The 4-digit thickness distribution is given by the following equation:

y = (t/c) \* ( A sqrt(x) + B x + C x2 + D x3 + E x4 )

where \*\* denotes exponentiation, (t/c) is the maximum thickness to chord ratio of the airfoil, x is the position as fraction of chord, and y is the half-thickness as fraction of chord.

The coefficients are:

A = 1.4845

B = -0.630

C = -1.758

D = 1.4215

E = -0.5075

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n NACA Report 492, the four and five digit airfoils were subjected to systematic variation of the thickness distribution to allow the airfoil designer to choose the position of maximum thickness and leading edge radius. These modifications are indicated by a suffix consisting of a dash and two digits, such as 0012-34 or 23015-64. These modifications change the leading edge radius and the position of maximum thickness. The first integer following the dash indicates the relative magnitude of the leading edge radius. The nominal leading edge radius is denoted 6 and a sharp leading edge would be zero. The leading edge radius varies as the square of this index number. The second integer indicates the position of the maximum thickness in tenths of chord.

For example, a 23015-64 airfoil would have a maximum thickness of 15 per cent, a 230 mean line, a leading edge radius corresponding to an index of 6, and the position of maximum thickness would be at 40 per cent chord.

The thickness distribution is given by the following equation ahead of the maximum thickness:

y = A0 sqrt(x) + A1 x + A2 x\*\*2 + A3 x\*\*3

where \*\* denotes exponentiation, (t/c) is the maximum thickness to chord ratio of the airfoil, x is the position as fraction of chord, and y is the half-thickness as fraction of chord.

and by the following equation from maximum thickness to trailing edge:

y = D0 + D1(1-x) + D2 (1-x)\*\*2 + D3 (1-x)\*\*3

There is a special page that describes the details of the calculation of A0, A1, A2, A3, D0, D1, D2, D3 from the values of thickness, position of maximum thickness, and leading edge radius that are specified by the user.

The thickness distribution is given by the following equation ahead of the maximum thickness:

y = A0 sqrt(x) + A1 x + A2 x\*\*2 + A3 x\*\*3

where \*\* denotes exponentiation, (t/c) is the maximum thickness to chord ratio of the airfoil, x is the position as fraction of chord, and y is the half-thickness as fraction of chord.

and by the following equation from maximum thickness to trailing edge:

y = D0 + D1(1-x) + D2 (1-x)\*\*2 + D3 (1-x)\*\*3

The constants A0, A1, A2, A3, D0, D1, D2, D3 are calculated from the values of maximum thickness, position of maximum thickness, and leading edge radius that are specified by the user.

The airfoil must satisfy the following constraints:

y = one half the maximum thickness when x/c = m, the specified location of maximum thickness (as fraction of chord).

The leading edge radius = 1.1019/36.0\*((t/c)\*leIndex))\*\*2 [ see p.117 in Abbott & von Doenhoff]

The first and second derivatives of the forward function and the aft function match exactly at the point of maximum thickness.

The coefficient D1 is given by the following table:

m D1

0.2 1.000 t

0.3 1.170 t

0.4 1.575 t

0.5 2.325 t

0.6 3.500 t

D1 is the negative of the trailing edge slope.

These conditions are sufficient to determine all of the A and D terms in the polynomial equations.

The NACA 6-series and 6A-series airfoils are defined by means of conformal transformations. These relate the flow over an airfoil to that of a near-circle and that to a circle. The basic reference is Theodorsen, NACA 411.

The NACA 6-series airfoils are calculated by a nonlinear mapping of a unit circle by a four-step algorithm that uses a pair of functions defined on [0,pi] named psi and epsilon that were chosen to satisfy a prescribed velocity distribution about the airfoil. The definition of the psi and epsilon functions is described in refs 7-8. Each of the five members of the 6-series family and the three members of the 6A-series family has its own psi and epsilon functions. These functions are multiplied by a scale factor to produce airfoils of various thickness to chord ratios. The mapping is shown in figure 1. A given value of the scale factor is used to multiply both basic parameters giving new values of the psi and epsilon functions that will be used in the mapping. A given value of scale factor will produce a certain thickness to chord ratio of the airfoil in the normalized physical plane. It is not known in advance just what thickness will result from a given value of the scale factor. The algorithms of references 1 and 3 use an iterative procedure to determine the scaling factor required to achieve an airfoil of a given thickness.

The algorithm used in the present method is based upon a study of the scaling factor required to achieve a given thickness. Calculations were made of the thickness resulting from a given value of scale factor for each of the eight airfoil families. The dependency is somewhat nonlinear but easily fitted as a polynomial with four coefficients. The fitting is done on the data as if scale factor c is a function of t/x.

c=K1(t/c) + K2(t/c)2 + K3(t/c)3 + K4(t/c)4

and the K-values for each family are given by:

K1 K2 K3 K4

63 8.1827700 1.3776209 -0.0928517 7.5942563

64 4.6535511 1.0380630 -1.5041794 4.7882784

65 6.5718716 0.4937629 0.7319794 1.9491474

66 6.7581414 0.1925377 0.8128826 0.852090

67 6.6272890 0.0989966 0.9675977 0.9053758

63A 8.1845925 1.0492569 1.3115094 4.4515579

64A 8.2125018 0.7685596 1.4922345 3.6130133

65A 8.2514822 0.4656936 1.5013018 2.0908904

Now, for a specified family and thickness, the thickness distribution may be determined without iteration. From the thickness, the scale factor is computed from the polynomial function shown above. Then, the scale factor is used to multiply the basic values of the psi and epsilon functions for this airfoil family. These scaled psi and epsilon functions are used in mapping the z-plane to the z'-plane shown in Figure 1. The Joukowski function zeta = z' + 1/z' then maps the z'-plane into the zeta-plane and these results are normalized so that the leading edge is at x=0 and the trailing edge is at x=1.

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The 2-digit camber line is designated by a 2-digit number. The first denotes the maximum camber in percent of chord. The second digit denotes the chordwise position of the maximum camber, expressed in tenths of chord. A 24 camber line (or mean line) has a maximum camber of 0.02 chord located at 0.4c. The camber line is formed by two parabolic segments that match in value and slope (=0) at the maximum camber position. From these conditions, one may derive that the equation of each line is

y/ymax = 2\*x/m -(x/m)2 if x ≤ m

y/ymax = (1-2\*m + 2\*m\*x - x2) / (1-m)2 if x > m

where m is the position of the maximum camber as a fraction of chord and x is the position as a fraction of chord.

The NACA airfoil designation requires that m be an integer multiple of one-one-hundredth chord and ymax be an integer multiple of one-tenth chord, but the program will allow any fraction of chord for either quantity.

You may note that both NASA TM 4741 and Abbott and von Doenhoff use the parameters m and p to express the shape of the camber line, but the meanings of m and p are reversed. If you downloaded the PDF from NASA several years ago, the equation in NASA 4741 is incorrect. The copy on the NASA document server has now been corrected.

To provide a camber line with a very far forward location of the maximum camber, the 3-digit camber line was developed and reported in NACA Report 537. The first digit of the 3-digit camber line designation is defined as two thirds of the design lift coefficient (in tenths); the second digit, as twice the longitudinal location of the maximum camber (in tenths of chord); and the third digit of zero indicates a non-reflexed trailing edge. A 210 camber line has a design lift coefficient of 0.3 with the maximum camber occurring at 0.05 chord and no reflex at the trailing edge.

The camber line is made up of two equations so that the second derivative decreases linearly to zero at a point r slightly aft of the maximum camber position and remains zero from this point to the trailing edge.

y'' = k\*(x-r) if x < r and

y'' = 0 if x ≥ r

y(0)=0 and y(1)=0 where x is fraction of chord.

These differential equations may be integrated to show that

y=(k/6)\*(x\*\*3 -3\*r\*x\*\*2 +r\*\*2\*(3-r)\*x) if x < r and

y=(k\*r\*\*3/6)\*(1-x) if x ≥ r

This camber line family was reported in NACA Report 537 and the following values of k and r for selected values of m were given as:

Camber-line

designation m r k

210 0.05 0.0580 361.400

220 0.10 0.1260 51.640

230 0.15 0.2025 15.957

240 0.20 0.2900 6.643

250 0.25 0.3910 3.230

Note: k is written as k1 in both NASA 4741 and Abbott and von Doenhoff. The notation in 4741 and AVD differ again. The variable r used here agrees with 4741, but is called m by AVD. Also note that m is not actually used in the equations.

The camber-line designation for the 3-digit-reflex camber line is the same as that for the 3-digit camber line except that the last digit is changed from 0 to 1 to indicate the reflex characteristic. The three digit reflex camber line was designed to have a zero pitching moment coefficient about the quarter-chord position. The principal application was to be sections for rotorcraft main rotors.

The camber line is made up of two equations so that the second derivative decreases linearly to zero at a point r aft of the maximum camber position and remains zero from this point to the trailing edge.

y'' = k\*(x-r) if x < r and

y'' = n\*k\*(x-r) if x ≥ r

y(0)=0 and y(1)=0

These differential equations may be integrated to show that

y=(k/6)\*(x3 -(k2/k1)\*(1-r)3\*x +r3\*(3-r)\*x + r3) if x < r and

y=(k/6)\*(n\*(x-r))3 - n\*(1-r)3\*x - r3\*x + r3) if x ≥ r

The constant n is given by:

n = (3/(1-r))\*(r-m)2 - r3

This camber line family was reported in NACA Report 537 and the following values of k and r for selected values of m were given as:

Camber-line

designation m r k n

221 0.10 0.1300 51.990 0.000764

231 0.15 0.2170 15.793 0.00677

241 0.20 0.3180 6.520 0.0303

251 0.25 0.4410 3.191 0.1355

The equations for the 6-series camber lines are rather complicated and there is little value in displaying them here.

They may be found

on p.8 of NASA TM 4741

as Eq. 4.26 and 4.27 on p.74 of Abbott and von Doenhoff, Theory of Wing Sections

as Eq. 6 on p.264 of NACA Report 824

6-series camber lines are specified with a parameter a, where a is a fraction of chord. The intent is to provide a uniform loading from the leading edge to the station x=a and a loading that decreases linearly to zero thereafter.