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CENG 462 – ARTIFICIAL INTELLIGENCE
Report

In the homework; we are asked to implement A* search algorithm with two different heuristic functions. The first one is the Manhattan Distance; and second is the optional function. I used straight line distance as my heuristic function; and, its admissiblity can be proved as:

Definition of Admissibility: An admissible heuristic function never overestimates the cost of reaching goal state(h(n) for A*). This fact can be represented as;

for every n value; $h(n) <= h^*(n)$

I will prove straight line distance heuristic's admissibility with the help of following lemma:

LEMMA: Every consistent heuristic is admissible. (Proof of this lemma will be explained later.)

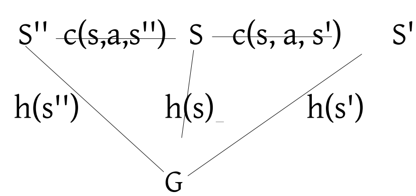
So, with the help of this lemma, if i prove the straight line distance is consistent, then that means that I proved this heuristic's admissibility.

Proof of straight line distance is consistent:
A consistent heuristic is formulated as;

$h(n) <= c(n, a, n') + h(n')$ where n is a state and n' is the one of the successor of it. And c(n, a, n') means the cost of going from state n to state n' with action a and cost. (In our homework, since an object is moving one direction only at a time, right, left, up and down, our cost to reach successor is always 1)

Now, let's prove straight line distance's consistency:

Let s be any state and s' and s'' be two successors of it. “g“ means goal state.



To prove it, we must show the mathematical expression above is satisfied for all states.

Since straight line distance computes the positions for any of object related to goal state, we can show h(n) values as a straight line.

Now, with the help of triangle equality in mathematic, we can easily say that:

$h(s) < h(s'') + c(s, a, s'')$
$h(s) < h(s') + c(s, a, s')$
..... → This equalities repeats for all successors of state s.

So, we have proved that for every successors of n,(namely n')

$h(s) <= c(n, a, n') + h(s')$ is satisfied.

It means that h(n) (*straight line distance*) is consistent.

Since h(n) is *consitent*, according to lemma; it is *admissible* also.

Hence; we proved that straight line distance's admissibility with using its consistency.

Proof of Lemma:

Any consistent heuristic is admissible.

Where consistency is: $h(n) <= c(n, a, n') + h(n')$

and admissibility is $h(n) <= h^*(n)$ for every n.

Proof by induction:

k is used for number of steps to achieving goal. (induction on path length)

Basis Step:

k =1

$h(n) <= c(n, a, n') + 0$

$h^*(n)$

Inductive step:

For a path to end same goal require k steps;

$h(n') <= h^*(n')$ → assume holds for k steps, state n

$c(n, a, n') + h(n') <= h^*(n') + c(n, a, n')$ → add c(n, a, n') both sides

The left side is greater than h(n) (from def. of consistency $h(n) <= c(n, a, n') + h(n')$)

The right side is equal to h*(n); then put them to inequality;

$h(n) <= h^*(n)$

Hence, we prove that any consistent heuristic is admissible.