td></tr>
<tr><td>clear;
x=0:pi/20:2*pi;
plot(x,sin(x))
title('Sin wave')
</td><td>clear;
x=0:pi/20:2*pi;
plot(x,sin(x))
title('Sin wave')
xlabel('x value')
ylabel('sin(x)')
</td></tr>
<tr><td>clear;
x=0:1:1000;
semilogx(x,x)
</td><td>clear;
x=0:pi/20:2*pi;
plot(x,sin(x))
axis([0 2*pi -1.5 1.5])
</td></tr>
</table>

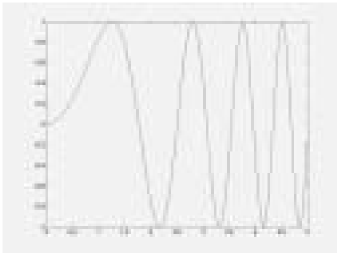
```
clear;
x=0:pi/20:2*pi;
plot(x,sin(x));
axis([0 2*pi -1.5 1.5]);
text(3*pi/4,sin(3*pi/4),...
    '\leftarrowsin(x)=.707');
text(pi,sin(pi),...
    '\leftarrowsin(x)=0');
text(5*pi/4,sin(5*pi/4),...
    'sin(x)=-.707\rightarrow',...
    'HorizontalAlignment','right');
```



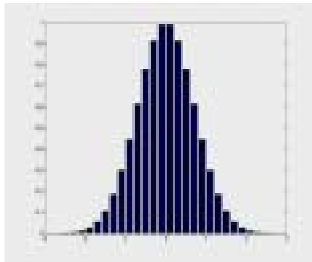
```
clear;
x=0:pi/20:2*pi;
plot(x,sin(x));
hold on
plot(x,sin(x)-0.1,'r--')
grid on
```



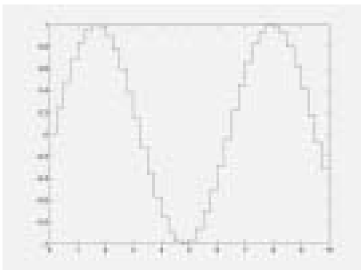
```
% Line plot of a chirp
x=0:0.05:5;
y=sin(x.^2);
plot(x,y);
```



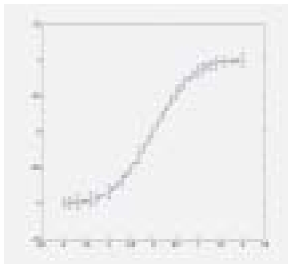
```
% Bar plot of a bell shaped curve
x = -2.9:0.2:2.9;
bar(x,exp(-x.*x));
```



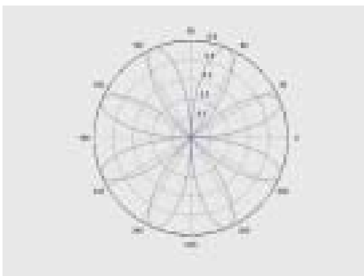
```
% Stairstep plot of a sine wave
x=0:0.25:10;
stairs(x,sin(x));
```



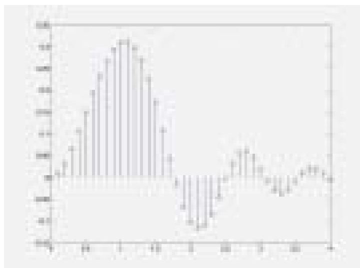
```
% Errorbar plot
x=-2:0.1:2;
y=erf(x);
e=rand(size(x))/10;
errorbar(x,y,e);
```



```
% Polar plot
t=0:.01:2*pi;
polar(t,abs(sin(2*t).*cos(2*t)));
```



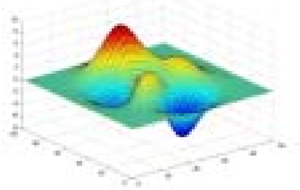
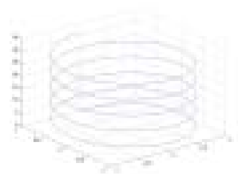
```
% Stem plot
x = 0:0.1:4;
y = sin(x.^2).*exp(-x);
stem(x,y)
```

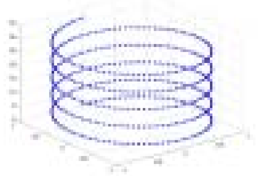
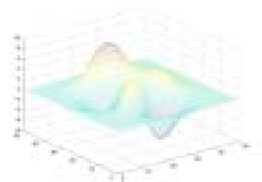


y	黃色	m	紫色	c	青色	r	紅色
g	綠色	b	藍色	w	白色	k	黑色

.	點	v	向下三角形	-	實線
0	圓	^	向上三角形	:	虛線
x	打叉	<	向左三角形	-.	中軸線
*	星號	>	向右三角形	—	斷折線
s	正方形	p	五角星形		
d	菱形	h	六角星形		

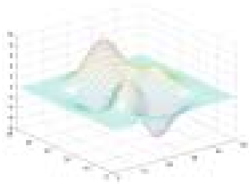
- 3D 繪圖

clear;
z=peaks(50);
h=surf(z);

	clear;
	x=0:pi/50:10*pi;
	plot3(sin(x),cos(x),x);
	grid on;
	

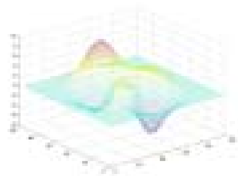
clear;
x=0:pi/50:10*pi;
plot3(sin(x),cos(x),x,...
'.','MarkerSize',15);
grid on;

	clear;
	z=peaks(50);
	h=mesh(z);
	

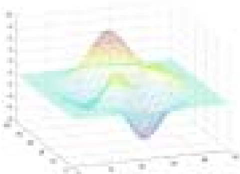
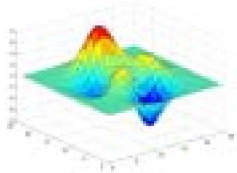
clear;
[x,y]=meshgrid([-4:.1:4]);
z=peaks(x,y);
plot3(x,y,z);

	clear;
	z=peaks(50);
	h=mesh(z);
	colormap hot;
	

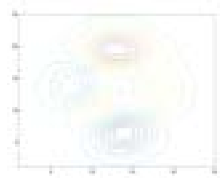
clear;
z=peaks(50);
h=mesh(z);
light('Position',[0 0 5]);

	clear;
	z=peaks(50);
	h=mesh(z);
	set(h,'FaceLighting','phone',...
	'FaceColor','interp');
	

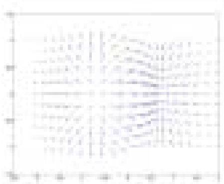
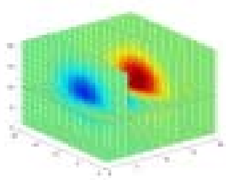
clear;
[x,y]=meshgrid([-3:.2:3]);
z=peaks(x,y);
mesh(x,y,z);
plot3(x,y,z,'x','MarkerSize',3);

	clear;
	z=peaks(50);
	mesh(z);
	hidden off;
	

clear;
z=peaks(50);
mesh(z);
hidden off;
view(-20,25);

	% Surface Plot of Peaks
	z=peaks(25);
	surf(z);
	colormap(jet);
	

% Surface Plot (with Shading) of Peaks
z=peaks(25);
surfl(z);
shading interp;
colormap(pink);

	% Contour Plot of Peaks
	z=peaks(25);
	contour(z,16);
	

x = -2:.2:2; y = -1:.2:1;
[xx,yy] = meshgrid(x,y);
zz = xx.*exp(-xx.^2-yy.^2);
[px,py] = gradient(zz,.2,.2);
quiver(x,y,px,py,2);

	[x,y,z] = meshgrid(-2:.2:2,-2:.2:2,-2:.2:2);
	v = x .* exp(-x.^2 - y.^2 - z.^2);
	slice(v,[5 15 21],21,[1 10])
	axis([0 21 0 21 0 21]);
	colormap(jet)
	

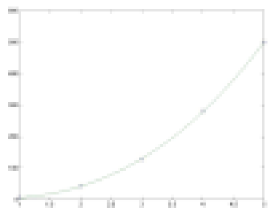
第六章　數值分析與統計

- 多項式：Matlab 使用列向量表示一個多項式，提供 roots 指令求解，poly 指令還原方程式，polyval 指令計算多項式的值，polyvalm 指令做多項式矩陣運算，conv 和 deconv 指令求迴旋折積和反回旋折積，polyder 指令求多項式微分，redisidue 指令求一個分式之部份分式分解，polyfit(x,y,n)指令求多項式的曲線揉合方程式。

以下[分別以 $p(x)=x^3-2x+5$、$\frac{4s-8}{s^2+6s+8}$、$p(X)=X^3-2X+5$ 多項式為例。

```
>> p=[1 0 -2 5]
p =
     1     0    -2     5
>> r=roots(p)
r =
  -2.0946
   1.0473 + 1.1359i
   1.0473 - 1.1359i
>> p2=poly(r)
p2 =
    1.0000         0   -2.0000    5.0000
>> clear
>> p=[1 0 -2 5]
p =
     1     0    -2     5
>> polyval(p,5)
ans =
   120
>> a=[1 2 4];b=[3 5 6];
>> c=conv(a,b)
c =
     3    11    28    32    24
>> [q,r]=deconv(c,a)
q =
     3     5     6
r =
     0     0     0     0     0
>> a=[2 4 6];b=[1 2 3];
>> c=polyder(a,b)
c =
     8    24    40    24
>> [q,d]=polyder(a,b)
q =
     0
d =
     1     4    10    12     9
```

```
>> A=[1 2 3;4 5 6; 7 8 0]
A =
     1     2     3
     4     5     6
     7     8     0
>> p3=poly(A)
p3 =
    1.0000   -6.0000  -72.0000  -27.0000
>> X=[1 0 3;2 4 5;7 0 5]
X =
     1     0     3
     2     4     5
     7     0     5
>> p
p =
     1     0    -2     5
>> Y=polyvalm(p,X)
Y =
   151     0   150
   430    61   460
   350     0   351
>> b=[4 -8];a=[1 6 8];
>> [r,p,k]=residue(b,a)
r =
    12
    -8
p =
    -4
    -2
k =
     []
>> x=[1 2 3 4 5];
>> y=[5 43 128 280 500];
>> p=polyfit(x,y,3)
p =
    1.7500   15.0357  -20.7143    9.2000
>> x2=1:.1:5;
>> y2=polyval(p,x2);
>> plot(x,y,'o',x2,y2)
```



● 資料分析

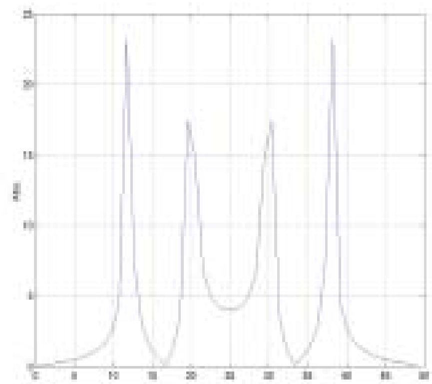
指令	功能
max	
min	
mean	
median	
std	
sort	
sortrows	
sum	
cumsum	
prob	
cumprob	
diff	
conv	
corrcoef	

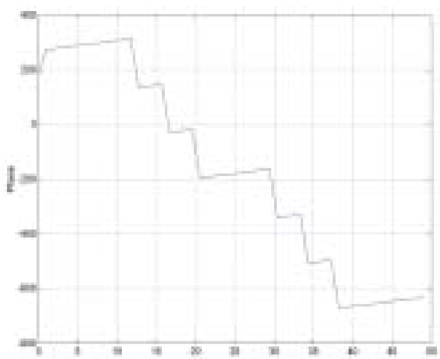
● 回歸分析與曲線揉合

- 快速傅立葉轉換

指令	功能
fft	離散傅立葉轉換
fft2	二維傅立葉轉換
fftn	N 維傅立葉轉換
ifft	逆傅立葉轉換
ifft2	二維逆傅立葉轉換
ifftn	N 維逆傅立葉轉換
abs	向量絕對值(magnitude)
angle	向量向位值(phase)
unwrap	使相位在 180 度不會產生不連續點

```
>> x=[4 3 9 -7 1 2 0 0]'
x =
     4
     3
     9
    -7
     1
     2
     0
     0
>> y=fft(x)
y =
  12.0000
   8.6569 - 4.7574i
  -4.0000 -12.0000i
  -2.6569 +13.2426i
  16.0000
  -2.6569 -13.2426i
  -4.0000 +12.0000i
   8.6569 + 4.7574i

>> t=0:1/50:1;
>> x=sin(2*pi*20*t)+sin(2*pi*12*t);
>> y=fft(x);
>> m=abs(y);
>> p=unwrap(angle(y));
>> f=(0:length(y)-1)'*50/length(y);
>> plot(f,m),ylabel('Abs.'),grid on
>> plot(f,p*180/pi),ylabel('Phase'),grid on
```





第七章　微分

第八章　積分

第九章　常微分方程

● 一階微分方程

1. If $y' = f(x,y)$ with $y(x_0) = y_0$, then $y(x) = y_0 + \int_{x_0}^{x} f(x)dx$。
2. If $y' = f(x,y)$ with $y(x_0) = y_0$, then formal solution can be solved interactively as $y^{[n+1]}(x) = y_0 + \int_{x_0}^{x} f(x, y^{[n]}(x))dx$，starting with $y^{[n+1]}(x) = y_0$。
3. If $y' + M(x)y = N(x)$，then $y(x) = e^{-\int Mdx}\left(\int Ne^{\int Mdx} dx + C\right)$，where C is a constant。

● 二階微分方程

If $p(x)y'' + q(x)y' + r(x)y = h(x)$ with $y(x_0) = y_0$ and $y'(x_0) = v_0$，then

$$y_1 \equiv y \qquad y_1(x_0) = y_0 \qquad y_1{}' = y'$$

$$y_2 \equiv y' \qquad y_2(x_0) = v_0 \qquad y_2' = \frac{h(x)}{p(x)} - \frac{q(x)}{p(x)}y' - \frac{r(x)}{p(x)}y = \frac{h(x)}{p(x)} - \frac{q(x)}{p(x)}y_2 - \frac{r(x)}{p(x)}y_1$$

● ODE 常用函數

Solver	Problem type	Type of alogorith
ode45	Nonstiff	Explicit Range-Kutta pair, orders 4 and 5.
ode23	Nonstiff	Explicit Range-Kutta pair, orders 2 and 3.
ode113	Nonstiff	Explicit linear multistep, orders 1 and 13.
ode15s	Stiff	Implicit linear multistep, orders 1 and 5.
ode23s	Stiff	Modified Rosenbrock pair (one-step), order 2 and 3.
ode23t	Mildly stiff	Trapezoidal rule (implicit), orders 2 and 3.
ode23tb	Stiff	Implicit Runge-Kutta type algorithm, order 2 and 3.

Function	Meaning
odeset	Create/alter ODE OPTIONS structure.
odeget	Get ODE OPTIONS parameters.
brussode	Stiff problem modelling a chemical reaction (the Brusselator).
rigidode	Euler equations of a rigid body without external forces.
ballode	Run a demo of a bouncing ball.
orbitode	Restricted three body problem.
fem1ode	Stiff problem with a time-dependent mass matrix, M(t)*y' = f(t,y).
fem2ode	Stiff problem with a constant mass matrix, M*y' = f(t,y).
batonode	Simulate the motion of a thrown baton.
hb1dae	Stiff differential-algebraic equation (DAE) from a conservation law.

amp1dae	Stiff differential-algebraic equation (DAE) from electrical circuit.

- Taylor series

The Taylor's series expansion for f(b) is following:

$$f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a) + \ldots + \frac{(b-a)^n}{n!}f^{(n)}(a) + \ldots$$

A **first-order** Taylor's series approximation uses the terms involving the function and its first derivative: $f(b) \approx f(a) + (b-a)f'(a)$

A **second-order** approximation uses the terms involving the function, its first derivative, and its second derivative: $f(b) \approx f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a)$

- Runge-Kutta Method

A **first-order** Runge-Kutta method: uses a first-order Taylor series expansion. And Euler's method is equivalent to a first-order Runge-Kutta method.

A **second-order** Runge-Kutta method: uses a second-order Taylor series expansion.

The Taylor's series expansion for f(b) is

$$f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a) + \ldots + \frac{(b-a)^n}{n!}f^{(n)}(a) + \ldots$$

We assume that the term (b-a) represents a step size h and $y_b = f(b)$, $y_a = f(a)$, $y_a{}' = f(a)$, and so on. So:

$$f(b) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \ldots + \frac{h^n}{n!}f^{(n)}(a) + \ldots$$

$$y_b = y_a + hy_a{}' + \frac{h^2}{2!}y_a{}'' + \ldots + \frac{h^n}{n!}y_a{}^{(n)} + \ldots$$

A first-order Runge-Kutta integration equation is the following: $y_b = y_a + hy_a{}'$. If we have determined the value of y_b, we can estimate the next value of the function f(c) using the following: $y_c = y_b + hy_b{}'$. And an initial value or boundary value is need to start the process of estimating other points of the function f(x), the Runge-Kutta methods (and Euler's method) are also called **initial-value solutions** or **boundary-value solution**.

A nth-order Runge-Kutta integration equation uses terms in the Taylor's series expansion that include the first, second, …, nth derivatives, and computes the function estimate using n tangent-line estimates.

- ode functions

1. ode23: using second-order and third-order Runge-Kutta methods.
   [T,Y] = ODE23(ODEFUN,TSPAN,Y0), TSPAN = [T0 TFINAL]
   [T,Y] = ODE23(ODEFUN,TSPAN,Y0,OPTIONS), OPTIONS, an argument created
   with the ODESET function

2. ode45: using fourth-order and fifth-order Runge-Kutta methods
   [T,Y] = ODE45(ODEFUN,TSPAN,Y0)
   [T,Y] = ODE45(ODEFUN,TSPAN,Y0,OPTIONS) , OPTIONS, an argument created
   with the ODESET function

- First-Order Ordinary Differential Equations

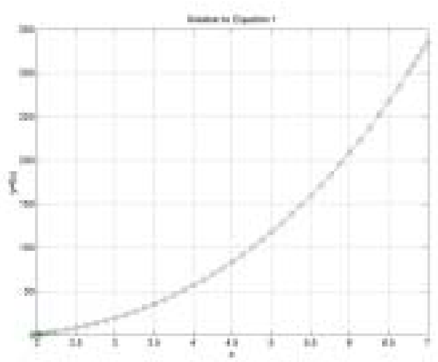
$$y' = \frac{dy}{dx} = g(x, y)$$

Equation 1: $y' = g_1(x, y) = 3x^2$
Equation 1 Solution: $y = x^3 - 7.5$

```
function dy = g1(x,y)
%MAXENTRY File: ../ch9/g1.m
%               Author: Ming-Kai Chen, 5 Dec 2002
%               Prupose: dy = g1(x,y)

dy = 3*x.^2;
```

```
% File: ../ch9/demo9_1.m
% Author: Ming-Kai Chen, 5 Dec 2002
% Prupose: ODE example

clear;
tspan = [2 7]; yzero = 0.5;
[x,num_y] = ode45(@g1,tspan,yzero);

y=x.^3-7.5;
disp([x x num_y y])
plot(x,num_y,x,y,'o'),...
    title('Solution to Equation 1'),...
    xlabel('x'), ylabel('y=f(x)'),grid
```

```
>>
    2.0000    2.0000    0.5000    0.5000
    2.0021    2.0021    0.5251    0.5251
    2.0042    2.0042    0.5503    0.5503
    2.0063    2.0063    0.5756    0.5756
    2.0084    2.0084    0.6009    0.6009
    2.0188    2.0188    0.7282    0.7282
```



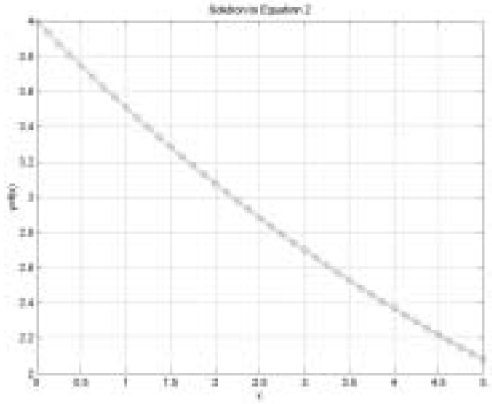
Equation 2: $y' = g_2(x,y) = -0.131y$

Equation 2 Solution: $y = 4e^{-0.131x}$

```
function dy = g2(x,y)
%MAXENTRY File: ../ch9/g2.m
%           Author: Ming-Kai Chen, 5 Dec 2002
%           Prupose: dy = g2(x,y)

dy = -0.131*y;
```

```
% File: ../ch9/demo9_2.m
% Author: Ming-Kai Chen, 5 Dec 2002
% Prupose: ODE example

clear;
tspan = [0 5]; yzero = 4;
[x,num_y] = ode45(@g2,tspan,yzero);

y = 4*exp(-0.131*x);
disp([x x num_y y])
plot(x,num_y,x,y,'o'),...
    title('Solution to Equation 2'),...
    xlabel('x'), ylabel('y=f(x)'),grid
```

```
>>
        0         0    4.0000    4.0000
   0.1250    0.1250    3.9350    3.9350
   0.2500    0.2500    3.8711    3.8711
   0.3750    0.3750    3.8082    3.8082
   0.5000    0.5000    3.7464    3.7464
   0.6250    0.6250    3.6855    3.6855
```



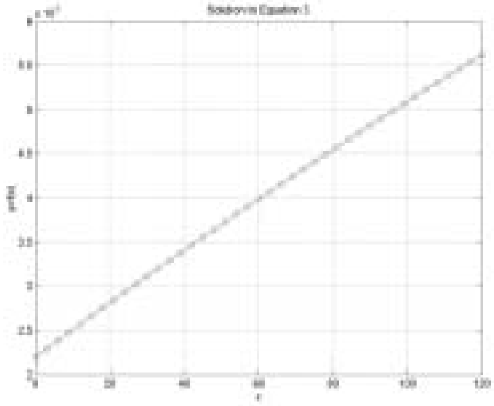
Equation 3: $y' = g_3(x,y) = 3.444E-05 - 0.0015y$

Equation 3 Solution: $y = 0.022963 - 0.020763e^{-0.0015x}$

```
function dy = g3(x,y)
%MAXENTRY File: ../ch9/g3.m
%           Author: Ming-Kai Chen, 5 Dec 2002
%           Prupose: dy = g3(x,y)

dy = 3.4444E-05 - 0.0015*y;
```

```
% File: ../ch9/demo9_3.m
% Author: Ming-Kai Chen, 5 Dec 2002
% Prupose: ODE example

clear;
tspan = [0 120]; yzero = 0.0022;
[x,num_y] = ode45('g3',tspan,yzero);

y = 0.022963 - 0.020763*exp(-0.0015*x);
disp([x x num_y y])
plot(x,num_y,x,y,'o'),...
    title('Solution to Equation 3'),...
    xlabel('x'), ylabel('y=f(x)'),grid
```

```
>>
        0         0    0.0022    0.0022
   3.0000    3.0000    0.0023    0.0023
   6.0000    6.0000    0.0024    0.0024
   9.0000    9.0000    0.0025    0.0025
  12.0000   12.0000    0.0026    0.0026
```



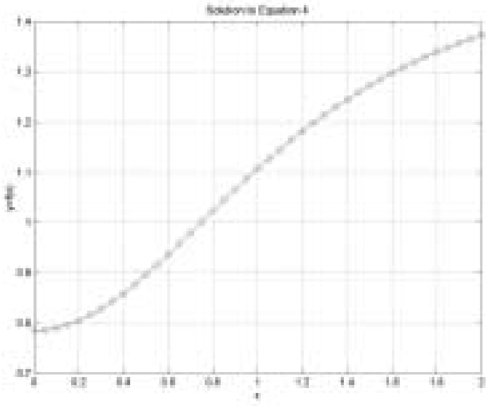
Equation 4: $y' = g_4(x,y) = 2 \cdot x \cdot \cos^2(y)$

Equation 4 Solution: $y = \tan^{-1}(x^2 + 1)$

```
function dy = g4(x,y)
%MAXENTRY File: ../ch9/g4.m
%           Author: Ming-Kai Chen, 5 Dec 2002
%           Prupose: dy = g4(x,y)

dy = 2*x*cos(y).^2;
```

```
% File: ../ch9/demo9_4.m
% Author: Ming-Kai Chen, 5 Dec 2002
% Prupose: ODE example

clear;
tspan = [0 2]; yzero = pi/4;
[x,num_y] = ode45('g4',tspan,yzero);

y = atan(x.*x+1);
disp([x x num_y y])
plot(x,num_y,x,y,'o'),...
    title('Solution to Equation 4'),...
    xlabel('x'), ylabel('y=f(x)'),grid
```

```
>>
         0         0    0.7854    0.7854
    0.0500    0.0500    0.7866    0.7866
    0.1000    0.1000    0.7904    0.7904
    0.1500    0.1500    0.7965    0.7965
```



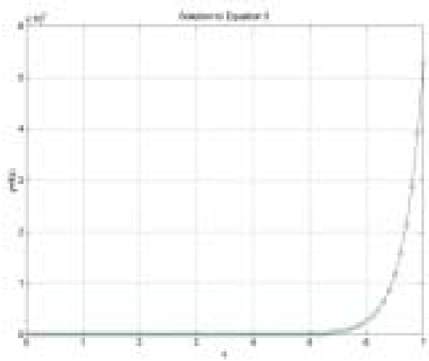
Equation 5: $y' = g_5(x,y) = 3y + e^{2x}$

Equation 5 Solution: $y = 4e^{3x} - e^{2x}$

```
function dy = g5(x,y)
%MAXENTRY File: ../ch9/g5.m
%           Author: Ming-Kai Chen, 5 Dec 2002
%           Prupose: dy = g5(x,y)

dy = 3*y+exp(2*x);
```

```
% File: ../ch9/demo9_5.m
% Author: Ming-Kai Chen, 5 Dec 2002
% Prupose: ODE example

clear;
tspan = [0:0.1:7]; yzero = 3;
[x,num_y] = ode45('g5',tspan,yzero);

y = 4*exp(3*x)-exp(2*x);
disp([x x num_y y])
plot(x,num_y,x,y,'o'),...
    title('Solution to Equation 5'),...
    xlabel('x'), ylabel('y=f(x)'),grid
```

```
>>
  1.0e+009 *

         0         0    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
```



- Higher-Order Differential Equations

The nth order differential equation: $y(n) = g(x, y, y', y'', y''',..., y^{(n-1)})$

Define n new unknown function with these equations:

$u_1(x) = y^{(n-1)}$

$u_2(x) = y^{(n-2)}$

$u_3(x) = y^{(n-3)}$

……………

$u_{n-2}(x) = y''$

$u_{n-1}(x) = y'$

$u_{n-1}(x) = y$

The following system of n first-order equations is equivalent to the nth order differential equation given above:

$u_1' = y^{(n)} = g(x, u_n, u_{n-1}, u_{n-2},..., u_1)$

$u_2' = y^{(n-1)} = u_1$

$u_3' = y^{(n-2)} = u_2$

…………………..

$u_{n-2}' = y' = u_{n-3}$

$u_{n-1}' = y' = u_{n-2}$

Define n new unknown function with these equations:

$u_1(x) = y$

$u_2(x) = y'$

$u_3(x) = y''$

……………

$u_{n-2}(x) = y^{(n-3)}$

$u_{n-1}(x) = y^{(n-2)}$

$u_n(x) = y^{(n-1)}$

The following system of n first-order equations is equivalent to the nth order differential equation given above:

$u_1'(x) = y' = u_2(x) \Rightarrow u_2$

$u_2'(x) = y'' = u_3(x) \Rightarrow u_3$

$u_3'(x) = y''' = u_4(x) \Rightarrow u_4$

…………………..

$u_{n-2}'(x) = y^{(n-2)} = u_{n-1}(x) \Rightarrow u_{n-1}$

$u_{n-1}'(x) = y^{(n-1)} = u_n(x) \Rightarrow u_n$

$u_n'(x) = y^{(n)} = g(x, u_n, u_{n-1}, u_{n-2},,..., u_2)$

Equation: $y'' = g(x, y, y') = y'(1 - y^2) - y$

Define: $u_1 = y'\ , u_2 = y$

Thus: $u_1{}' = y'' = g(x, u_2, u_1)$

$u_2{}' = y' = u_1$

Obtain: $u_1{}' = y'' = u_1(1 - u_2{}^2) - u_2$

Define: $u_1 = y\ , u_2 = y'$

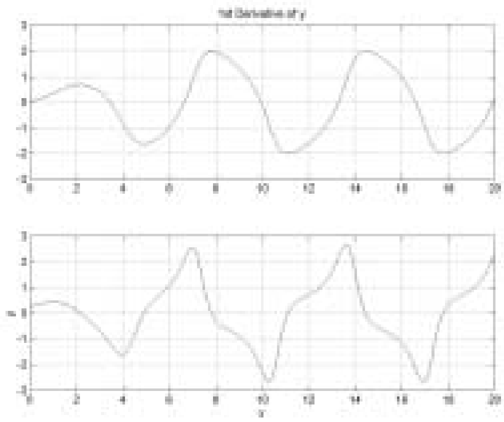
Thus: $u_1{}' = y' = u_2$

$u_2{}' = y'' = y'(1 - y^2) - y = u_2(1 - u_1{}^2) - u_1$

Now: initial condition of $y'(0) = 0.0$ and $y(0) = 0.25$。

```
function uprime = eqns2(x,u)
%MAXENTRY File: ../ch9/eqns2.m
%                Author: Ming-Kai Chen, 5 Dec 2002
%                Prupose: u_prime = eqns2(x,u)

uprime = [u(2) ; u(2)*(1-u(1)^2)-u(1)];

%uprime(1) = u(1)*(1-u(2)^2)-u(2);
%uprime(2) = u(1) ;
```

```
% File: ../ch9/demo9_6.m
% Author: Ming-Kai Chen, 5 Dec 2002
% Prupose: ODE example

clear;
tspan = [0 20]; initial = [0 0.25];
[x,num_y] = ode23('eqns2',tspan,initial);

subplot(2,1,1),plot(x,num_y(:,1)),...
     title('1st Derivative of y'),grid,...
subplot(2,1,2),plot(x,num_y(:,2)),...
    xlabel('x'), ylabel('y'),grid
```



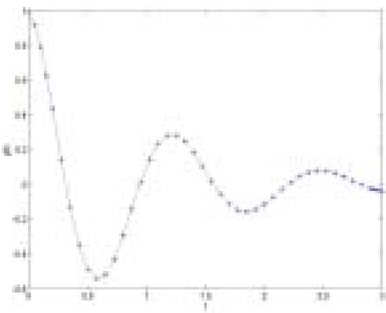
- **實例演練一** (chap9_1.m)

$$\frac{d}{dt}y(t)=-y(t)-5e^{-t}\sin 5t\text{ ， } y(0)=1\text{ ， } 0\le t\le 3\text{。}$$

```
function yprime = myf(t,y)
%MAXENTRY File: ../ch9/myf.m
%              Author: Ming-Kai Chen, 16 Nov 2002
%              Prupose: yprime = myf(t,y)
% =====>   ODE example function
%              YPRIME = MYF(T,Y) evaluates derivative

yprime = -y -5*exp(-t)*sin(5*t);
```

```
% File: ../ch9/ch9_1.m
% Author: Ming-Kai Chen, 16 Nov 2002
% Prupose: ODE example

clear;
tspan = [0 3]; yzero = 1;
[t,y] = ode45(@myf,tspan,yzero);
plot(t,y,'*--')
xlabel t, ylabel y(t)

tspan2 = 0:4;
[t2,y2] = ode45(@myf,tspan2,yzero);
disp([t2 y2])

tspan3 = [0 -0.5 -1];
[t3,y3] = ode45(@myf,tspan3,yzero);
disp([t3 y3])
```

```
>>
         0    1.0000
    1.0000    0.1043
    2.0000   -0.1136
    3.0000   -0.0378
    4.0000    0.0075

         0    1.0000
   -0.5000   -1.3209
   -1.0000    0.7711
```



- 實例演練二 (chap9_2.m)

Equation: $\dfrac{d^2}{dt^2}\theta(t) + \sin\theta(t) = 0$, $0 \le t \le 10$

Define: $y_1 = \theta(t)$, $y_2 = \dfrac{d}{dt}\theta(t)$

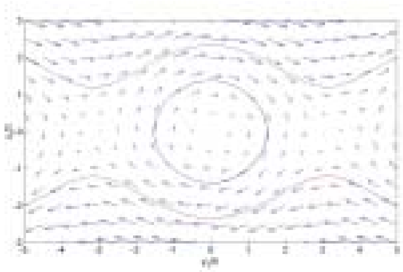
Thus: $y_1{}' = \dfrac{d}{dt}\theta(t) = y_2$

$$y_2{}' = \frac{d^2}{dt^2}\theta(t) = -\sin\theta(t) = -\sin y_1$$

Obtain $\dfrac{d}{dt}y_1(t) = y_2(t)$, $\dfrac{d}{dt}y_2(t) = -\sin y_1(t)$

```
function yprime = pend(t,y)
%MAXENTRY File: ../ch9/pend.m
%            Author: Ming-Kai Chen, 16 Nov 2002
%            Prupose: yprime = pend(t,y)

yprime = [y(2); -sin(y(1))];
```

```
% File: ../ch9/ch9_2.m
% Author: Ming-Kai Chen, 16 Nov 2002
% Prupose: ODE example

clear;
tspan = [0 10];
yazero = [1;1]; ybzero = [-5;2]; yczero = [5;-2];
[ta,ya] = ode45(@pend,tspan,yazero);
[tb,yb] = ode45(@pend,tspan,ybzero);
[tc,yc] = ode45(@pend,tspan,yczero);

[y1,y2] = meshgrid(-5:.5:5,-3:.5:3);
Dy1Dt = y2; Dy2Dt = -sin(y1);
quiver(y1,y2,Dy1Dt,Dy2Dt)
hold on
plot(ya(:,1),ya(:,2),yb(:,1),yb(:,2),yc(:,1),yc(:,2))
axis equal, axis([-5 5 -3 3])
xlabel y_1(t), ylabel y_(2), hold off
```



- 實例演練三 (chap9_3.m)

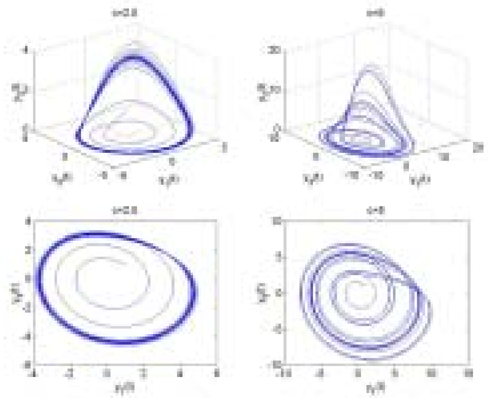
$$\frac{d}{dt}y_1(t) = -y_2(t) - y_3(t)$$

$$\frac{d}{dt}y_2(t) = y_1(t) + ay_2(t)$$

$$\frac{d}{dt}y_3(t) = b + y_3(t)(y_1(t) - c)$$

```
function yprime = rossler(t,y,a,b,c)
%MAXENTRY File: ../ch9/rossler.m
%            Author: Ming-Kai Chen, 16 Nov 2002
%            Prupose: yprime = rossler(t,y,a,b,c)

yprime = [-y(2)-y(3); y(1)+a*y(2); b+y(3)*(y(1)-c)];
```

```
% File: ../ch9/ch9_3.m
% Author: Ming-Kai Chen, 16 Nov 2002
% Prupose: ODE example

clear;
tspan = [0 100];yzero = [1;1;1];
options = odeset('AbsTol',1e-7,'RelTol',1e-4)

a = 0.2; b = 0.2; c = 2.5;
[t,y] = ode45(@rossler,tspan,yzero,options,a,b,c);
subplot(221),plot3(y(:,1),y(:,2),y(:,3)),title('c=2.5'),grid
xlabel y_1(t), ylabel y_2(t), zlabel y_3(t)
subplot(223),plot(y(:,1),y(:,2)),title('c=2.5')
xlabel y_1(t), ylabel y_2(t)

c = 5;
[t,y] = ode45(@rossler,tspan,yzero,options,a,b,c);
subplot(222),plot3(y(:,1),y(:,2),y(:,3)),title('c=5'),grid
xlabel y_1(t), ylabel y_2(t), zlabel y_3(t)
subplot(224),plot(y(:,1),y(:,2)),title('c=5')
xlabel y_1(t), ylabel y_2(t)
```



● 實例演練四 (chap9_4.m)

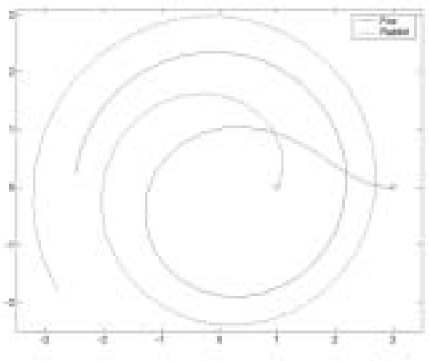
$$\frac{d}{dt}y_1(t) = s(t)(r_1(t) - y_1(t))\text{，}\quad \frac{d}{dt}y_2(t) = s(t)(r_2(t) - y_2(t))\text{，}$$

$$\text{where}\quad s(t) = \frac{k\sqrt{\left(\frac{d}{dt}r_1(t)\right)^2 + \left(\frac{d}{dt}r_2(t)\right)^2}}{\sqrt{(r_1(t) - y_1(t))^2 + (r_2(t) + y_2(t))^2}}$$

$$\begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \sqrt{1+t}\begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$

starting the fox at $y_1(0) = 3, y_2(2) = 0, k = 0.75$。

```
function yprime = fox1(t,y)
%MAXENTRY File: ../ch9/fox1.m
%            Author: Ming-Kai Chen, 16 Nov 2002
%            Prupose: yprime = fox1(t,y)

k = 0.75;
r = sqrt(1+t)*[cos(t);sin(t)];
r_p= (0.5/sqrt(1+t))*[cos(t)-2*(1+t)*sin(t);sin(t)+2*(1+t)*cos(t)];
dist = norm(r-y);
if dist > 1e-4
    factor = k*norm(r_p)/dist;
    yprime = factor*(r-y);
else
    error('ODE model ill-defined.')
end
```



```
% File: ../ch9/ch9_4.m
% Author: Ming-Kai Chen, 16 Nov 2002
% Prupose: ODE example

clear;
tspan = [0 10];yzero = [3;0];
[tfox,yfox] = ode45(@fox1,tspan,yzero);
plot(yfox(:,1),yfox(:,2)), hold on
plot(sqrt(1+tfox).*cos(tfox),sqrt(1+tfox).*sin(tfox),'--')
plot([3 1],[0 0],'o')
axis equal, axis([-3.5 3.5 -2.5 3.1])
legend('Fox','Rabbit',0),                                    hold
offsubplot(222),plot3(y(:,1),y(:,2),y(:,3)),title('c=5'),grid
xlabel y_1(t), ylabel y_2(t), zlabel y_3(t)
subplot(224),plot(y(:,1),y(:,2)),title('c=5')
xlabel y_1(t), ylabel y_2(t)
```

第十章　傅立葉轉換

- 傅立葉轉換與逆轉換數學方程式

FFT: $X(k)=\sum_{n=1}^{N} x(n)e^{-j2\Pi(k-1)\left(\frac{n-1}{N}\right)}$，$1\le k\le N$。

IFFT: $X(n)=\frac{1}{N}\sum_{k=1}^{N} x(k)e^{-j2\Pi(k-1)\left(\frac{n-1}{N}\right)}$，$1\le n\le N$。

- 快速傅立葉轉換(FFT)常用函數

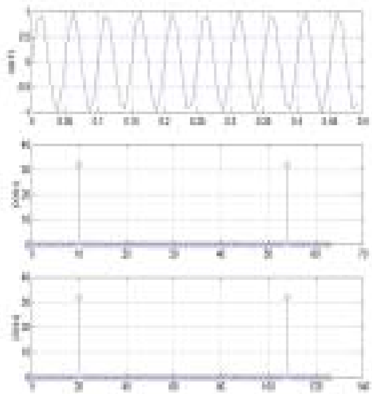
函數	意義
fft	離散傅立葉轉換
fft2	二維傅立葉轉換
fftn	N 維傅立葉轉換
ifft	逆傅立葉轉換
ifft2	二維傅立葉轉換
iffn	N 維逆傅立葉轉換
abs	向量絕對值(magnitude)
angle	向量的相位角(phase)
unwrap	使相位在 180 度不會產生不連續點

- The fast Fourier transform (FFT), use fft function by y=fft(x). The inverse FFT, $x=F_n*y$, is carried out by ifft function using x=ifft(t).

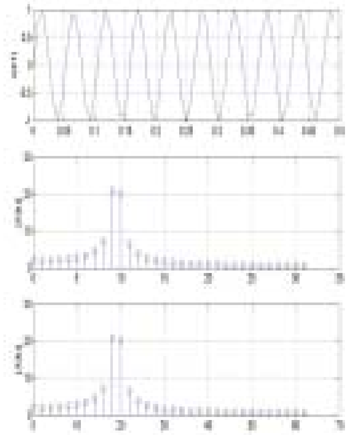
```
>> y=fft([1 1 -1 -1])
y =
         0          2.0000 - 2.0000i          0          2.0000 + 2.0000i
>> x=ifft(y)
x =
     1     1    -1    -1
>> y2=fftn([1 2 3 4])
y2 =
   10.0000          -2.0000 + 2.0000i   -2.0000          -2.0000 - 2.0000i
>> x2=ifftn(y2)
x2 =
     1     2     3     4
```

- 波函式經傳立葉轉換後，觀察其純量和相位。

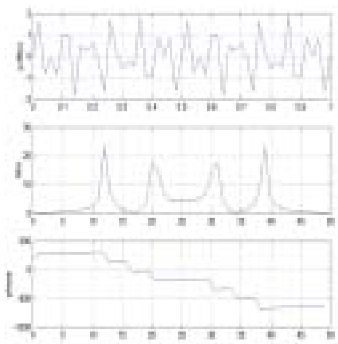
```
% File: ../ch10/demo10_1.m
% Author: Ming-Kai Chen, 7 Dec 2002
% Prupose: Gnerate a 20 Hz sinnsoid samples at 128 Hz.

clear;
N = 64;
T = 1/128;
k = 0:N-1;
x = sin(2*pi*20*k*T);
X = fft(x);
magX = abs(X);
hertz = k*(1/(N*T));

figure;
subplot(311);plot (0:T:T*(N-1),x), ylabel ('x(kT)'), grid on
subplot(312);stem (k(1:N),magX(1:N)), ylabel ('|X(k)|'), grid on
subplot(313);stem (hertz(1:N),magX(1:N)), ylabel ('|X(k)|'), grid on
```



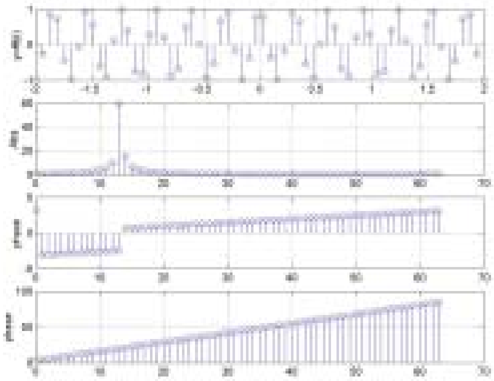
```
% File: ../ch10/demo10_1.m
% Author: Ming-Kai Chen, 7 Dec 2002
% Prupose: Gnerate a 20 Hz sinnsoid samples at 128 Hz.

clear;
N = 64;
T = 1/128;
k = 0:N-1;
x = sin(2*pi*19*k*T);
X = fft(x);
magX = abs(X);
hertz = k*(1/(N*T));
sum(x-ifft(fft(x)))

figure;
subplot(311);plot (0:T:T*(N-1),x), ylabel ('x(kT)'), grid on
subplot(312);stem (k(1:N/2),magX(1:N/2)), ylabel ('|X(k)|'), grid on
subplot(313);stem (hertz(1:N/2),magX(1:N/2)), ylabel ('|X(k)|'), grid
on
```



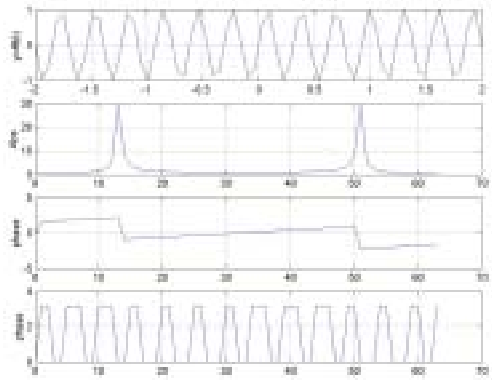
```
% File: ../ch10/demo10_3.m
% Author: Ming-Kai Chen, 7 Dec 2002
% Prupose: FFT example

clear;
t = 0:1/50:1;
x = sin(2*pi*20*t)+sin(2*pi*12*t);
y = fft(x);
m = abs(y);
p = unwrap(angle(y));
f = (0:length(y)-1)'*50/length(y);

figure;
subplot(311);plot (t,x), ylabel ('y=fft(x)'), grid on
subplot(312);plot (f,m), ylabel ('Abs'), grid on
subplot(313);plot (f,p*180/pi), ylabel ('phase'), grid on
```



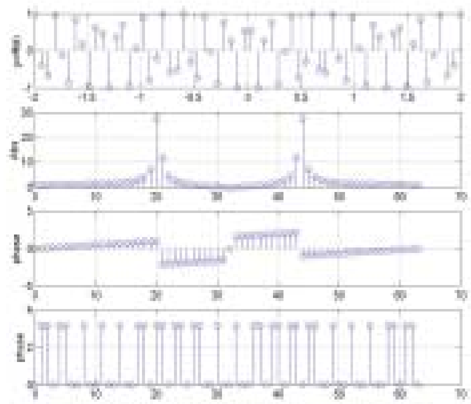
```
% File: ../ch10/demo10_4.m
% Author: Ming-Kai Chen, 7 Dec 2002
% Prupose: FFT example

clear;
n=63; L=2;
t = -L: 2*L/n: L;
y = exp(13*i*pi*t/L);
z = fft(y);
p1 = angle(z);
p2 = unwrap(angle(y));

figure;
subplot(411);stem (t,y), ylabel ('y=fft(t)'), grid on
subplot(412);stem (0:n,abs(z)), ylabel ('Abs'), grid on
subplot(413);stem (0:n,p1), ylabel ('phase'), grid on
subplot(414);stem (0:n,p2), ylabel ('phase'), grid on
```



```
% File: ../ch10/demo10_5.m
% Author: Ming-Kai Chen, 7 Dec 2002
% Prupose: FFT example

clear;
n=63; L=2;
t = -L: 2*L/n: L;
y = sin(13*pi*t/L);
z = fft(y);
p1 = angle(z);
p2 = unwrap(angle(y));

figure;
subplot(411);plot (t,y), ylabel ('y=fft(t)'), grid on
subplot(412);plot (0:n,abs(z)), ylabel ('Abs'), grid on
subplot(413);plot (0:n,p1), ylabel ('phase'), grid on
subplot(414);plot (0:n,p2), ylabel ('phase'), grid on
```



```
% File: ../ch10/demo10_6.m
% Author: Ming-Kai Chen, 7 Dec 2002
% Prupose: FFT example

clear;
n=63; L=2;
t = -L: 2*L/n: L;
y = cos(20*pi*t/L);
z = fft(y);
p1 = angle(z);
p2 = unwrap(angle(y));

figure;
subplot(411);plot (t,y), ylabel ('y=fft(t)'), grid on
subplot(412);plot (0:n,abs(z)), ylabel ('Abs'), grid on
subplot(413);plot (0:n,p1), ylabel ('phase'), grid on
subplot(414);plot (0:n,p2), ylabel ('phase'), grid on
```

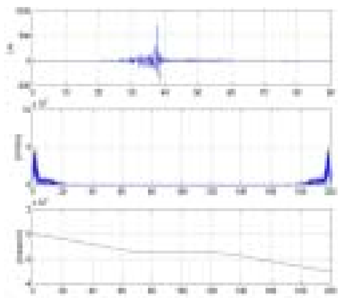


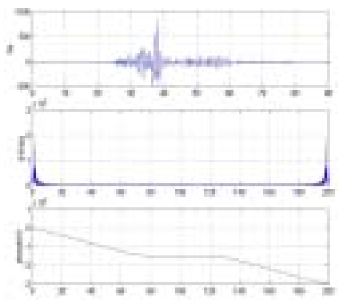
- 實例演練一 (chap10_1.m)

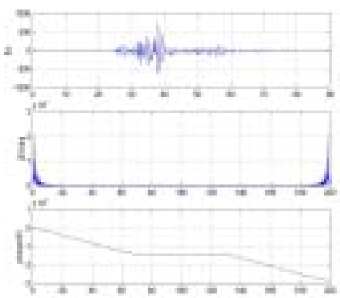
　　本範例所取得的資料來源為中央氣象局網站(http://www.cwb.gov.tw/V3.0/index.htm)，選取 1999/09/21，921 大地震中主震原始資料(ASCI)的草嶺站資料(CHN5,CHY080)，本筆資料相關資訊如下：

```
StationCode: CHY080
LocationLongitude(°E): 120.678
LocationLatitude (°N):  23.597
LocationElavation(M):   840.0
InstrumentKind: A900A(T362002.263 )
StartTime: 1999/ 9/20 17:47: 2.0
SampleRate(Hz): 200
AmplitudeUnit:  gal
RecordLength(sec):   90.0
DataSequence: U(+); N(+); E(+)
Data: 3F10.3
     5.323    -13.757    -15.612
```

```
% File: ../ch10/ch10_1.m
% Author: Ming-Kai Chen, 7 Dec 2002
% Prupose: FFT example

clear;
N = 18000;
T = 1/200;
k = 0:N-1;
hertz = k * (1/(N*T));

load 921_CHY080.dat;
Ux = X921_CHY080(:,1)';
UX = fft(Ux);
UP = unwrap(angle(UX));
figure(1);
subplot(311);plot (0:T:T*(N-1),Ux), ylabel ('Ux'), grid on
subplot(312);plot (hertz(1:N),abs(UX)), ylabel ('|UX(k)|'), grid on
subplot(313);plot (hertz(1:N),UP), ylabel ('phase(U)'), grid on

Nx = X921_CHY080(:,2)';
NX = fft(Nx);
NP = unwrap(angle(NX));
figure(2);
subplot(311);plot (0:T:T*(N-1),Nx), ylabel ('Nx'), grid on
subplot(312);plot (hertz(1:N),abs(NX)), ylabel ('|NX(k)|'), grid on
subplot(313);plot (hertz(1:N),NP), ylabel ('phase(N)'), grid on

Ex = X921_CHY080(:,3)';
EX = fft(Ex);
EP = unwrap(angle(EX));
figure(3);
subplot(311);plot (0:T:T*(N-1),Ex), ylabel ('Ex'), grid on
subplot(312);plot (hertz(1:N),abs(EX)), ylabel ('|EX(k)|'), grid on
subplot(313);plot (hertz(1:N),EP), ylabel ('phase(E)'), grid on
```







第十一章　向量微分

第十二章　向量積分

第十三章　偏微分方程

第十四章　常用工具箱

一. 符號工具箱

function	Meaning	Example
sym	宣告任一符號變數	sym('x');sym('d1')
syms	宣告諸多符號變數	syms a b c d3
pretty	一般數學化外形	F=(1+x)^4/(1+x^2)+4/(1+x^2);pretty(F)
simplify	簡化浮式	simplify(F)
expand	展開數學式	p=expand((1+x)^4)
horner	巢狀排列	horner(p)
factor	分解數學式	factor(a^3+b^3-a*b-a*c-b*c+b^2+c^2)
simple	嘗式不同方法以簡化數學式，並傳回最簡化式子	syms x y;y=sqrt(cos(x)+i*sin(x)); simple(y)
subs	代換不同變數	subs(p,x,2)
collect	整理同次幕	a=(x+3)(x-4);collect(a)
double	將符號表示化簡為標準數值	double(1/4*pi)
vpa	取得任何十進位數目	vpa(sqrt(16),100)
taylor	泰勒式展開開始	taylor(exp(x),8)
gamma	求取 gamma(Γ)函數之加合	gamma(5)
symsum	指數函數的級數展開之加合	symsum(1/gamma(r),1,inf)
zeta	求取 Riemann Zeta 函數之加合	zeta(2)
solve	求取多項式的解	solve(x^3-7/2*x^2-17/2*x+5)
diff	對任一函數進行符號式(偏)微分	syms z,k; f=k*cos(z^4); diff(f,z)
int	對函數進行符號式積分	syms u; f=u^2*cos(u); int(f)
dsolve	求取微分方程的解	s=dsolve('(1+t^2)*Dy+2*t*y=cos(t),t(0)=0')
laplace	求取函數之拉式轉換	syms t; laplace(t^4)
ztrans	求取函數 Z 轉換	syms z n; ztrans(1/4^n)
fourier	求取傅立葉轉換值	y=fourier(cos(3*x))
ifourier	求取反傅立葉轉換值	z=ifourier(y)

- 符號式變數與表示法

```
>> x=sym('x')
x =
x
>> d1=sym('d1')
  d1 =
  d1
>> syms a b c d3
>> 1/(x+1)
ans =
1/(x+1)
>> syms x y
>> d=[x+1 x^2 x-y;1/x 3*y/x 1/(1+x); 2-x x/4 3/2]
  d =
  [         x+1,          x^2,          x-y]
  [         1/x,         3*y/x, 1/(x+1)]
  [         2-x,        1/4*x,          3/2]
>> d(2,2)
  ans =
3*y/x
>> c=d(2,:)
c =
[         1/x,      3*y/x, 1/(x+1)]
>> e=(1+x)^4/(1+x^2)+4/(1+x^2)
e =
(x+1)^4/(1+x^2)+4/(1+x^2)
>> pretty(e)
                   4
         (x + 1)           4
         -------- + ------
                 2             2
           1 + x         1 + x
>> simplify(e)
ans =
x^2+4*x+5
```

```
>> p=expand((1+x)^4)
p =
x^4+4*x^3+6*x^2+4*x+1
>> horner(p)
ans =
1+(4+(6+(4+x)*x)*x)*x
>> syms x;y=sqrt(cos(x)+i*sin(x));
>> simple(y)
simplify:
(cos(x)+i*sin(x))^(1/2)
radsimp:
(cos(x)+i*sin(x))^(1/2)
combine(trig):
(cos(x)+i*sin(x))^(1/2)
factor:
(cos(x)+i*sin(x))^(1/2)
  expand:
(cos(x)+i*sin(x))^(1/2)
combine:
(cos(x)+i*sin(x))^(1/2)
convert(exp):
exp(i*x)^(1/2)
convert(sincos):
(cos(x)+i*sin(x))^(1/2)
convert(tan):
((1-tan(1/2*x)^2)/(1+tan(1/2*x)^2)+2*i*tan(1/2*x)/(1+
tan(1/2*x)^2))^(1/2)
  collect(x):
(cos(x)+i*sin(x))^(1/2)
ans =
exp(i*x)^(1/2)
```

```
>> syms u v w
>> fmv=pi*v*w/(u+v+w)
fmv =
pi*v*w/(u+v+w)
>> subs(fmv,u,2*v)
ans =
pi*v*w/(3*v+w)
>> subs(ans,v,1)
ans =
pi*w/(3+w)
>> subs(ans,w,3)
ans =
       1.5708
>> syms x y
>> f=3x+2*y^2
>> f=3*x+2*y^2
f =
3*x+2*y^2
>> subs(f,x,2*y)
ans =
6*y+2*y^2
```

```
>> subs(f,x,3)
ans =
9+2*y^2
>> subs(ans,y,2)
ans =
       17
>>  f
f  =
3*x+2*y^2
>> subs(f,y,x-3)
ans =
3*x+2*(x-3)^2
>> collect(ans)
ans =
2*x^2-9*x+18
>> factor(f)
ans =
3*x+2*y^2
```

- 符號計算的變數精準度 (vpa)

- 級數展開與加合 (raylor, gamma, zeta)

$$e^{0.1}=\sum_{r=1}^{\infty}\frac{0.1^{r-1}}{(r-1)!}\text{ 或 }\sum_{r=1}^{\infty}\frac{0.1^{r-1}}{\Gamma(r)}$$

$$S=1+2^2+3^2+4^2+......+n^2$$

$$S=1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4_2}......$$

Riemann Zeta 函數(zeta)：$\xi(k)=1+\frac{1}{2^k}+\frac{1}{3^k}+\frac{1}{4^k}+.....+\frac{1}{r^k}+......$

$$S=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+......$$

$$S=1+\frac{1}{1!}+\frac{1}{(2!)^2}+\frac{1}{(3!)^2}+\frac{1}{(4!)^2}+......$$

```
>> taylor(cos(exp(x)),4)
ans =
cos(1)-sin(1)*x+(-1/2*cos(1)-1/2*sin(1))*x^2-1/2*cos(1)*x^3
>> s=taylor(exp(x),8)
s =
1+x+1/2*x^2+1/6*x^3+1/24*x^4+1/120*x^5+1/720*x^6+1/5040*x^7
>> syms r
>> symsum((0.1)^(r-1)/gamma(r),1,8)
ans =
55700614271/50400000000
>> double(ans)
ans =
     1.1052
>> double(subs(s,x,0.1))
ans =
     1.1052
>> symsum(r*r,1,n)
ans =
1/3*(n+1)^3-1/2*(n+1)^2+1/6*n+1/6
```

```
>> factor(ans)
ans =
1/6*n*(n+1)*(2*n+1)
>> symsum(r^3,1,n)
ans =
1/4*(n+1)^4-1/2*(n+1)^3+1/4*(n+1)^2
>> factor(ans)
ans =
1/4*n^2*(n+1)^2
>> symsum(1/r^2,1,inf)
ans =
1/6*pi^2
>> zeta(2)
ans =
     1.6449
>> zeta(3)
ans =
     1.2021
```

- 符號式矩陣運算

- 符號式的解方程式 (solve)

>> syms x >> f1=x^2-4*x+3 f1 = x^2-4*x+3 >> solve(f1) ans = [1] >> f2=x^3-7/2*x^2-17/2*x+5 f2 = x^3-7/2*x^2-17/2*x+5 >> solve(f2) ans = [5] [1/2] [-2]	>> f3=x^3-7/(2*x^2)-17/(2*x)+5 f3 = x^3-7/2/x^2-17/2/x+5 >> [x,y]=solve('x^2+y^2=a','x^2-y^2=b') x = [1/2*(2*b+2*a)^(1/2)] [-1/2*(2*b+2*a)^(1/2)] [1/2*(2*b+2*a)^(1/2)] [-1/2*(2*b+2*a)^(1/2)] y = [1/2*(-2*b+2*a)^(1/2)] [1/2*(-2*b+2*a)^(1/2)] [-1/2*(-2*b+2*a)^(1/2)] [-1/2*(-2*b+2*a)^(1/2)]	>> x.^2+y.^2,x.^2-y.^2 ans = [a] [a] [a] [a] ans = [b] [b] [b] [b]

- 符號式微分(diff)

$$f1 = x^n\ ,\ \frac{df1}{dx} = nx^{n-1}\ ,\ \int f1dx = \frac{x^{n+1}}{n+1} + k$$

$$f2 = x^{-1}\ ,\ \frac{df2}{dx} = -x^{-2}\ ,\ \int f2dx = \ln(x) + k$$

$$f3 = \ln(x)\ ,\ \frac{df3}{dx} = nx^{n-1}\ ,\ \int f3dx = x\ln(x) - x + k$$

$$f4 = e^a x$$

$$f5 = e^{ax}$$

$$f6 = a^x$$

$$f7 = \sin(ax)$$

$$f8 = \cos(ax)$$

$$f9 = \sin^{-1}(\frac{x}{a})$$

$$f10 = \sinh(ax)$$

>> syms x n >> f1=x^n f1 = x^n >> diff(f1) ans = x^n*n/x. >> f2=x^-1 f2 = 1/x >> diff(f2) ans = -1/x^2 >> f3=log(x) f3 = log(x) >> diff(f3) ans = 1/x	>> syms e a >> f4=e^a*x f4 = e^a*x >> diff(f4) ans = e^a >> f5=e^(a*x) f5 = e^(a*x) >> diff(f5) ans = e^(a*x)*a*log(e) >> f6=a^x f6 = a^x >> diff(f6) ans = a^x*log(a) >> f7=sin(a*x) f7 = sin(a*x)	>> diff(f7) ans = cos(a*x)*a >> f8=cos(a*x) f8 = cos(a*x) >> diff(f8) ans = -sin(a*x)*a >> f9=asin(x/a) f9 = asin(x/a) >> diff(f9) ans = 1/a/(1-x^2/a^2)^(1/2) >> f10=sinh(a*x) f10 = sinh(a*x) >> diff(f10) ans = cosh(a*x)*a

- 符號式積分 (int)

>> int(f1) ans = x^(n+1)/(n+1) >> int(f2) ans = log(x) >> int(f3) ans = x*log(x)-x	>> int(f4) ans = 1/2*e^a*x^2 >> int(f5) ans = 1/a/log(e)*e^(a*x) >> pretty(ans) (a x) e -------- a log(e) >> int(f6) ans = 1/log(a)*a^x	>> int(f7) ans = -1/a*cos(a*x) >> int(f8) ans = sin(a*x)/a >> int(f9) ans = a*(x/a*asin(x/a)+(1-x^2/a^2)^(1/2)) >> int(f10) ans = 1/a*cosh(a*x)

- 符號式偏微分 (diff)

```
>> syms u v w
>> fmv=u*v*w/(u+v+w)
fmv =
u*v*w/(u+v+w)
>> d=[diff(fmv,u) diff(fmv,v) diff(fmv,w)]
d =
[ v*w/(u+v+w)-u*v*w/(u+v+w)^2, u*w/(u+v+w)-u*v*w/(u+v+w)^2, u*v/(u+v+w)-u*v*w/(u+v+w)^2]
>> diff(d(3),u)
ans =
v/(u+v+w)-u*v/(u+v+w)^2-v*w/(u+v+w)^2+2*u*v*w/(u+v+w)^3
>> pretty(ans)
        v              u v            v w               u v w
    --------  -   ------------  -  ------------  +  2  ------------
    u + v + w               2               2                   3
                  (u + v + w)      (u + v + w)         (u + v + w)
>> diff(d(2),w)
ans =
u/(u+v+w)-u*w/(u+v+w)^2-u*v/(u+v+w)^2+2*u*v*w/(u+v+w)^3
>> diff(ans,u)
ans =
1/(u+v+w)-u/(u+v+w)^2-w/(u+v+w)^2+2*u*w/(u+v+w)^3-v/(u+v+w)^2+2*u*v/(u+v+w)^3+2*v*w/(u+v+w)^3-
6*u*v*w/(u+v+w)^4
```

- 符號式解常微分方程 (dsolve)

　　函數 dsolve 解微分方程式的一般呼叫如下：

$sol = dsolve('de1, de2, de3, ..., den, in1, in2, in3, ..., inn')$

其中參數 de1, de2, de3, ……den 代表各別微分方程，需使用符號式變數；標準的 matlab 運算元與代表一階、二階、及高階的微分運算子為 D1, D2, D3, ……Dn，以符號式表示；初值設定條件為 in1, in2, in3, ……inn，代表微分方程的初值條件，若應變數為 y，初始條件為 $y(0)=1, Dy(0)=0, D2y(3)=9.1$，表示 t=0 時，y 值為 1，$dy/dt=0, d^2/dt^2=9.1$。值得注意的是 matlab 最大接受 12 個輸入參數。

　　下面舉一些微分方程為例，並執行之！

$(1+t^2)\dfrac{dy}{dt}+2ty=\cos t$。

$\dfrac{d^2y}{dx^2}+y=\cos 2x$， $y(0)=0$。

$\dfrac{d^4y}{dt^4}=y$，when $x=0$時，則 $y=0, \dfrac{dy}{dx}=1$。

```
>> syms t y
>> s=dsolve('(1+t^2)*Dy+2*t*y=cos(t)')
s =
(sin(t)+C1)/(1+t^2)
>> pretty(s)
      sin(t) + C1
      -----------
              2
        1 + t
>> s=dsolve('(1+t^2)*Dy+2*t*y=cos(t),y(0)=0')
s =
1/(1+t^2)*sin(t)
>> pretty(s)
      sin(t)
      -----
          2
      1 + t
>> dsolve('D2y+y=cos(2*x), Dy(0)=1, y(0)=0','x')
ans =
(1/6*cos(3*x)-1/2*cos(x))*cos(x)+(1/2*sin(x)+1/6*sin(3*x))*sin(x)+1/3*cos(x)+sin(x)
>> simplify(ans)
ans =
-2/3*cos(x)^2+1/3*cos(x)+sin(x)+1/3
```

$\frac{dy}{dx}=\frac{e^{-x}}{x}$，when $x=1$時，則$y=1$。

$\frac{dy}{dx}=\cos(\sin x)$。

$\frac{d^2x}{dt^2}+\frac{a}{b}\sin t=0$，when $t=0$時，則$x=1, \frac{dx}{dt}=0$。

>> dsolve('D4y=y,y(pi/2)=1,Dy(pi/2)=0,D2y(pi/2)=-1,D3y(pi/2)=0') ans = sin(t) >> dsolve('Dy=exp(-x)/x,y(1)=1','x') ans = -Ei(1,x)+Ei(1,1)+1	>> vpa('Ei(1,1)',10) ans = .2193839344 >> dsolve('Dy=cos(sin(x))','x') ans = Int(cos(sin(x)),x)+C1 >> x=dsolve('D2x+g*sin(t)/L,x(0)=1,Dx(0)=0') x = (g*sin(t)-g*t+L)/L

$\frac{du}{dt}=-\frac{a}{b}\sin t$

$\frac{dx}{dt}=u$

>> [u x]=dsolve('Du+g*sin(t)/L=0,Dx=u,x(0)=1,u(0)=0') u = (-g+g*cos(t))/L x = (g*sin(t)-g*t+L)/L	>> s=dsolve('Du+g*sin(t)/L=0,Dx=u,x(0)=1,u(0)=0') s = u: [1x1 sym] x: [1x1 sym] >> s.u ans = (-g+g*cos(t))/L >> s.x ans = (g*sin(t)-g*t+L)/L

- 符號式拉普拉斯轉換 (laplace)

- 符號式 Z 轉換 (ztrans,iztrans)

- 符號式傅立葉轉換 (fourier,ifourier)

函數 *f(x)*的傅立葉轉換定義， $F(s)=\zeta\left[f(x)\right]=\int_{-\infty}^{\infty} f(x)e^{-sx}dx$，其中 $s=j\omega$(虛數)，亦即 $F(s)=\zeta\left[f(x)\right]=\int_{-\infty}^{\infty} f(x)e^{-j\omega x}dx$，其中 F(ω)為複數，稱 *f(x)*的頻譜。反傅立葉轉換定義為 $f(x)=\zeta^{-1}\left[F(\omega)\right]=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{-j\omega x}d\omega$，分別以 ζ、ζ^{-1}表示傅立葉轉換與反傅立葉轉換。

```
>> syms x
>> y=fourier(cos(3*x))
y =
pi*Dirac(w-3)+pi*Dirac(w+3)
>> z=ifourier(y)
z =
1/2*exp(3*i*x)+1/2*exp(-3*i*x)
>> simplify(z)
ans =
cos(3*x)
```

二．地圖工具箱

三．虛擬實境工具箱