CS 353 Spring 2020
Homework 2 Solutions
Due: 4 March, Wednesday till 17:00

Q.1 [60 pts]

(Each part, except (e), (g), and (h) are 5 points; (e), (g), and (h) are 10 points.

Consider the computer product database schema below.

Product(maker, model, type)
PC(model, speed, ram, hd, price)
Laptop(model, speed, ram, hd, screen, price)
Printer(model, color, type, price)

Maker of a product is the manufacturer firm. models are numbers for PCs, Laptops, and Printers. Type of a product is "pc", "laptop", or "printer." Color for printer is true for color printers, false for black and white printers. Printer type is "laser" or "inkjet". PC models are four-digit numbers 1XXX. Laptop models are four-digit numbers 2XXX. Printer models are four-digit numbers 3XXX.

Write the following queries in Relational Algebra:

a) Find the model number and price for all color laser printers.

b) Find the manufacturers that produce PC's or Laptops, but not printers.

c) Find the manufacturers that produce laptops who has a hard disk higher than 120 GB and a memory (RAM) of at least 1024 MB and a screen size of at least 17.0 inch, together with the laptop models and prices of the models.

d) Find the manufacturer pairs that produce the same PC models with the speeds of at least 2.5 MHz. Report the pairs only once.

e) Find the Laptop models that are produced by at least three different manufacturers. Do this i) without aggregate operators, ii) with aggregate operators.

f) Find the manufacturers that produce all PCs and laptops whose speed is at least 2.50.

g) Find the manufacturers who produce exactly one PC, one Laptop, and one Printer model.

   Do this i) without aggregate operators, ii) with aggregate operators.

h) Find the manufacturer(s) who produce Laptops with the highest speed.

   Do this i) without aggregate operators, ii) with aggregate operators.

i) Find the pairs of laptops (model) with the same speed, RAM, and screen size. Report the pairs only once.

Solution:

(a) $\Pi_{\text{model, price}} (\sigma_{\text{color = “true” } \wedge \text{ type = “laser”}} (\text{Printer}))$

(b) $\rho(T1, (\Pi_{\text{maker}} \sigma_{\text{type = “printer”}} (\text{Product})))$
$\Pi_{\text{maker}} (\text{Product}) - T1$

(c) $\Pi_{\text{maker, Product.model, price}} (\text{Product} \bowtie (\sigma_{\text{hd > 120 } \wedge \text{ ram >= 1024 } \wedge \text{ screen >= 17.0}} (\text{Laptop}))$

(d)
$\rho(T1, \Pi_{\text{model}} (\sigma_{\text{speed >= 2.50}} (\text{PC}))$
$\rho(P1, \text{Product} \bowtie T1)$
$\rho(P2, \text{Product} \bowtie T1)$

$\Pi_{\text{P1.maker, P2.maker}} (\sigma_{\text{P1.maker < P2.maker } \wedge \text{ P1.model = P2.model}} (P1 \times P2))$

(e)

(i)
$\rho(P1, \text{Product})$
$\rho(P2, \text{Product})$
$\rho(P3, \text{Product})$

$\Pi_{\text{P1.model}} (\sigma_{\text{P1.type = “laptop” } \wedge \text{ P1.model = P2.model } \wedge \text{ P2.model = P3.model } \wedge \text{ P1.maker != P2.maker } \wedge \text{ P2.maker != P3.maker } \wedge \text{ P1.maker != P3.maker}} (P1 \times P2 \times P3))$

(ii)
$t_1 \leftarrow \text{model} \mathcal{G}_{\text{count(*) as cnt}} (\sigma_{\text{type = “laptop”}} (\text{Product}))$
$\Pi_{\text{P1.model}} (\sigma_{\text{cnt>=3}} (t_1))$

(f)
$\rho(T1, \Pi_{\text{model}} (\sigma_{\text{speed >= 2.50}} (\text{PC}) \cup \Pi_{\text{model}} (\sigma_{\text{speed >= 2.50}} (\text{Laptop}))$
$\Pi_{\text{maker, model}} (\text{Product}) / T1$

(g)

(i)

$\rho(P1, \sigma_{type = \text{“pc”}} (Product))$

$\rho(P2, \sigma_{type = \text{“pc”}} (Product))$

$\rho(P3, \sigma_{type = \text{“laptop”}} (Product))$

$\rho(P4, \sigma_{type = \text{“laptop”}} (Product))$

$\rho(P5, \sigma_{type = \text{“printer”}} (Product))$

$\rho(P6, \sigma_{type = \text{“printer”}} (Product))$

$$\rho(R1, \Pi_{maker} (\sigma_{type = \text{“pc”}}(Product)) - \Pi_{P1.maker} (\sigma_{P1.maker = P2.maker \wedge P1.model != P2.model} (P1 \times P2))))$$

$$\rho(R2, \Pi_{maker} (\sigma_{type = \text{“laptop”}}(Product) - \Pi_{P3.maker} (\sigma_{P3.maker = P4.maker \wedge P3.model != P4.model} (P3 \times P4))))$$

$$\rho(R3, \Pi_{maker} (\sigma_{type = \text{“printer”}}(Product) - \Pi_{P5.maker} (\sigma_{P5.maker = P6.maker \wedge P5.model != P6.model} (P3 \times P4))))$$

$R1 \cap R2 \cap R3$

(ii)

$t_1 \leftarrow \text{type, maker } \mathcal{G}_{count(*) \text{ as } cnt} ((Product))$

$\rho(R1, \Pi_{maker} (\sigma_{type = \text{“laptop”} \wedge cnt = 1} (t_1))$

$\rho(R2, \Pi_{maker} (\sigma_{type = \text{“pc”} \wedge cnt = 1} (t_1))$

$\rho(R3, \Pi_{maker} (\sigma_{type = \text{“printer”} \wedge cnt = 1} (t_1))$

$R1 \cap R2 \cap R3$

(h)

(i)

$\rho(L1, (Laptop))$

$\rho(L2, (Laptop))$

$$\rho(HighestSpeedLaptops, \Pi_{model} (Laptop) - \Pi_{L1.model} (\sigma_{L1.speed < L2.speed} (L1 \times L2))$$

$$\Pi_{maker} (Product \bowtie HighestSpeedLaptops)$$

(ii)

$\rho(\text{HighestLaptopSpeed}, \mathcal{G}_{\text{max(speed) as maxspeed}}(\text{Laptop}))$

$\rho(\text{HighestSpeedLaptops}, \Pi_{\text{speed= HighestLaptopSpeed.maxspeed}}(\text{Laptop}))$

Π_{maker} (Product $\bowtie$ HighestSpeedLaptops)

i) Find the pairs of laptops (model) with the same speed, RAM, and screen size. Report the pairs only once.

ρ(L1, (Laptop))

ρ(L2, (Laptop))

$\Pi_{\text{L1.model,L2.model}}(\sigma_{\text{L1.model} < \text{L2.model} \wedge \text{L1.ram} = \text{L2.ram} \wedge \text{L1.speed} = \text{L2.speed} \wedge \text{L1.screen} = \text{L2.screen}}(\text{L1} \times \text{L2}))$

Q.2 [20 pts, 5 pts each]

Suppose the relations *R* and *S* have *n* tuples and *m* tuples, respectively. Give the minimum and maximum numbers of tuples that the results of the following expressions can have.

a) $R \cup S$
b) $R \bowtie S$
c) $\sigma_c(R) \times S$, for some condition *C*.
d) $\pi_L(R) - S$, for some list of attributes *L*. Assume the projection operator eliminates duplicates.

Solution:

a) min = min(*n*, *m*); max = *n* + *m*
b) min = 0; max = min(*n*, *m*)
c) min = 0; max = *n* × *m*
d) min = 0; max = *n*

Q.3 [20 pts]

a) **[5 pts]** Let *R* be a relation with schema

(A_1, A_2, ..., A_n, B_1, B_2, ..., B_m)

and let *S* be a relation with schema (B_1, B_2, ..., B_m); that is, the attributes of *S* are a subset of the attributes of *R*. The quotient of *R* and *S*, denoted by, $R \div S$ (or R / S), is the set of tuples *t* over attributes A_1, A_2, ..., A_n (i.e., the attributes of *R* that are not attributes of *S*) such that for every tuple *s* in *S*, the tuple *ts*, consisting of the components of *t* for A_1, A_2, ..., A_n and the components of *s* for B_1, B_2, ..., B_m, is a member of *R*. Give an expression in Relational Algebra, using the basic operators of Relational Algebra, that is equivalent to $R \div S$.

b) **[15 pts]** Show the result of the division operations A/B1, A/B2, and A/B3 for the following relations:

A

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s2	p4
s3	p2
s4	p1
s4	p3

B1

pno
p1
p2
p3
p4

B2

pno
p1
p3

B3

pno
p1
p2
p4

Solution:

a)

Idea: For $R \div S$, compute all t values that are not `disqualified' by some s value in S. t value is *disqualified* if by attaching s value from S, we obtain an st tuple that is not in R.

Disqualified *t* values: $\Pi_t ((\Pi_t (R) \times S - R)$

$R \div S = \Pi_t (R)$ – all disqualified tuples

$R \div S = \Pi_t (R) - \Pi_t ((\Pi_t (R) \times S - R)$

b)

A / B1

sno
s1

A / B2

sno
s1
s4

A / B3

sno
s1
s2