

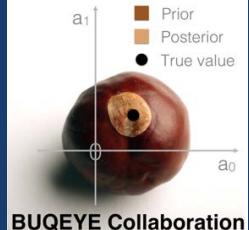
Equation of State of Neutron-Rich Matter and Implications for Neutron Stars

OHIO
UNIVERSITY

Christian Drischler

14th Conference on the Intersections of Particle and Nuclear Physics 2022

September 1, 2022



Facility for Rare Isotope Beams
at Michigan State University

Samuel L. Stanley
President of MSU

Jennifer M. Granholm
Secretary of Energy

Recent neutron star observations

What is the maximum neutron star mass?

GW170817
GRB170817A
AT2017gfo

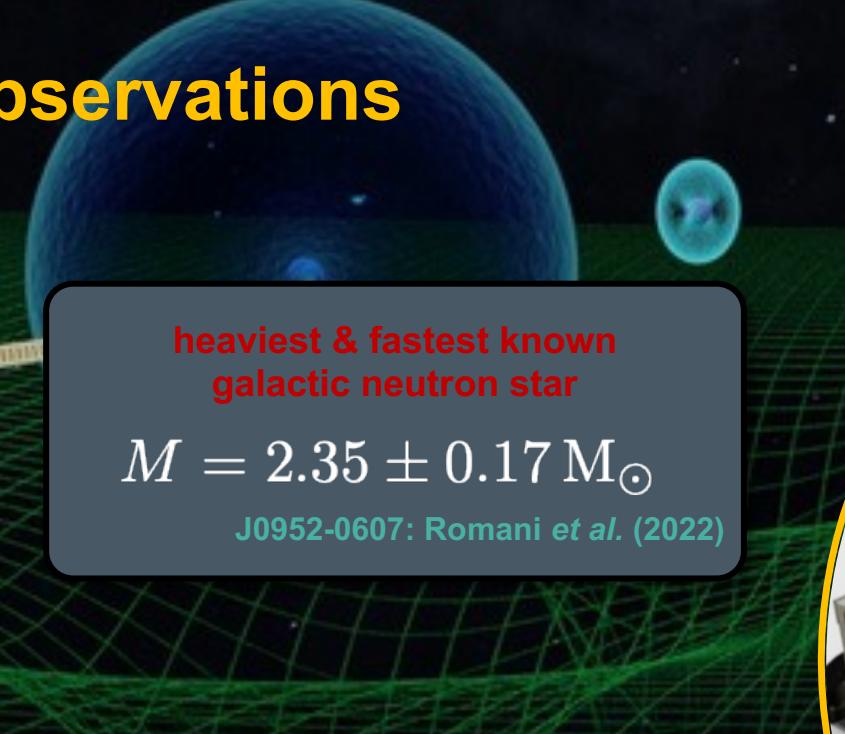


+ Virgo (Italy)
+ (KAGRA)

heaviest & fastest known
galactic neutron star

$$M = 2.35 \pm 0.17 M_{\odot}$$

J0952-0607: Romani et al. (2022)



multi-messenger
astronomy

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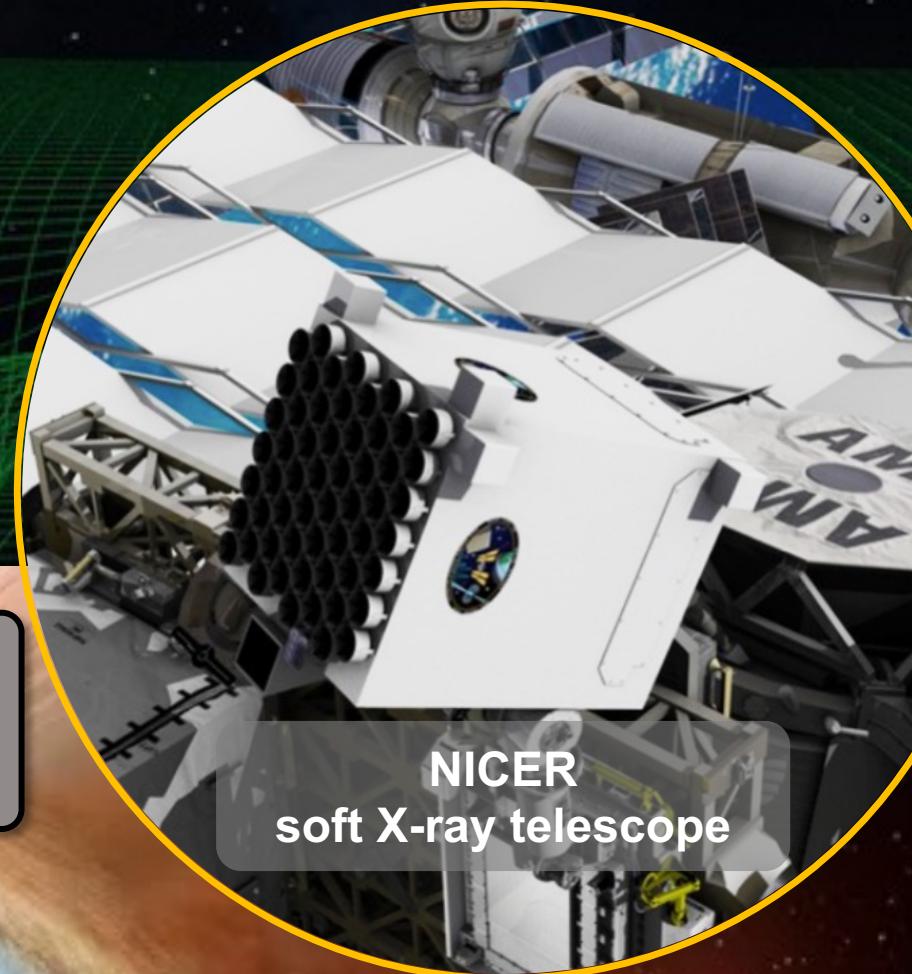
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J0952-0607: Romani et al. (2022)

See also Ben Margalit's talk (this session):
Mergers, kilonovae, and two-solar-mass neutron stars: What does astrophysics tell us about the nuclear EOS?

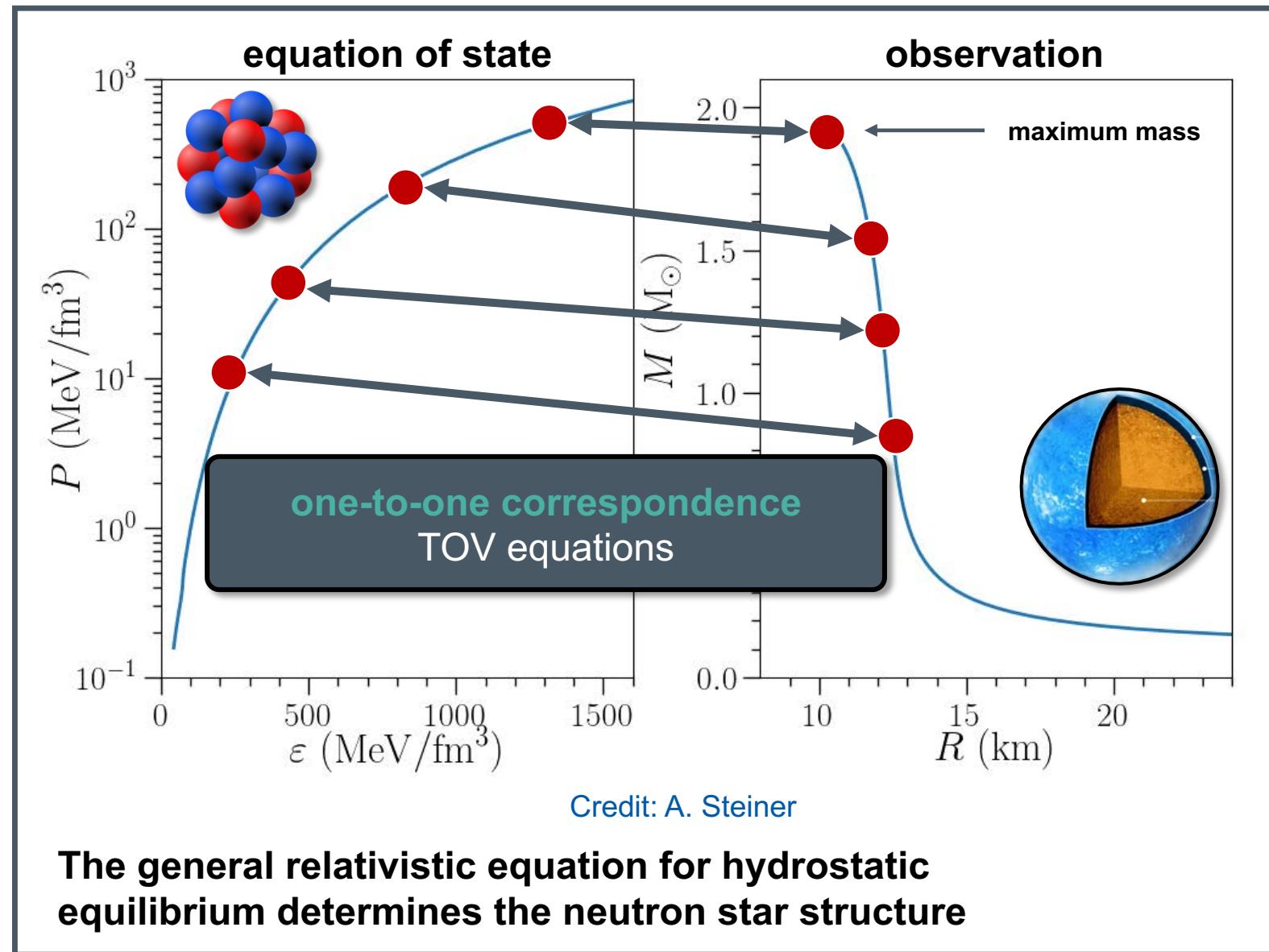
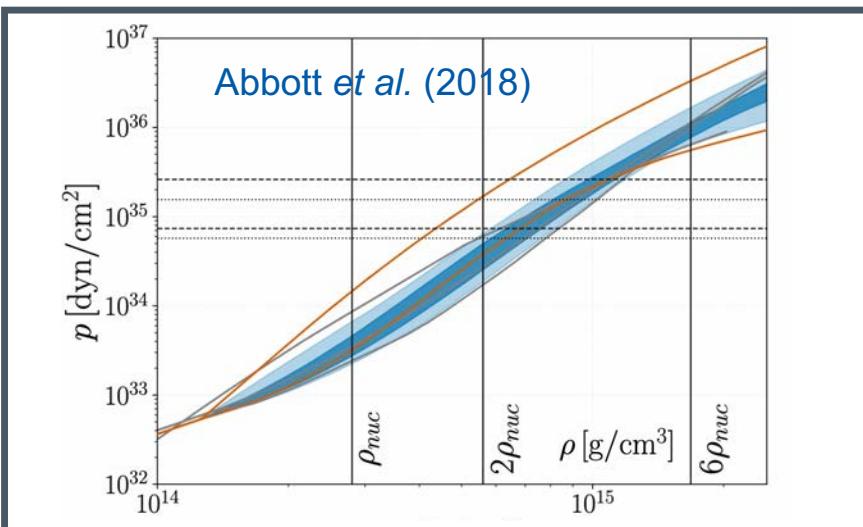
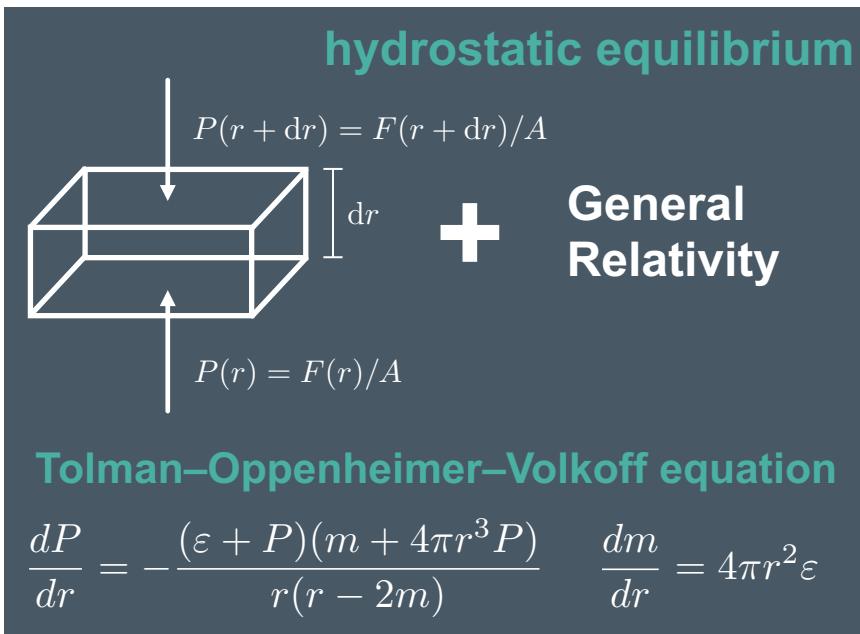


See also K. Chatzilioannou's talk (Saturday):
GWs from binary neutron stars and neutron star black-hole mergers

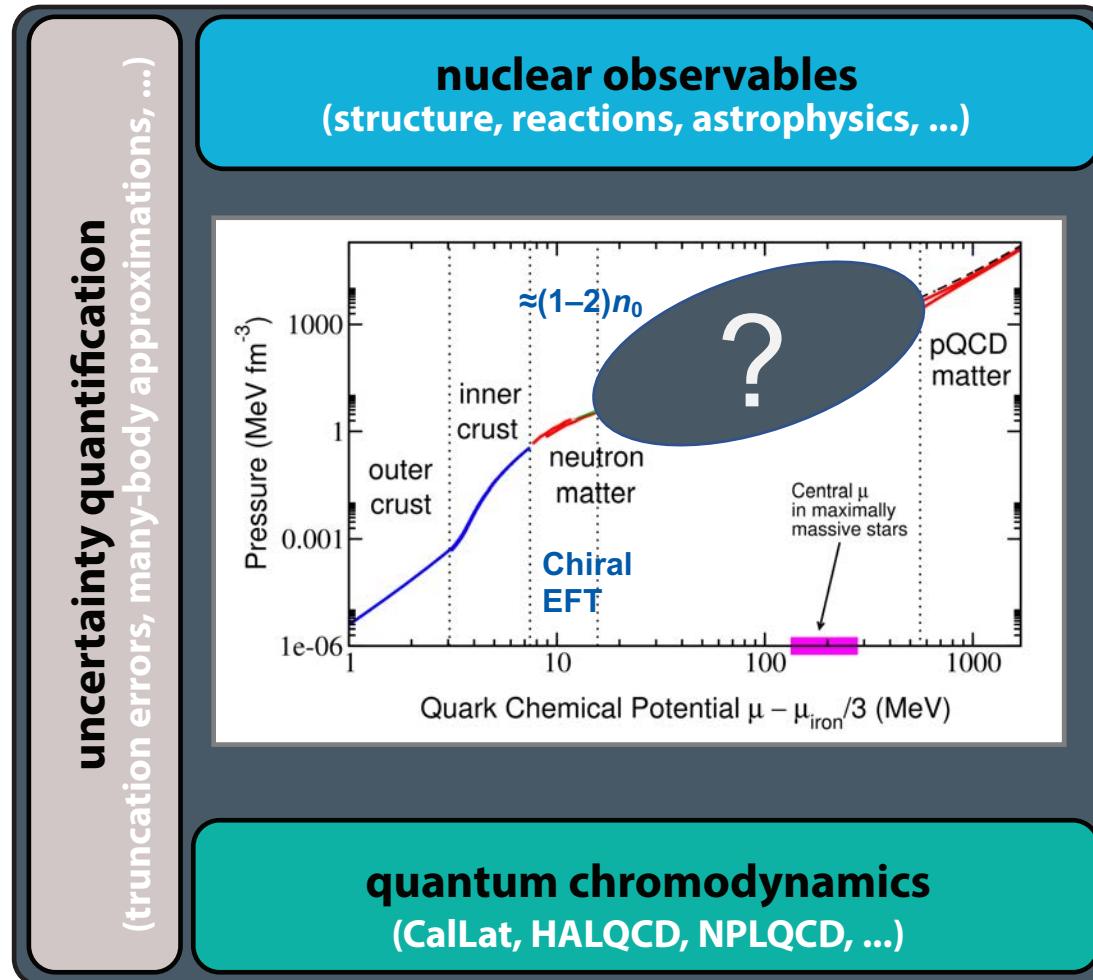


multi-messenger astronomy

Structure of cold neutron stars



Ab initio workflow (idealized)

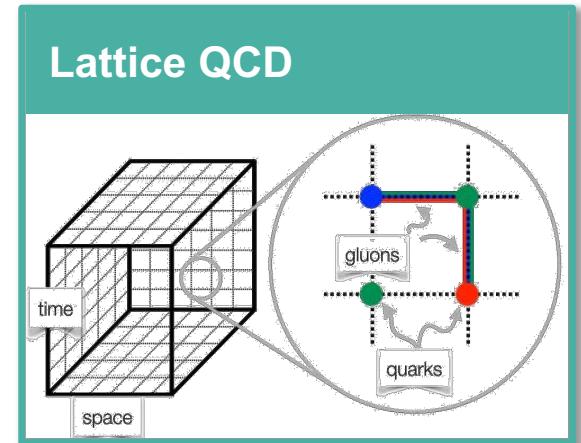
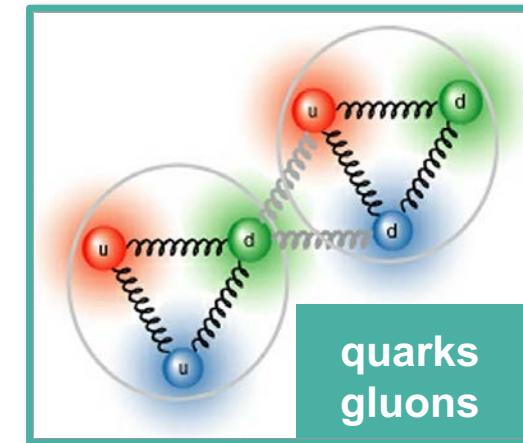


CD & Bogner, Few Body Syst. **62**, 109

Here: nuclear equation of state (EOS)
energy per particle (and derived quantities)

$$\frac{E}{A}(n, \delta, T)$$

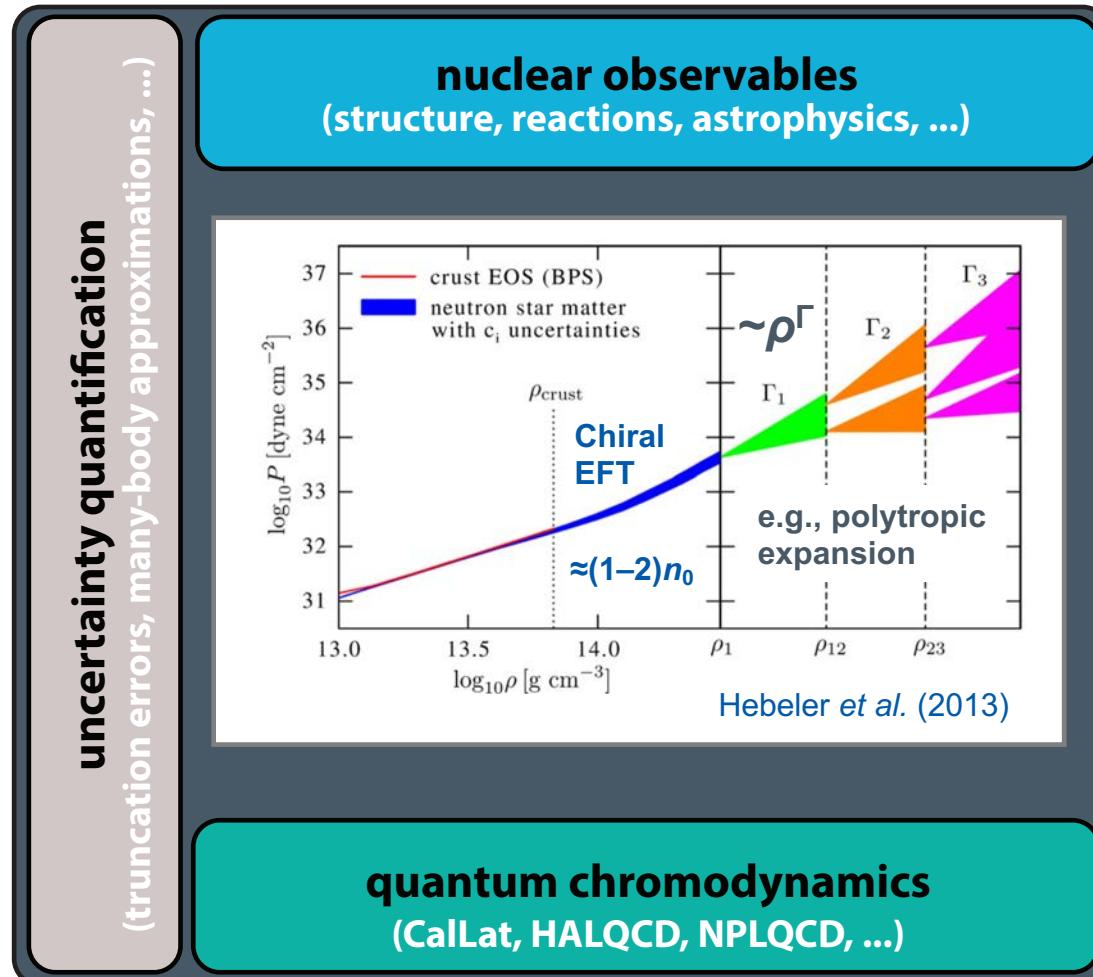
baryon density n
neutron excess δ
temperature $T (= 0)$



theory of strong interactions
QCD is nonperturbative at the low energies
relevant for nuclear physics (cf. pQCD & LQCD)

CD, Haxton, McElvain, Mereghetti *et al.*, PPNP **121**, 103888

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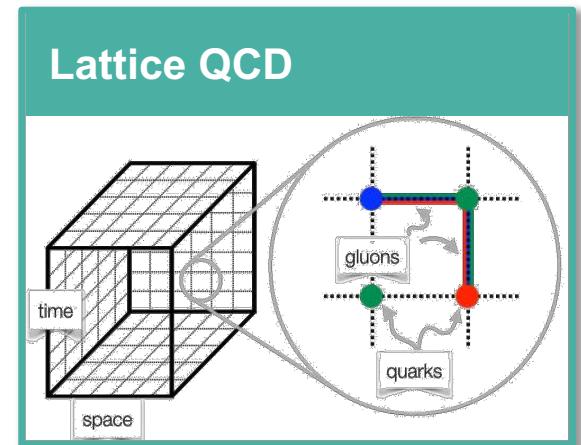
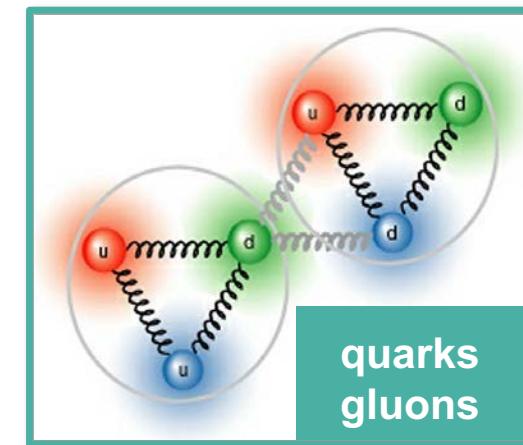


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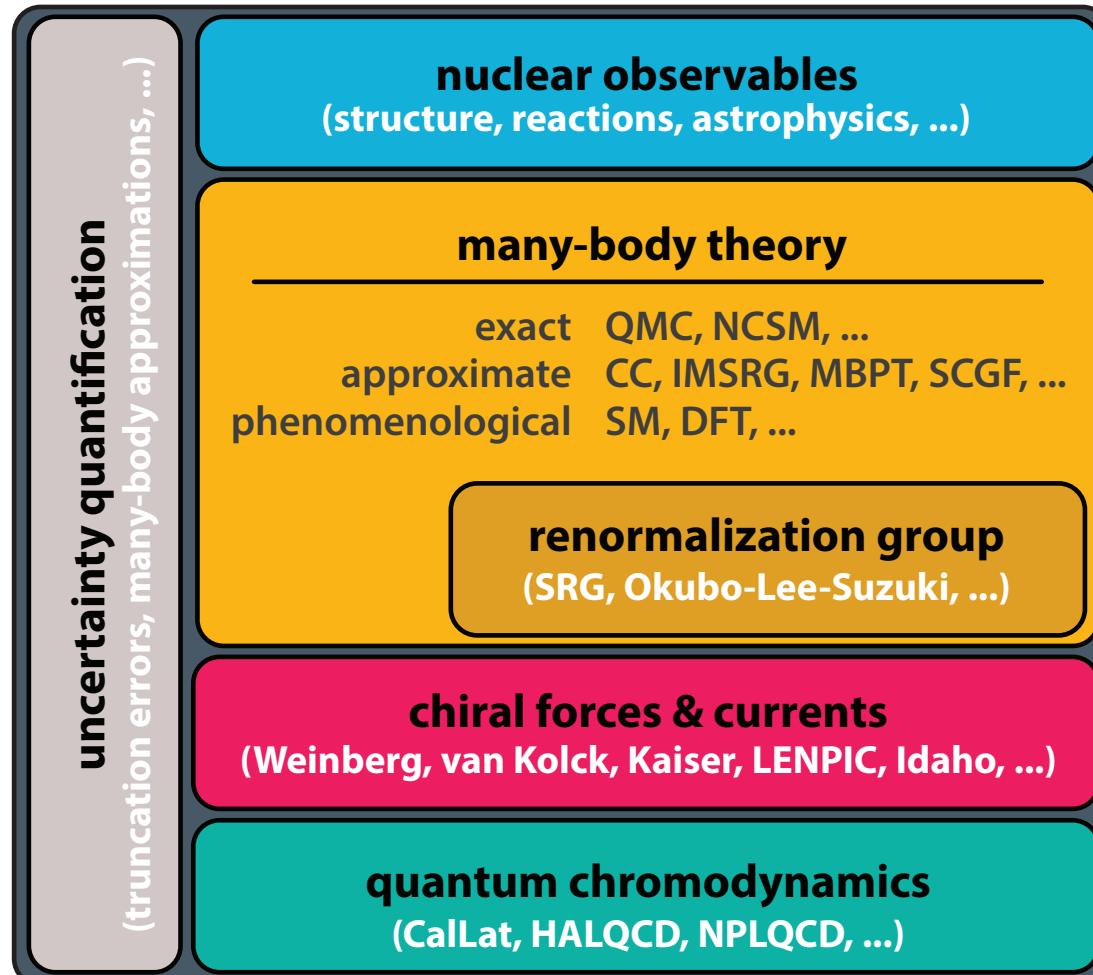
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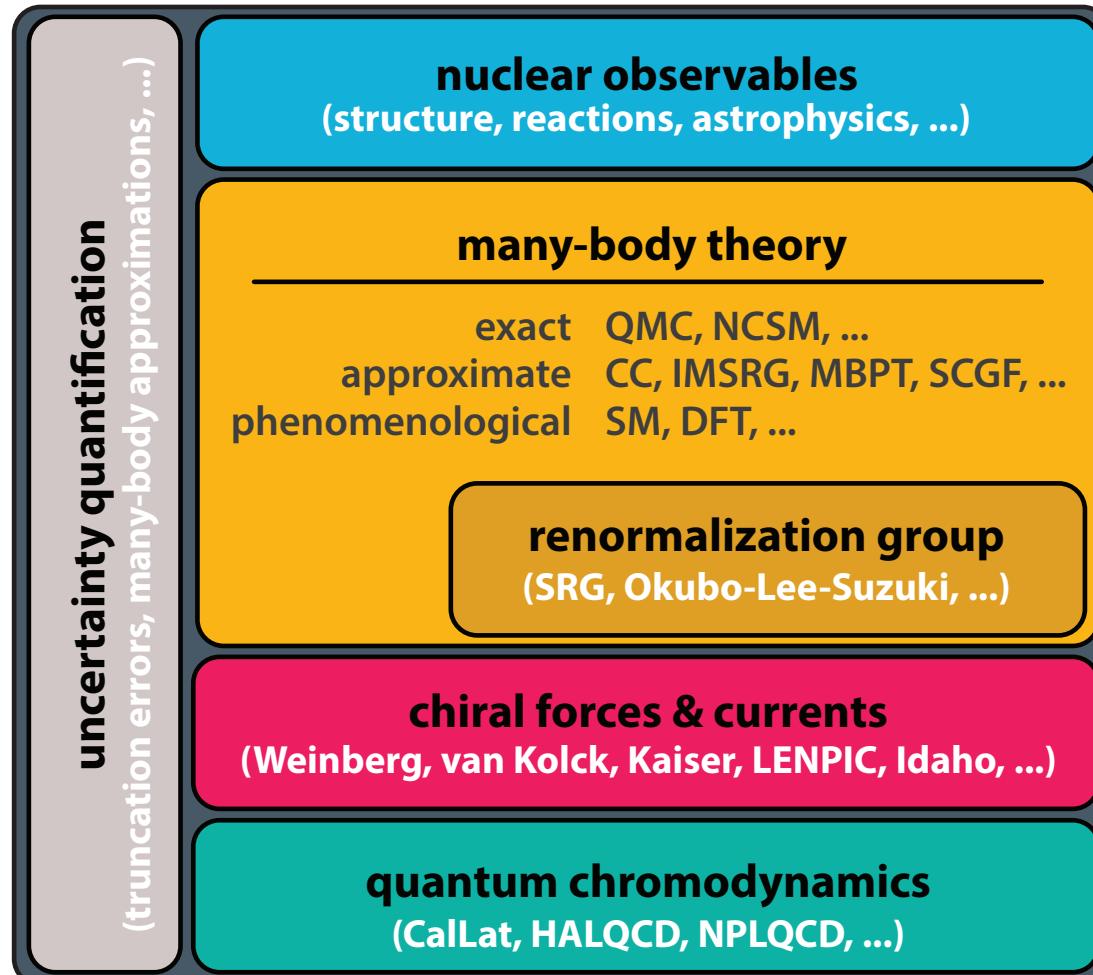
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computational framework
solves the (many-body) Schrödinger equation
requires a nuclear potential as input

chiral effective field theory
provides microscopic interactions consistent with
the symmetries of *low-energy* QCD

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QCD is nonperturbative at the low energies
relevant for nuclear physics (cf. pQCD & LQCD)

Ab initio workflow (idealized)



Recent highlight: *Ab initio* predictions link the neutron skin of ^{208}Pb to nuclear forces by Hu, Jiang *et al.*, arXiv:2112.01125

Here: nuclear equation of state (EOS)
energy per particle (and derived quantities)

See also Heiko Hergert's slides:
*A Status Update on
Ab Initio Calculations in Nuclear Physics*

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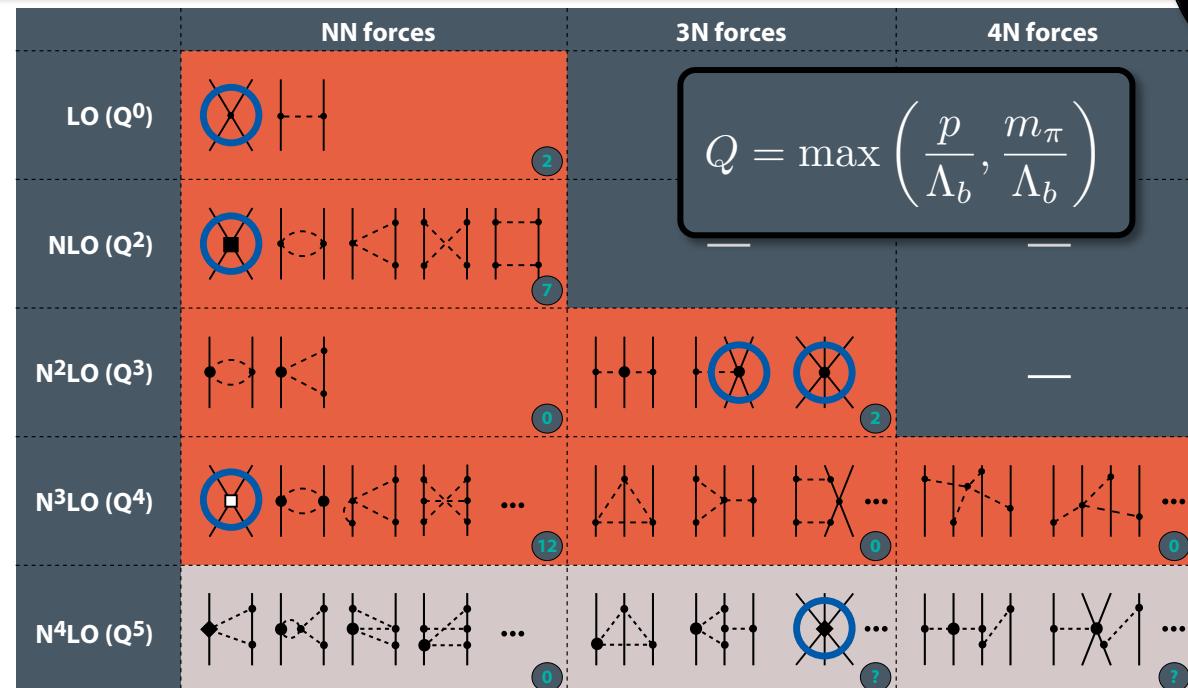
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Rigorous UQ for nuclear matter



CD, Furnstahl, Melendez,
Phillips, PRL 125, 202702

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Chiral Effective Field Theory (nucleons & pions)

dominant approach for deriving *microscopic* interactions
consistent with the symmetries of *low-energy* QCD

three- and four-neutron forces predicted through N^3LO

See also Evgeny Epelbaum's slides (LENPIC):
Chiral EFT for low-energy nuclear physics

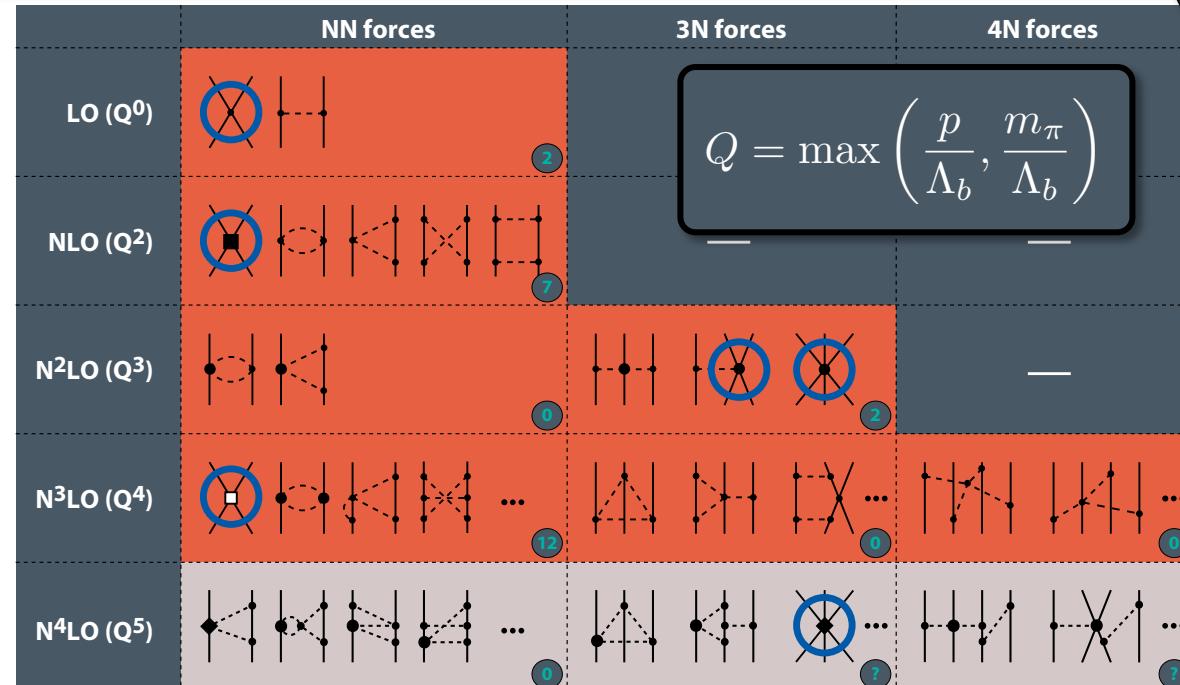
See also Maria Piarulli's slides:
*Analyzing the nuclear interaction: challenges
and new opportunities*

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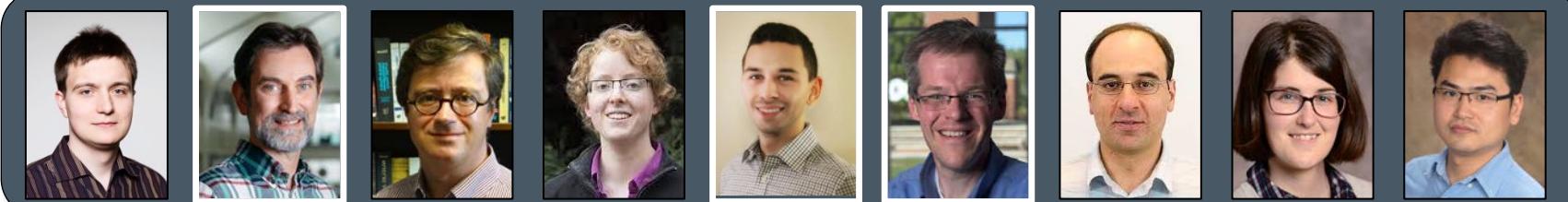
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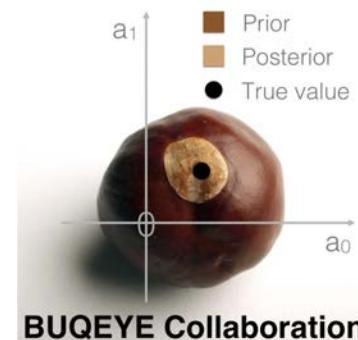
three- and four-neutron forces predicted through N^3LO

enables **uncertainty quantification** (EFT truncation)

Bayesian methods are powerful tools for quantifying &
propagating EFT uncertainties based on *falsifiable* model
assumptions



Open-source software & tutorials (Jupyter): <https://bugeye.github.io>



BUQEYE Collaboration

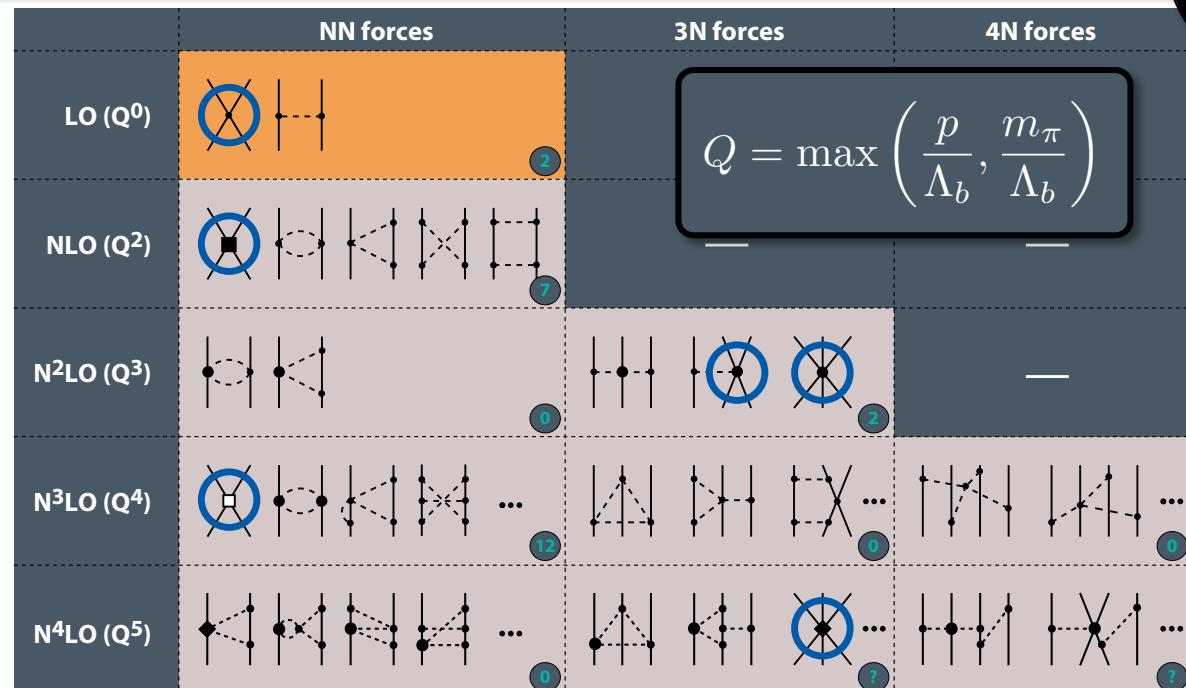
Bayesian
Uncertainty
Quantification:
Errors for
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EFT

Rigorous UQ for nuclear matter



CD, Furnstahl, Melendez,
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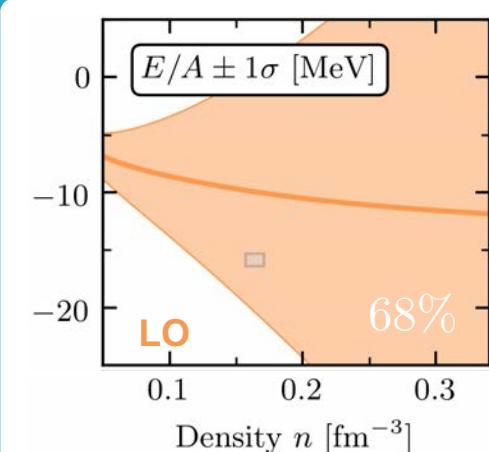
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An example: symmetric matter

$$y = \frac{E}{A}, \quad k = 4 \quad (\text{N}^3\text{LO})$$

Uncertainty bands depict
68% credibility regions

$$y = y_k + \delta y_k$$

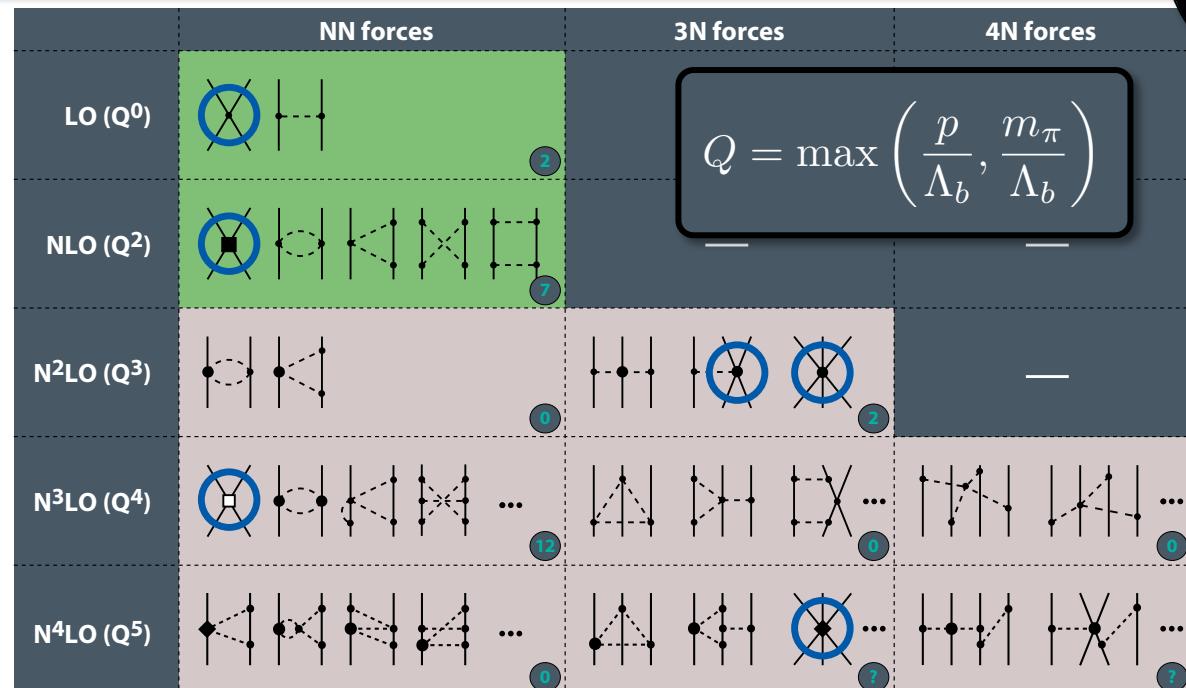


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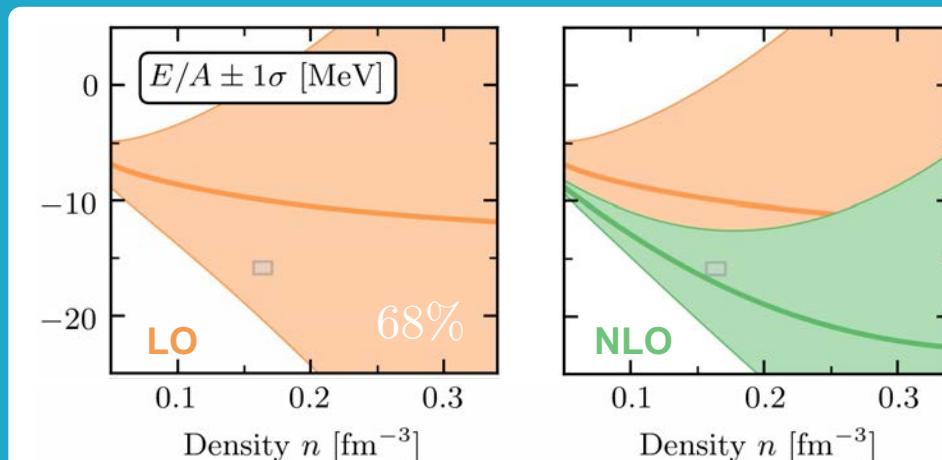
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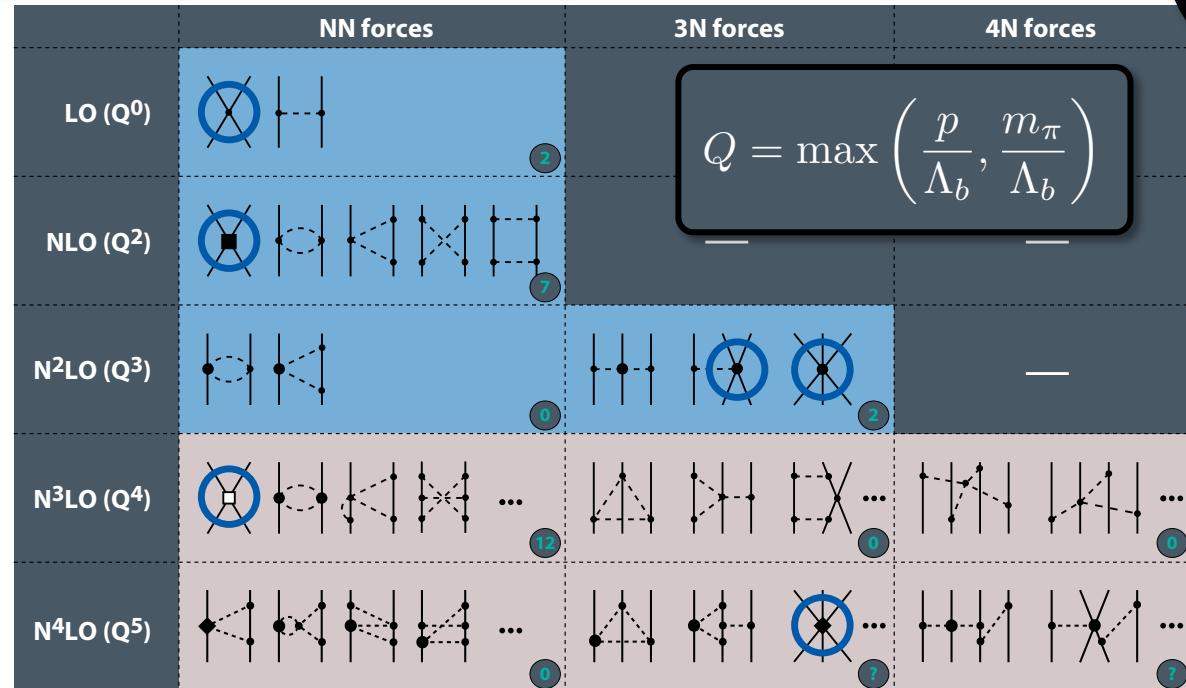


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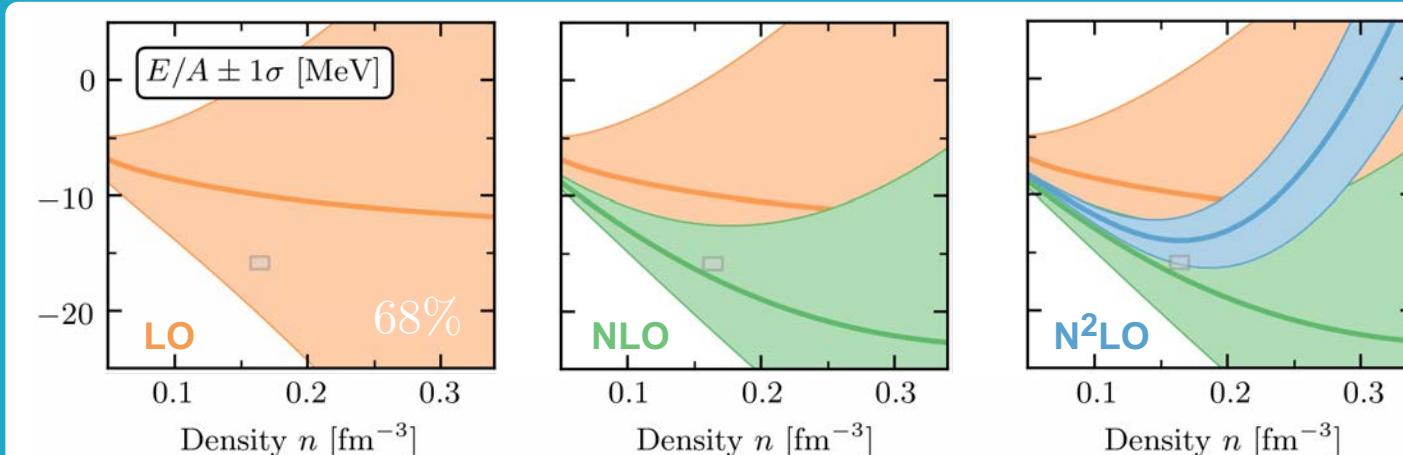
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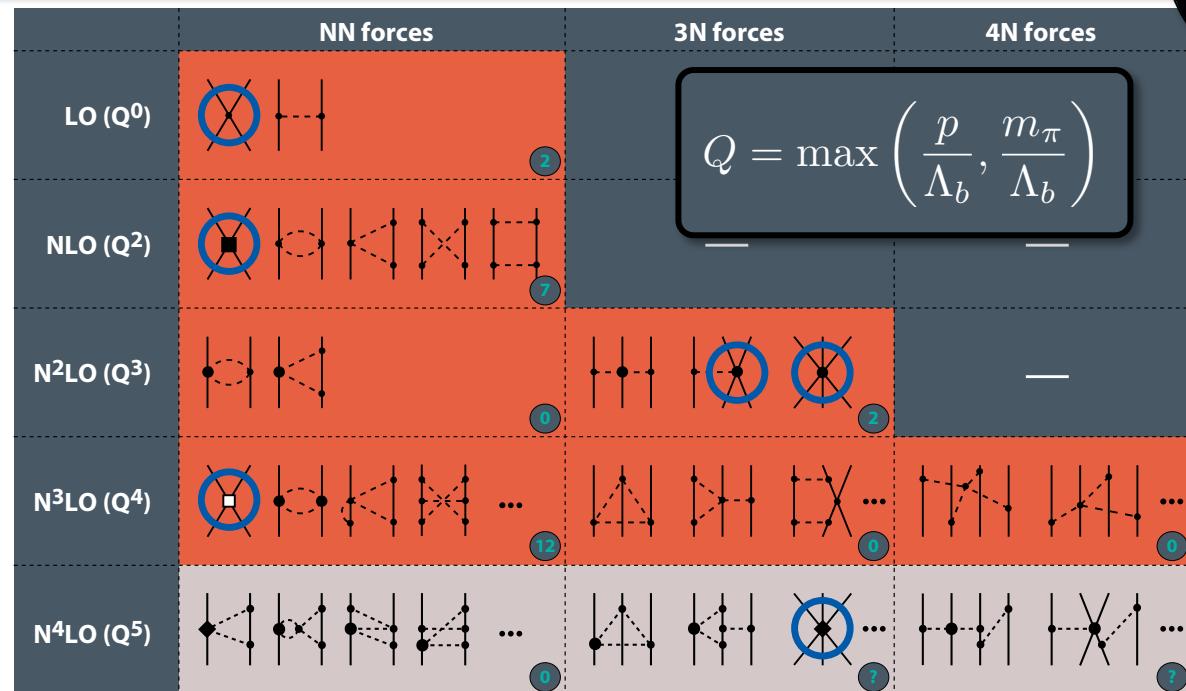


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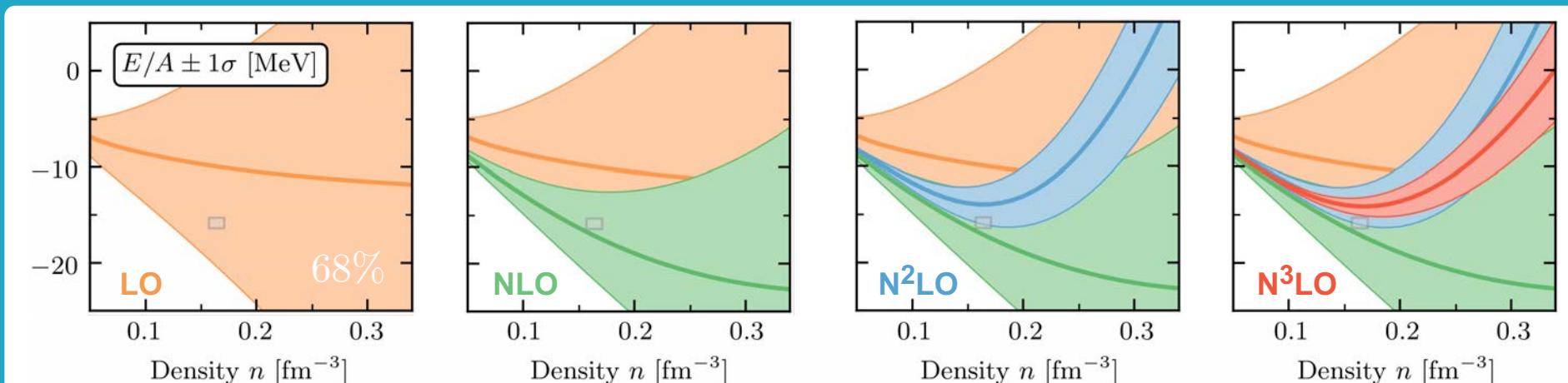
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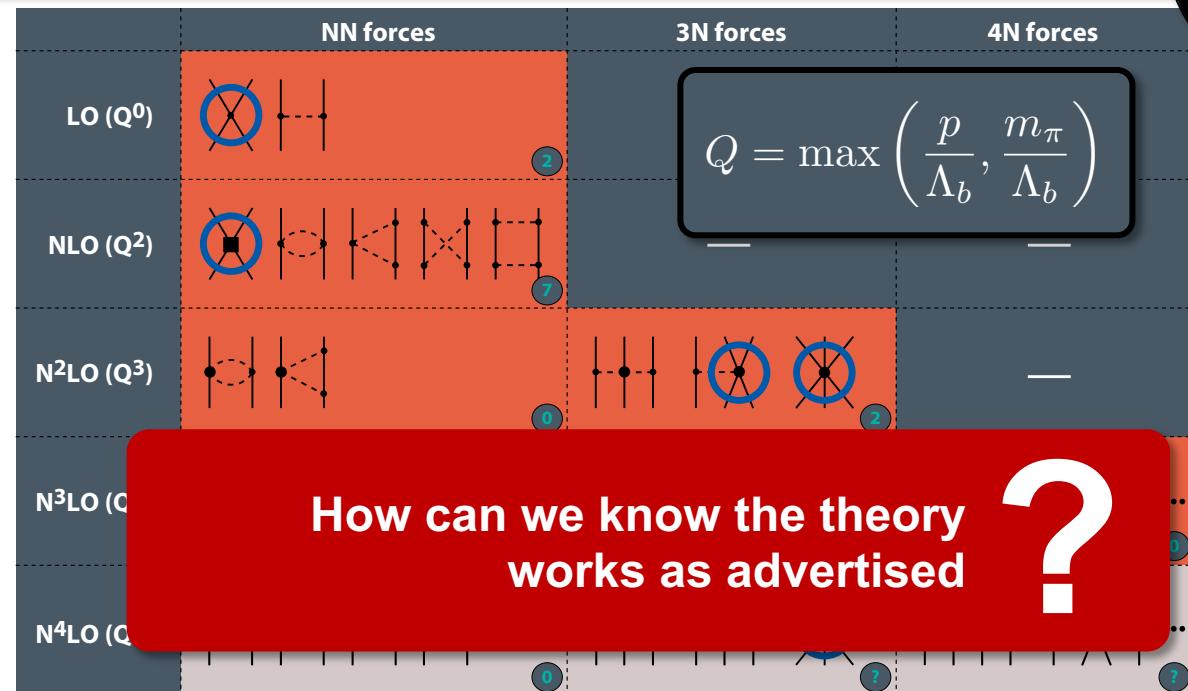


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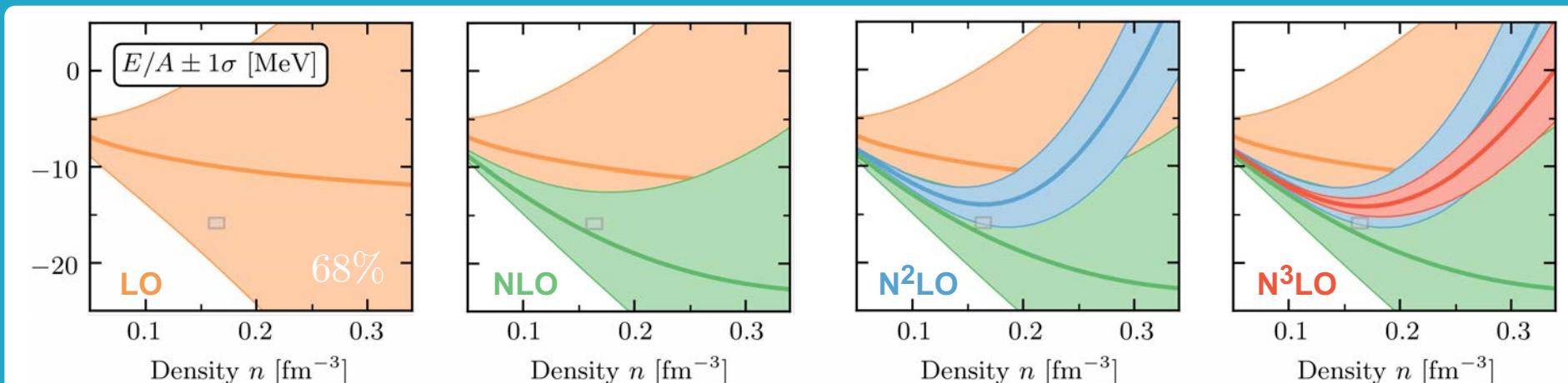
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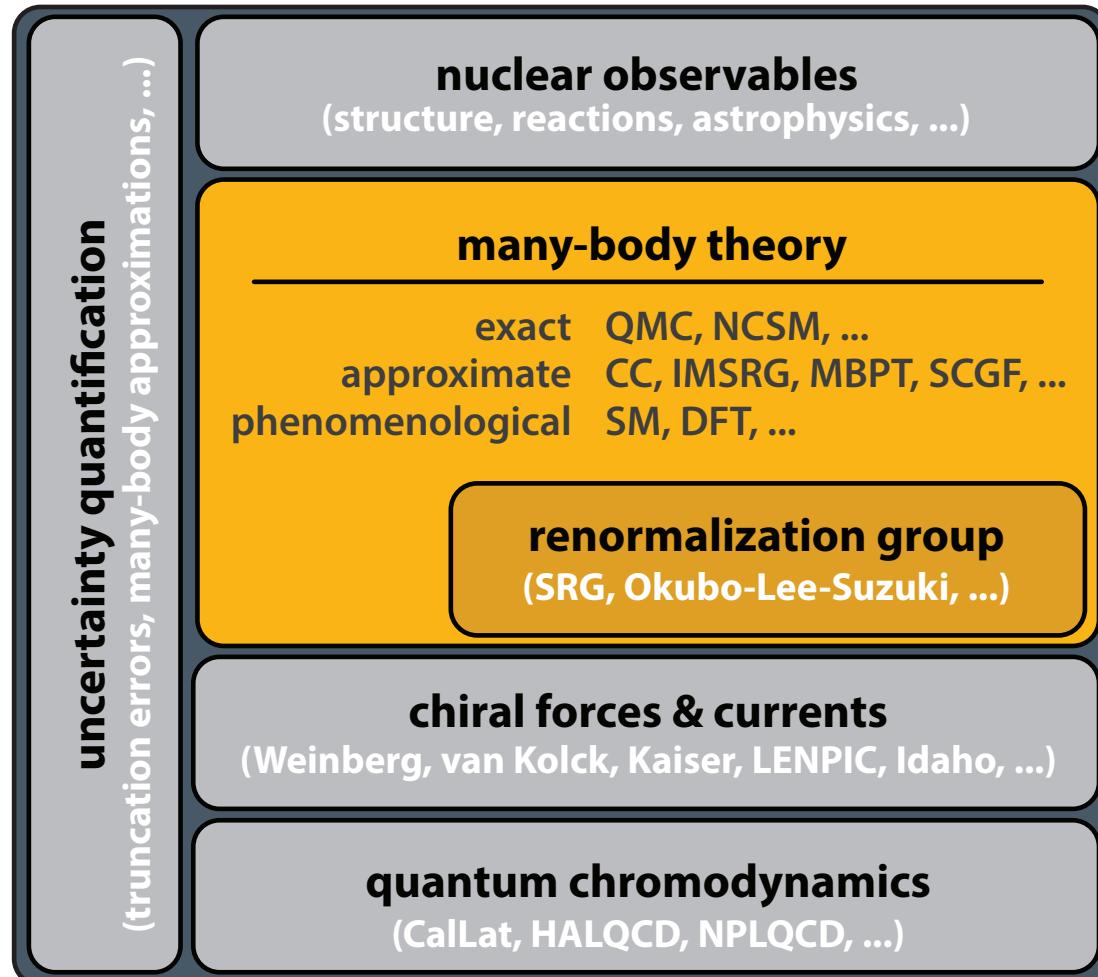
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Here: nuclear equation of state (EOS)
energy per particle (and derived quantities)

$$\frac{E}{A}(n, \delta, T)$$

baryon density n
neutron excess δ
temperature $T (= 0)$

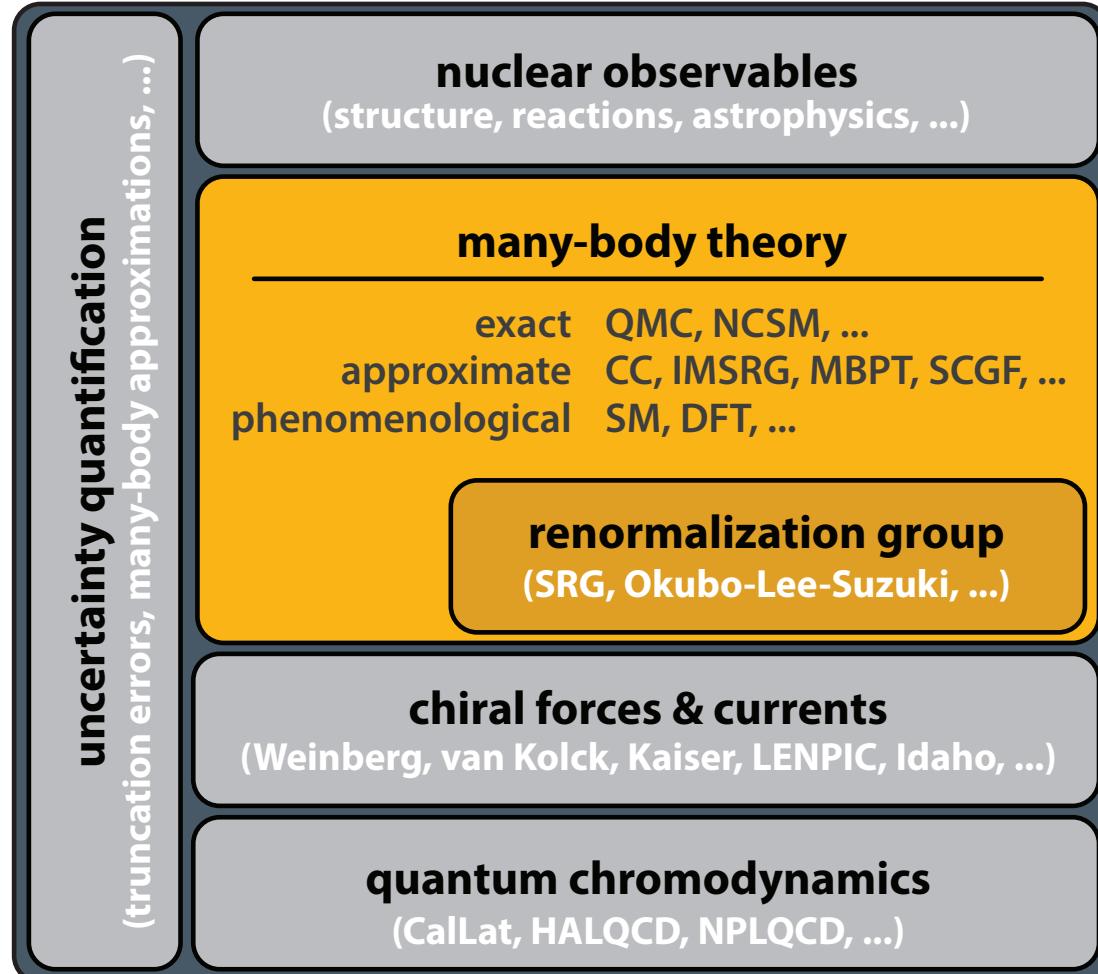
Here: many-body perturbation theory (MBPT)

computationally efficient method (HPC-friendly)
allows to estimate many-body uncertainties

Widely applicable:

- ✓ **arbitrary proton fractions**
- ✓ finite temperature
- ✓ optical potentials, linear response, nuclei, ...

Other frameworks include **quantum Monte Carlo**,
coupled cluster, and self-consistent Green's functions



Here: nuclear equation of state (EOS)

energy

See also Stefano Gandolfi's slides:
Recent QMC calculations of properties
of nuclei & nuclear matter

A

temperature $T (= 0)$

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computationally efficient method (HPC-friendly)
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Widely applicable:

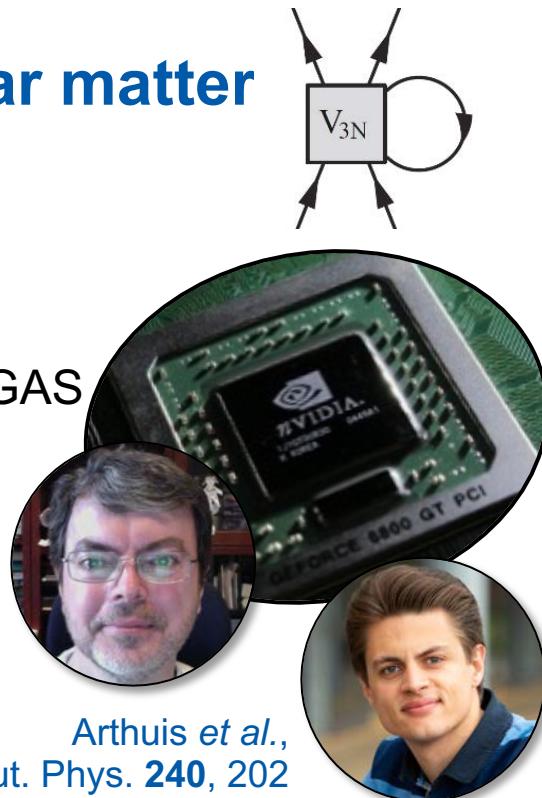
- ✓ arbitrary proton fractions
- ✓ finite temperature
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Other frameworks include **quantum Monte Carlo**,
coupled cluster, and self-consistent Green's functions



automated approach to MBPT for nuclear matter with NN, 3N, and 4N forces

- **implementation of arbitrary diagrams** has become **straightforward** (numerically exact)
- multi-dimensional momentum integrals: improved VEGAS
- GPU-accelerated normal ordering of **3N interactions**
- propagation of importance sampling distributions
- **controlled evaluation of 1000s of MBPT diagrams**



high-order MBPT
calculations of the EOS

automated code
generation

analytic expressions
interaction & MBPT diagrams

automated diagram
generation

The number of diagrams increases rapidly!

1, 3, 39, 840, 27 300, 1 232 280, ...

$n =$ 2 3 4 5 6 7

Integer sequence A064732:
Number of labeled Hugenholtz diagrams with n nodes.

with automated diagram generation



automated approach
to MBPT for nuclear matter

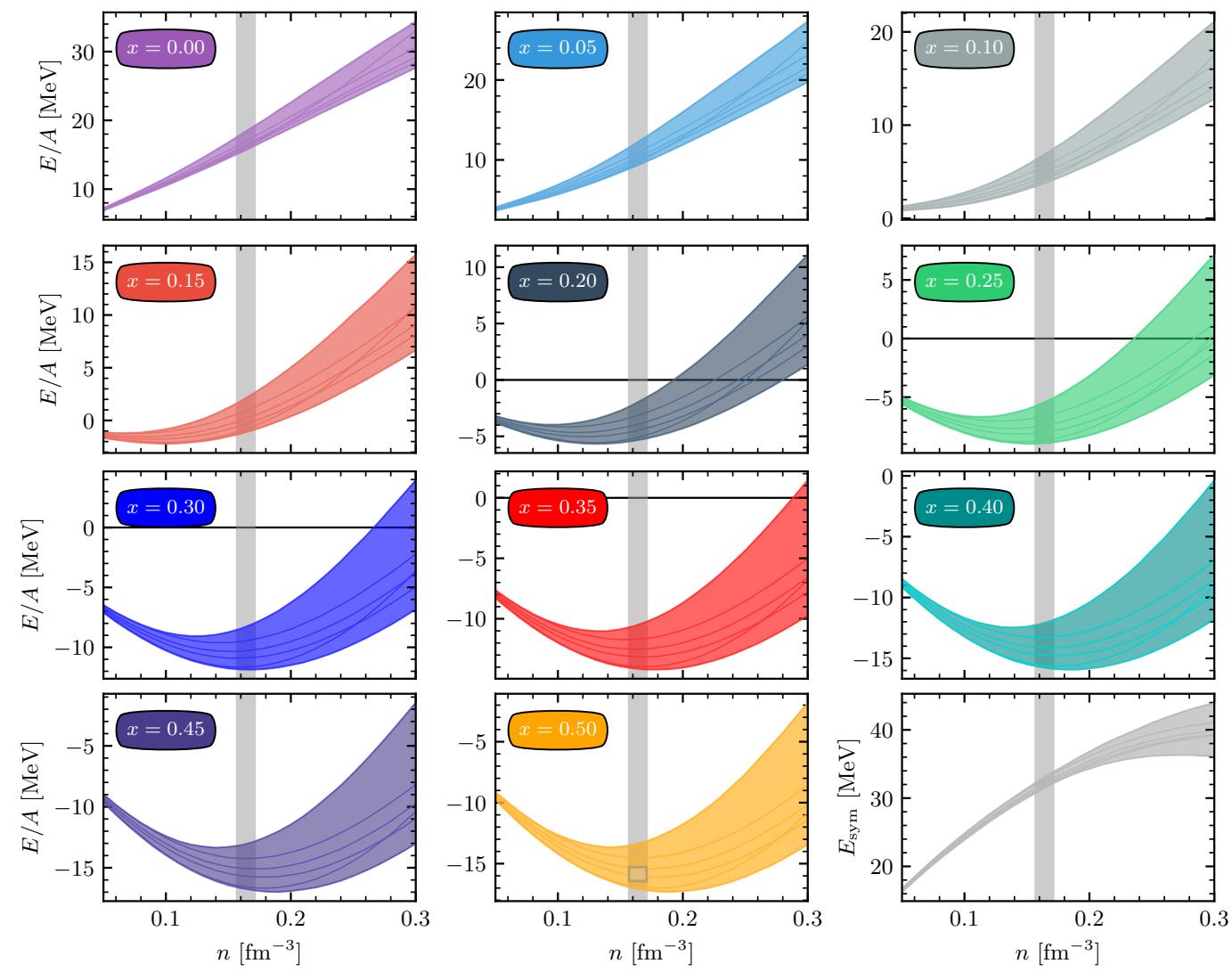
MBPT: an HPC application

#2 (U.S.)

Summit @ Oak Ridge Leadership Computing Facility

202 752 CPU Cores
27 648 Nvidia GPUs
122.3 peta flops

MBPT: an HPC application

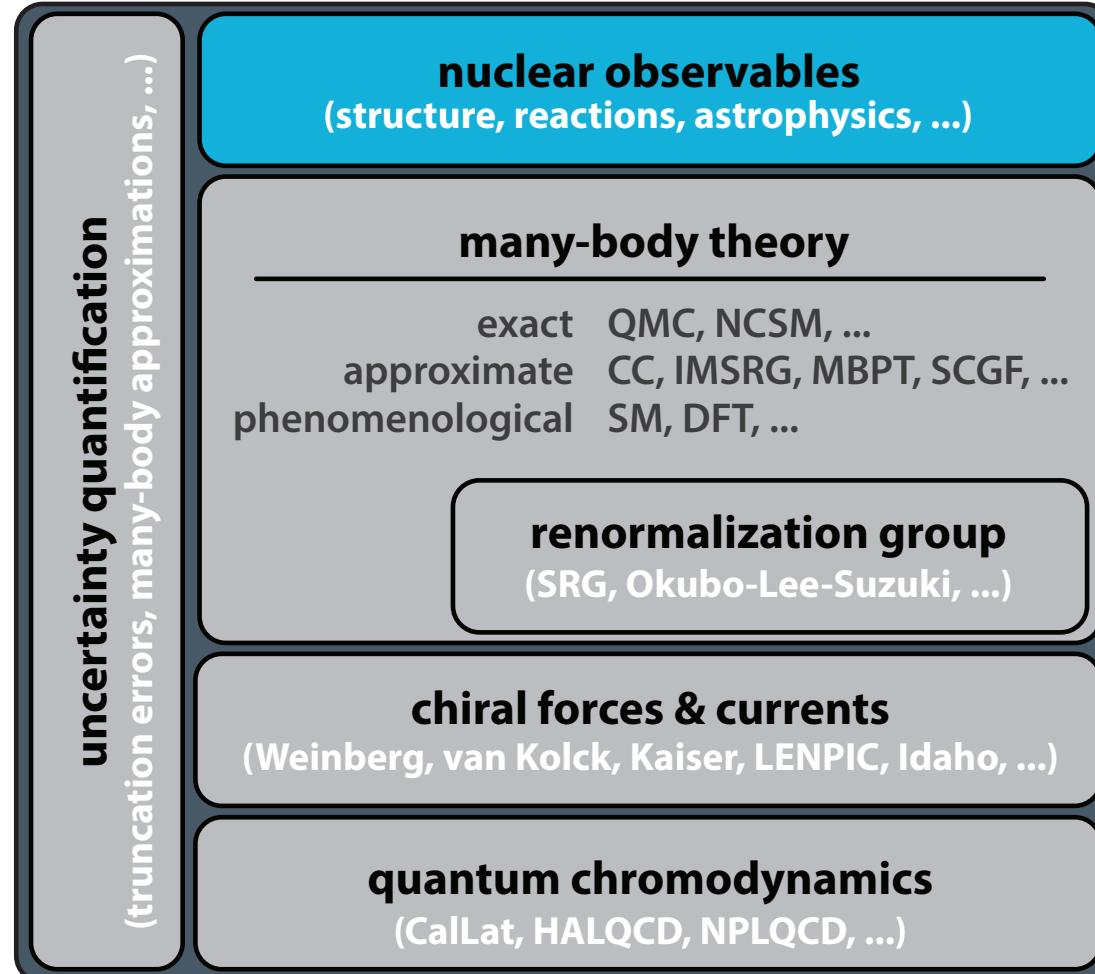


Drischler, McElvain *et al.*, in prep.

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nuclear EOS with quantified uncertainties
energy per particle (and derived quantities)

$$\frac{E}{A}(n, \delta, T)$$

baryon density n
neutron excess δ
temperature $T (= 0)$

Uncertainty quantification

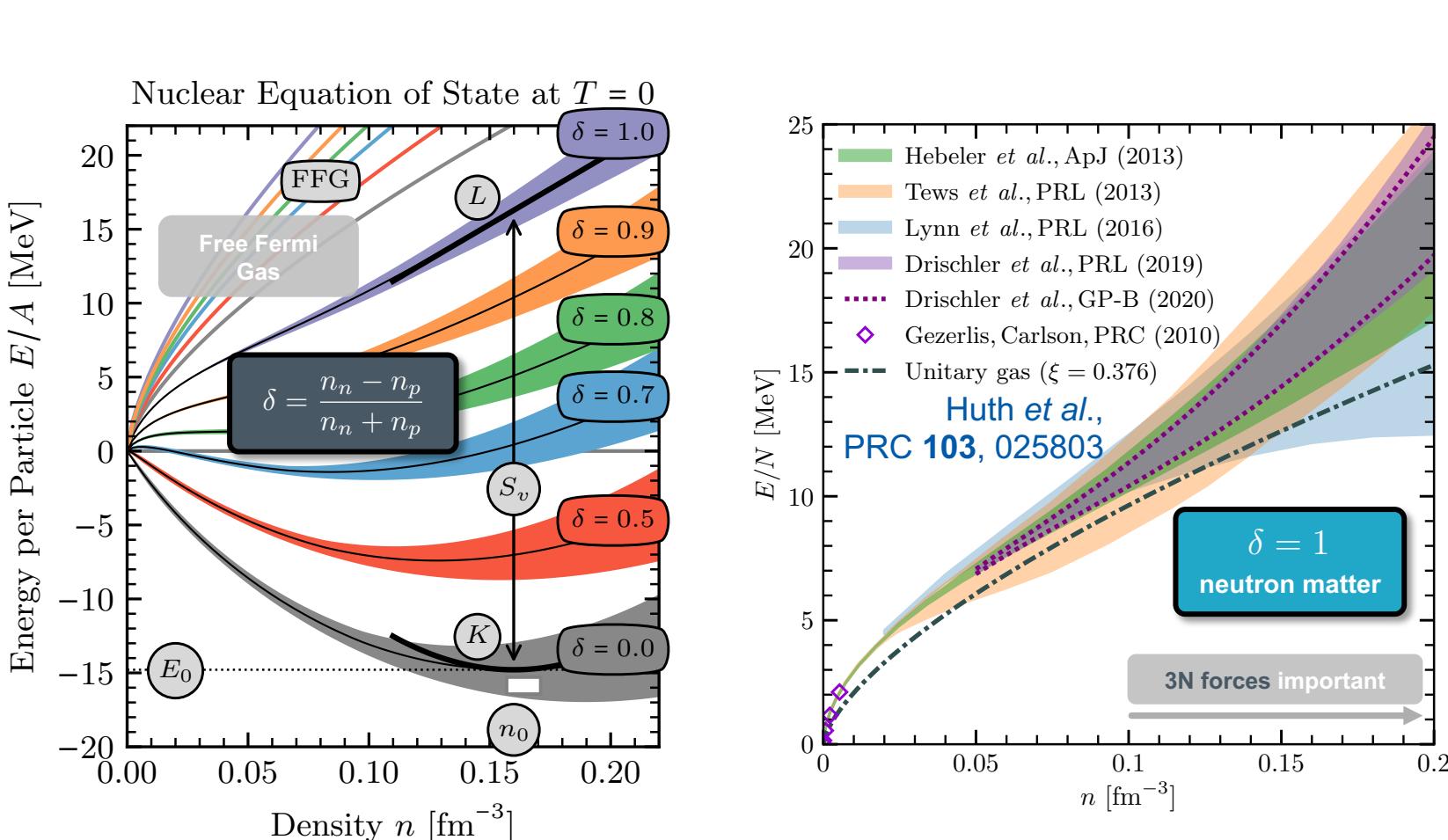
robust estimates of theoretical uncertainties using
Bayesian machine learning via Gaussian Processes
uncertainties in EFT-based calculations due to:

- **truncating the EFT expansion**
- **applying many-body (and other) approximations**
- **fitting LECs to experimental data**

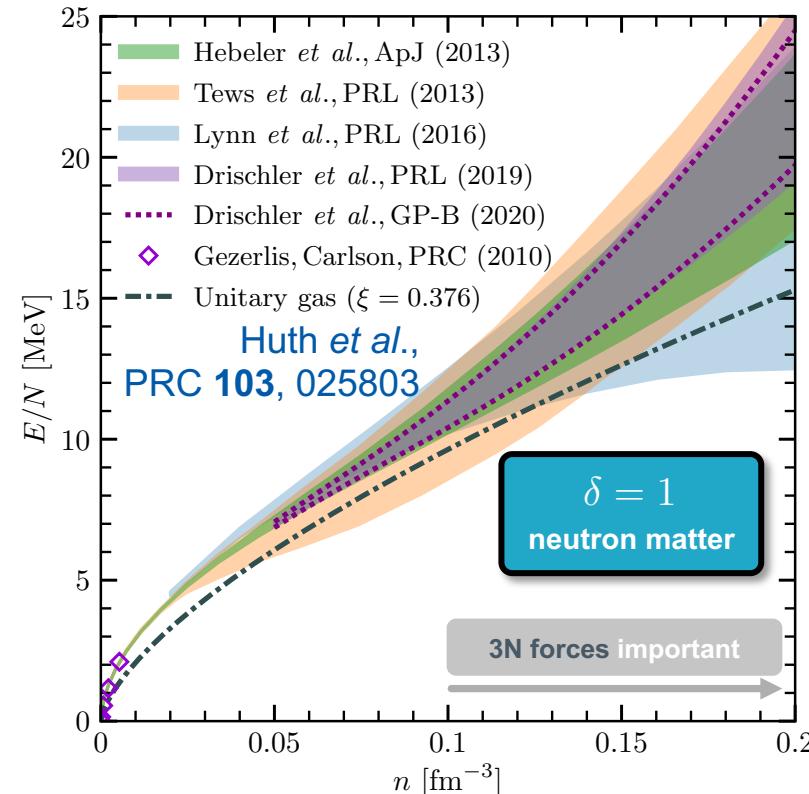
First chiral potentials with uncertainties
fully quantified and their applications:

Wesolowski, Svensson *et al.*, PRC **104**, 064001
Djärv, Ekström *et al.*, PRC **105**, 014005

Neutron matter | saturation in symmetric matter

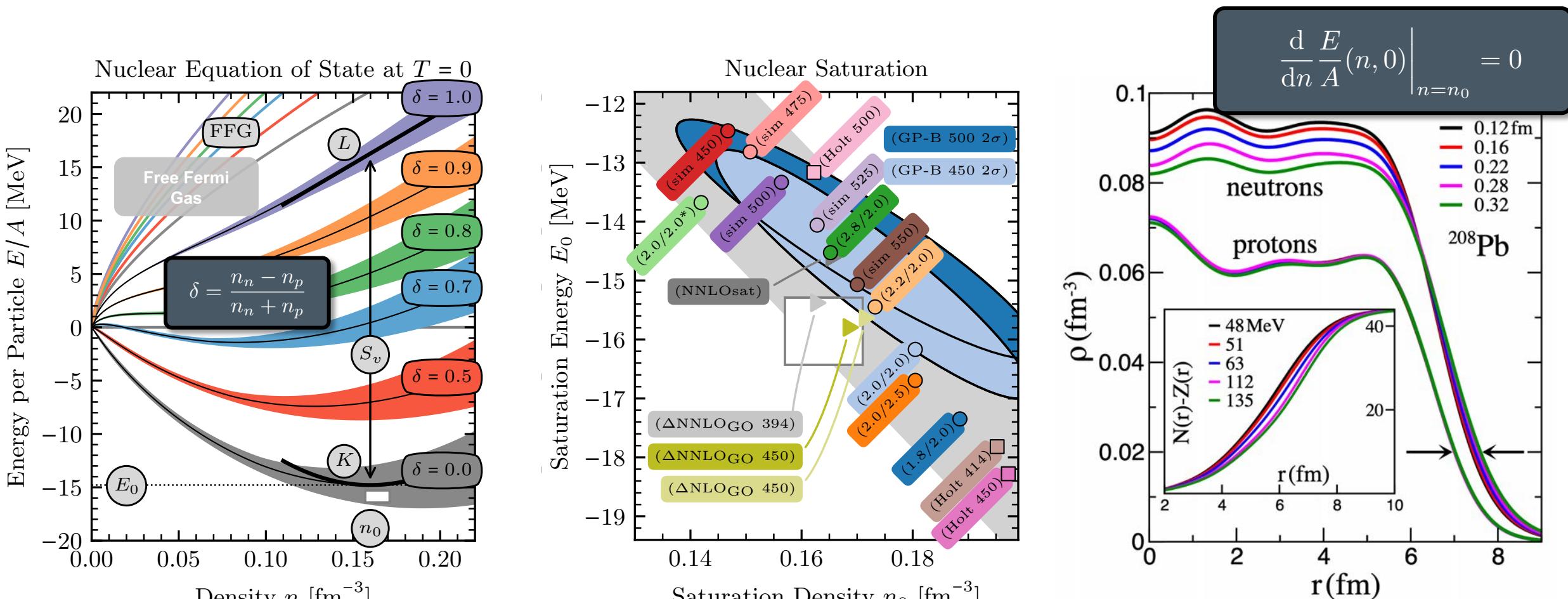


CD, Holt, and Wellenhofer, Annu. Rev. Nucl. Part. Sci. **71**, 403



$$\left. \frac{d}{dn} \frac{E}{A}(n, 0) \right|_{n=n_0} = 0$$

Neutron matter | saturation in symmetric matter



CD, Holt, and Wellenhofer, Annu. Rev. Nucl. Part. Sci. **71**, 403

Piekarewicz & Fattoyev, Phys. Today **72**, 7

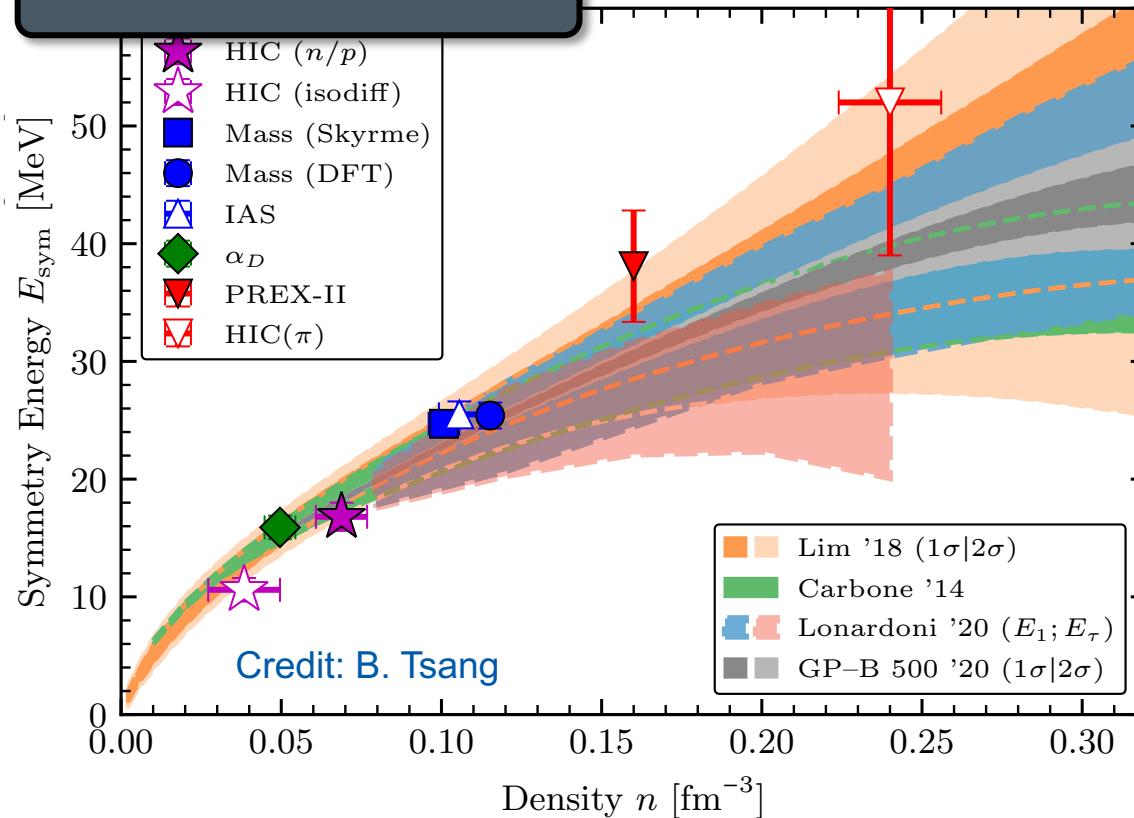
saturation point: fine-tuned cancellation
between the kinetic and interaction
contributions (ideal testbed for chiral EFT)

Coester band overlaps with the empirical
box (but limited meaning without errors)
Annotations: (λ / Λ_{3N}) in fm^{-1} or (Λ) in MeV

needed: improved predictions with
novel NN+3N interactions and
robust uncertainty quantification

Nuclear symmetry energy

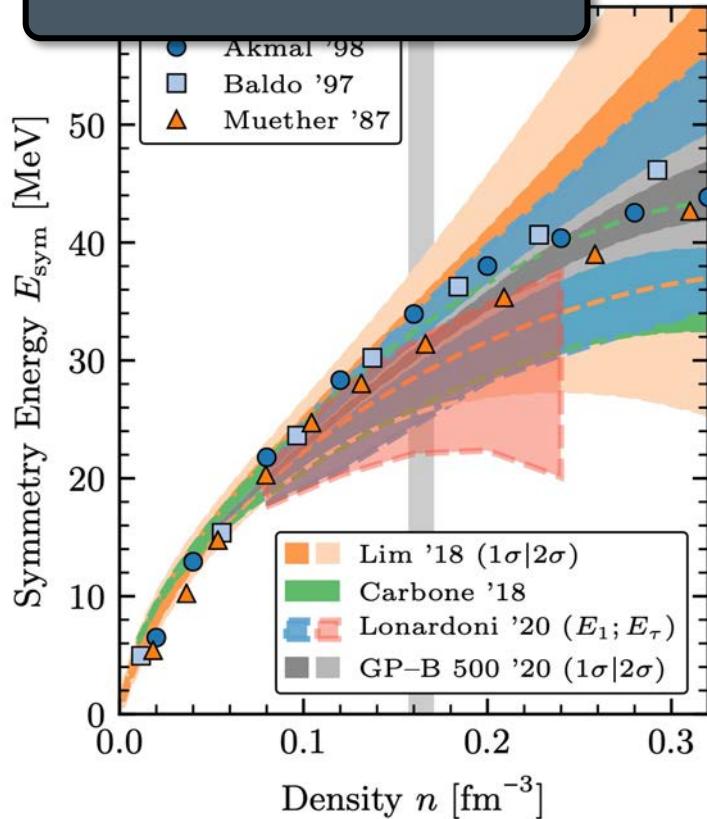
$$E_{\text{sym}}(n) \approx \frac{E}{N}(n) - \frac{E}{A}(n)$$



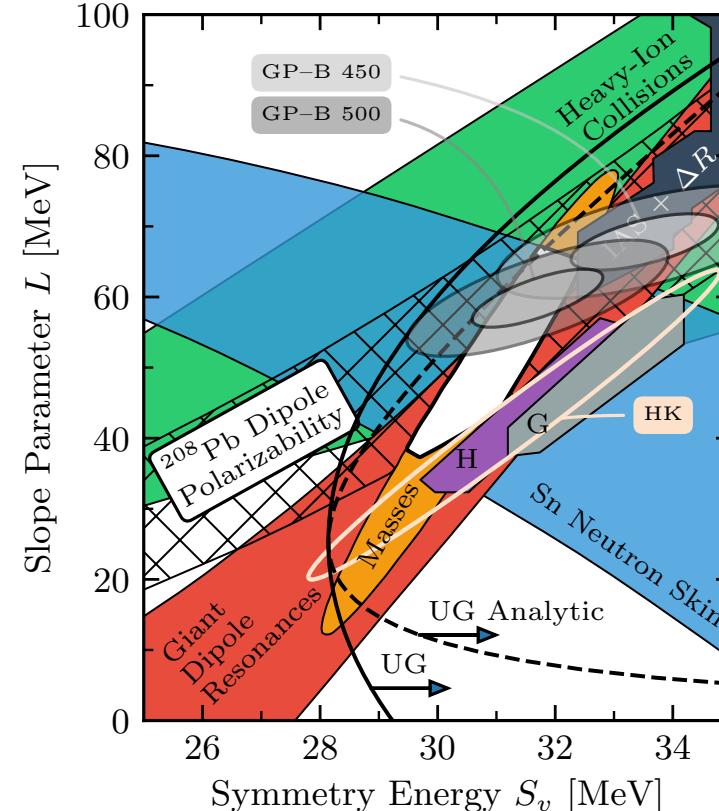
See also Betty Tsang's talk:
Symmetry Energy (Saturday)

Nuclear symmetry energy

$$E_{\text{sym}}(n) \approx \frac{E}{N}(n) - \frac{E}{A}(n)$$



excellent agreement with experiment



$$S_2(n) \equiv S_v + \frac{L}{3} \left(\frac{n - n_0}{n_0} \right) + \dots$$

$$\text{pr}(S_v, L \mid \mathcal{D}) = \int \text{pr}(S_v, L \mid \mathcal{D}, n_0) \text{pr}(n_0 \mid \mathcal{D}) dn_0$$

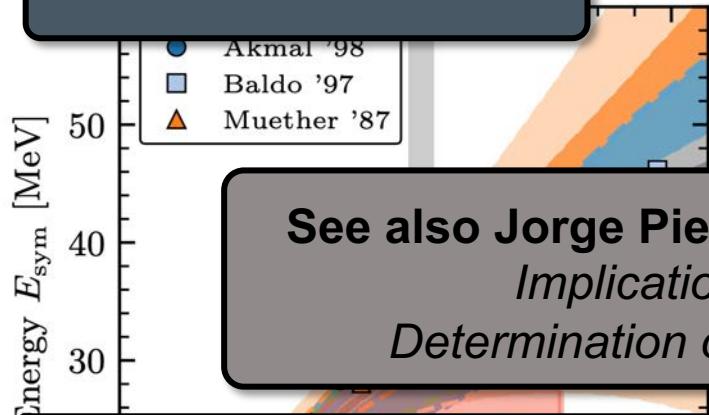
$$\text{pr}(n_0 \mid \mathcal{D}) \approx 0.17 \pm 0.01 \text{ fm}^{-3}$$

Correlations are important:
uncertainties can be smaller
than one *might* naively think

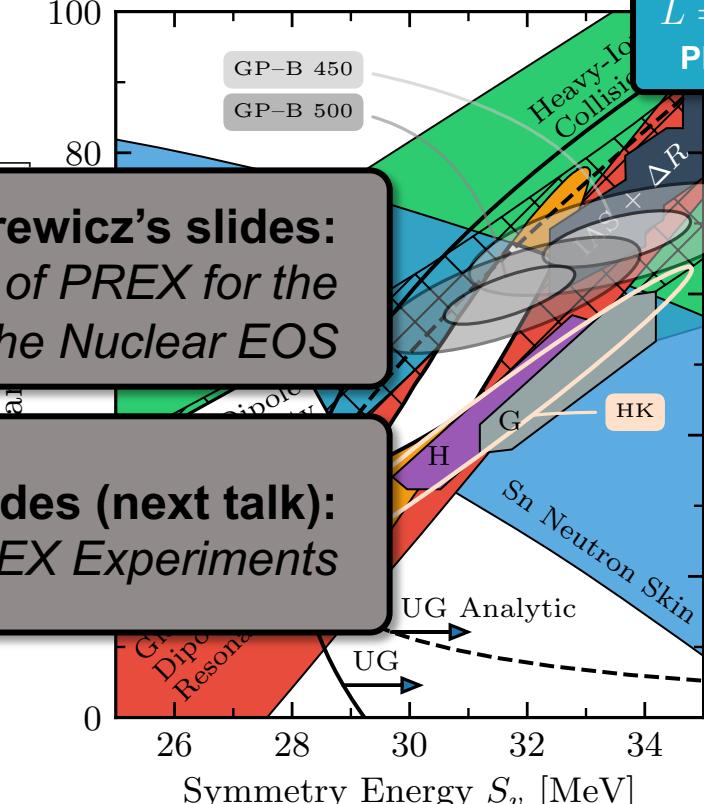
CD, Holt *et al.*, ARNPS **71**, 403
Lattimer & Lim, APJ **771**, 51

Nuclear symmetry energy

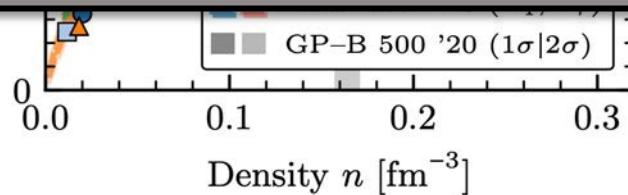
$$E_{\text{sym}}(n) \approx \frac{E}{N}(n) - \frac{E}{A}(n)$$



excellent agreement with experiment



See also Juliette Mammei's slides (next talk):
The PREX and CREX Experiments

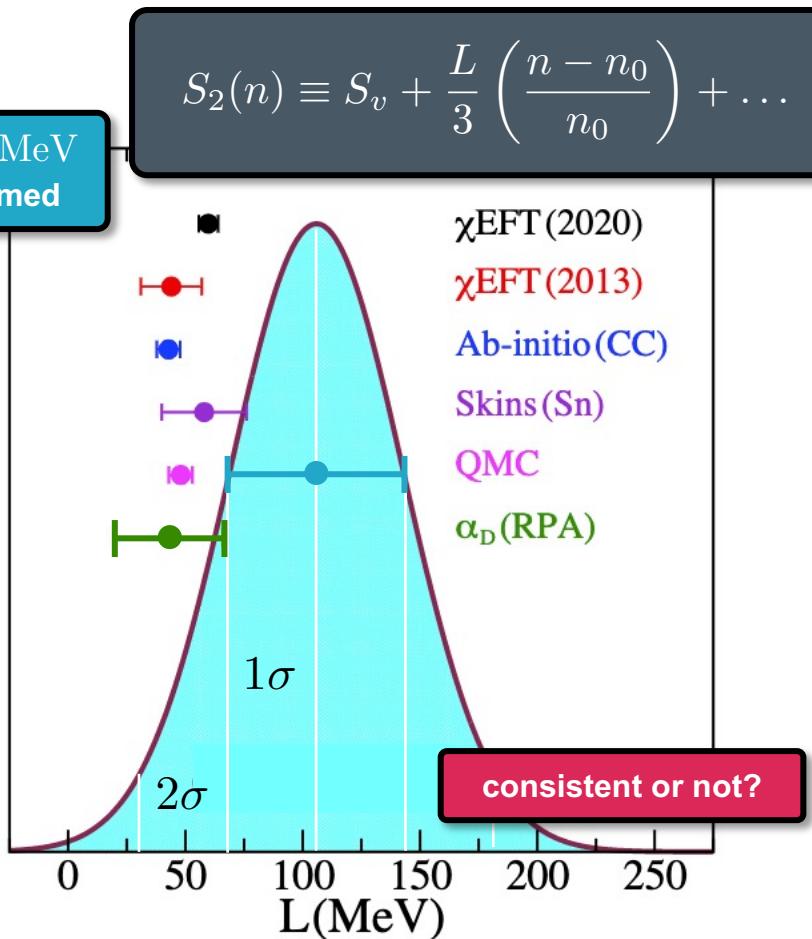


$$\text{pr}(S_v, L | \mathcal{D}) = \int \text{pr}(S_v, L | \mathcal{D}, n_0) \text{pr}(n_0 | \mathcal{D}) dn_0$$

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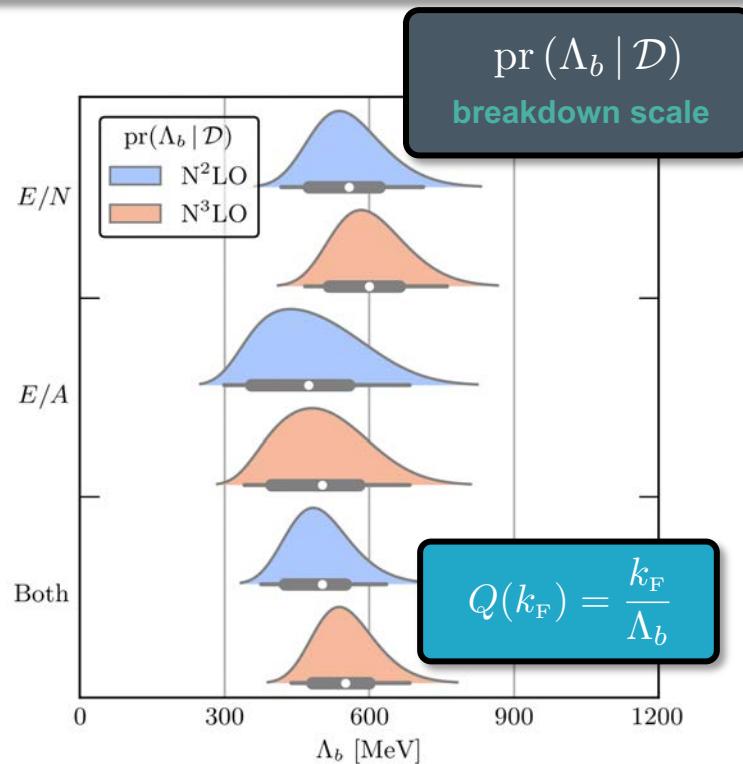
$$S_2(n) \equiv S_v + \frac{L}{3} \left(\frac{n - n_0}{n_0} \right) + \dots$$



Reinhard *et al.*, PRL 127, 232501
Reed, Fattoyev *et al.*, PRL 126, 172503
Piekarewicz, PRC 104, 024329

“Tension” between PREX-II and different theoretical approaches at the ~68-95% level

Exploring the limits of chiral EFT

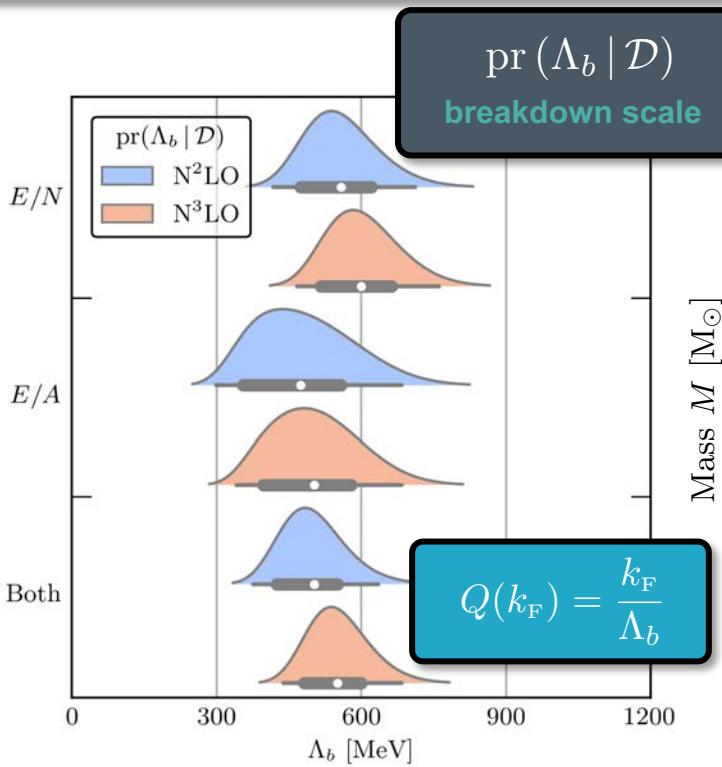


CD, Melendez *et al.*, PRC 102, 054315

Bayesian inference of the
in-medium breakdown scale

But: at what **density** does
chiral EFT break down?

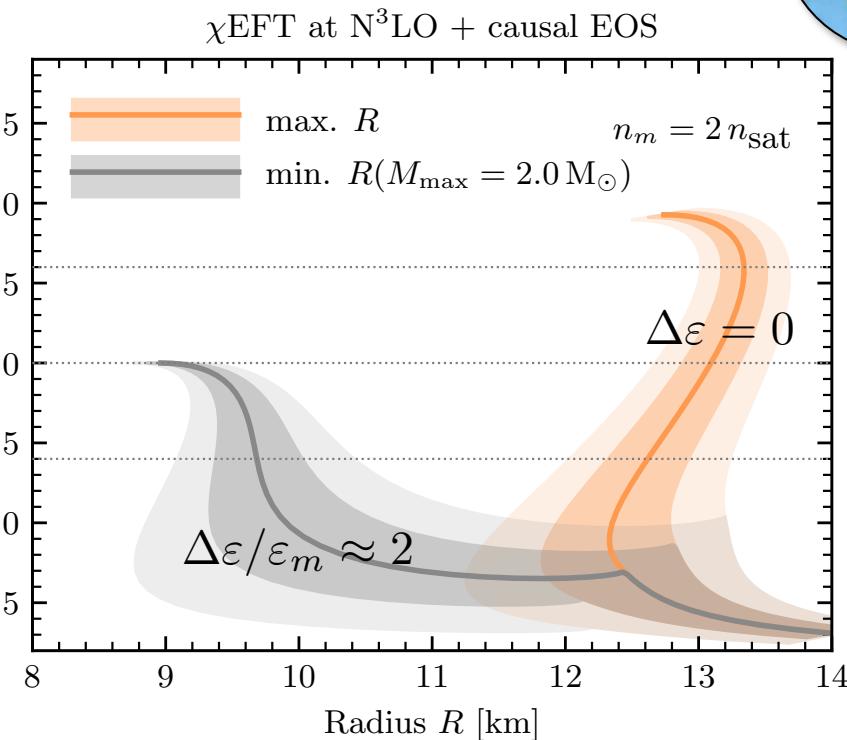
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CD, Melendez *et al.*, PRC **102**, 054315

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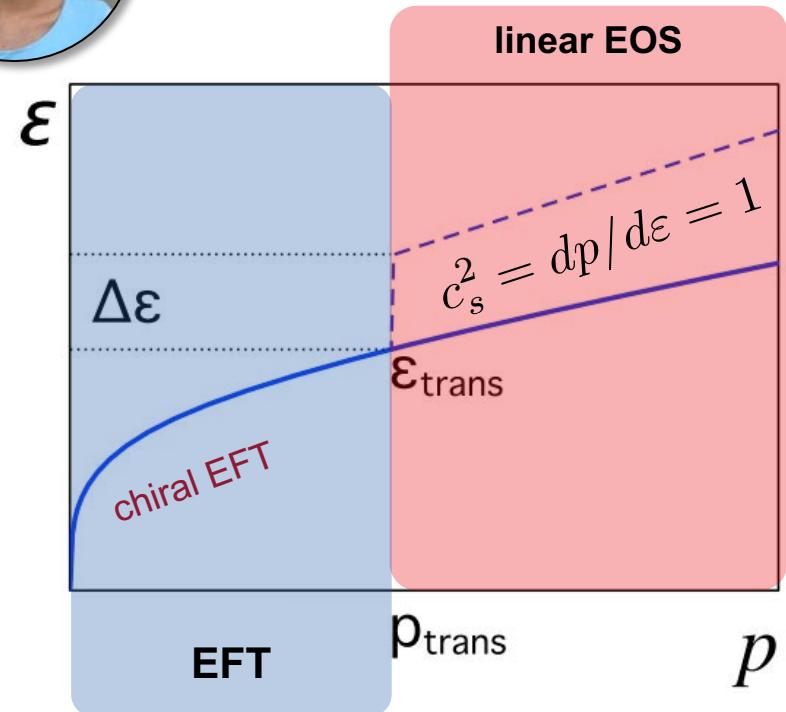
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CD, Han, Lattimer *et al.*, PRC **103**, 045808
CD, Han, and Reddy, PRC **105**, 035808

derived **bounds on the neutron star radius** (and sound speed) assuming chiral EFT is valid up to a given critical density (here: $2n_0$) could already be challenged by NICER

$R_{2.0} = (11.4 - 16.1)$ km Riley *et al.*, AJL **918**, L27
Miller *et al.*, AJL **918**, L28



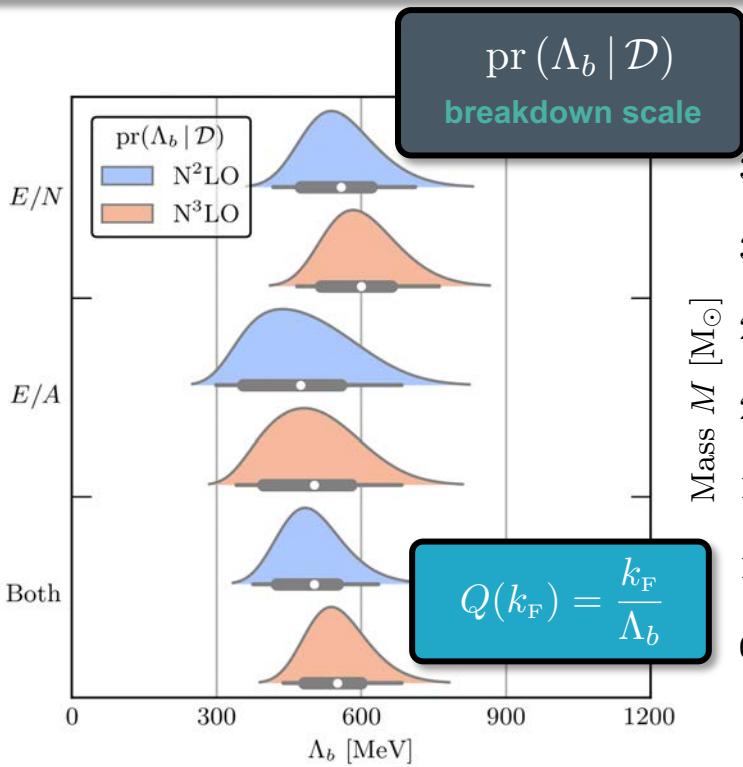
Han & Prakash, APJ **899**, 2
Alford *et al.*, JPG: NPP **46**, 114001

extend EFT EOS at n_m to linear EoS with finite discontinuity (softening)

continuous match sets upper bound

use **lower limit on M_{\max}** from observation to adjust $\Delta\epsilon$ and constrain R_{\min}

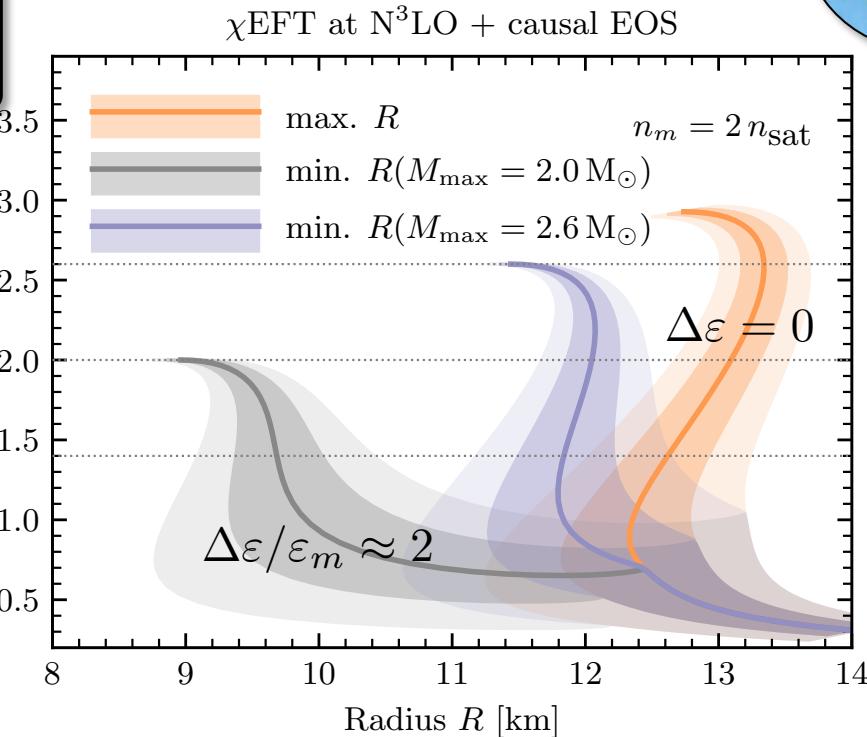
Exploring the limits of chiral EFT



CD, Melendez *et al.*, PRC **102**, 054315

Bayesian inference of the in-medium breakdown scale

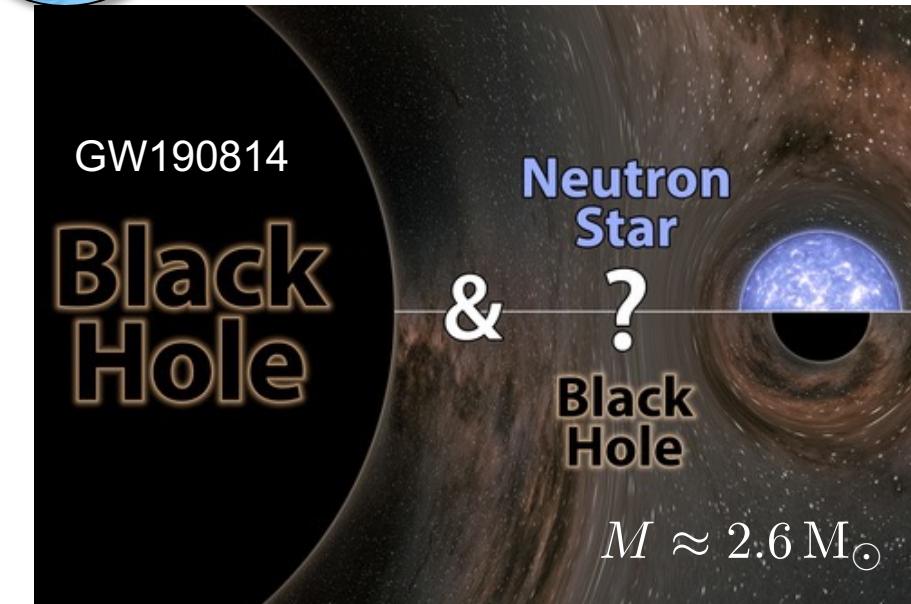
But: at what density does chiral EFT break down?



CD, Han, Lattimer *et al.*, PRC **103**, 045808
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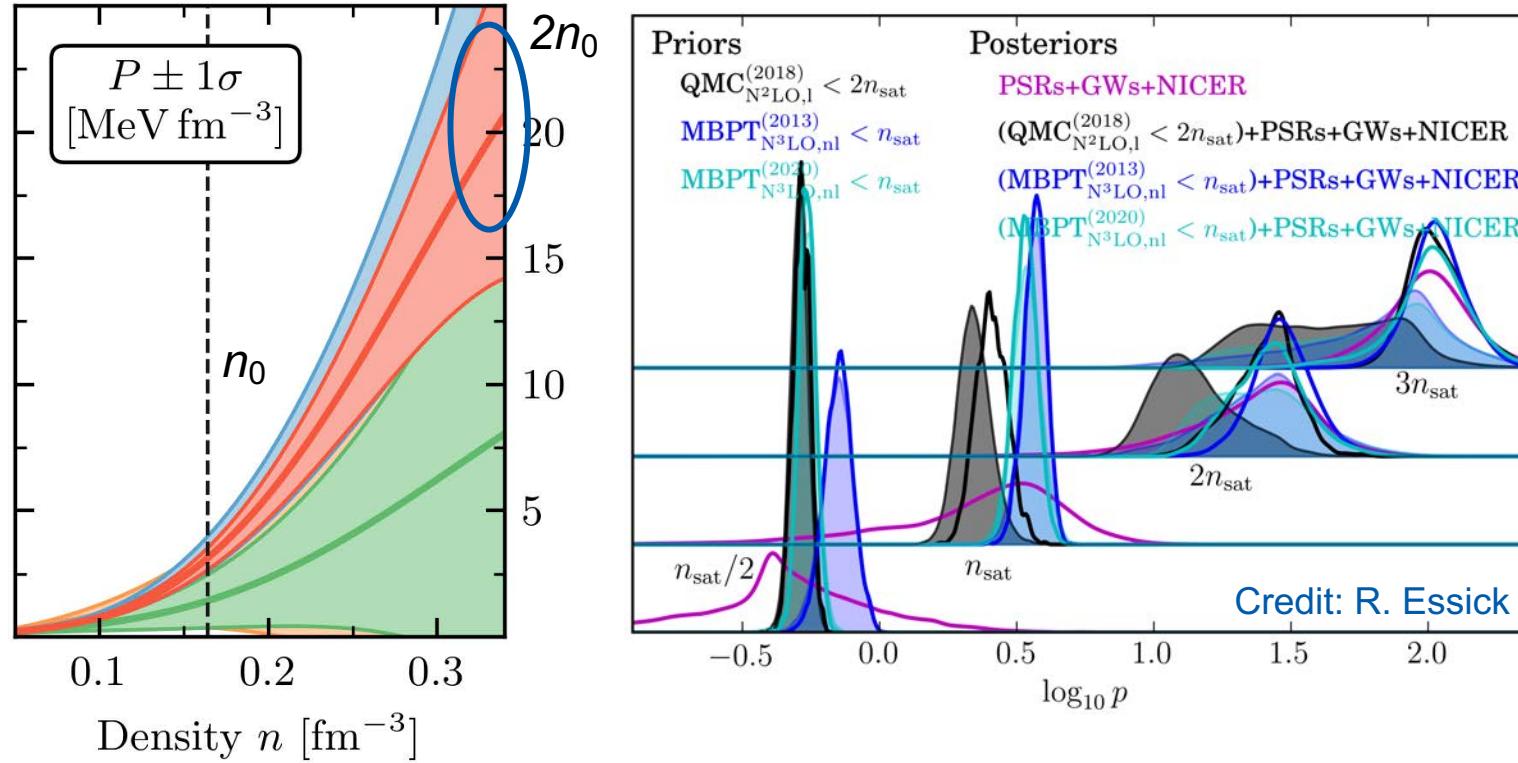
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Direct astrophysical tests at supranuclear densities



CD, Furnstahl *et al.*, PRL 125, 202702

see also: Essick *et al.*, PRC 102, 055803



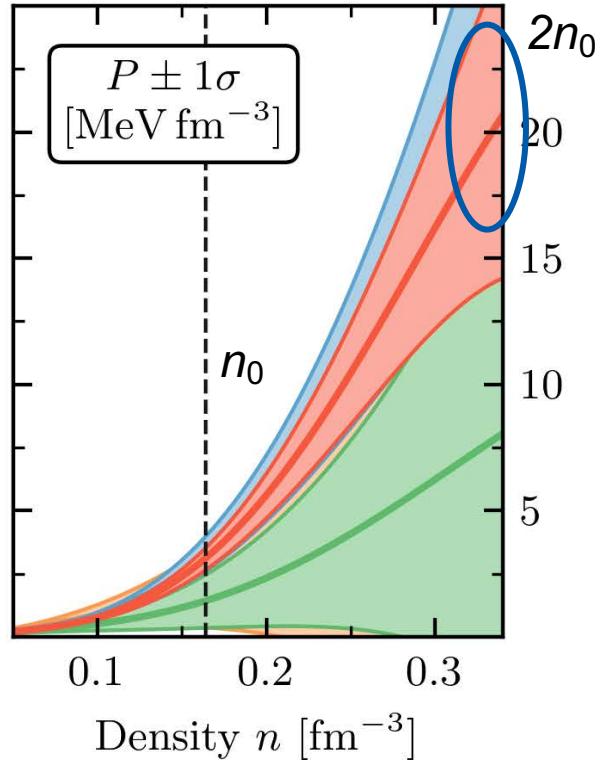
Neutron star observations could be used for:

Model checking & selection of chiral interactions

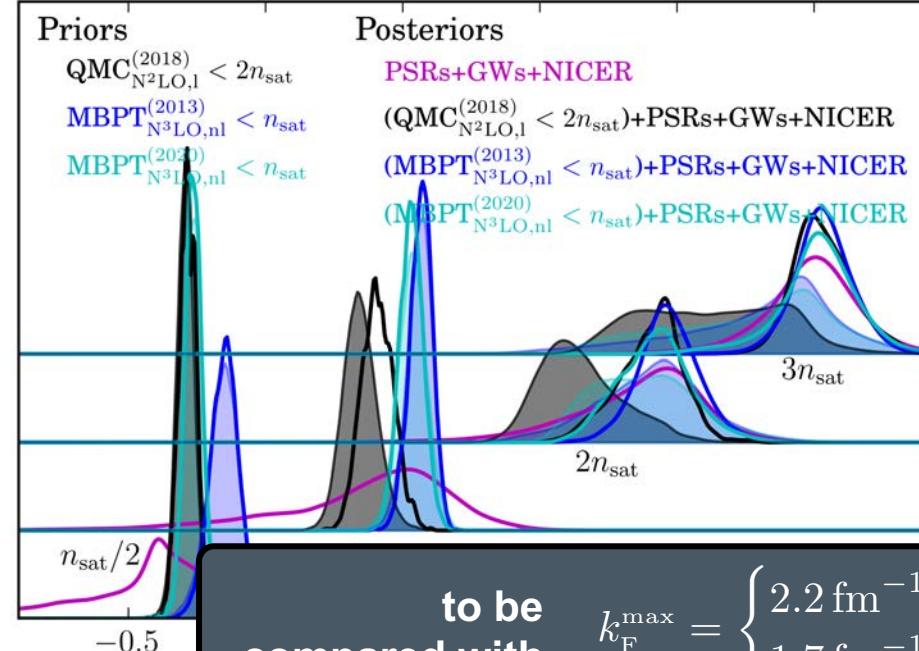
Constraints on coupling constants in nuclear forces

$$P(n = 0.32 \text{ fm}^{-3}) \approx \begin{cases} 20 \pm 6 \text{ MeV fm}^{-3} & \text{MBPT: nonlocal} \\ 15 \pm 5 \text{ MeV fm}^{-3} & \text{QMC: local } V_{E,1} \end{cases}$$

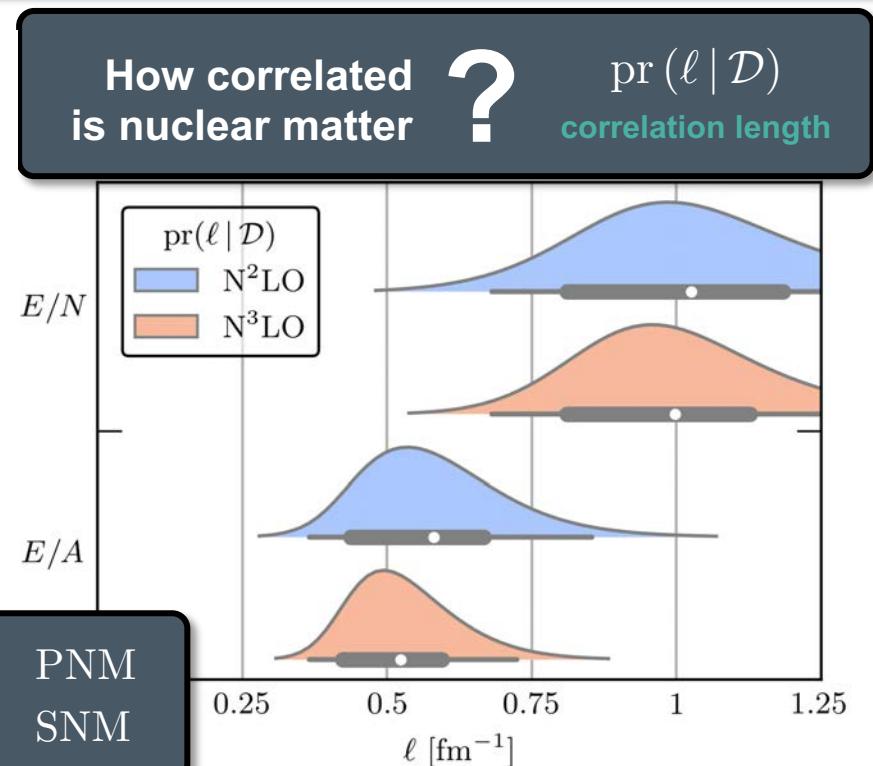
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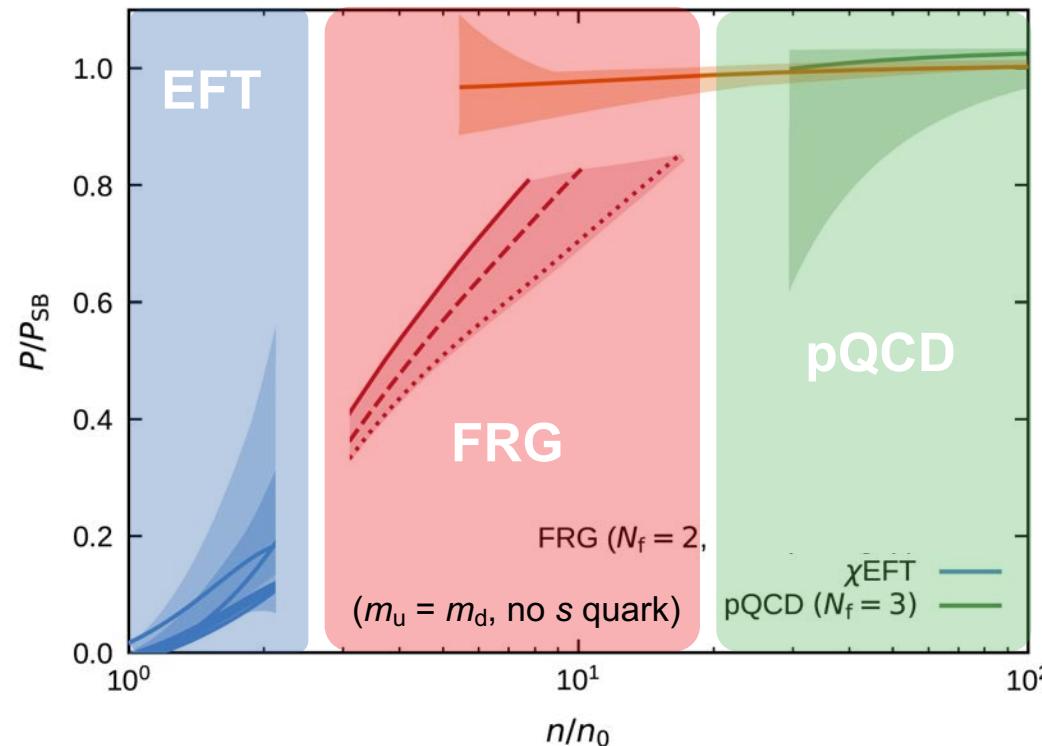
CD, Melendez *et al.*, PRC 102, 054315

Neutron star observations could be used for:
Model checking & selection of chiral interactions
Constraints on coupling constants in nuclear forces

EFT truncation error is *highly* correlated

$$P(n = 0.32 \text{ fm}^{-3}) \approx \begin{cases} 20 \pm 6 \text{ MeV fm}^{-3} & \text{MBPT: nonlocal} \\ 15 \pm 5 \text{ MeV fm}^{-3} & \text{QMC: local } V_{E,1} \end{cases}$$

New predictions in SNM at intermediate densities

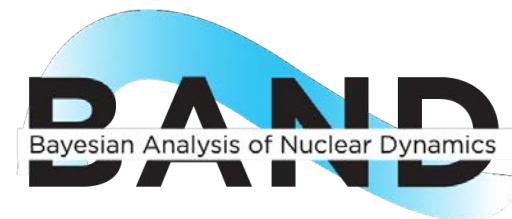


Leonhardt, Pospiech, Schallmo, Braun, CD,
Hebeler, and Schwenk, PRL **125**, 142502

Functional Renormalization Group (based on QCD action):
ab initio constraints at intermediate densities (~ 3 — $10n_0$)

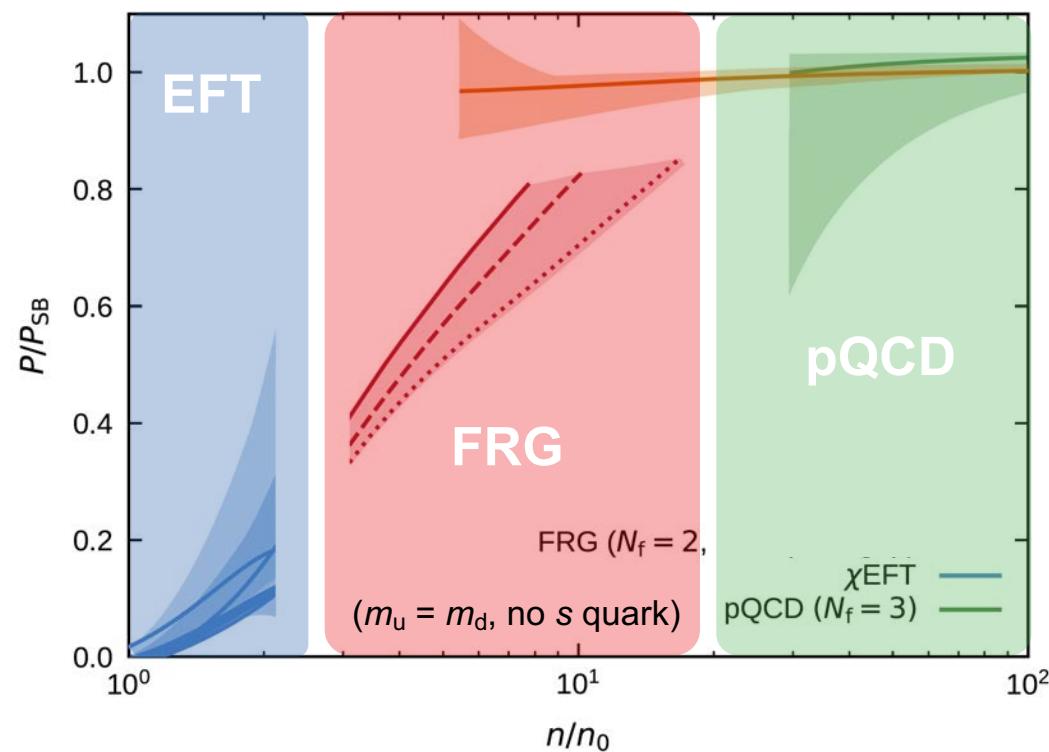
suggests that the different density regions can be
straightforwardly combined

for neutron star matter, see: Braun & Schallmo; arXiv:2204.00358



BAND Manifesto,
Phillips, Furnstahl, Heinz *et al.*,
JGP: NP **48** 07200

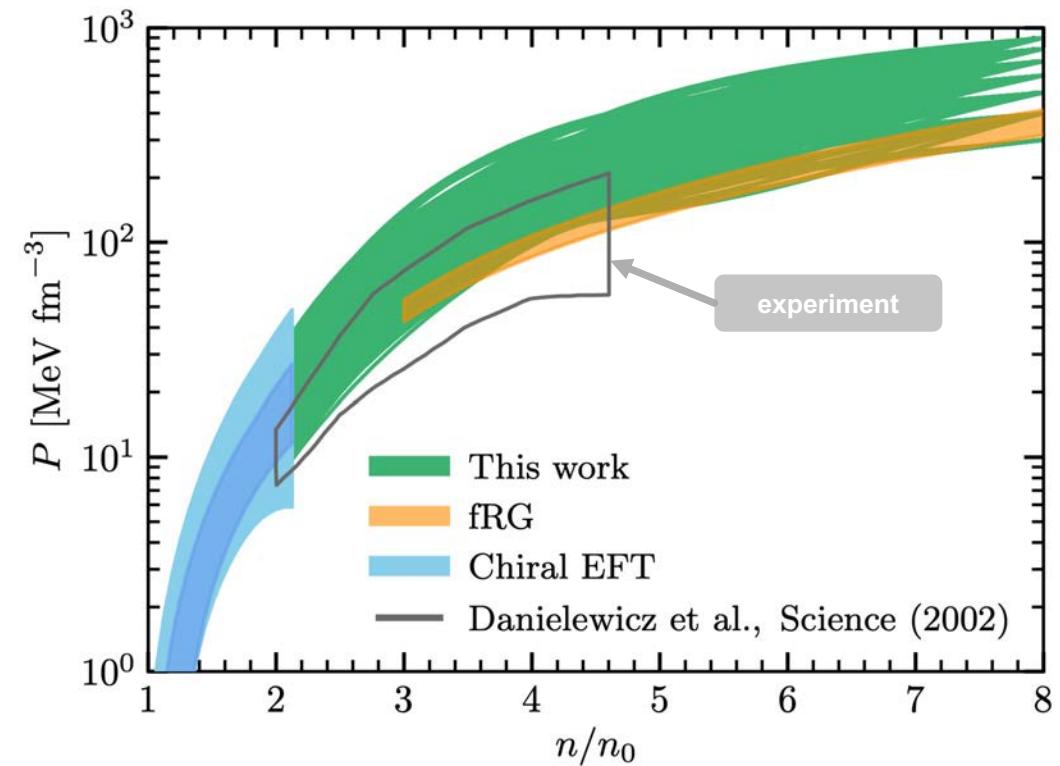
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Leonhardt, Pospiech, Schallmo, Braun, CD,
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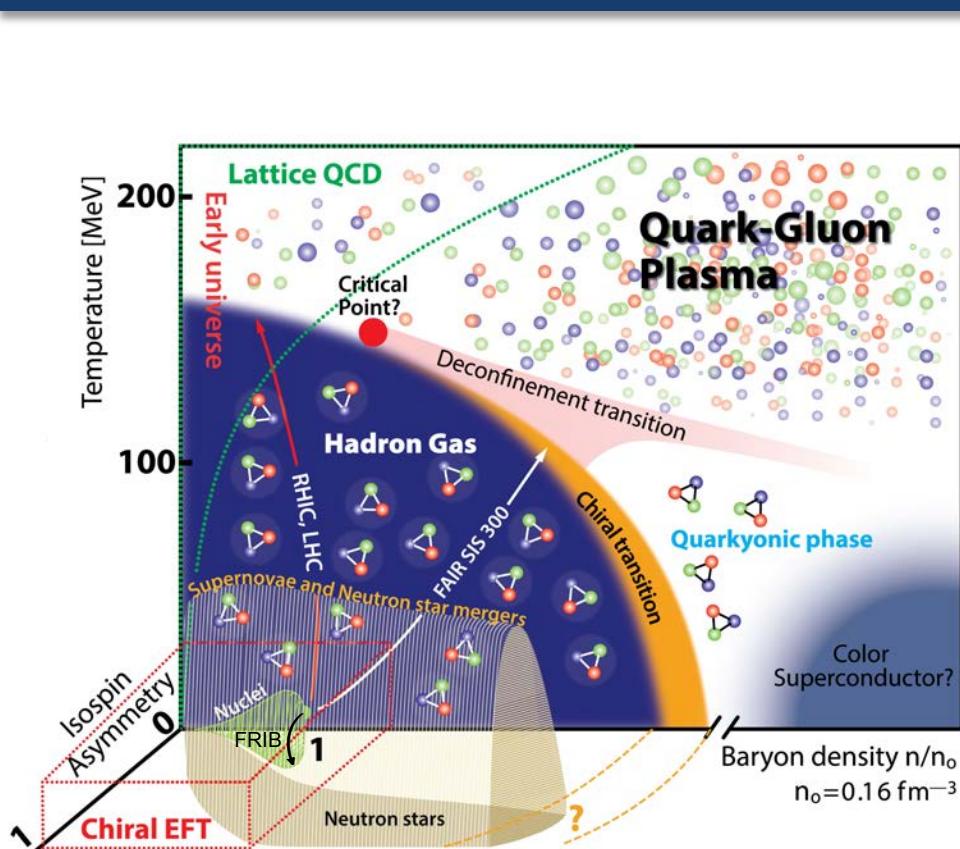
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Huth, Wellenhofer, and Schwenk, PRC **103**, 025803

remarkable consistency between theory
predictions, experiment, and astrophysics



Chiral Effective Field Theory and the High-Density Nuclear Equation of State

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<https://doi.org/10.1146/annurev-nucl-102419-041903>



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Keywords:

Chiral EFT | neutron stars | MBPT
nuclear matter at zero and finite temperature
Bayesian uncertainty quantification
recent neutron star observations

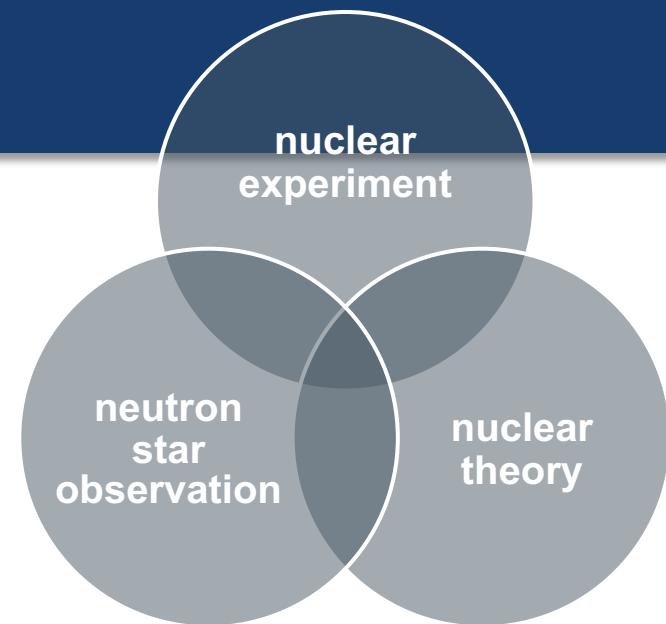
see also in the same journal:
James Lattimer, Annu. Rev. Nucl. Part. Sci. 71, 433

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Take-away points

multi-messenger
nuclear precision
FRIB

} era



unique opportunity to obtain a
fundamental understanding of
strongly interacting matter, with
great **potential for discoveries**

- 1 Upcoming observational (and experimental) campaigns will provide **stringent constraints** on the properties of neutron stars.
- 2 Chiral EFT enables **microscopic predictions** of nuclear matter (and nuclei) with **quantified uncertainties** to interpret these empirical constraints.
- 3 **Automated MBPT**: efficient EOS calculations across a wide range of densities, isospin asymmetries, and temperatures, as well as nuclear interactions.
- 4 Bayesian methods: powerful tools for quantifying & propagating **correlated uncertainties** in EFT-based calculations (*model checking* is important).

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R. Furnstahl S. Han J. W. Holt J. Lattimer K. McElvain
J. Melendez D. Phillips M. Prakash S. Reddy C. Wellenhofer T. Zhao

