Matrix Models with Quantum Computers

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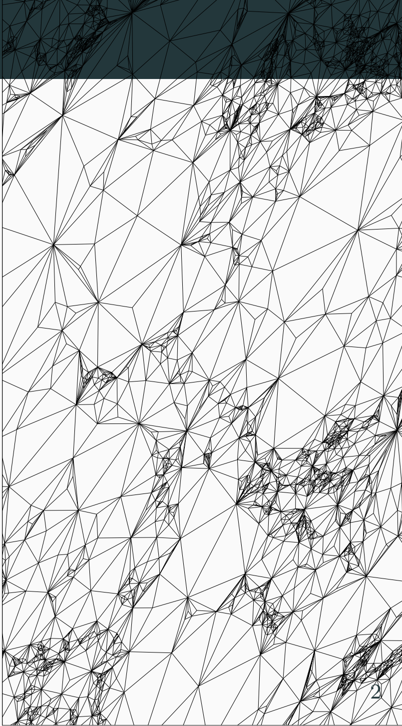
May 16, 2024

CQTA-DESY Zeuthen

Outline



Introduction and Theory

Discretization on Quantum Computer

Results

Conclusion and Outlook

Introduction and Theory

Introduction and Underlying Theory

- End Goal: Simulate Matrix Models of the kind:

$$e^{Z} = \int \prod_{\alpha=1}^{\nu-1} dM^{(\alpha)} e^{-\operatorname{Tr}\left[V(M^{(\alpha)})\right]}$$

with $V(M^{(\alpha)}) = \sum_{\alpha=1}^{\nu-1} V_\alpha M^{(\alpha)} - \sum_{\alpha=1}^{\nu-2} c_\alpha M^{(\alpha)} M^{(\alpha+1)}$

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- Tessellations of the Space
- Duality to 2D Quantum Gravity!

Connection to Pure Gravity - One Matrix Model

- Consider for $N \times N$ Hermitian Matrices:

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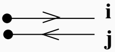

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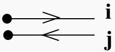
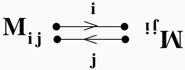


- Propagator: $\langle M_{ij} M_{ji} \rangle$



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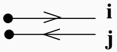

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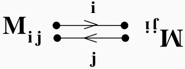
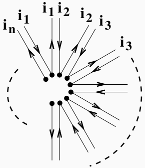
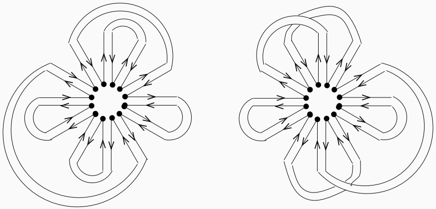


- Vertex: $\mathrm{Tr}[M^n]$



One Matrix Model

- $\langle \mathrm{Tr}\{M^n\} \rangle \leftrightarrow$ Contractions of Graphs:



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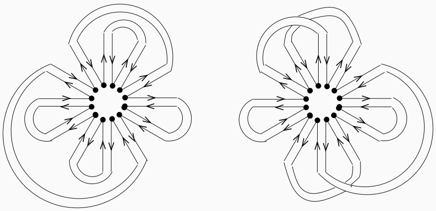

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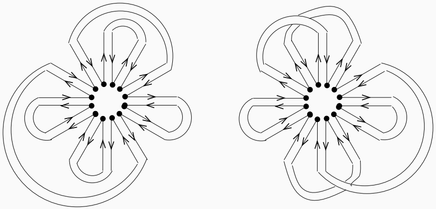

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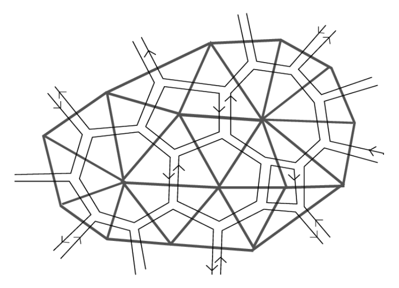
- $Z = \sum_{\text{Connected } \Gamma} \frac{N^{V-E+F}}{|Aut(\Gamma)|} \prod_i g_i^{n_i}$
- Euler Characteristic: $\chi = V - E + F = 2 - 2h \longrightarrow$ Connection to Topology!

Connection to Gravity

- Assign Vertices $\leftrightarrow$ Faces: Dual Graph, $n_3 = A$



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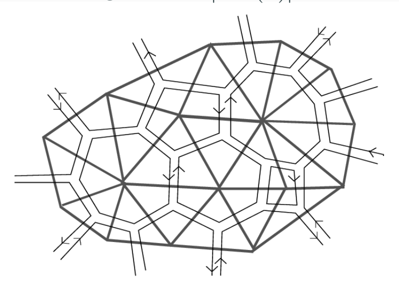

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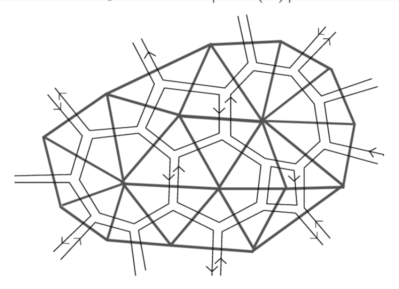

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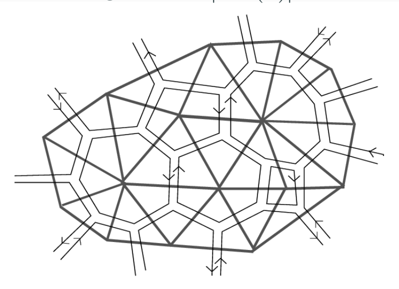

- $g = e^{-\Lambda}$ and $N = e^{\mathcal{N}} \implies Z = \sum_{\text{Connected Triangulations}} e^{-S_E}$
- $S_E = \Lambda A + \mathcal{N}\chi = \Lambda \int_\Sigma \sqrt{g} d^2 + \mathcal{N} \int_\Sigma \sqrt{g} R d^2$ [3]

Discretization on Quantum Computer

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- $\hat{k} = \sigma_z^\pm \sum_{m=0}^{\infty} \hat{k}_m 2^m$, $\hat{k}_m := \frac{1-\sigma_z^{(m)}}{2} \implies \hat{k}\,|\pm k\rangle = \pm k\,|\pm k\rangle$

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- $\hat{x} = a\left(\frac{1}{2}\sigma_z^{(\pm)} + \hat{k}\right)$
- Hamiltonian: $H = \hat{x}^2$ with $H\,|k\rangle = a^2(\frac{1}{2}+k)^2\,|k\rangle$

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- Expectation Values:

$$\int dx f(x) \rho(x, \sigma) \approx \sum_k a\, f(ak) e^{-\frac{a^2 k^2}{\sigma^2}} = a \mathrm{Tr} f(\hat{x}) \rho_{\frac{1}{\sigma^2}}$$

Step 2: One Matrix Model

- Ensemble of Hermitian Random Matrices with $V(M) = M^2 + \sum_{k\geq 3} \alpha_k M^k$. How to discretisize?

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- Forbenius norm:
  $\|M\|^2 = tr(MM^\dagger) \longrightarrow tr(d\Lambda^2 - [\Lambda, dT][\Lambda, dT])$,
  T Hermitian

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$$Z = \int \prod_i^N d\lambda_i \Delta(\Lambda)^2 \exp\left\{(-\sum_i V(\lambda_i))\right\}$$

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- Hamiltonian: $H = \sum_{i=0}^{N-1} \mathbb{1}^{\otimes i} \otimes V(\hat{x}_\lambda) \otimes \mathbb{1}^{\otimes((N-1)-i)}$

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- Transform:

$$Z \propto \int \prod^{N} d\lambda_{1,i} d\lambda_{2,j} \Delta^2(\Lambda_1)\Delta^2(\Lambda_2) e^{[-(V(\Lambda_1)+V(\Lambda_2))]} I(\Lambda_1, \Lambda_2)$$

with $I(M_1, M_2; \beta) \propto \beta^{\frac{-N(N-1)}{2}} \frac{det(e^{(\beta\lambda_{1,i}\lambda_{2,j})})}{\Delta(\Lambda_1)\Delta(\Lambda_2)}$ [4]

Multi Matrix Model

- Complete transformation [2]:

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- Implementation similar

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- Maximally mixed state:

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- $\rho^{Gibbs} = \mathrm{Tr}_B\,|\psi\rangle\langle\psi| \approx \frac{e^{-\beta H}}{Z}$

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$$e^{\beta H}\frac{1}{\sqrt{2^N}}\left|000...\right\rangle + \left|100...\right\rangle + ... \left|111...\right\rangle)$$
$$= \frac{1}{\sqrt{2^N}}(e^{\beta E_{000...}}\left|000...\right\rangle + e^{\beta E_{100...}}\left|100...\right\rangle + ... + e^{\beta E_{111..}}\left|111...\right\rangle)$$

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- Works with pure state!

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- Approximate:

$$\left|\phi(\tau)\right\rangle = V(\theta(\vec{\tau}))\left|\phi(0)\right\rangle \approx \left|\psi(\tau)\right\rangle \text{ [6]}$$

VarQITE

- After a timestep:

$$|\psi(\tau + \delta\tau)\rangle \approx \left|\phi(\theta(\vec{\tau}))\right\rangle - \delta\tau(H - E_\tau)\left|\phi(\theta(\vec{\tau}))\right\rangle$$

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- Parameter Space:

$$|\phi(\tau + \delta\tau)\rangle \approx \left|\phi(\theta(\vec{\tau}))\right\rangle + \sum_j \frac{\partial\left|\phi(\theta(\vec{\tau}))\right\rangle}{\partial\theta_j}\delta\theta_j$$

VarQITE

- After a timestep:

$$|\psi(\tau + \delta\tau)\rangle \approx \left|\phi(\theta(\vec{\tau}))\right\rangle - \delta\tau(H - E_\tau)\left|\phi(\theta(\vec{\tau}))\right\rangle$$

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- Minimize the distance with $\left\|\left|\phi(\theta(\vec{\tau}))\right\rangle\right\| = 1$

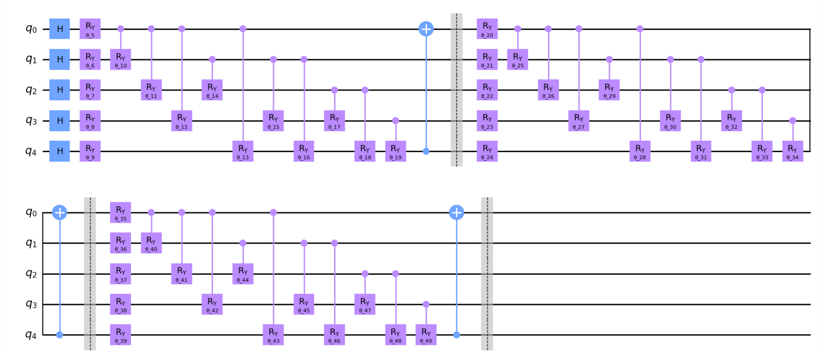
Results

Ansatz

- Generate Combinations:

$$H^{\otimes n}\left|0\right\rangle^{\otimes n} = \sum_{i=0}^{2^n-1} \left|i\right\rangle = \frac{1}{\sqrt{2^n}} \left|000...\right\rangle + \left|100...\right\rangle + ... \left|111...\right\rangle$$

- Parameterized Circuit:



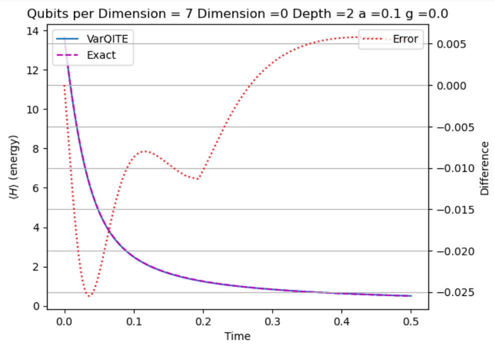
What are we Calculating?

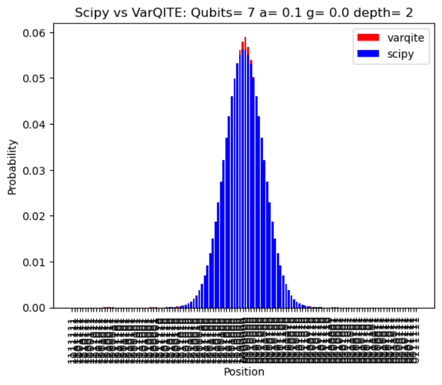
- Transformed:

$$Z = \int \prod_{i}^{N} d\lambda_i \Delta(\Lambda)^2 \exp\left\{(-\sum_i V(\lambda_i))\right\}$$

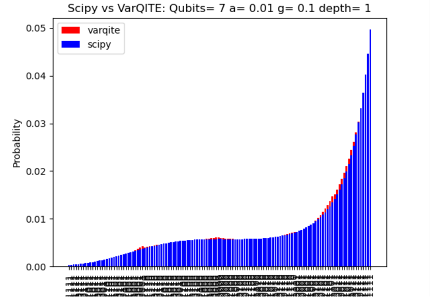
- $Z_{phys} \longrightarrow Z_{QC} = \left\langle \Delta(\Lambda)^2 \right\rangle$
- Expectation Values: $\langle f(ak) \rangle_{phys} \longrightarrow \left\langle \Delta(\Lambda)^2 f(ak) \right\rangle$

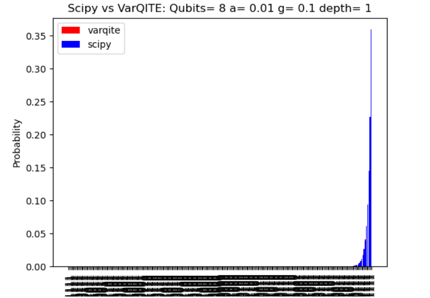
Gaussian Distribution $V(x) = x^2$

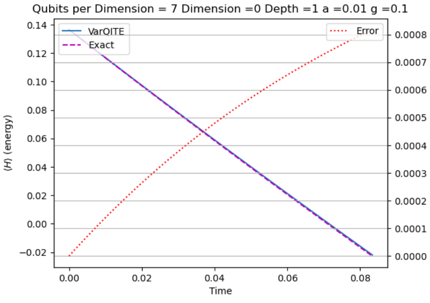


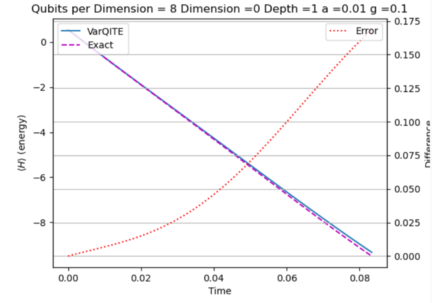


with gx^3 Term









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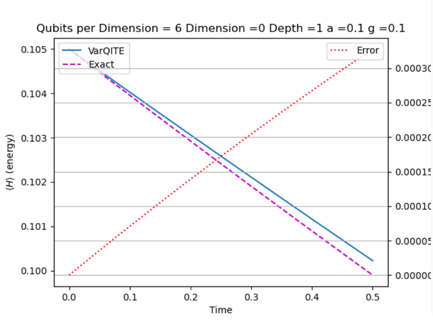
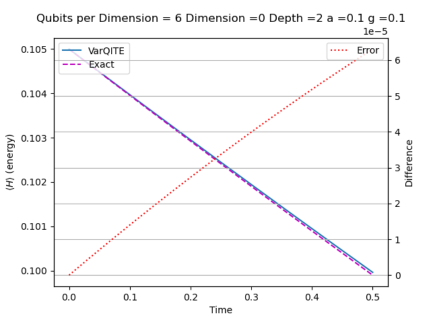
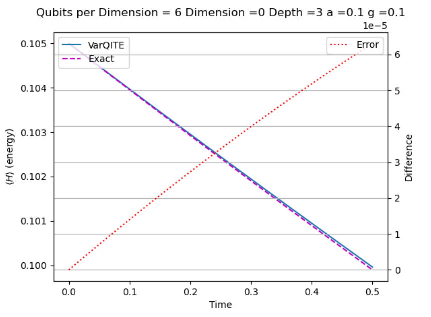
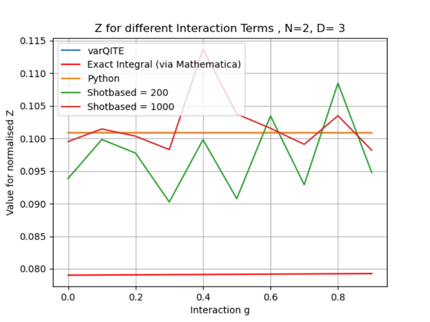
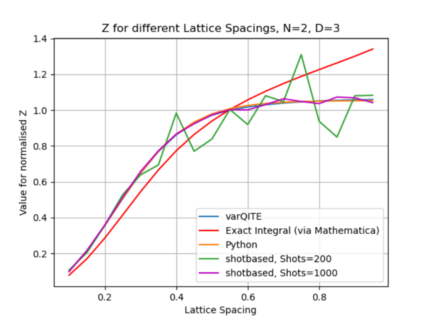


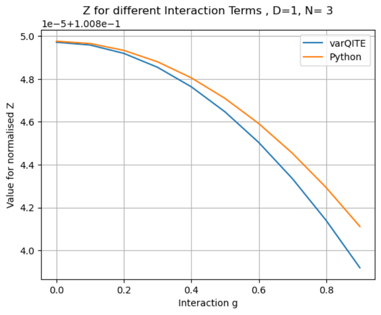




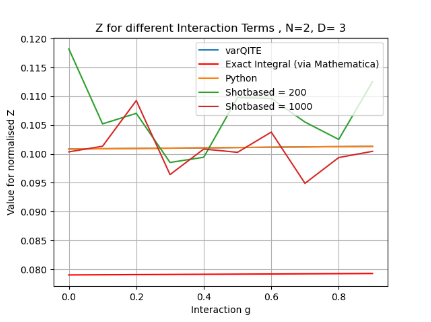
One Matrix Model: $V(M) = M^2 - gM^3$

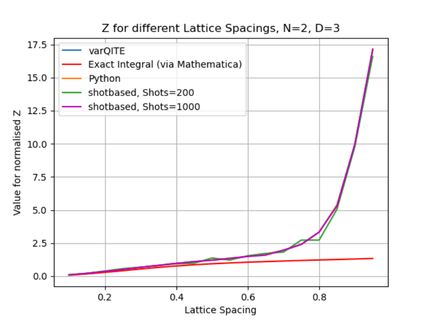


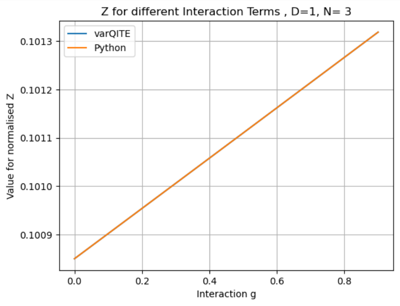




One Matrix Model: with $V(M) = M^2 - gM^4$







Conclusion and Outlook

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