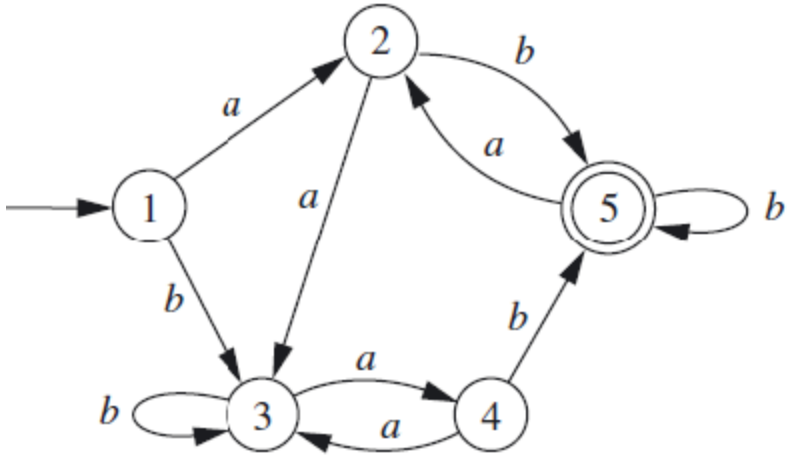
Az informatika számítástudományi alapjai gyakorlat

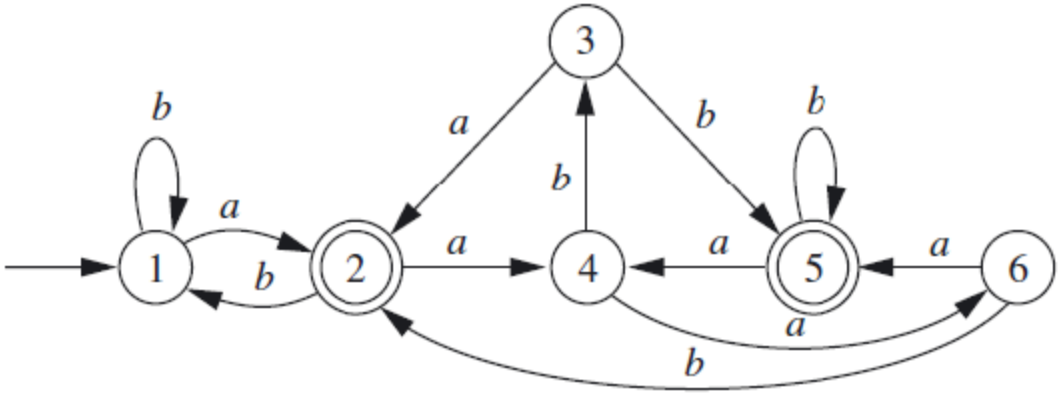
4. feladatsor

(beszéljük meg, hogy miként megy a minimalizálás algoritmusa, az előadáson nem szerepelt)

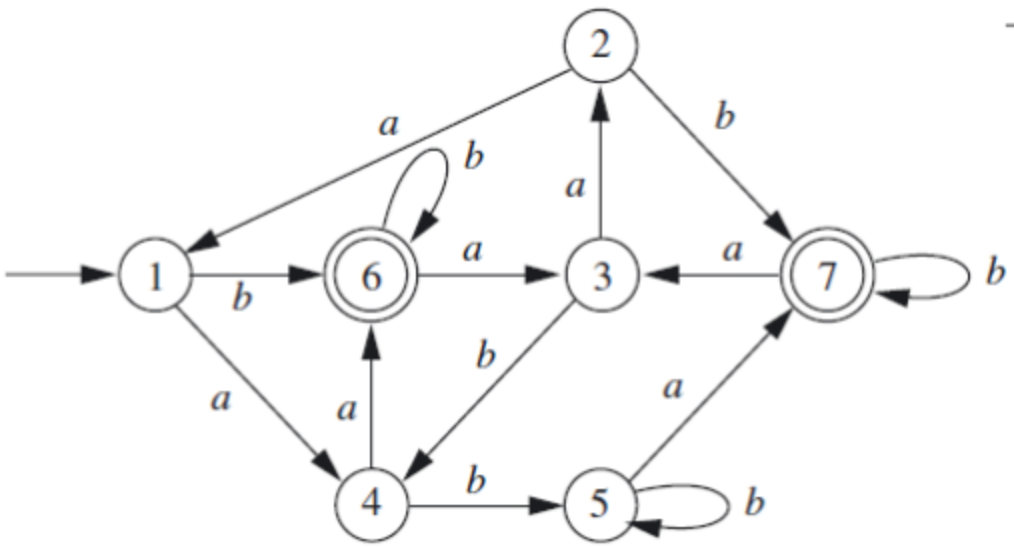
2.55. For each of the FAs pictured in Fig. 2.45, use the minimization algorithm described in Section 2.6 to find a minimum-state FA recognizing the same language. (It's possible that the given FA may already be minimal.)



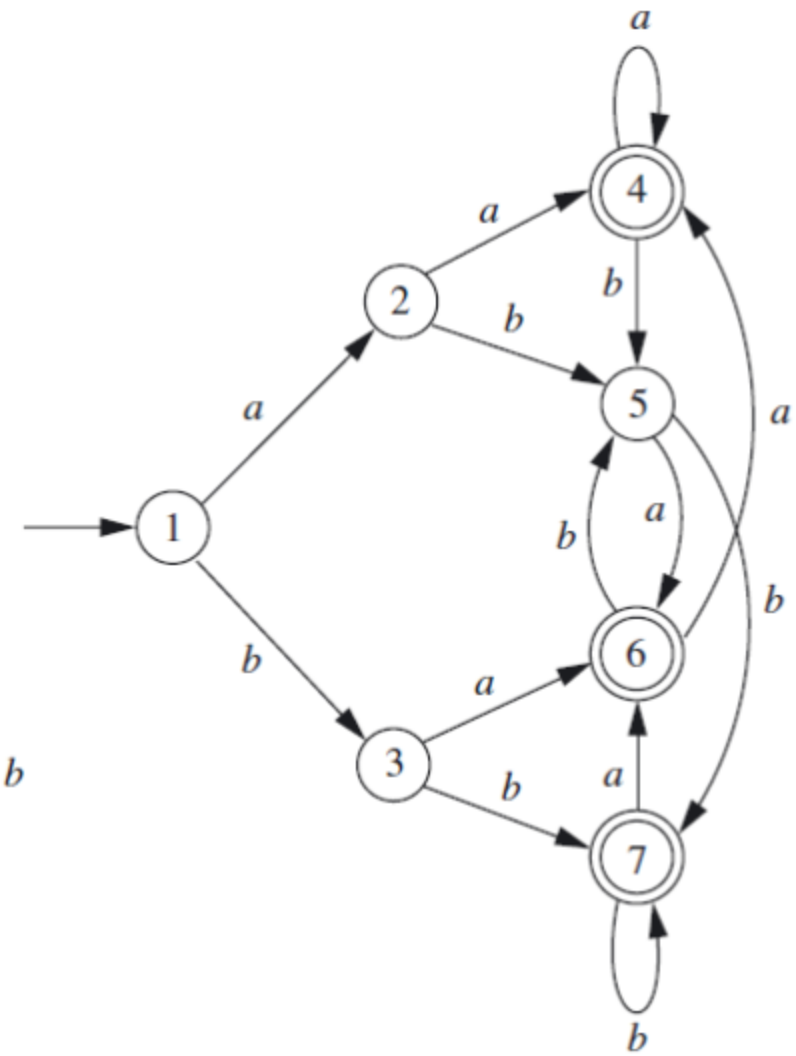
(*a*)



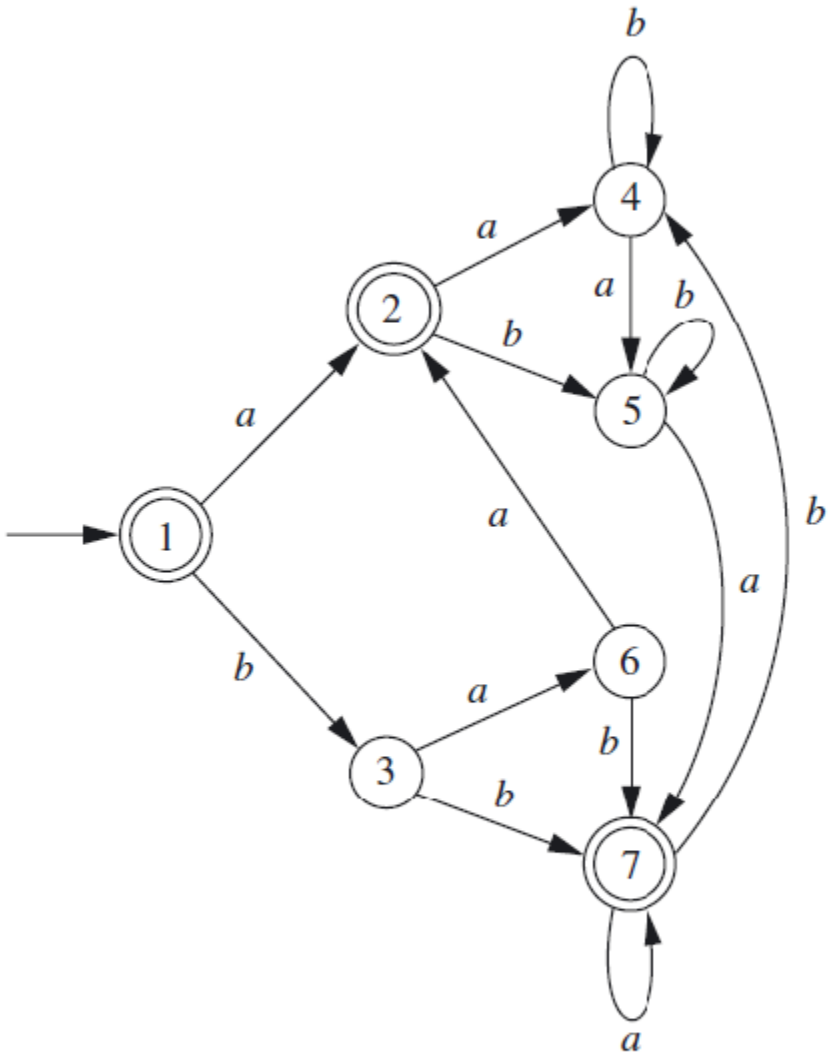
(*b*)



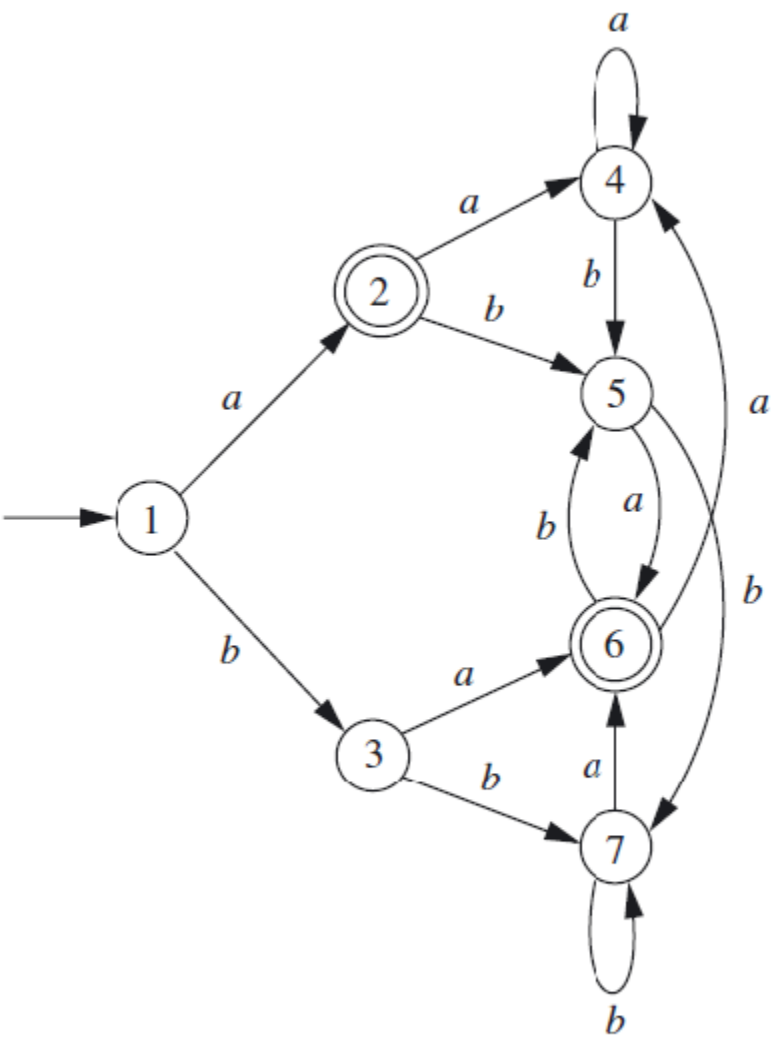
(c)



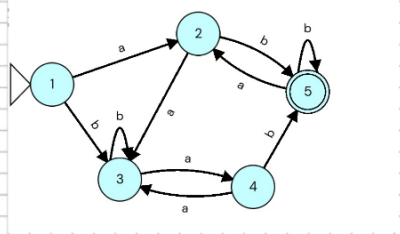
(d)



(e)



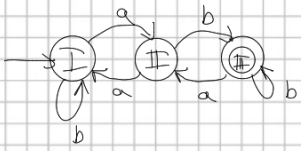
(f)



	1	2	3	4	5
	I				II
a	I	I	I	I	I
b	I	II	I	II	II

	1	3	2	4	5
	I		II		III
a	II	II	I	I	II
b	I	I	III	III	III

ha egyenértékű van a sorban nem kell tovább bontani



	1	3	4	6	2	5
	I				II	
a	II	II	I	II	I	I
b	II	II	I	II	II	II

külön kell a szétválasztani

	1	3	6	4	2	5
	I	II	III	IV	V	
a	IV	IV	V	IV	III	III
b	I	V	III	I	I	V

nem lehet minimalizálni

	3	4	5	6	1	2	7
	I				II		
a	II	II	II	II	II	I	II
b	II	I	I	II	I	I	I

	3	4	5	6	1	7	2
	I	II	III	IV	V		VI
a	IV	III	I	II	VI	V	I
b	V	I	II	V	I	IV	III

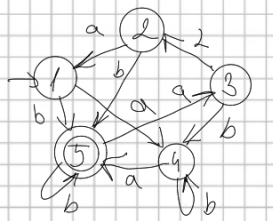
nem minimalizálható !!

c)

	1	2	3	4	5	6	7
	I					II	
a	I	I	I	II	II	I	I
b	II	II	I	I	I	II	II

	1	2	3	4	5	6	7
	I	II	III			IV	
a	III	I	I	IV	IV	I	I
b	IV	IV	III	III	III	IV	III

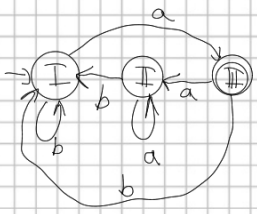
	1	2	3	4	5	6	7
	I	II	III	IV		V	
a	IV	I	II	V	V	II	II
b	V	V	IV	IV	IV	V	V



f)

	1	3	4	5	7	2	6
	I					II	
a	I	II	I	II	I	I	I
b	I	I	I	I	I	I	I

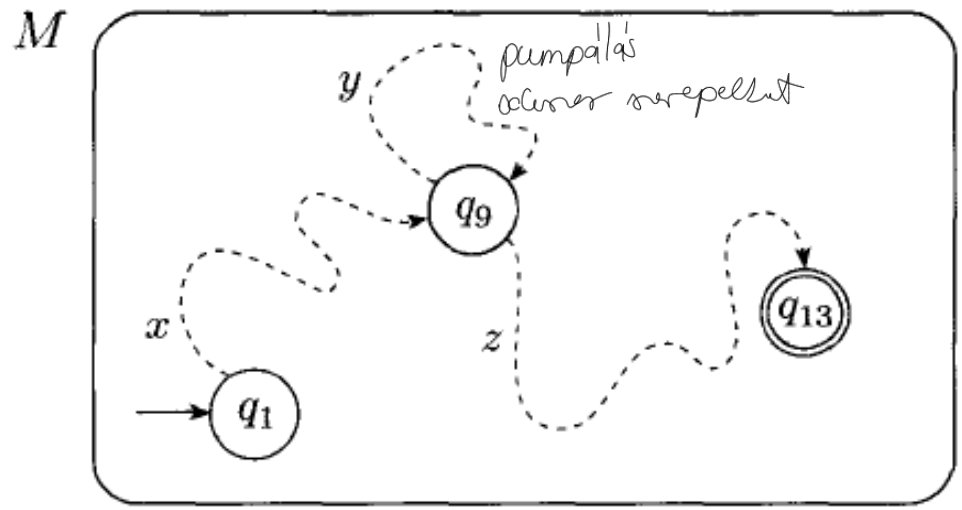
	1	3	5	7	4	2	6
		I			I		II
a	III	III	III	III	I	I	IV
b	I	I	I	I	I	I	I



Pumpálási lemma

Ha egy véges automata által elfogadott nő elégtelen, akkor az automata hány lehet legalább egy állapotot kétszer is felvenni.

$$s = \underset{q_1}{s_1}\ \underset{q_3}{s_2}\ \underset{q_{20}}{s_3}\ \underset{q_9}{s_4}\ \underset{q_{17}}{s_5}\ \underset{q_9}{s_6}\ \underset{q_6}{\cdots}\ \underset{q_{35}}{s_n}\ \underset{q_1}{}$$



Azaz:

Ha $L \subseteq \Sigma^*$ nyelvet elfogadja $M = (Q, \Sigma, q_0, A, \delta)$ véges automata és $n = |Q|$, akkor minden olyan $x \in L$ L-beli szó, amelyre $|x| \geq n$, felírható

állapothalmaz

↳ az állapothalmaz számossága

$$x = \overset{x}{u}\,\overset{y}{v}\,\overset{z}{w}$$ → részszavak

alakban, ahol:

1. $|uv| \leq n$
2. $|v| > 0$ (azaz $v \neq \lambda$)
3. Minden $i \geq 0$-ra, $uv^iw \in L$

$$\underbrace{aa\ldots bb\ldots}_{u}\underbrace{c\ldots aa\ldots}_{v}\underbrace{b}_{w}$$

1. $|uv| \leq n$
   ha $n=5$, akkor max(5)
2. $|v|>0$ → megjelenhet mégegyszer pl. uv^2w

2.22. For each of the languages in Exercise 2.21, use the pumping lemma to show that it cannot be accepted by an FA.

a. $L = \{a^n b a^{2n} \mid n \geq 0\}$

b. $L = \{a^i b^j a^k \mid k > i + j\}$

c. $L = \{a^i b^j \mid j = i \text{ or } j = 2i\}$

d. $L = \{a^i b^j \mid j \text{ is a multiple of } i\}$

e. $L = \{x \in \{a, b\}^* \mid n_a(x) < 2n_b(x)\}$

f. $L = \{x \in \{a, b\}^* \mid \text{no prefix of } x \text{ has more b's than a's}\}$

g. $L = \{a^{n^3} \mid n \geq 1\}$

h. $L = \{ww \mid w \in \{a, b\}^*\}$

$S = 00\ldots0\,11\ldots1 = 0^p1^p$

$S = xyz$ $\qquad xy^iz \in L$

I. $y = 0\ldots0$ → nem jó mivel egy adott szám több 0-ra van mint 1

$y = 1\ldots1$

$y = 00\ldots11$

↳ ellentétes az szimmetriával $0^2y = 0^2y$

$x = 000$
$y = 11$
$z = 1$

xy^2z : 000 11 1

a) $L = \{a^n b a^{2n} \mid n \geq 0\}$

$S = a\ldots ab\,aaaa\ldots a$

$= a^p b a^{2p}$

I. $|uv| \leq 1$

II. $|v| > 0$

III. $uvw \in L$

① $\overbrace{a^p\,b\,a^{2p}}^{uvw}$

↳ A v nem tartalmazhat b-t, mert abból nem lehet 1-el több

② $\underbrace{a^p}_{u}\,\underbrace{a^n}_{v}\,\underbrace{b\,a^{2p}}_{w}$

↳ nem a 2x-ese lesz az utolsó a sorozat az első részben

b) $L = \{a^i b^j a^k \mid k \geq i+j\}$

I. $|uv| \leq 1$

II. $|v| > 0$

III. $uvw \in L$

$i = 2$
$j = 3$
$k = p$

$aa\underbrace{bbb}_{v}aaaaaa$

① nem kell jó

② $ab = v$ ×

③ a^i ×

④ a^k ✓

c) $L = \{a^i b^j \mid j = i \text{ or } j = 2i\}$

↓ not good

aaa bbbbbb ×

I $v = a\ldots a$

$v = aaabb\ldots$ → rossz sorrend

$v = b\ldots b$

2.29. For each statement below, decide whether it is true or false. If it is true, prove it. If it is not true, give a counterexample. All parts refer to languages over the alphabet $\{a, b\}$.

a. If $L_1 \subseteq L_2$, and L_1 cannot be accepted by an FA, then L_2 cannot.

b. If $L_1 \subseteq L_2$, and L_2 cannot be accepted by an FA, then L_1 cannot.

c. If neither L_1 nor L_2 can be accepted by an FA, then $L_1 \cup L_2$ cannot.

e. If L cannot be accepted by an FA, then L' cannot.

(L' az L komplementerét, vagyis az {a,b}*-L nyelvet jelenti.)

a) $0^n 1^n$ → nem működik a pumpálási lemma → nem automatás

$L_1 = \{a^n b^n \mid n \geq 0\} \leftarrow \times$

hamis

$L_1 \subseteq L_2$ ✓

$L_2 = (a+b)^* \leftarrow$ ✓

e) igaz

az állapotainak száma utáni is tetszőleges sok

b) hamis

$L_1 = \{ab\} \rightarrow$ ✓

$L_2 = \{a^n b^n \mid n \geq 1\} \rightarrow$ nincs automata