

Quantifying uncertainties in the nuclear-matter equation of state

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Christian Drischler

Online S@INT | Zoom conference | May 28, 2020

nuclear matter

astrophysics
neutron stars,
supernovae, ...

EFT

nuclear physics
structure & reactions
of nuclei

correlated EFT uncertainties

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U.S. DEPARTMENT OF
ENERGY

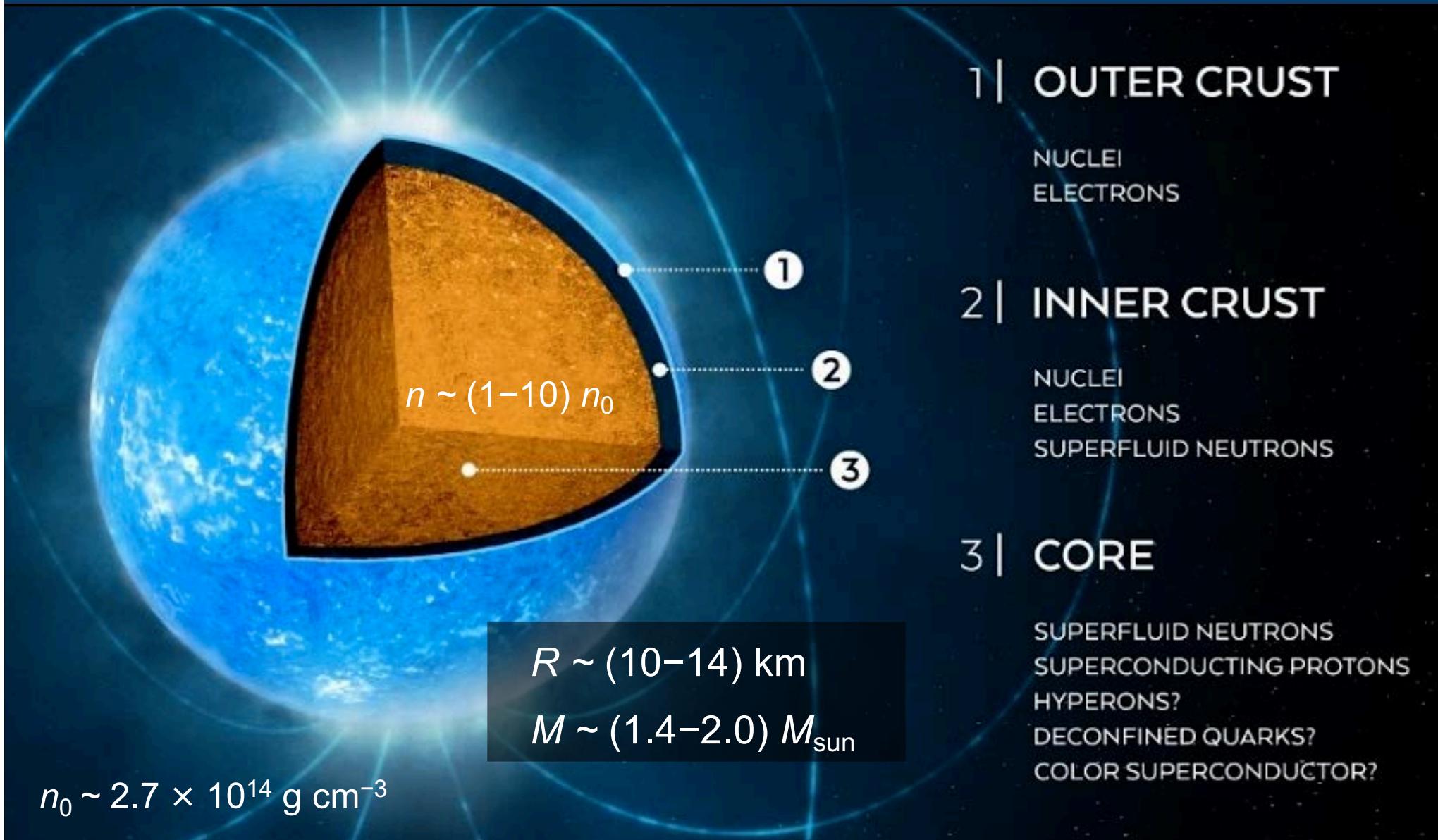


Quantifying uncertainties in the nuclear-matter equation of state

Neutron stars

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e.g., Watts *et al.*, Rev. Mod. Phys. **88**, 021001

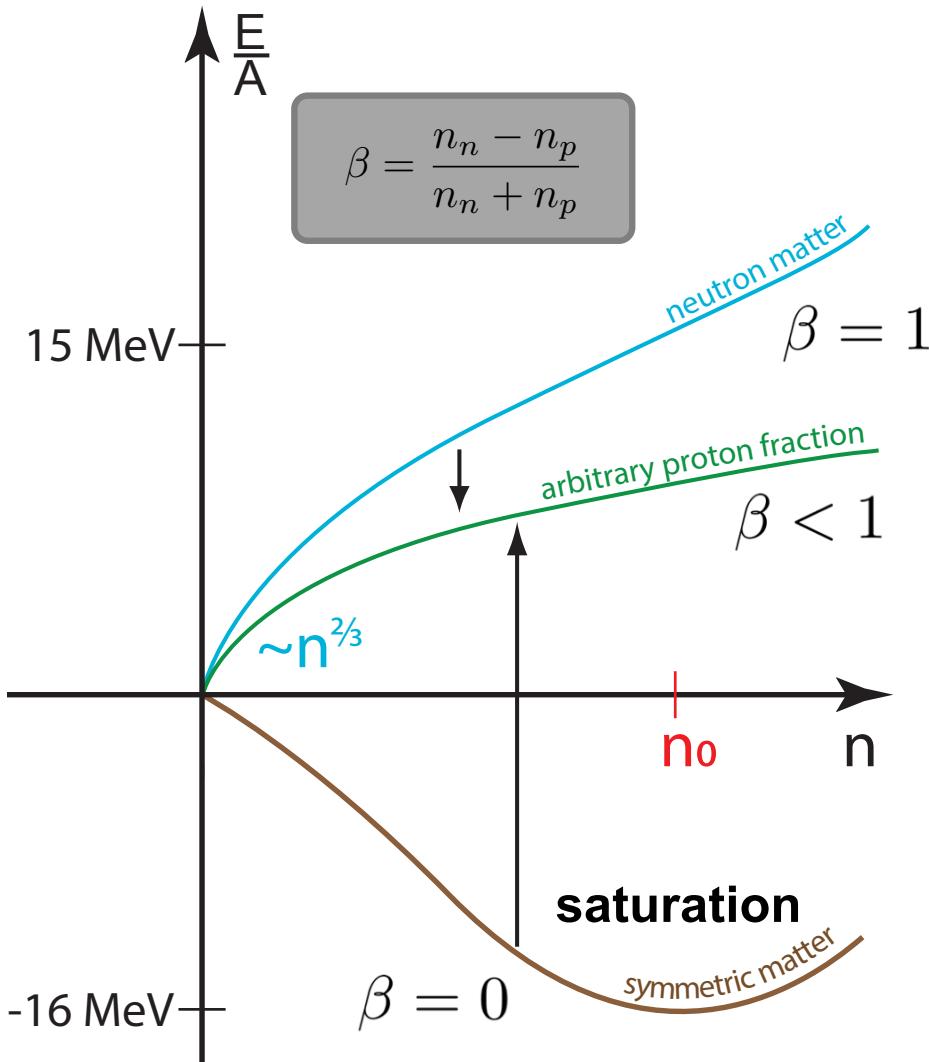


Quantifying uncertainties in the nuclear-matter equation of state

Landscape of nuclear matter

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e.g., Hebeler, Holt *et al.*, ARNP 65, 457



great progress in predicting the **EOS** of infinite matter and the structure of **neutron stars** at densities $\lesssim n_0$

Hebeler, Lattimer *et al.*, APJ 773, 11
Carbone, Rios *et al.*, PRC 88, 044302
Hagen, Papenbrock *et al.*, PRC 89, 014319
Coraggio, Holt *et al.*, PRC 89, 044321
Wellenhofer, Holt *et al.*, PRC 89, 064009
CD, Carbone *et al.*, PRC 94, 054307
Ekström, Hagen *et al.*, PRC 97, 024332
Roggero, Mukherjee *et al.*, PRL 112, 221103
CD, Hebeler *et al.*, PRL 122, 042501
Lonardoni, Tews *et al.*, PRR 2, 022033(R)
Piarulli, I. Bombaci *et al.*, PRC 101, 045801
...

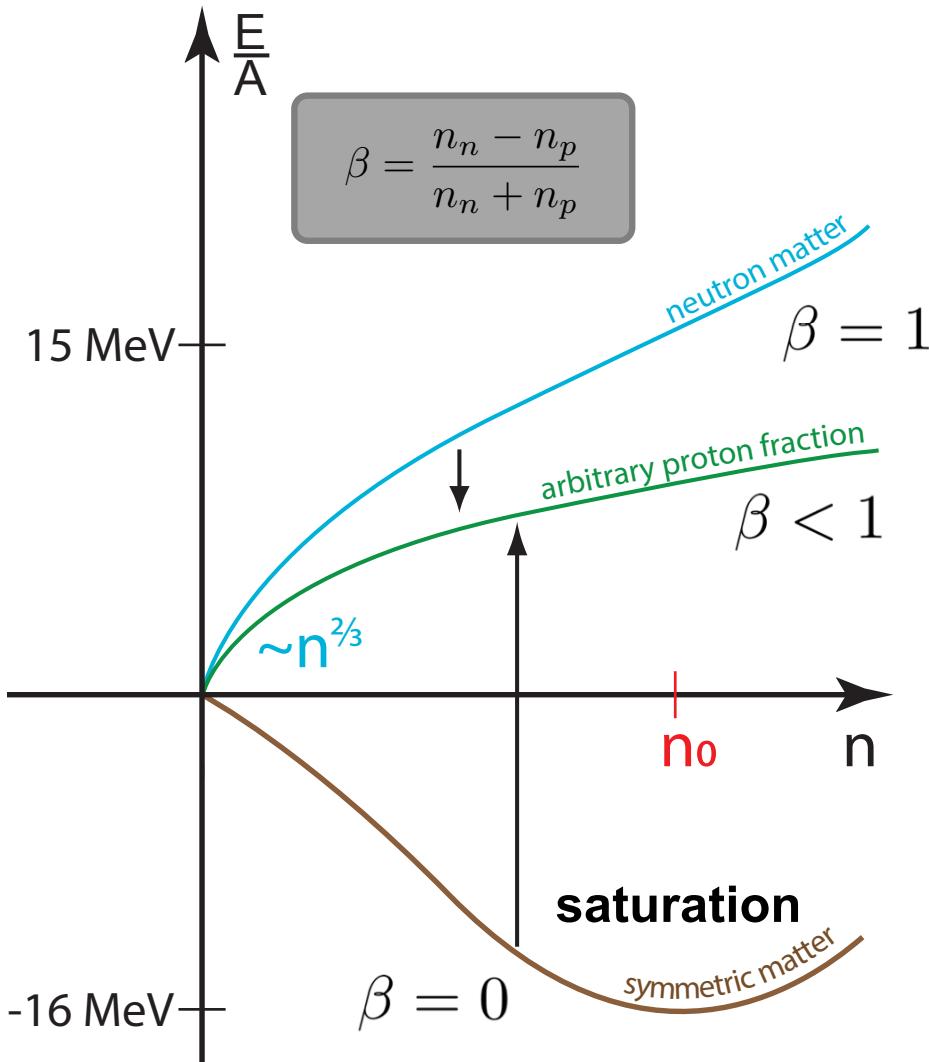
But: existing predictions **only** provided **rough estimates** for the with-density-growing EFT truncation error, without accounting for **correlations**

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needed: *statistically meaningful comparisons* between nuclear **theory** and recent **observational constraints**

Lonardoni, Tews *et al.*, PRR 2, 022033(R)
Piarulli, I. Bombaci *et al.*, PRC 101, 045801

...

But: existing predictions **only** provided **rough estimates** for the with-density-growing EFT truncation error, without accounting for **correlations**

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Direct detection of gravitational waves

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ligo.caltech.edu



- + Virgo
- + GEO600
- + ...

Binary Neutron-Star Merger

multi-messenger
astronomy



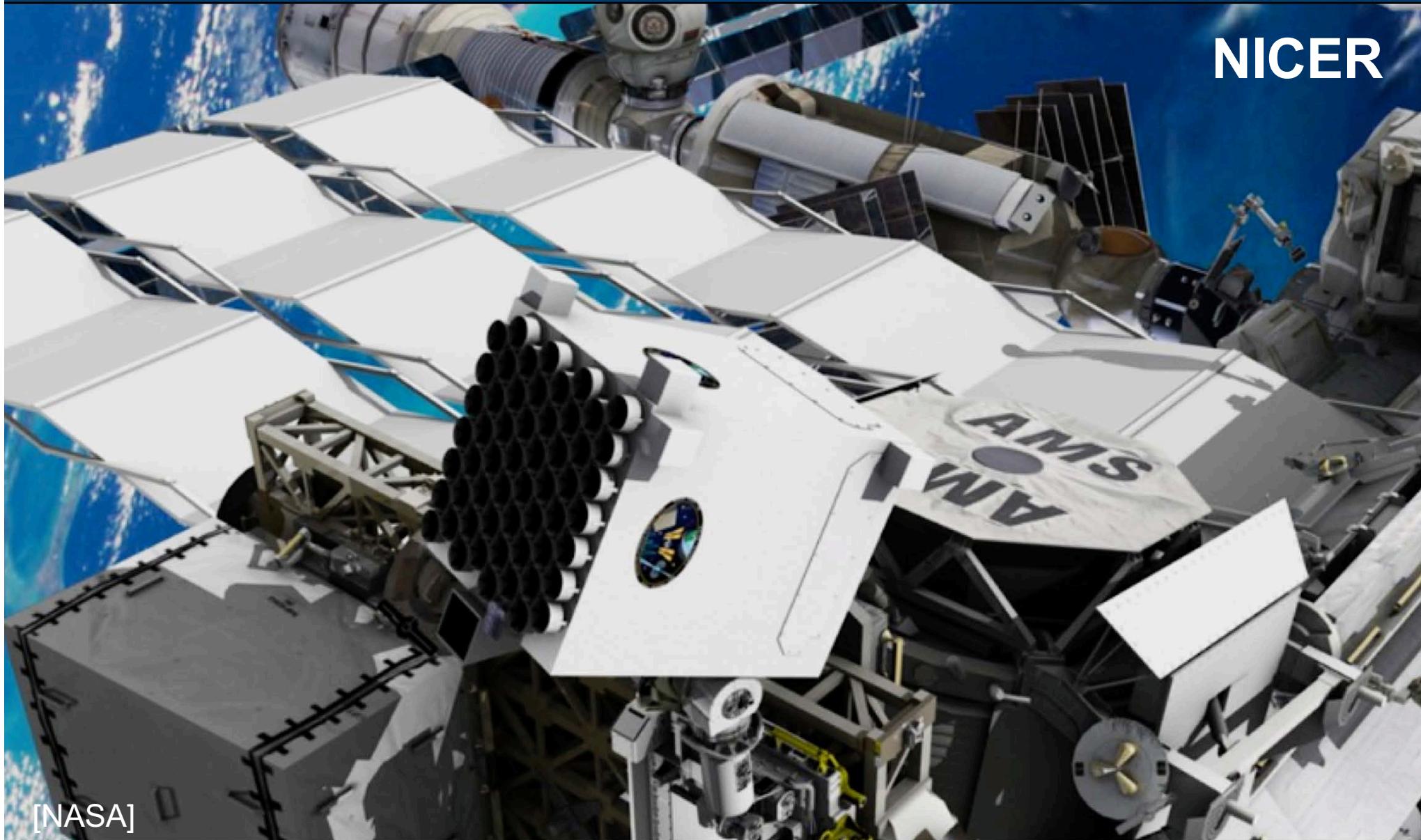
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(Simultaneous) Mass-radius constraints

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see also Greif *et al.*, MNRAS 485, 4

NICER

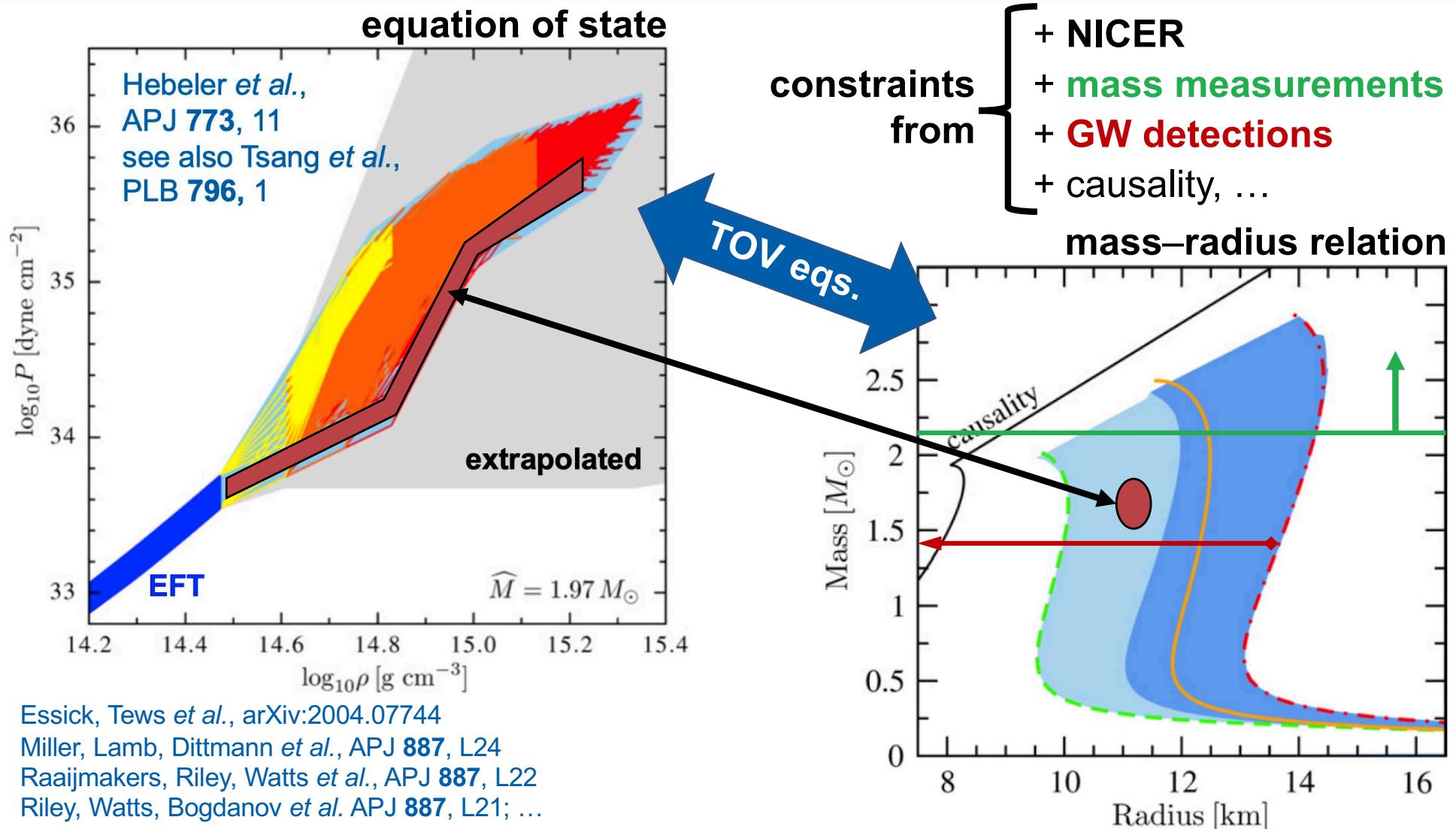


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(Simultaneous) Mass-radius constraints

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see also Greif *et al.*, MNRAS 485, 4



Essick, Tews *et al.*, arXiv:2004.07744

Miller, Lamb, Dittmann *et al.*, APJ 887, L24

Raaijmakers, Riley, Watts *et al.*, APJ 887, L22

Riley, Watts, Bogdanov *et al.* APJ 887, L21; ...

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ISNET: Bringing statisticians and physicists together!

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frib.msu.edu



Information and Statistics in Nuclear Experiment and
Theory (ISNET 8)

Facility for Rare Isotope Beams
Michigan State University
December 14 to 18, 2020

Local Organizing Committee:

Pawel Danielewicz	Morten Hjorth-Jensen
Dean Lee	Witek Nazarewicz
Scott Pratt	Betty Tsang
Frederi Viens	



Bayesian Inference in Subatomic Physics – A Marcus Wallenberg Symposium at Chalmers
September 17-20, 2019, organized by: A. Ekström, C. Forssén, and S. Wesolowski
INT Program INT-16-2a: *Bayesian Methods in Nuclear Physics*

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Thanks to my collaborators!

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buqeye.github.io

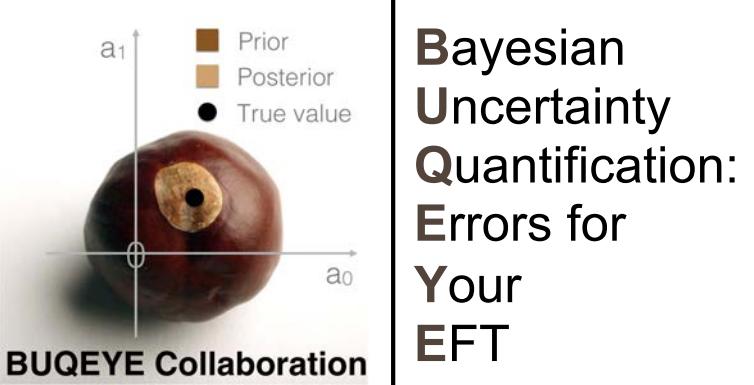


CD, Furnstahl, Melendez, and Phillips

How well do we know the neutron-matter equation of state at the densities inside neutron stars? A Bayesian approach with correlated uncertainties, [arXiv:2004.07232](https://arxiv.org/abs/2004.07232).

CD, Melendez, Furnstahl, and Phillips

Effective Field Theory Convergence Pattern of Infinite Nuclear Matter, [arXiv:2004.07805](https://arxiv.org/abs/2004.07805).



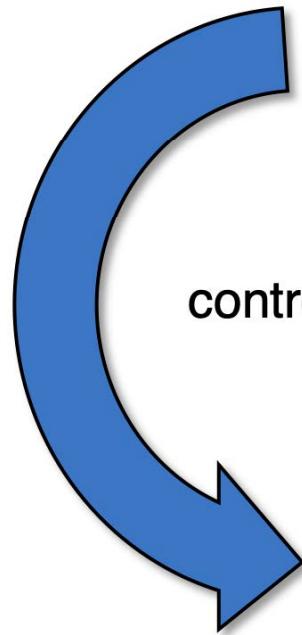
Bayesian
Uncertainty
Quantification:
Errors for
Your
EFT

UQ framework available at
<https://buqeye.github.io>

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CD, Haxton, McElvain, Mereghetti *et al.*, arXiv:1910.07961



nuclear-matter equation of state
statistically rigorous UQ for chiral EFT

many-body perturbation theory
controlled and computational efficient method
many-body uncertainty estimates

chiral effective field theory
systematic expansion of nuclear forces
order-by-order nuclear potentials



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...

observables

many-body
framework

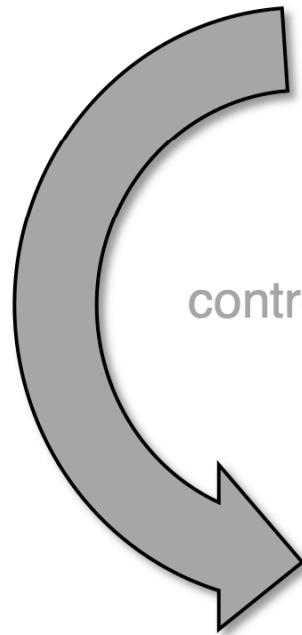
effective
field theory

quantum
chromodynamics

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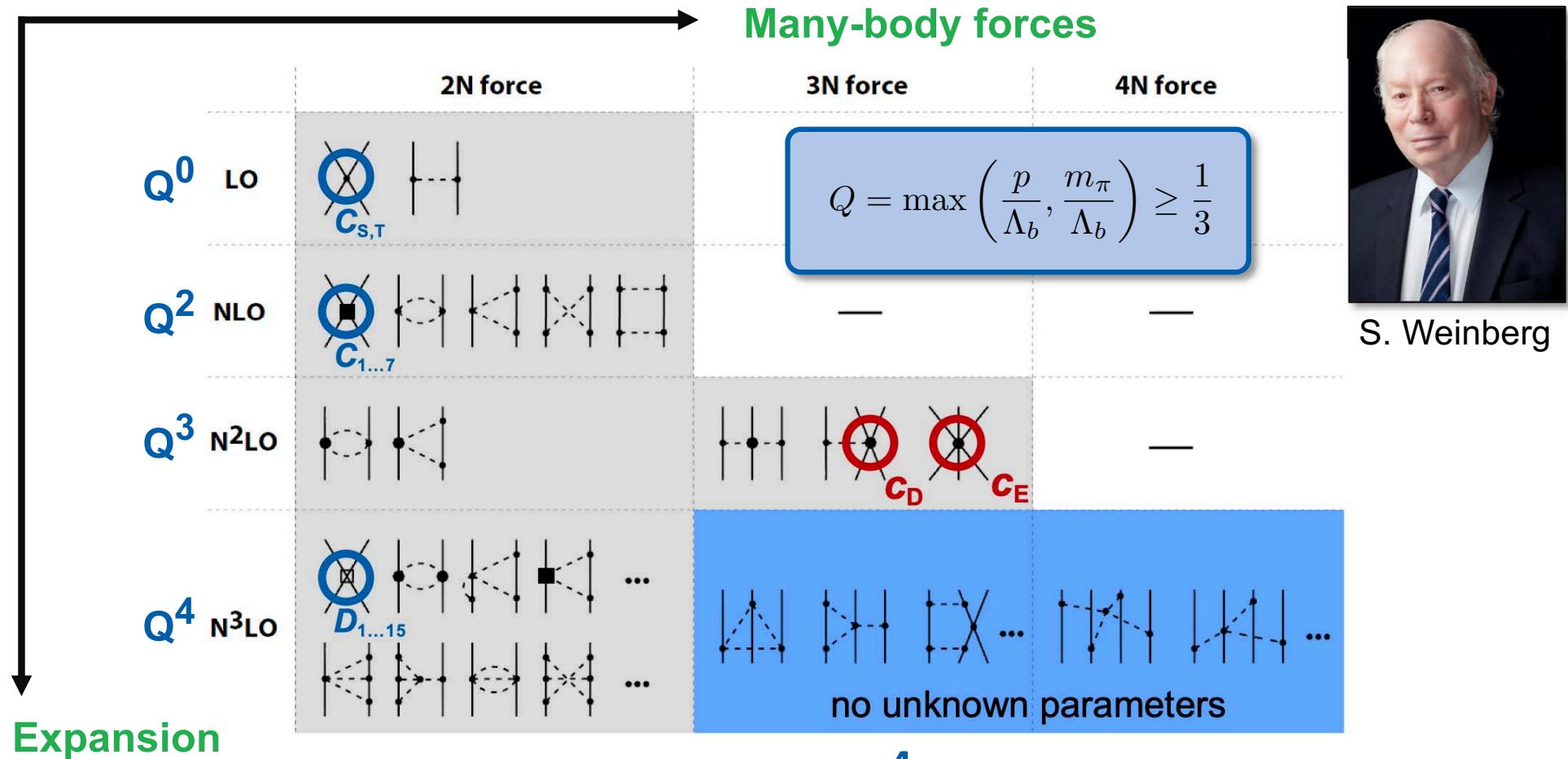
quantum
chromodynamics

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Hierarchy of nuclear forces in chiral EFT

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e.g., Machleidt, Entem, Phys. Rep. 503, 1



Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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Including correlations in UQ is important!

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CD, Melendez *et al.*, arXiv:2004.07805

$$\delta y_{i \geq 2}(x) = \max_{2 \leq j \leq i} (Q^{i+1}(x)|y_0(x)|, Q^{i+1-j}(x) |\Delta y_j(x)|)$$
$$\delta y_0(x) = Q^2(x) |y_0(x)| \quad \text{Epelbaum, Krebs, Mei\ss{}ner, EPJ A 51, 53}$$

x : generic sampling point
 $y_n(x)$: n -th order prediction

The “standard EFT” uncertainty does *not* account for correlations:

Type-x:

Correlations between $y(x)$ and $y(x')$; e.g., $y = E/A(n^{'})$;
quantified and propagated using the ML
truncation-error model proposed by BUQEYE



Melendez, Furnstahl *et al.*, PRC **100**, 044001

Melendez's GSUM, github.com/buqeye/gsum

Type-y:

Correlations between discrete observables $y_i(x)$ and $y_j(x')$;
e.g., an observable i and its derivative j
extension of the truncation error model: now **quantified and propagated** using **multi-output Gaussian Processes** (GPs)

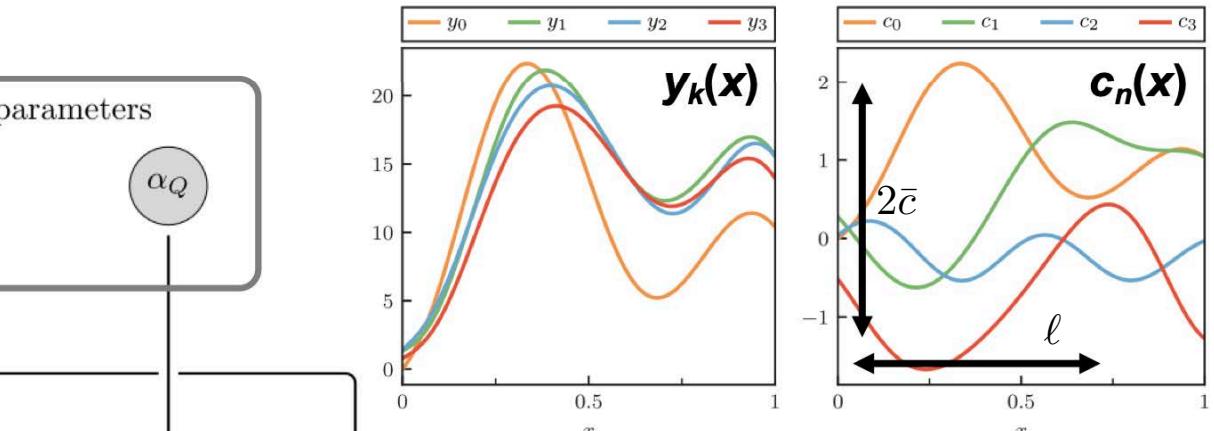
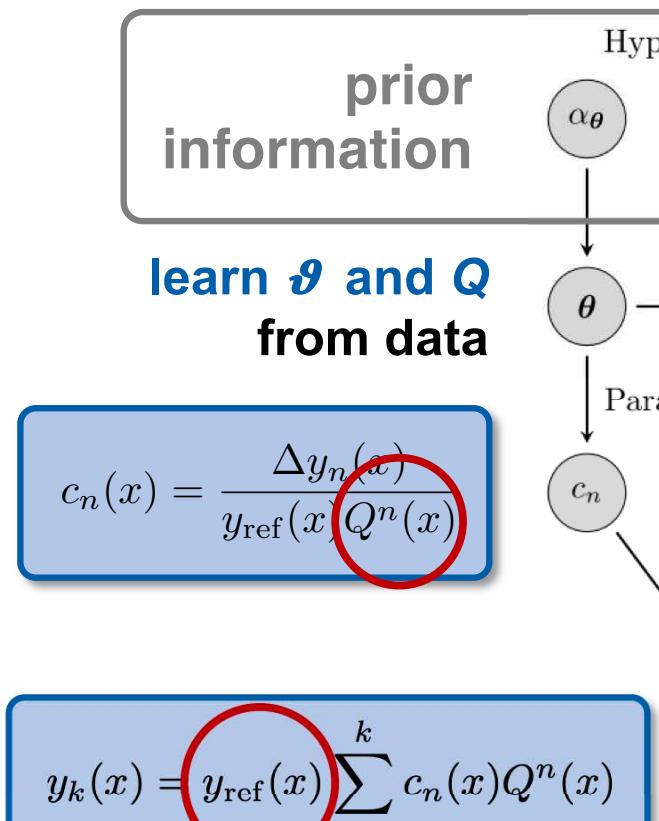
github.com/buqeye/nuclear-matter-convergence

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BUQEYE's EFT truncation-error model

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Melendez, Furnstahl *et al.*, PRC 100, 044001



$$c_n(x) \mid \bar{c}^2, \ell \sim \text{GP}[0, \bar{c}^2 r(x, x'; \ell)]$$

model all $c_n(x)$ as independent draws from a single GP

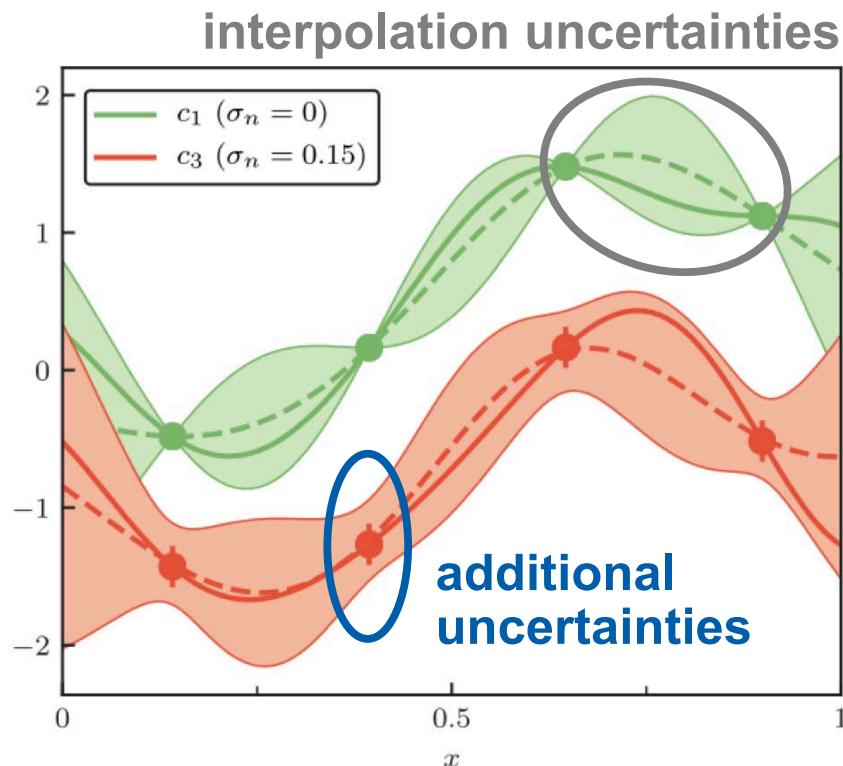
$$\delta y_k(x) = y_{\text{ref}}(x) \sum_{n=k+1}^{\infty} c_n(x) Q^n(x)$$

Quantifying uncertainties in the nuclear-matter equation of state

Using GPs for interpolation

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CD, Melendez *et al.*, arXiv:2004.07805



additional (many-body) **uncertainties** can be **easily included**

GPs are **closed** under **differentiation**
straightforward to compute derivatives
with correlations accounted for

e.g., **pressure** and **speed of sound**

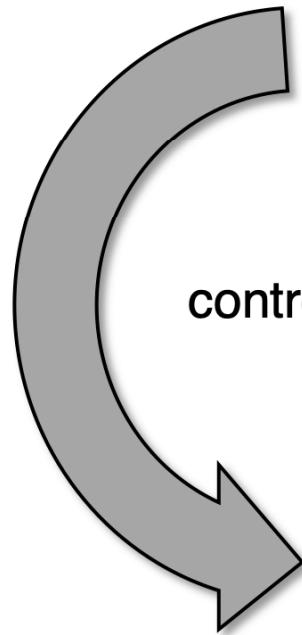
» **in contrast to other methods**

Gaussian Process interpolates the EOS while quantifying uncertainties, including **to-all-orders EFT truncation errors**

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CD, Haxton, McElvain, Mereghetti *et al.*, arXiv:1910.07961



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Efficient Monte Carlo framework

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CD, Hebeler, Schwenk, PRL 122, 042501



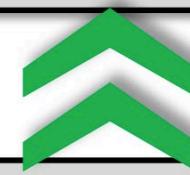
efficient evaluation of **MBPT diagrams**
with **NN**, **3N**, and **4N forces** (single-particle basis)

- **implementing diagrams** has become **straightforward** (incl. particle-hole terms)
- multi-dimensional momentum integrals:
(improved) Lepage's VEGAS algorithm
- **controlled computation** of arbitrary interaction and many-body diagrams

EOS up to high orders



automatic code generation



analytic form of the diagrams

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Significant challenges are past!

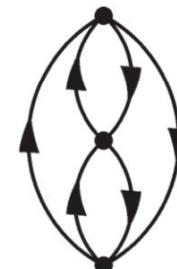
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CD, Hebeler, Schwenk, PRL 122, 042501



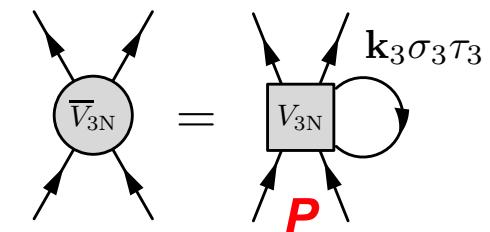
Higher orders: particle-hole contributions

Coraggio *et al.*, PRC 89, 044321; Holt, Kaiser, PRC 95, 034326



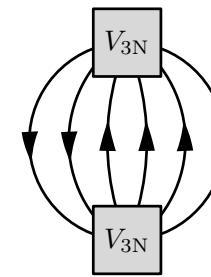
No approximations for 3N normal ordering

CD *et al.*, PRC 93, 054314; Holt *et al.*, PRC 81, 024002



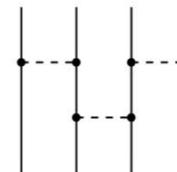
Include residual 3N diagram(s)

Hagen *et al.*, PRC 89, 014319; Kaiser, EPJ A 48, 58



Higher many-body forces

Hebeler *et al.*, PRC 91, 044001



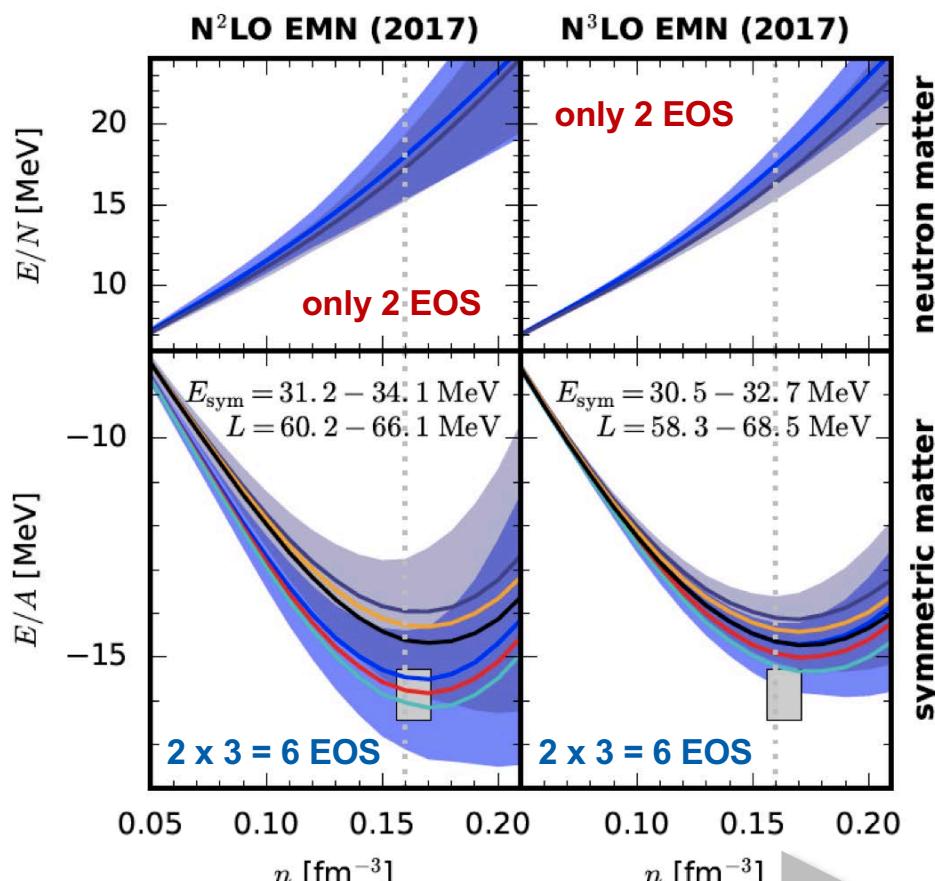
development of a novel
Monte Carlo framework

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Many-body input of our UQ

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CD, Hebeler, Schwenk, PRL 122, 042501



extended up to $2n_0$

use the Monte Carlo framework to evaluate the EOS up to N^3LO

bare N^2LO / N^3LO EMN potentials

Entem, Machleidt, Nosyk, PRC 96, 024004
Hoppe, CD, Furnstahl *et al.*, PRC 96, 054002

fit 3N interactions to the triton and nuclear **saturation point**

identified (c_D, c_E) which **saturate** close to the empirical **saturation point**

3 Hamiltonians available for each cutoff and chiral order; **in total: 12**

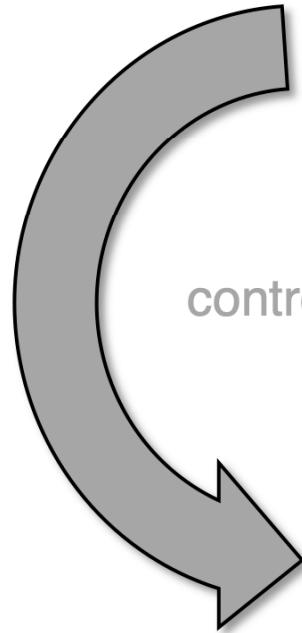
Λ/c_D [MeV]/[1]	
450/2.25	500/ - 1.75
450/2.50	500/ - 1.50
450/2.75	500/ - 1.25

Λ/c_D [MeV]/[1]	
450/0.00	500/ - 3.00
450/0.25	500/ - 2.75
450/0.50	500/ - 2.50

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CD, Haxton, McElvain, Mereghetti *et al.*, arXiv:1910.07961



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NPLQCD

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observables

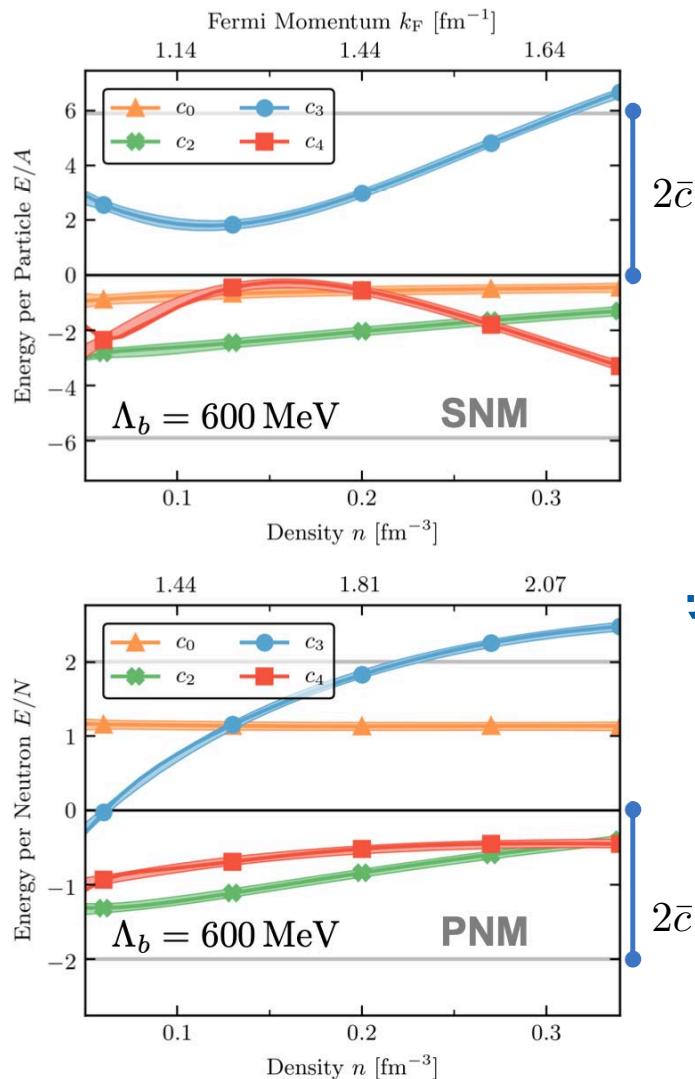
many-body framework

effective field theory

quantum chromodynamics

Quantifying uncertainties in the nuclear-matter equation of state

Extracting the observable coefficients



assumptions

We need to choose:

$$c_n(x) = \frac{\Delta y_n(x)}{y_{\text{ref}}(x)Q^n(x)}$$

\mathbf{x} \vdots γk_F or $n = g \frac{k_F^3}{6\pi^2}$ $\gamma > 0$
stationary kernel: same curviness across entire range

$y_{\text{ref}}(\mathbf{x})$ \vdots $y_{\text{ref}}(k_F) = 16 \text{ MeV} \times \left(\frac{\gamma k_F}{k_{F,0}} \right)^2$
naturalness: overall scale for $c_n(x)$

$Q(\mathbf{x})$ \vdots $Q(k_F) = \frac{p}{\Lambda_b}$ $p = \gamma k_F$
What is the soft scale of nuclear matter?

Trust, but verify
model-checking diagnostics $\gamma = 1$

feedback

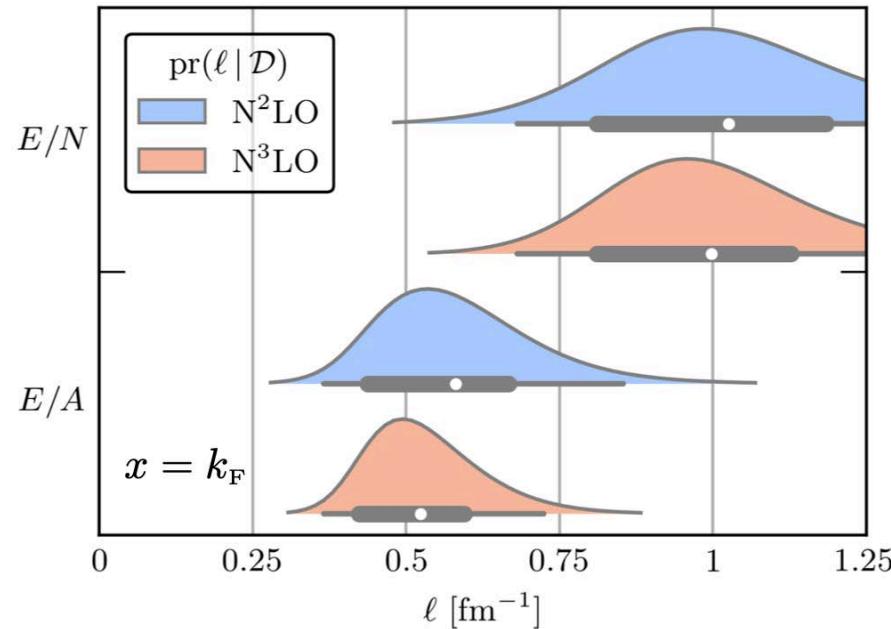
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Bayesian inference from the data

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How correlated
is nuclear matter ?

$\text{pr}(\ell | \mathcal{D})$
correlation length

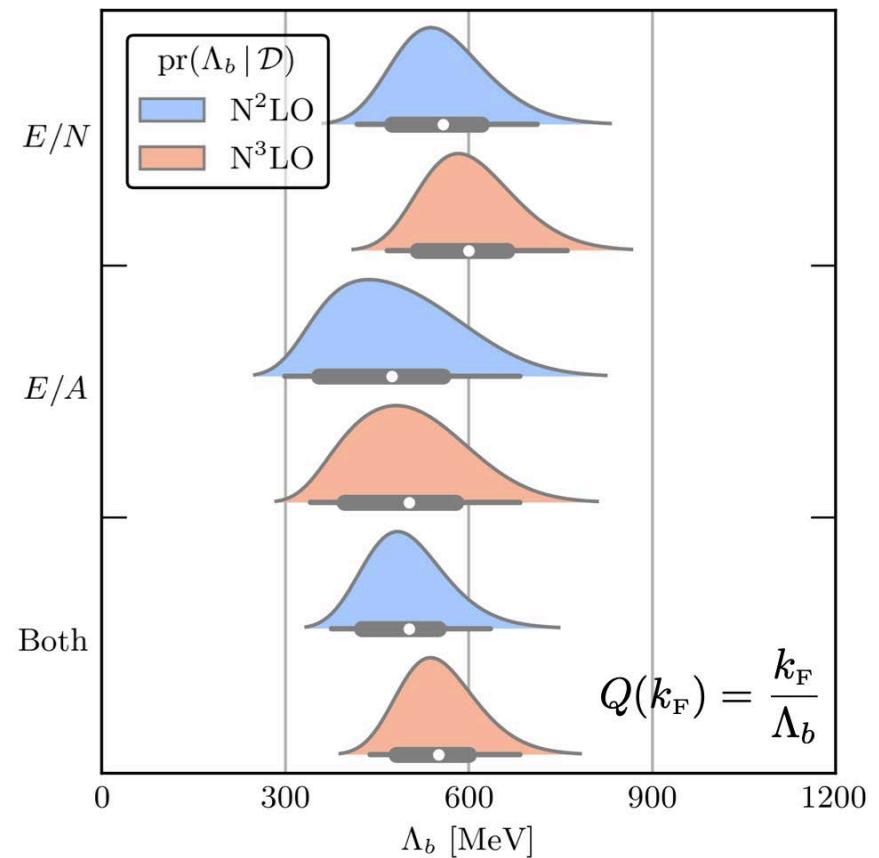


to be
compared with

$$k_F^{\max} = \begin{cases} 2.2 \text{ fm}^{-1} & \text{PNM} \\ 1.7 \text{ fm}^{-1} & \text{SNM} \end{cases}$$

Where does the
EFT break down ?

$\text{pr}(\Lambda_b | \mathcal{D})$
breakdown scale



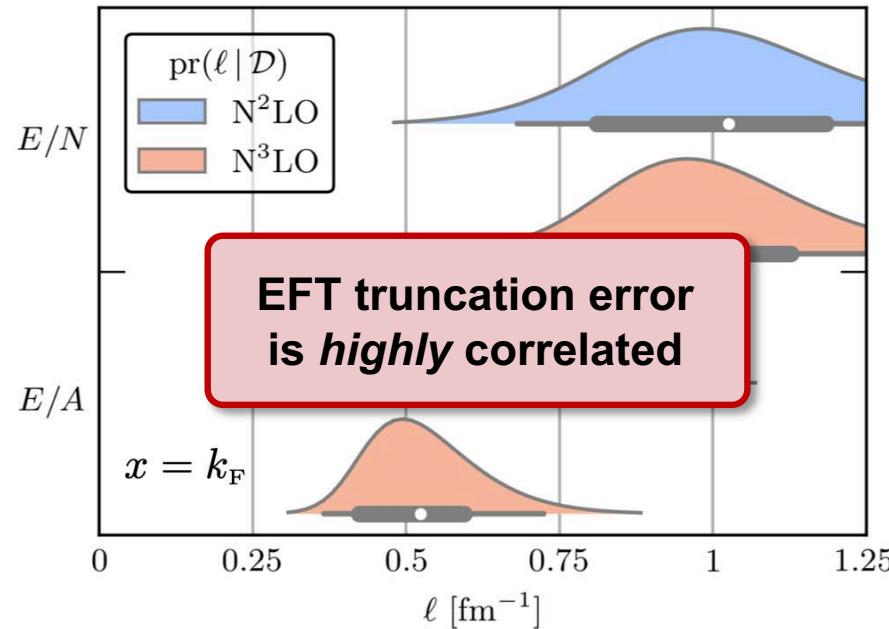
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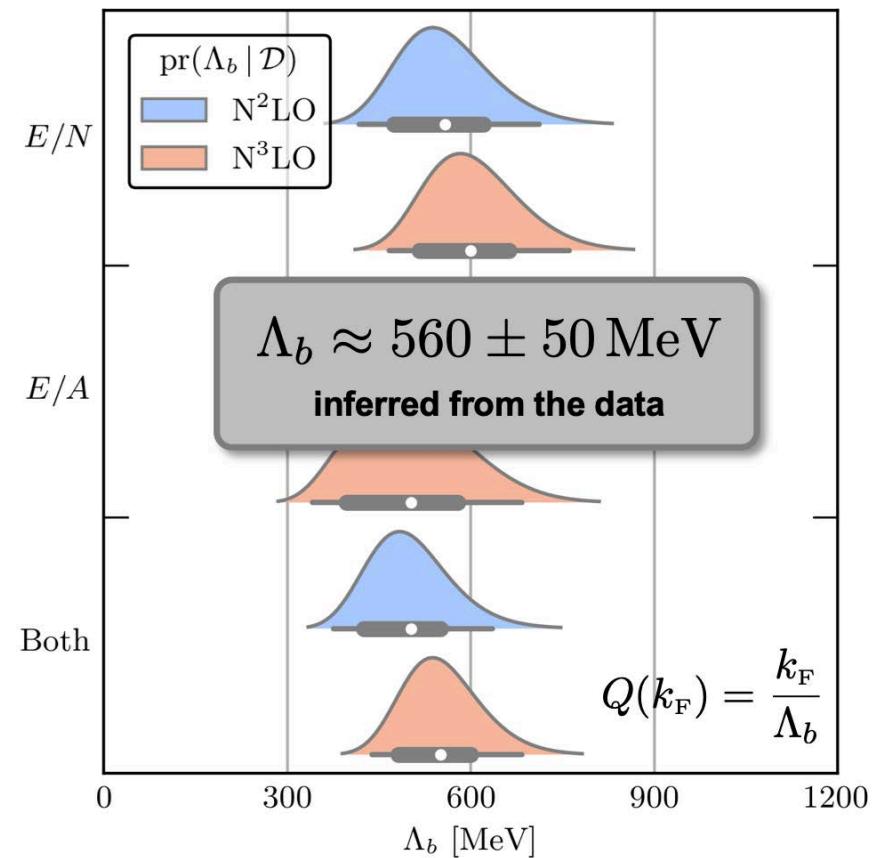


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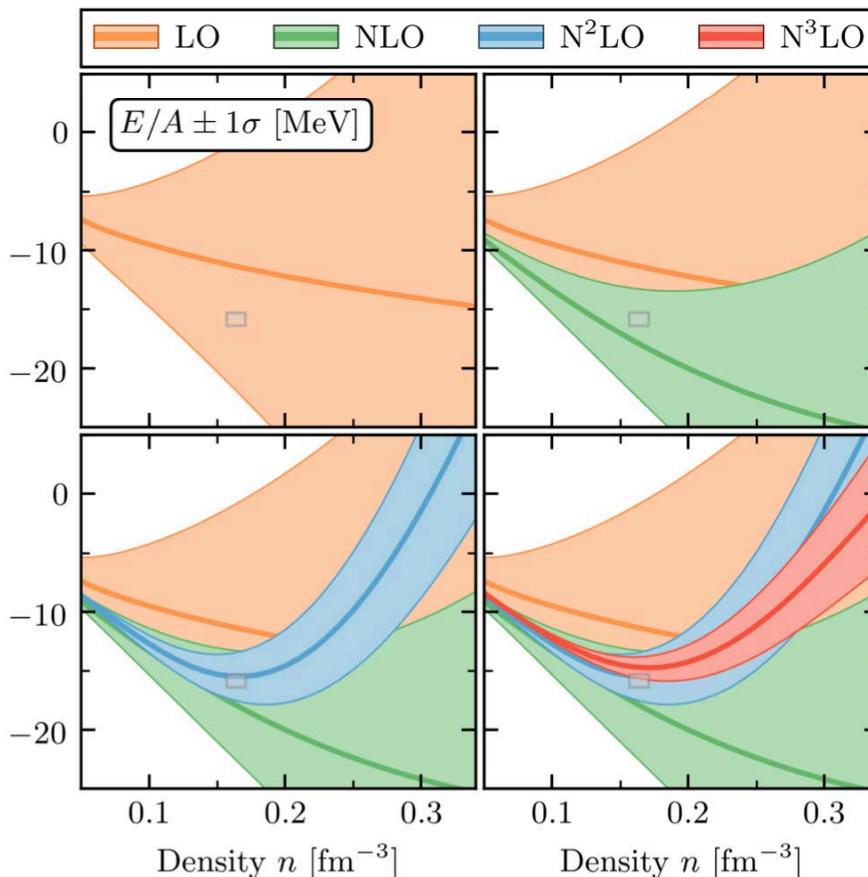
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SNM and the saturation point

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CD, Melendez *et al.*, arXiv:2004.07805

$\Lambda = 450 \text{ MeV}$



$$\text{pr} \left(\frac{E}{A}(n_0), n_0 \mid \mathcal{D} \right)$$

see also CD, Hebeler, Schwenk, PRL 122, 042501

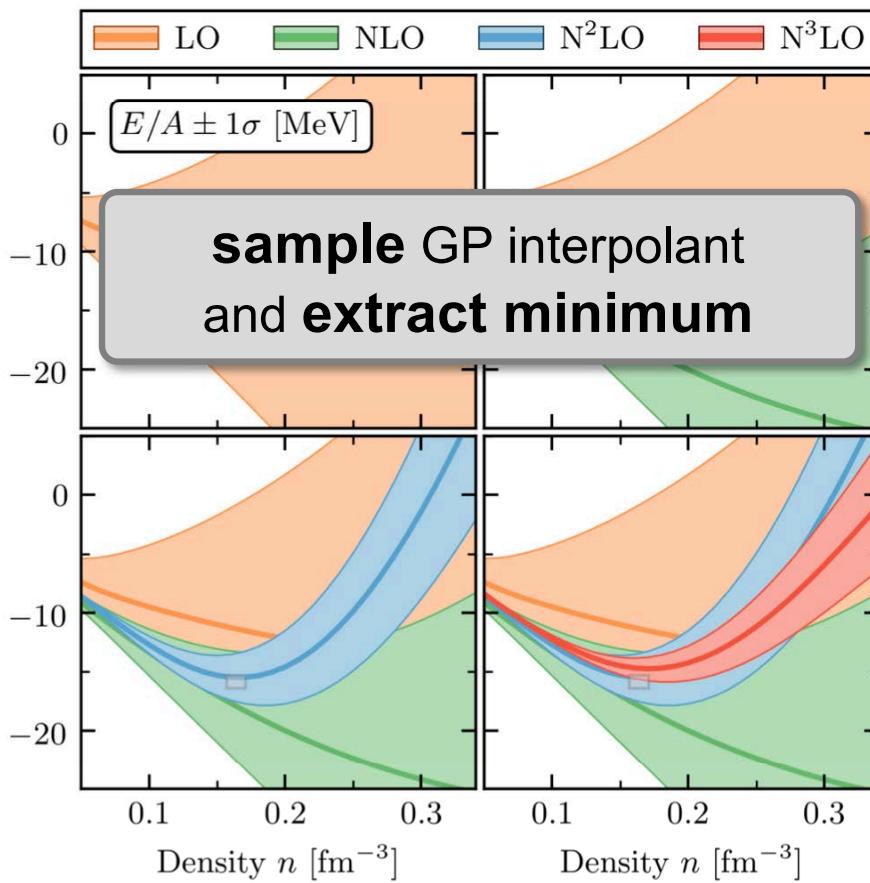
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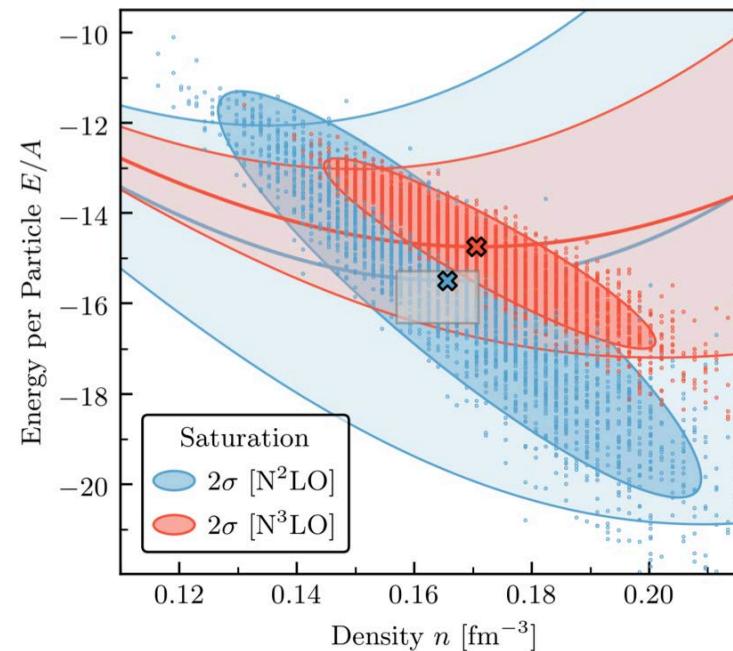
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$\Lambda = 450 \text{ MeV}$



$$\text{pr} \left(\frac{E}{A}(n_0), n_0 \mid \mathcal{D} \right)$$



two-dimensional Gaussian

$$\begin{bmatrix} n_0 \\ \frac{E}{A}(n_0) \end{bmatrix} \approx \begin{bmatrix} 0.173 \\ -14.9 \end{bmatrix} \quad \Sigma \approx \begin{bmatrix} 0.014^2 & -0.014 \\ -0.014 & 1.1^2 \end{bmatrix}$$

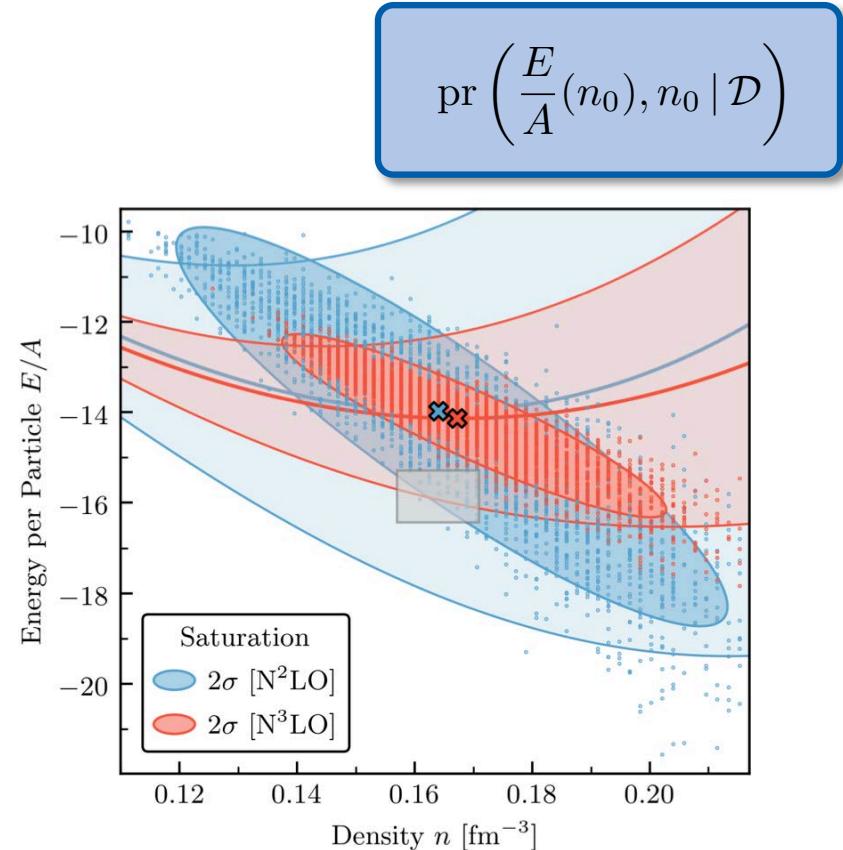
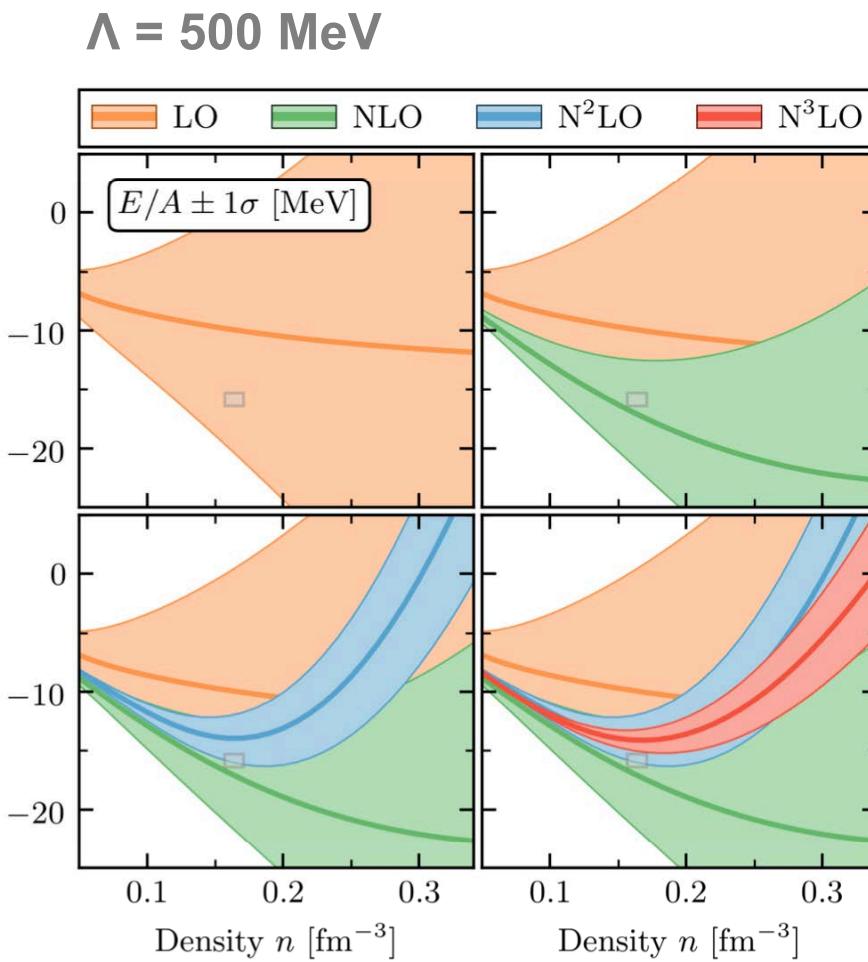
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two-dimensional Gaussian

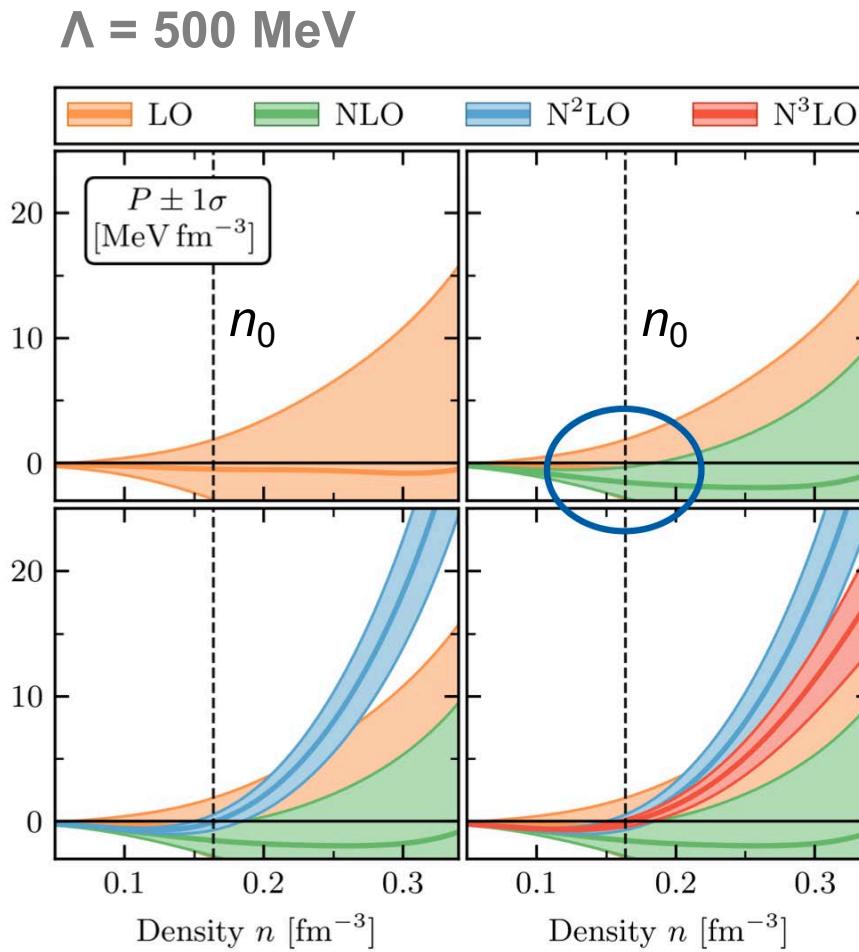
$$\begin{bmatrix} n_0 \\ \frac{E}{A}(n_0) \end{bmatrix} \approx \begin{bmatrix} 0.170 \\ -14.3 \end{bmatrix} \quad \Sigma \approx \begin{bmatrix} 0.016^2 & -0.015 \\ -0.015 & 1.0^2 \end{bmatrix}$$

Quantifying uncertainties in the nuclear-matter equation of state

Pressure of SNM

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CD, Melendez *et al.*, arXiv:2004.07805



$$P(n) = n^2 \frac{d}{dn} \frac{E}{A}(n)$$

! **LO / NLO:** nuclear saturation at $\sim n_0$ could be achieved within the large uncertainties possible zero-crossing over a wide range of densities suggests that the nuclear **saturation point is *fine tuned***

$N^2\text{LO} / N^3\text{LO}$: consistently within the bands of the previous orders, $n \lesssim n_0$, but diverge beyond

be careful: bands are difficult to gauge due to the *long* correlation length (small effective sample size)

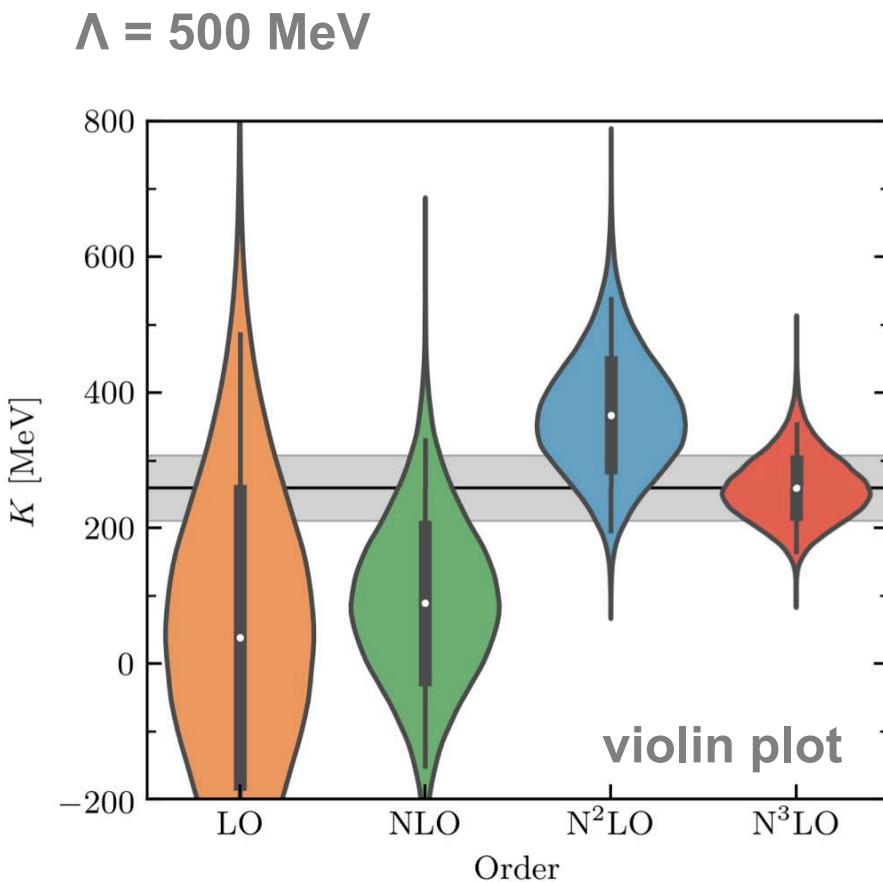
» **model-checking diagnostics**

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Incompressibility of SNM

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CD, Melendez *et al.*, arXiv:2004.07805



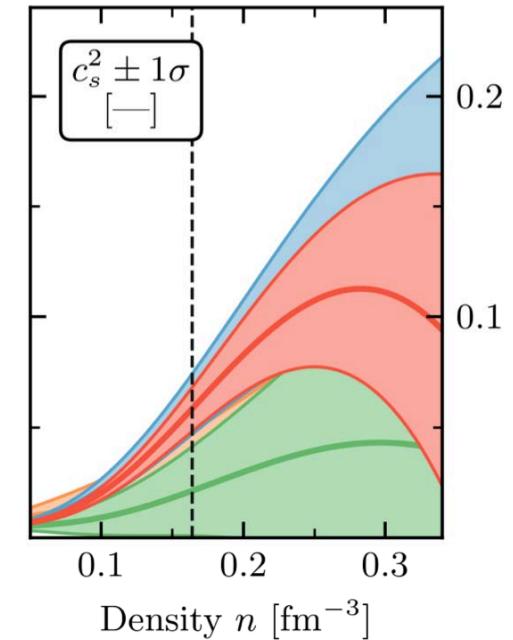
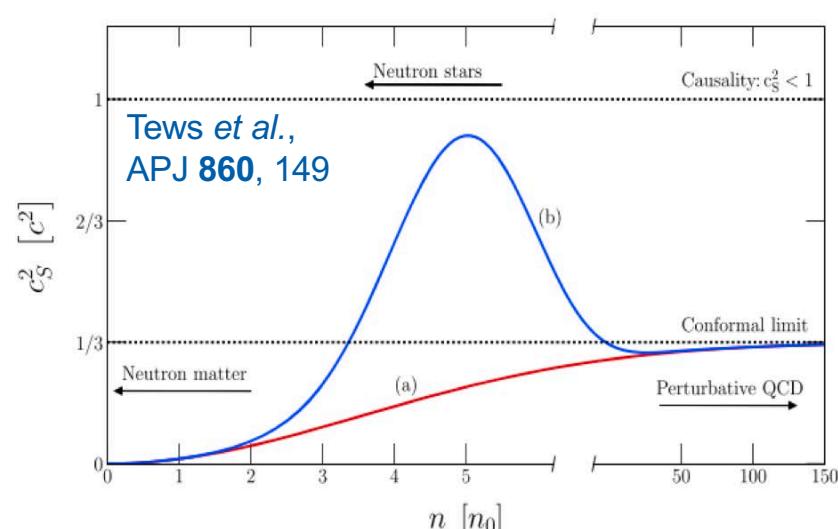
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Speed of sound and pressure in PNM

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CD, Furnstahl *et al.*, arXiv:2004.07805

$$c_s^2(n) = \frac{\partial P(n)}{\partial \varepsilon(n)}$$



LO NLO N²LO N³LO

Quantifying uncertainties in the nuclear-matter equation of state

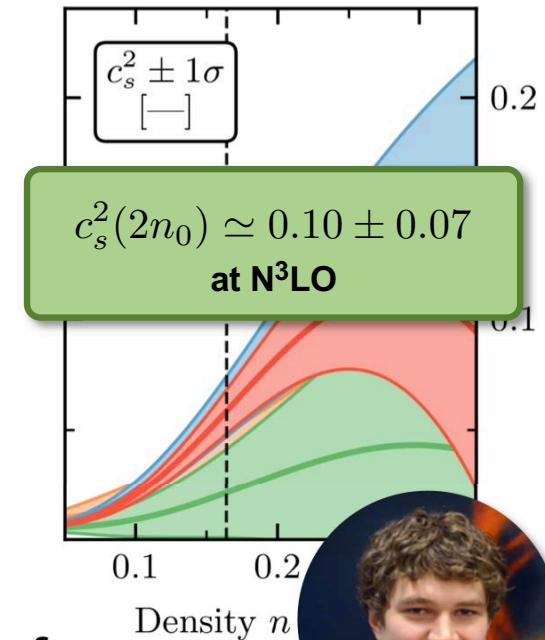
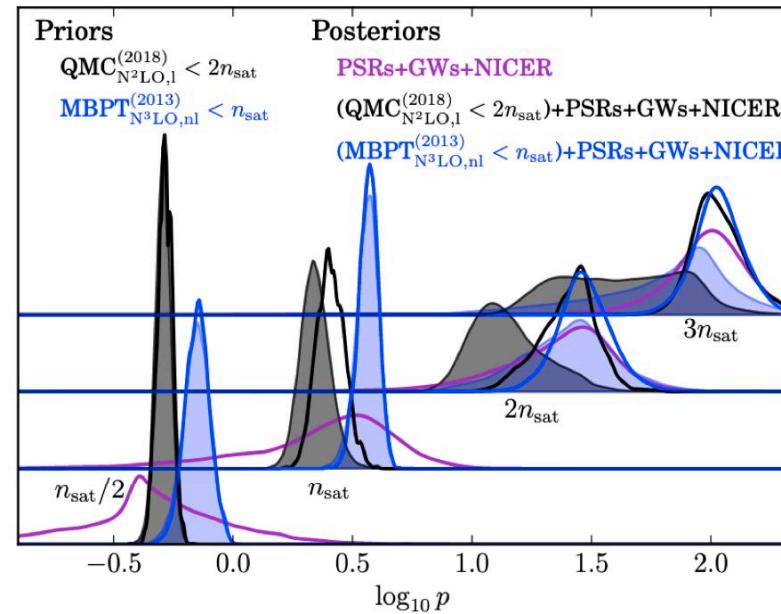
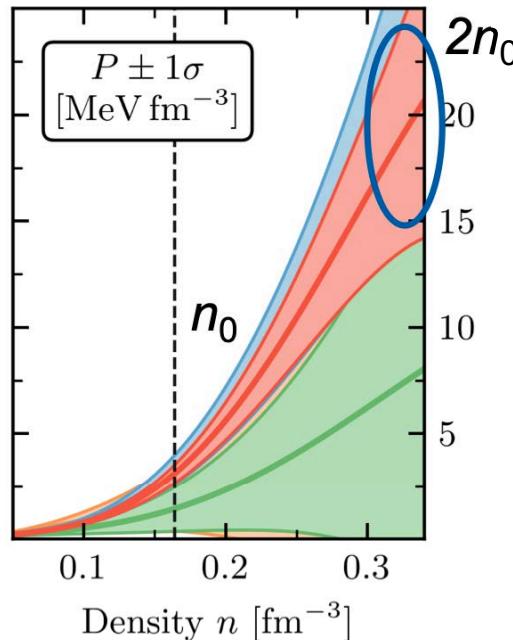
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CD, Furnstahl *et al.*, arXiv:2004.07805

$$P(n = 0.32 \text{ fm}^{-3}) \approx \begin{cases} 20 \pm 6 \text{ MeV fm}^{-3} & \text{MBPT: nonlocal} \\ 15 \pm 5 \text{ MeV fm}^{-3} & \text{QMC: local } V_{E,1} \end{cases}$$

$$c_s^2(n) = \frac{\partial P(n)}{\partial \varepsilon(n)}$$



$$P(n) = n^2 \frac{d}{dn} \frac{E}{N}(n)$$

Direct astrophysical tests of chiral EFT at supranuclear densities
Essick, Tews, Landry, Reddy, Holz, arXiv:2004.07744

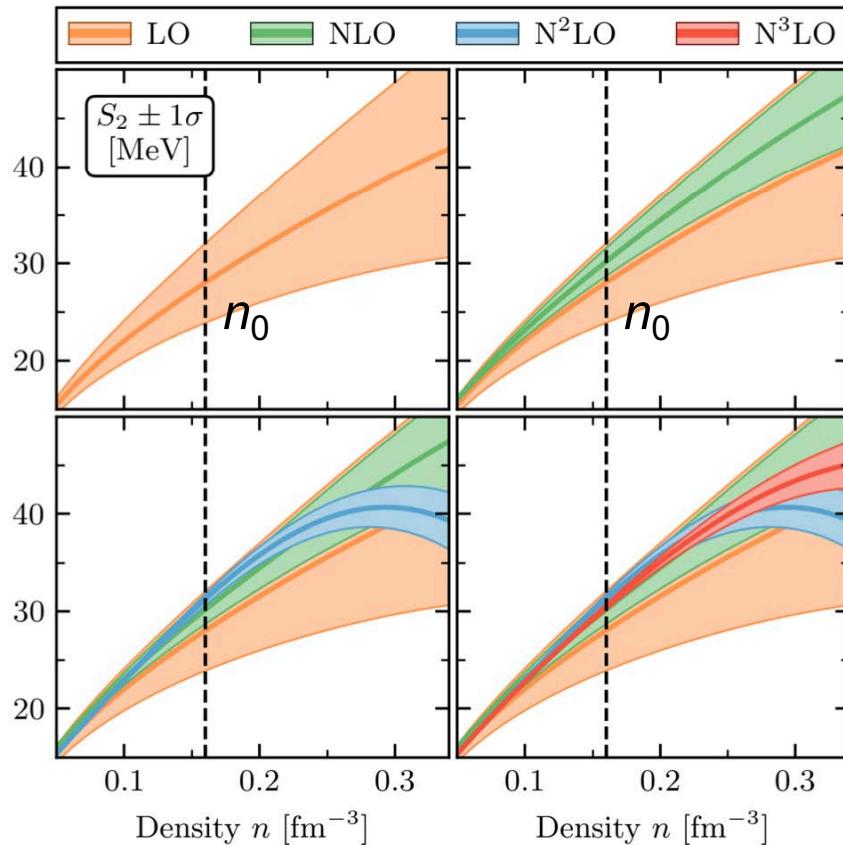
LO NLO N²LO N³LO

Quantifying uncertainties in the nuclear-matter equation of state

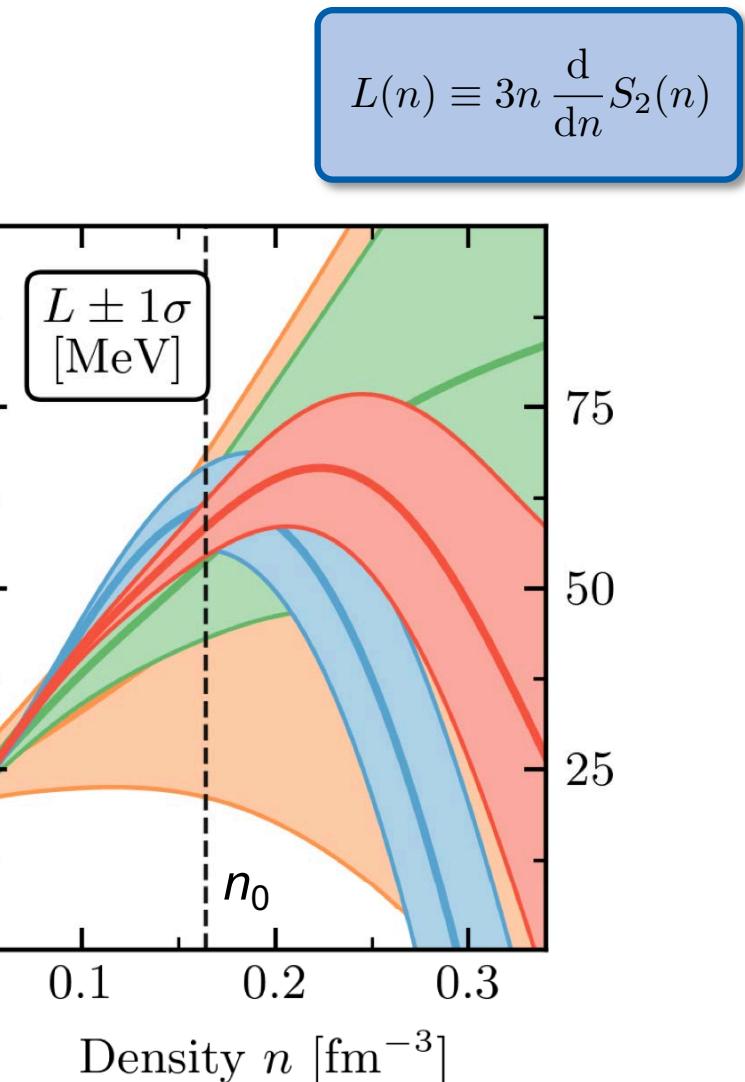
Nuclear symmetry energy

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CD, Furnstahl *et al.*, arXiv:2004.07805



$$S_2(n) \approx \frac{E}{N}(n) - \frac{E}{A}(n)$$

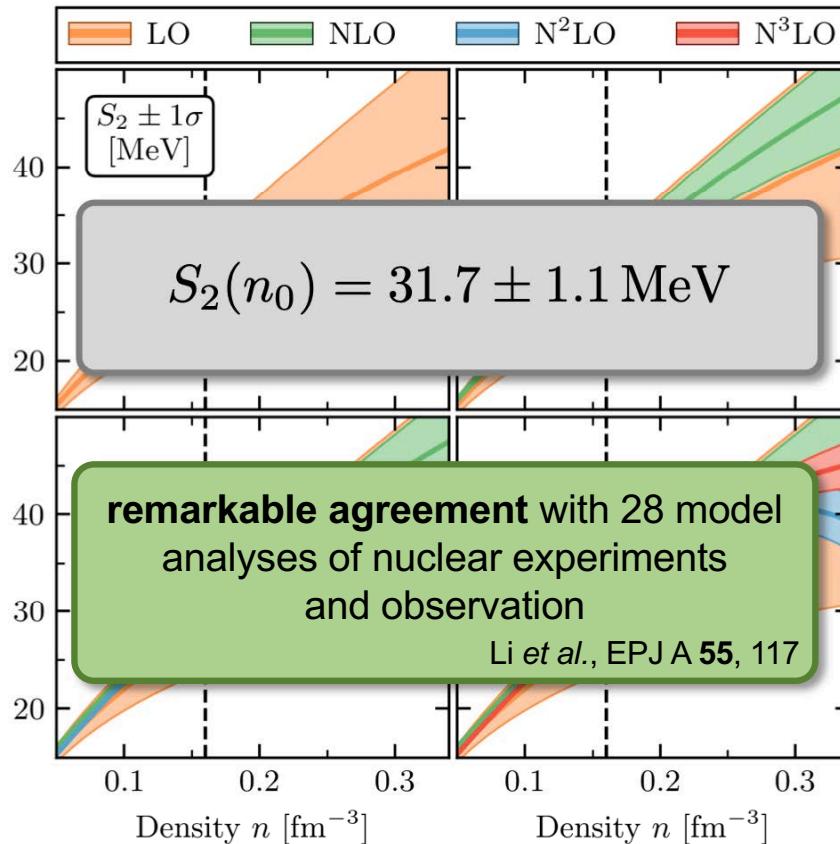


Quantifying uncertainties in the nuclear-matter equation of state

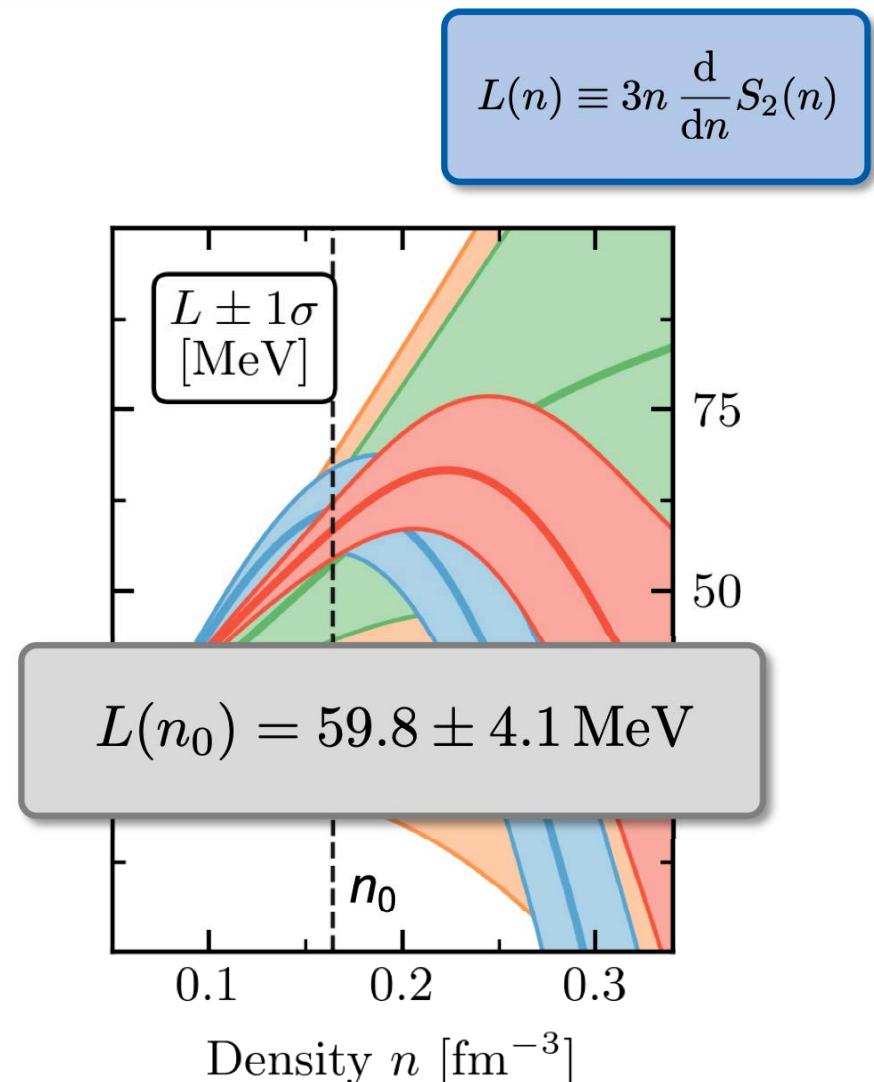
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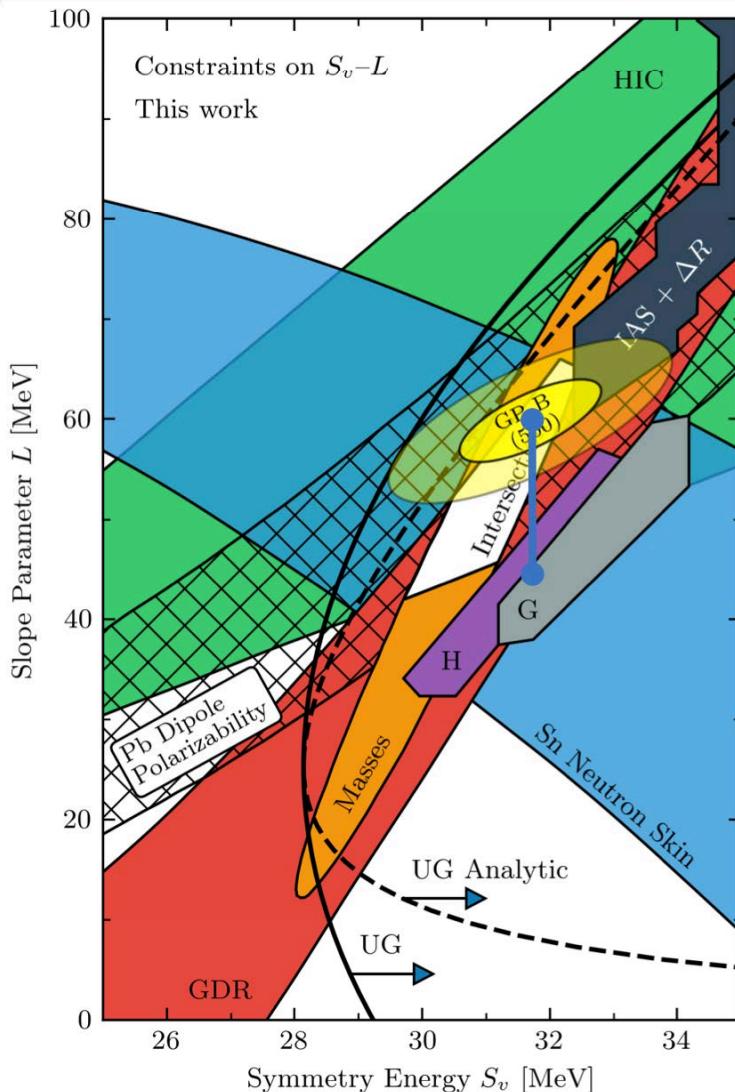


Quantifying uncertainties in the nuclear-matter equation of state

S_v – L correlation (as compiled by Lattimer *et al.*)

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CD, Furnstahl *et al.*, arXiv:2004.07805



$$S_2(n) \equiv S_v + \frac{L}{3} \left(\frac{n - n_0}{n_0} \right) + \dots$$



Excellent agreement with experiment
Lattimer and Lim, APJ 771, 51

$$\text{pr}(S_v, L | \mathcal{D}) = \int dn_0 \text{pr}(S_2, L | n_0, \mathcal{D}) \text{pr}(n_0 | \mathcal{D})$$

$$\text{pr}(n_0 | \mathcal{D}) \approx 0.17 \pm 0.01 \text{ fm}^{-3}$$

2σ ellipse (light yellow) is completely within the conjectured unitary gas limit
predicted range in **S_v agrees** with other **theoretical constraints**; but ~ 15 MeV stronger density-dependence of $S_2(n_0)$

two-dimensional Gaussian

$$\begin{bmatrix} \mu_{S_v} \\ \mu_L \end{bmatrix} = \begin{bmatrix} 31.7 \\ 59.8 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1.24 & 3.27 \\ 3.27 & 16.95 \end{bmatrix}$$

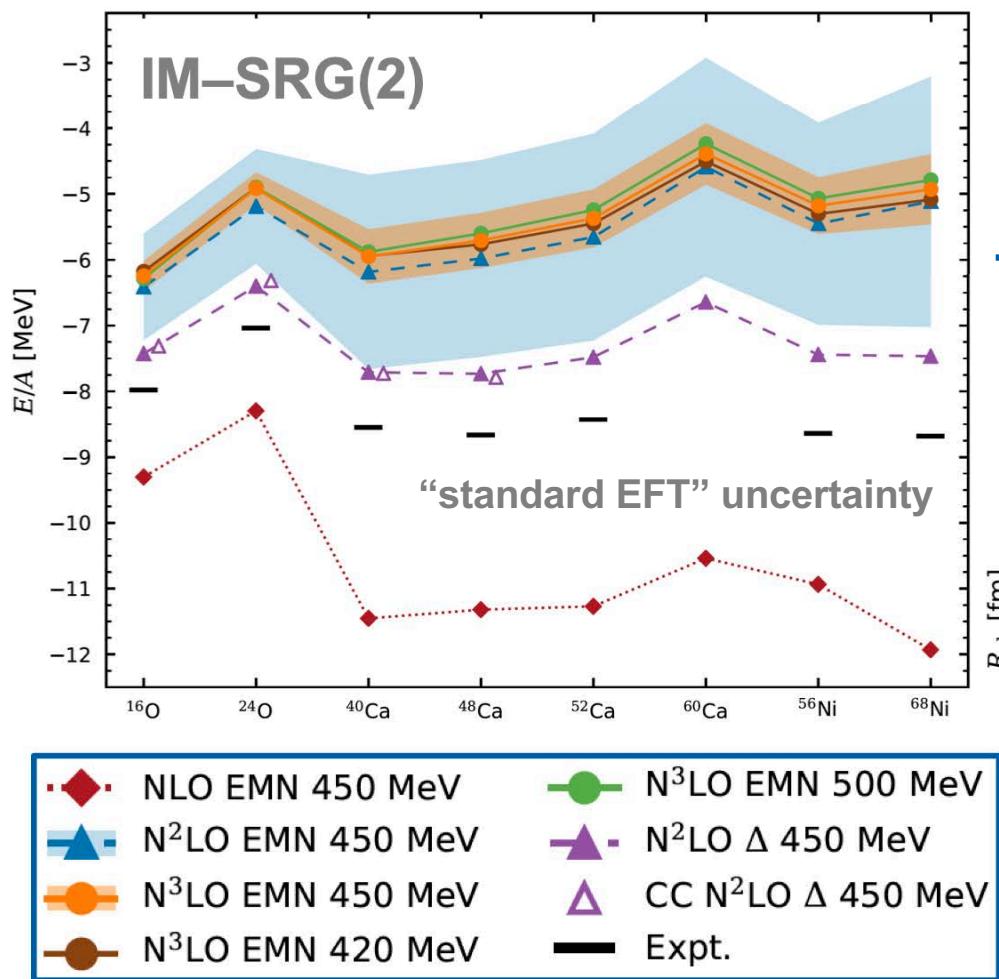
Quantifying uncertainties in the nuclear-matter equation of state

Results for nuclei with these potentials

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Hoppe, CD *et al.*, PRC 100, 024318

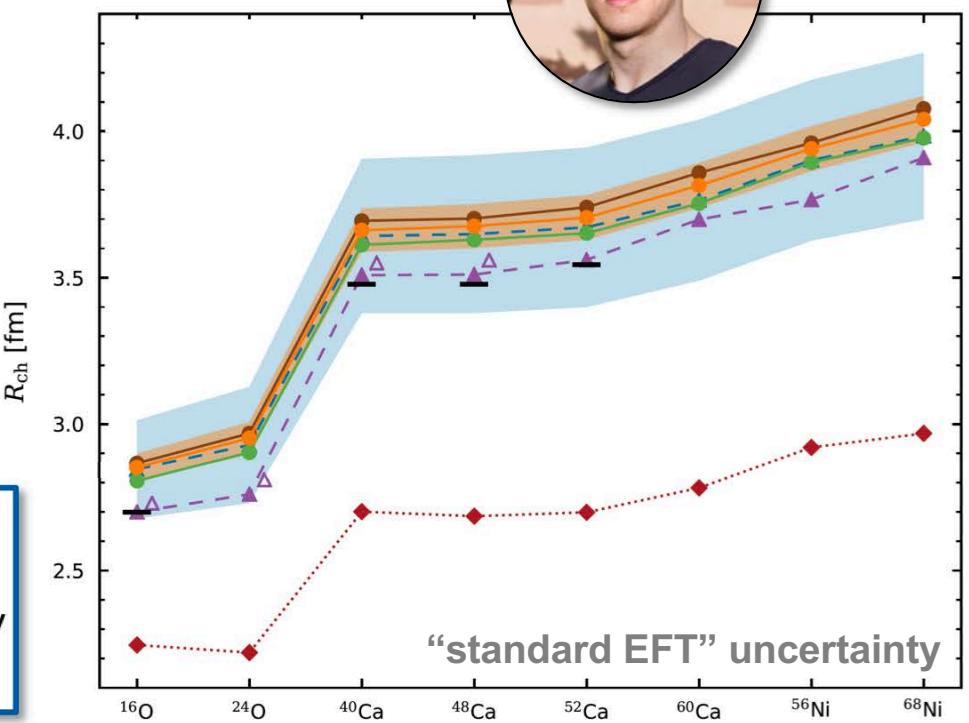
binding energies



realistic **saturation properties** may
not suffice for medium-mass nuclei !



charge radii



Quantifying uncertainties in the nuclear-matter equation of state

Summary and outlook



1

set a new standard for UQ in infinite-matter calculations

- correlations within *and* between observables are crucial for reliable UQ
- need for *statistically* meaningful comparisons between theory and observation
- efficiently quantify and propagate EOS uncertainties to derived quantities

2

statistically rigorous analysis of the EOS up to N^3LO

- excellent agreement of predicted S_v-L correlation with experiment
- PNM and SNM show a regular EFT convergence pattern with increasing order
- extracted Λ_b is consistent with NN scattering • N^2LO coefficient may be an outlier

3

improved NN+3N potentials up to N^3LO are needed

- Hüther *et al.*, arXiv:1911.04955; Hoppe *et al.*, PRC **100**, 024318; ...

4

full Bayesian UQ: MCMC for LECs & hyperparameters

- consistently include uncertainties in the LECs of chiral interactions
- compute nuclear saturation properties using Bayesian optimization

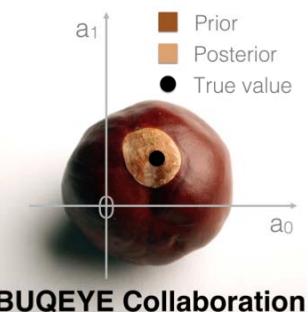
Unterstützt von / Supported by



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thanks to my collaborators:

R. Furnstahl K. Hebeler J. Hoppe J. Melendez
K. McElvain D. Phillips A. Schwenk C. Wellenhofer



Quantifying uncertainties in the nuclear-matter equation of state

Next-generation MBPT calculations



MBPT(n)*	2	3	4	5
NN only	✓(1)	✓(3)	✓(24)	✓(375)
norm. ord. 3N	✓	✓	✓	✓
residual 3N	✓(1)	✓(14)	-	-

scalable multi-CPU/-GPU application

complete third order

MC framework 2.0

SQL-based data repository

asymmetric-matter EOS

openMP, MPI, and CUDA

MPI-JM

self error detecting

significantly improved accuracy



Automated generation and evaluation of many-body diagrams I. Bogoliubov many-body perturbation theory

Pierre Arthuis, Thomas Duguet, Alexander Tichai, Raphaël-David Lasseri, Jean-Paul Ebran
Comput. Phys. **240**, 202

2

highly optimized: > 100x faster

3N partial-wave calculations

