Random Forest for Classification Problems

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Overview

Intro

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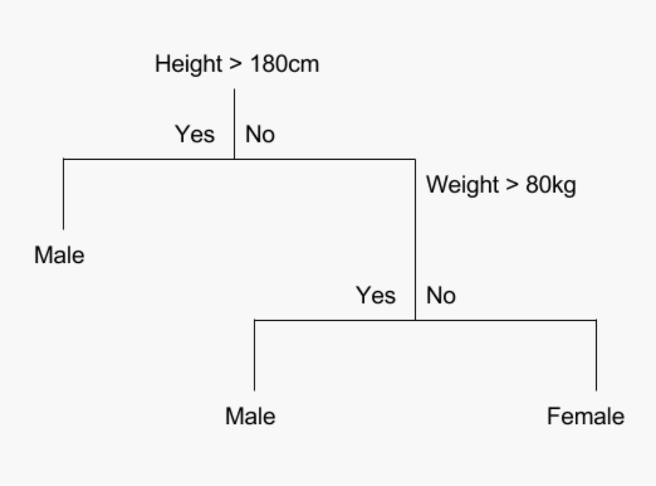
Decision Tree: Example



Decision Tree: Tree Building Process

A tree is grown starting from the root node by repeatedly using the following steps on each node (also called binary splitting):

(i) **Find best split s for each feature X_m:** For each feature X_m, there exist $K - 1$-many potiential splits whereas K is the number of different values for the respective feature. Evaluate each value $X_{m,i}$ at the current node t as a candidate split point (for $x \in X_m$, if $x \leq X_{m,i} = s$, then x goes to left child node t_L else to right child node t_R). The best split point is the one that maximize the splitting criterion $\Delta i(s,t)$ the most when the node is split according to it. The different splitting criteria will be covered in the next chapter.

(ii) **Find the node's best split:** Among the best splits for each feature from Step (i) find the one s^*, which maximizes the splitting criterion $\Delta i(s,t)$.

(iii) **Satisfy stopping criterion:** Split the node t using best node split s^* from Step (ii) and repeat from Step (i) until stopping criterion is satisfied.

Decision Tree: Purity Measures

Gini Measure

$$i(t) = \sum_{c \in C} p(c|t)(1 - p(c|t)) = 1 - \sum_{c \in C} p_c^2 \tag{1}$$

Information Entropy

$$i(t) = \sum_{c \in C} p(c|t) log(p(c|t)) \tag{2}$$

Expected Generalization Error

Let $D = \{(x_1, y_1), (x_2, y_2), ..., (x_N, y_N)\}$ and $y = f(x) + \epsilon$.
The decomposition of a model's expected generalization error is

$$\boldsymbol{Err}(f(x)) = Noise(x) + [Bias(\hat{f}(x))]^2 + Var(\hat{f}(x)) \tag{3}$$

Noise is irreducible and independent of the model.
There is a trade-off between $bias^2$ and variance.
Adjusting parameters to decrease variance increases $bias^2$ and vice versa.

Bias-Variance Trade-off

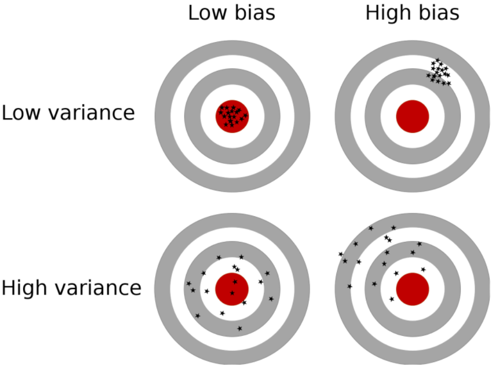
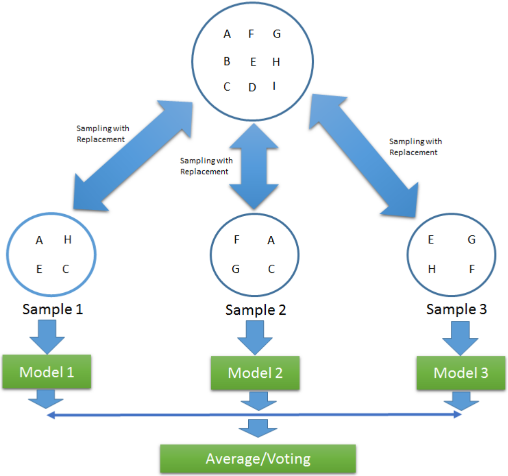


Figure: Illustration of bias-variance trade-off [2]

Decision trees generally have low bias and high variance [1].

Bagging

1. created for methods with high variance
2. reduces variance and gives better predictions
3. improvement of bagging: Random Forest



Random Forest

An ensemble of randomly trained decision trees, so in other words random forest was defined by L. Breiman:

Theorem

A random forest is a classifier consisting of a collection of tree-structured classifiers $\hat{T}_{\theta_b}(\mathbf{x}), b = 1, ..., B$ where the θ_b are independent identically distributed random vectors and each tree casts a unit vote for the most popular class at input $\mathbf{x}$.

Random Forest is an extension and improvement over bagging:

1. Like in bagging, multiple decision trees are built
2. Improvement: an injection of randomness is made

Random Forest: randomness in the model

Two key concepts that makes decision forest "random" are:

1. Random sampling of training data points when building trees
2. Random subsets of features considered when splitting nodes. Recommended number of variables:
   a. For classification: $\lfloor \sqrt{n} \rfloor$
   b. For regression: $\lfloor \frac{n}{3} \rfloor$

Random Forest: algorithm

Algorithm 1: Random Forest for Regression or Classification

1. For $b = 1$ to B:
   a. Draw a bootstrap sample θ_b of size N from the training data.
   b. Grow the Random Forest tree T_{θ_b} to the bootstrapped data, by recursively repeating the following steps for each terminal node of the tree, until the minimum node size n_{min} is reached:
      i. Select m variables at random from the n variables
      ii. Pick the best variable/split-point among the m
      iii. Split the node into two daughter nodes
2. Output the ensemble of trees $\{T_{\theta_b}\}_1^B$

Mathematical Concept

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
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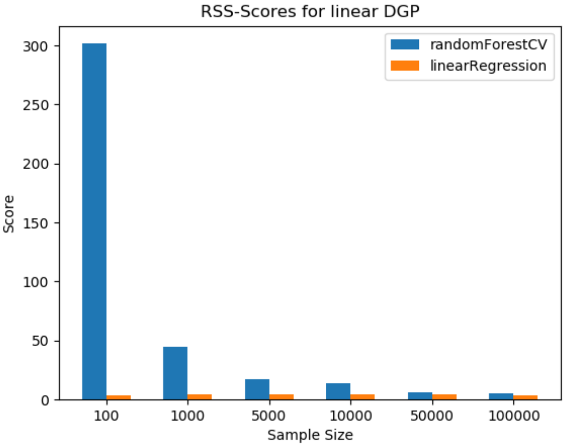
Simulation Study: Linear DGP

The linear DGP generates the data tuples (y, x_1, x_2, x_3) as follows:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon, \tag{4}$$

whereas $(\beta_0, \beta_1, \beta_2, \beta_3) = (0.3, 5, 10, 15)$, $x_1, x_2, x_3 \sim \mathcal{N}(0,\ 3)$, and $\epsilon \sim \mathcal{N}(0,\ 1)$.

Decision Tree: Linear DGP Results



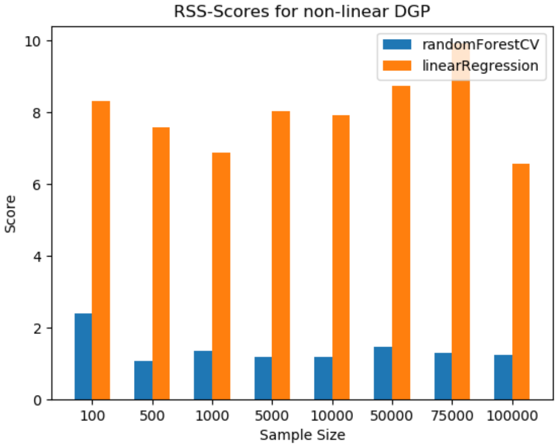
Simulation Study: Non-Linear DGP

The non-linear DGP generates the data tuples (y, x_1, x_2) as follows:

$$y = \beta_0 + \beta_1 \mathbb{1}(x_1 \geq 0, x_2 \geq 0) + \beta_2 \mathbb{1}(x_1 \geq 0, x_2 < 0) + \beta_3 \mathbb{1}(x_1 < 0) + \epsilon, \quad (5)$$

whereas $(\beta_0, \beta_1, \beta_2, \beta_3)$, x_1, x_2 and ϵ are the same as in the previous DGP.

Simulation Study: Non-Linear DGP Results



Real Data

Data: Titanic data

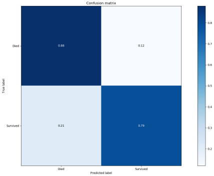
Method used:

1. Random Forest
2. AdaBoost
3. Gradient Boosting Classifier

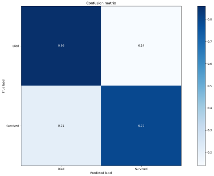
Goal: Given features of passengers predict which passengers survived the Titanic shipwreck

Real Data: results

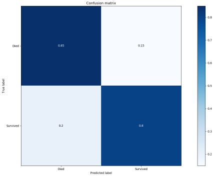
Random Forest
Accuracy: 84,32%



AdaBoost
Accuracy: 82.8%



Gradient Boosting
Accuracy: 82,8%



The End

References

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Daan van der Valk and Stjepan Picek. *Bias-variance decomposition in machine learning-based side-channel analysis*. Tech. rep. Cryptology ePrint Archive, Report 2019/570, 2019.