Modeling a Variable-Mass Rocket Trajectory

"It's Not Like it's Rocket Surgery"

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ABSTRACT

Rockets pose a particularly interesting dilemma as far as equations of motion are concerned due to their variable mass. In addition, the desire for particular orbital shapes is key for real-world missions, making control over the orbital path of a standard multi-stage rocket an incredibly relevant challenge. This project aims to model the Atlas LV-3B two-stage rocket, which achieved a simple circular orbit historically in the 1960s, and addresses some of the inherent difficulties that arise when modeling these variable mass systems and controlling orbital paths.

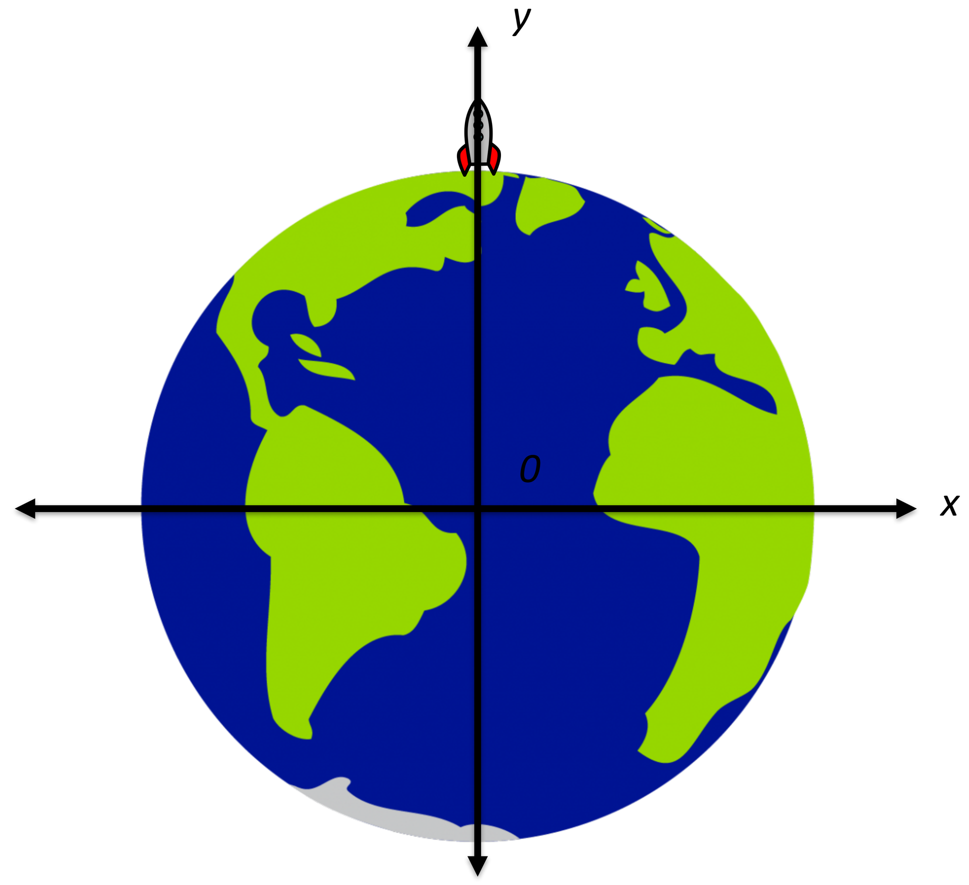


Fig. 1
Orientation of the axes system, as performed in the computation. Rectangular coordinates were used for simplicity of gridding. Figure is not to scale.

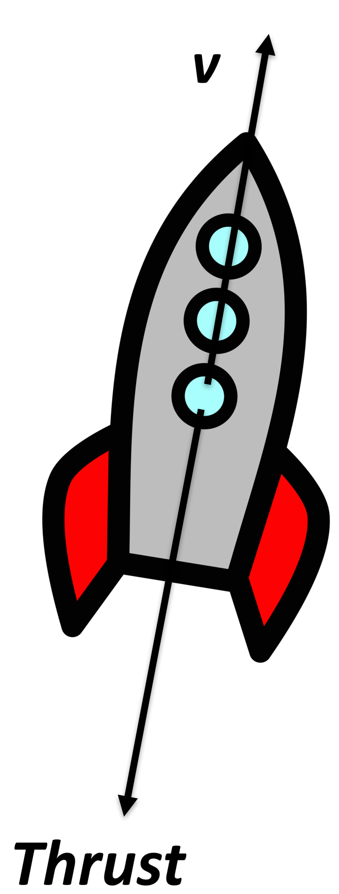
INTRODUCTION

In this project, we consider a two-stage rocket, modeled after the Atlas LV-3B rocket that flew the Mercury missions in the early 1960s. Because of the spherical symmetry in the gravitational potential function

$$U = -\frac{GM_1M_2}{r}$$

the problem is reduced to a 2D space. We seek the equations of motion for the rocket, which leads to the issue of variable mass as rocket fuel leaves the system as propellant. The burn rate for this fuel is assumed to be linear, which so happens to not be far from reality.

THRUST FORCE



The thrust from a rocket's engines is calculated from Newton's second law, where

$$F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

This can be manipulated such that we are left with a quantity called "specific impulse" of a rocket booster, in units of time, which relate to the force put out by the thruster by:

$$F = (Isp)\frac{dm}{dt}g_0$$

where g_0 represents standard gravity, 9.8 m/s^2. If we recall our assumption that the fuel burns linearly, this allows us to calculate our thrust force in the direction of the rocket velocity.

TRAJECTORIES

Once thrust parameters of the system are established, the equations of motion are numerically integrated using fourth-order Runge-Kutta integration. The given parameters for specific impulse of Atlas LV-3B rockets showed variation and little true liftoff, so ultimately some manual adjustments considering the engine efficiency were quickly tested. The results of some of this manual testing are shown in fig. 2 below.

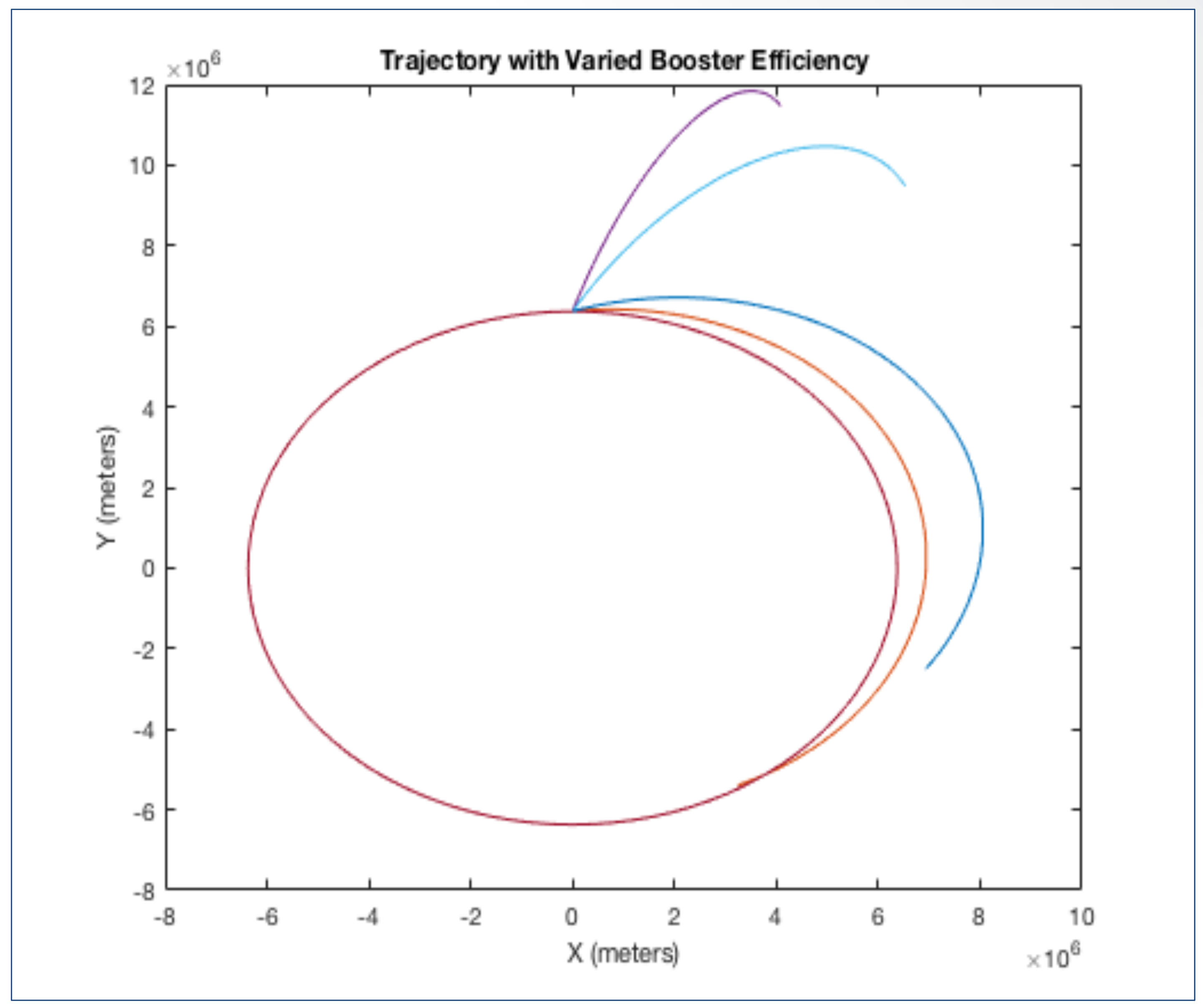


Fig. 2
Plots of several trajectories of the first and brief second stage of the Atlas LV-3B model with decreasing levels of ISP, specific impulse. Trajectories from top to right show decreasing values of the ISP varying from those provided from sources, and as these values converge on the value required, a more circular orbit appears.

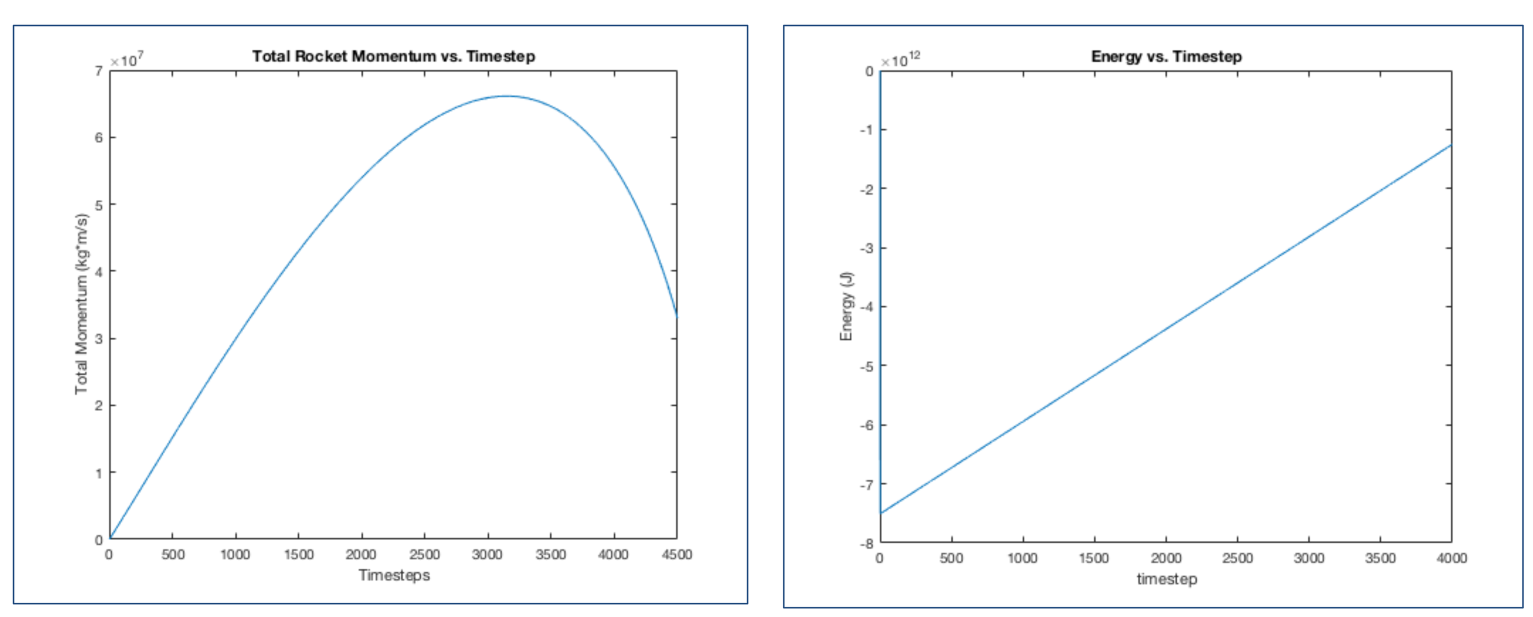


Fig. 3
Plots total momentum (left) and total energy (right) of the rocket over time. The system seems to show no conservation, however, this is somewhat explained by a linear loss of mass throughout the rocket launch, and potential issues in the relationship between mass and thrust.

ORBIT CONTROL

The primary criterion for a circular orbit is given by balancing the centripetal acceleration caused by the tangential velocity and the gravitational force, resulting in the equation:

$$v^2 = \frac{GM_e}{R}$$

where v^2 is the tangential velocity, R is the distance from the center of the earth, G is the gravitational constant, and M_e is the mass of the earth. Checking for this value was somewhat difficult, and this difficulty coupled with a lack of control over the thrust direction caused a large amount of difficulty in determining when to separate masses and shift to stage one, and when to cease acceleration. Several attempts resulted in plots similar to those shown in fig.2, but no circular orbits.

Ultimately, shifting to a polar coordinate space may cause a simple method for checking for circular orbit conditions. In addition, implementing controls over thrust vector orientation may result in more careful movement into circular orbit (and are not unrealistic).

ACCOMPLISHMENTS

This project accomplished several key goals:

- Utilize fourth-order Runge-Kutta to integrate Newton's second law and retrieve the equations of motion for a rocket.
- Modeled variable mass trajectory in a gravitational potential about earth.
- Properly introduced rocket thrust as a vector force to be numerically added in calculations.

While acknowledging these accomplishments, the project has several areas that require more work and investigation to make it successful:

- Proper control schemes and methods to guarantee a circular orbit shape.
- Implementation of control schemes to detach stage zero booster and shift to stage one rocket.
- Return from orbit calculations.

REFERENCES

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