Igaz-Hamis

1. $\int (3x+4)e^x\,dx = (3x+1)e^x + C$

$\underset{f(x)}{(3x+4)}\ \underset{g'(x)}{e^x}$

$f(x) = 3x+4 \qquad f'(x) = 3$

$g'(x) = e^x \qquad g(x) = e^x$

$f(x)\cdot g'(x) = f(x)\cdot g(x) - \int f'(x)\cdot g(x)\,dx + C$

$(3x+4)\cdot e^x - \int 3e^x\,dx + C = (3x+4)e^x - 3e^x + C =$

$= 3xe^x + 4e^x - 3e^x + C = 3xe^x + e^x + C = (3x+1)e^x + C$

2. $\int \frac{1}{(2x-4)^6}\,dx = \frac{\frac{(2x-4)^{-5}}{-5}}{2} + C$

$\int 1\cdot(2x-4)^{-6}\,dx =$

$\text{„}f'(x)\text{"} \quad f^{\alpha}(x)$

$f(x) = 2x-4 \qquad f'(x) = 2$

$f'(x) \to \text{„}f'(x)\text{"}$, ha $\frac{f'(x)}{2}$

$\int \frac{f'(x)}{f^{\alpha}(x)}\,dx = \frac{f^{\alpha+1}(x)}{\alpha+1}$

$\alpha = -6$

$\frac{1}{2}\cdot\frac{(2x-4)^{-5}}{-5} + C = \frac{\frac{(2x-4)^{-5}}{-5}}{2}$

3. $\int \frac{1}{(2x-4)^6}\,dx = -\frac{1}{10(2x-4)^5} + C$

$\int 1\cdot(2x-4)^{-6}\,dx =$

$\text{„}f'(x)\text{"} \quad f^{\alpha}(x)$

$f(x) = 2x-4 \qquad f'(x) = 2$

$f'(x) \to \text{„}f'(x)\text{"}$, ha $\frac{f'(x)}{2}$

$\int \frac{f'(x)}{f^{\alpha}(x)}\,dx = \frac{f^{\alpha+1}(x)}{\alpha+1}$

$\alpha = -6$

$\frac{1}{2}\cdot\frac{(2x-4)^{-5}}{-5} + C = \frac{\frac{(2x-4)^{-5}}{-5}}{2} =$

$= \frac{(2x-4)^{-5}}{-5}\cdot\frac{1}{2} = \frac{1\cdot(2x-4)^{-5}}{-10} = -\frac{1}{10(2x-4)^5}$

$\frac{1}{(2x-4)^5}$

4. $\int \frac{1}{\sqrt{1-x^2}}\,dx = -\arccos(x) + C$

alapintegrál!

5. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$

alapintegrál!

6. $\int \frac{1}{\cos^2(x)}\,dx = \mathrm{tg}(x) + C$

alapintegrál!

7. $\int \frac{1}{\sin^2(x)}\,dx = -\mathrm{ctg}(x) + C$

$\int \frac{dx}{\sin^2(x)} = -\mathrm{ctg}(x) + C$ alapintegrál!

$\int \frac{1}{\sin^2(x)}\cdot dx = -\mathrm{ctg}(x) + C$

8. $\int \frac{1}{\sinh^2(x)}\,dx = -\coth(x) + C$

$\int \frac{dx}{\sinh^2(x)} = -\coth(x) + C$ alapintegrál!

$\int \frac{1}{\sinh^2(x)}\cdot dx = -\coth(x) + C$

9. $\int \frac{1}{1+x^2}\,dx = -\mathrm{arcctg}(x) + C$ alapintegrál!

10. $\int \frac{1}{1+x^2}\,dx = \mathrm{arctg}(x) + C$ alapintegrál!

11. $\int (2x+2)e^x\,dx = 2xe^x + C$

$\underset{f(x)}{(2x+2)}\ \underset{g'(x)}{e^x}$

$f(x) = 2x+2 \qquad f'(x) = 2$

$g'(x) = e^x \qquad g(x) = e^x$

$f(x)\cdot g(x) - \int f'(x)\cdot g(x) = (2x+2)e^x - \int 2e^x\,dx =$

$= 2xe^x + 2e^x - 2e^x + C = 2xe^x + C$

12. $\int \frac{4}{3x-5}\,dx = \frac{4}{3}\ln(|3x-5|) + C$

$\int 4\cdot(3x-5)^{-1}$

$\text{„}f'(x)\text{"} \quad f^{\alpha}(x)$

$f(x) = 3x-5 \qquad f'(x) = 3$

$\alpha = -1 \qquad f'(x) \to \text{„}f'(x)\text{"}$, úgy $4\cdot\frac{1}{3} = \frac{4}{3}$

$\int \frac{f'(x)}{f^{\alpha}(x)}\,dx = \ln(|f(x)|)$

$\int \frac{4}{3x-5}\,dx = \frac{4}{3}\ln(|3x-5|) + C$

13. $\int \frac{7}{3x+4}\,dx = \frac{7}{3}\ln(|3x+4|) + C$

$\int 7\cdot(3x+4)^{-1}$

$\text{„}f'(x)\text{"} \quad f^{\alpha}(x)$

$f(x) = 3x+4 \qquad f'(x) = 3$

$\alpha = -1 \qquad f'(x) \to \text{„}f'(x)\text{"}$, úgy $7\cdot\frac{1}{3} = \frac{7}{3}$

$\int \frac{f'(x)}{f^{\alpha}(x)}\,dx = \ln(|f(x)|)$

$\int \frac{7}{3x+4}\,dx = \frac{7}{3}\ln(|3x+4|) + C$

14. $\int \cos(x)\,dx = \sin(x) + C$

alapintegrál!

15. $\int \cosh(x)\,dx = \sinh(x)+C$

alapintegrál!

16. $-\int \cosh(x)\,dx = -\sinh(x)+C$

$\int \cosh(x)\,dx = \sinh(x)+C \quad /\cdot -1$

$-\int \cosh(x)\,dx = -\sinh(x)+C$

17. $\int \ln(x)\,dx = x\ln(x) - x + C$

alapintegrál!

18. $\int \sin(x)\,dx = -\cos(x)+C$

alapintegrál!

19. $\int \sinh(x)\,dx = \cosh(x)+C$

alapintegrál!

20. $-\int \sinh(x)\,dx = -\cosh(x)+C$

$\int \sinh(x)\,dx = \cosh(x)+C \quad /\cdot -1$

$-\int \sinh(x)\,dx = -\cosh(x)+C$

21. $\int \frac{dx}{\cosh^2(x)} = \tanh(x)+C$

alapintegrál!

22. $\int \frac{dx}{\sqrt{x^2+1}} = \operatorname{arcsinh}(x)+C$

alapintegrál!

23. $\int e^x\,dx = e^x + C$

alapintegrál!

24. $\int e^x\,dx = e^x - C$

mindegy, hogy $-/+$ mert $C'=0$

25. $\int \sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}\,dx = \frac{2}{3}x^{\frac{3}{2}} + \frac{3}{5}x^{\frac{4}{3}} + \frac{4}{5}x^{\frac{5}{4}} + C$

$\int x^{\frac{1}{2}} + x^{\frac{1}{3}} + x^{\frac{1}{4}}\,dx = \frac{2}{3}x^{\frac{3}{2}} + \frac{3}{4}x^{\frac{4}{3}} + \frac{4}{5x}x^{\frac{5}{4}} + C$

$\hookrightarrow \frac{x^{\alpha+1}}{\alpha+1}$

26. $\int \frac{1}{x^2} + \frac{1}{x^4}\,dx = -\frac{1}{x} - \frac{1}{3x^3} + C$

$\int x^{-2} + x^{-4}\,dx = \frac{x^{-1}}{-1} + \frac{x^{-3}}{-3} = -\frac{1}{x} - 3\cdot\frac{1}{x^3} = -\frac{1}{x} - \frac{1}{3x^3} + C$

$\int c\,dx = c\cdot x$

27. $\int 2x + 5\,dx = x^2 + 5x + C$

$\int x\,dx = x^{\alpha+1}$

28. $\int 3x^2 + 2x + 5\,dx = x^3 + x^2 + 5x + C$

$\cancel{3}\cdot\frac{x^3}{\cancel{3}} + \cancel{2}\cdot\frac{x^2}{\cancel{2}} + 5\cdot x + C = x^3 + x^2 + 5x + C$

29. $\int -3x^2 - 2x - 5\,dx = -(x^3 + x^2 + 5x) + C$

$-\cancel{3}\cdot\frac{x^3}{\cancel{3}} - \cancel{2}\cdot\frac{x^2}{\cancel{2}} - 5\cdot x + C = x^3 - x^2 - 5x + C =$

$= -(x^3 + x^2 + 5x) + C$

30. $\int 5x^4 - 4x^3 + 3x^2\,dx = x^5 - x^4 + x^3 + C$

$\cancel{5}\cdot\frac{x^5}{\cancel{5}} - \cancel{4}\cdot\frac{x^4}{\cancel{4}} + \cancel{3}\cdot\frac{x^3}{\cancel{3}} + C = x^5 - x^4 + x^3 + C$

31. $\int 7x^9 - 3x^8 + 5x^6 + 3x - 9\,dx = \frac{7x^{10}}{10} - \frac{x^9}{3} + \frac{5x^7}{7} + \frac{3x^2}{2} - 9x + C$

$7\cdot\frac{x^{10}}{10} - \cancel{3}\cdot\frac{x^9}{\cancel{9}_3} + 5\cdot\frac{x^7}{7} + 3\cdot\frac{x^2}{2} - 9x + C =$

$= \frac{7x^{10}}{10} - \frac{x^9}{3} + \frac{5x^7}{7} + \frac{3x^2}{2} - 9x + C$

32. $\int x\cdot\sin(x)\,dx = \sin(x) - x\cos(x) + C$

$f(x) \quad g'(x)$

$f(x) = x \qquad f'(x) = 1$

$g'(x) = \sin(x) \qquad g(x) = -\cos(x)$

$f(x)\cdot g(x) - \int f'(x)\cdot g(x)\,dx = x\cdot -\cos(x) - \int 1\cdot -\cos(x)\,dx =$

$= -x\cos(x) - (-\sin(x)) + C = -x\cos(x) + \sin(x) + C$

33. $\int x\sqrt{x}\,dx = \frac{2x^{\frac{5}{2}}}{5} + C$

$\int \underbrace{x\cdot x^{\frac{1}{2}}}_{x\cdot x^{\frac{1}{2}} = x^{\frac{3}{2}}}\,dx = \int x^{\frac{3}{2}}\,dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2x^{\frac{5}{2}}}{5} + C$

34. $\int x\cdot e^x\,dx = (x-1)\cdot e^x + C$

$f(x) \quad g'(x)$

$f(x) = x \qquad f'(x) = 1$

$g'(x) = e^x \qquad g(x) = e^x$

$f(x)\cdot g(x) - \int f'(x)\cdot g(x) = x\cdot e^x - \int 1\cdot e^x =$

$= xe^x - e^x + C = (x-1)e^x + C$