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1) a) $T(n) = 9T(n/4) + n^2 \longrightarrow \frac{a}{b^d} = \frac{9}{4^2} < 1 \rightarrow T(n) \in \Theta(n^d) = \Theta(n^2)$

$\Rightarrow T(n) = \Theta(n^2)$

b) $T(n) = 3T(n/3) + \log n \longrightarrow$ Derste gösterilen Master Theorem'de $O(n^d)$ teriminin $d \geq 0$ iken polinom olarak aldık. Bu sebeple bize derste gösterilen şekliyle bir alt sınır belirleyebiliriz diye düşündüm.

- $d=0$ için $P(n) = 3P(n/3) + 1$ bağıntısı olur. $\frac{a}{b^d} = \frac{3}{3^0} > 1$

$\rightarrow P(n) \in \Theta(n^{\log_b a}) = \Theta(n^{\log_3 3})$

$P(n) = \Theta(n)$

* Sonuç olarak $T(n)$ için $d=0$ olduğu durumdan $\Omega(n)$ alt sınırı koyabiliriz.

$\boxed{T(n) \in \Omega(n)}$ Çünkü $P(n)$'de her çağrıda fonksiyon çağrıları dışında sabit iş varken, $T(n)$'de $\log n$'lik bir iş yapılıyor.

c) $T(n) = 3T(n/2) + n \rightarrow \frac{a}{b^d} = \frac{3}{2^1} > 1 \rightarrow T(n) \in \Theta(n^{\log_b a}) = \Theta(n^{\log_2 3})$

$\rightarrow T(n) = \Theta(n^{\log_2 3})$

2) F1 $\rightarrow \boxed{O(n)}$

F2 $\rightarrow \sum_{i=0}^{N}\sum_{j=0}^{i}\sum_{k=0}^{j} 1 = \sum_{i=0}^{N}\sum_{j=0}^{i} j = \sum_{i=0}^{N}\left(\frac{i.(i+1)}{2}\right) = \frac{1}{2}\cdot\left(\sum_{i=0}^{N} i^2 + \sum_{i=0}^{N} i\right)$

$= \frac{1}{2}\left(\frac{N.(N+1).(2N+1)}{6} + \frac{N.(N+1)}{2}\right) = \boxed{O(N^3)}$

$F3 \rightarrow T(N) = N.T(N-1) + N = N.[(N-1).T(N-2) + N-1] + N$

$= N.(N-1).T(N-2) + (N-1).N + N = N.(N-1).(N-2).T(N-3) + N.(N-1).(N-2) + N.(N-1) + N$

$= N.(N-1) \ldots 2.T(1) + N! + \frac{N!}{2!} + \frac{N!}{3!} + \frac{N!}{4!} + \ldots + \frac{N!}{(N-1)!}$

$= N! + N! + \frac{N!}{2!} + \ldots + \frac{N!}{(N-1)!} = O(N!)$

$F4 \rightarrow T(N) = 2T(N/2) + O(N) \rightarrow \frac{a}{b^d} = \frac{2}{2^1} = 1 \rightarrow T(N) \in \Theta(n^d . \log n)$

$T(N) = O(N \log N)$

3)

	$F(n)$	$g(n)$	
O	n^2	n^3	$\rightarrow \lim_{n \to \infty} \frac{n^2}{n^3} = 0 \rightarrow O$
Ω	$n \log n$	n	$\rightarrow \lim_{n \to \infty} \frac{n \log n}{n} = \infty \rightarrow \Omega$
Θ	1	$3 + \sin n$	$\rightarrow \lim_{n \to \infty} \frac{1}{3 + \sin n} = \text{sabit} \rightarrow \Theta$
Ω	3^n	2^n	$\rightarrow \lim_{n \to \infty} \frac{3^n}{2^n} = \infty \rightarrow \Omega$
Θ	4^{n+4}	2^{2n+2}	$\rightarrow \lim_{n \to \infty} \frac{4^{n+4}}{2^{2n+2}} = \lim_{n \to \infty} \frac{2^{2n+8}}{2^{2n+2}} = \text{sabit} \rightarrow \Theta$
O	$n \log n$	$n^{\frac{105}{100}}$	$\rightarrow \lim_{n \to \infty} \frac{n \log n}{n^{\frac{105}{100}}} = 0 \rightarrow O$
Θ	$\log(\sqrt{10}\,n)$	$\log n^3$	$\rightarrow \lim_{n \to \infty} \frac{\log(\sqrt{10}\,n)}{\log n^3} = \lim_{n \to \infty} \frac{\frac{1}{2} + \log n}{3 \log n} = \text{sabit} \rightarrow \Theta$
O	$n!$	$(n+1)!$	$\rightarrow \lim_{n \to \infty} \frac{(n)!}{(n+1)!} = 0 \rightarrow O$

4) a) $T(n) = 2^{n+1} + 3^{n-1} = 2.2^n + \frac{3^n}{3} \rightarrow 3^n$, 2^n 'den daha hızlı büyüyor

$\boxed{T(n) \in \Theta(3^n)} \rightarrow c_1 . 3^n \leq 2^{n+1} + 3^{n-1} \leq c_2 . 3^n$

$c_1 = 1$, $c_2 = 2$, $n_0 = 1$ iken sağlar

$3 \leq 5 \leq 6$

b) $T(n) = 2n\log(n+2)^2 + (n+2)^2 \cdot \log\frac{n}{2} = 4n\log(n+2) + (n+2)^2 \log\frac{n}{2}$

$n^2\log n$ domine ettiği için $\boxed{T(n) \in \Theta(n^2\log n)}$ //

$c_1 \cdot n^2\log n \leq 4n\log(n+2) + (n+2)^2\log\frac{n}{2} \leq c_2 \cdot n^2\log n$

$c_1 = 1,\ c_2 = 2,\ n_0 = 5$

$17.47 \leq 31.65 \leq 34.94$

5) $F(n) = \sum_{i=1}^{n} (i+1) \cdot 2^{i-1} = 2 \cdot 2^0 + 3 \cdot 2^1 + \dots + (n+1) \cdot 2^n$

$\boxed{F(n) \in \Theta(n \cdot 2^n)}$ //

$F(n) = \sum_{i=1}^{n} 2^{i-1} + \sum_{i=1}^{n} i\,2^{i-1} = 2^n - 1 + (n-1) \cdot 2^n + 1$

$= n \cdot 2^n$

6) $T(n) = T(n-2) + 2n$

$T(n) = T(n-4) + 2 \cdot (n-2) + 2n = T(n-6) + 2 \cdot (n-4) + 2 \cdot (n-2) + 2n$

$= T(0) + 2 \cdot 0 + 2 \cdot 2 + \dots + 2 \cdot (n-2) + 2 \cdot n = 2 \cdot \left(\frac{n}{2} \cdot \left(\frac{n}{2} + 1\right)\right) = n \cdot \left(\frac{n}{2} + 1\right)$

$= \frac{n^2}{2} + n \rightarrow T(n) = \frac{n^2}{2} + n \rightarrow T(n) \in \Theta(n^2)$