EE 464 Homework 3

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1 Derivation of Input to Output TF

To derive TF, first I will find small signal circuit equivalent. Forward converter can be treated as a buck converter with $V_{in} = n \cdot V_{in}$ where n is transformer ratio.

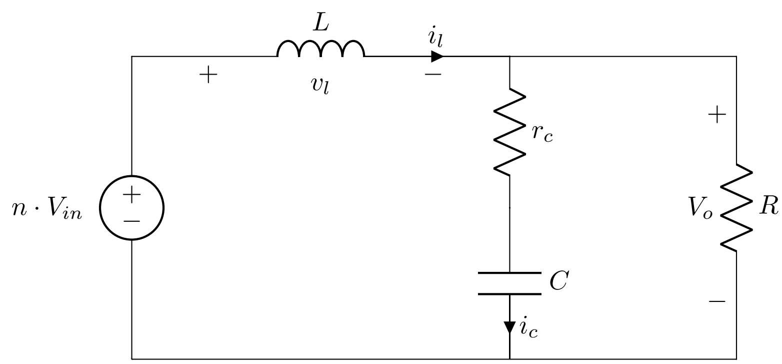

Figure 1: Simplified Forward Converter when switch is on

KVL at the outer path,

$$-nV_{in} + L\frac{di}{dt} + i_o R = 0 \tag{1}$$

Since,

$$i_o = i_l - i_c \tag{2}$$

$$L\frac{di}{dt} = nV_{in} - Ri_l + Ri_c \tag{3}$$

Find i_c by KVL on inductor and capacitor

$$-nV_{in} + L\frac{di}{dt} + i_c r_c + V_c = 0 \tag{4}$$

$$i_c = \frac{nV_{in} - L\frac{di}{dt} - V_c}{r_c} \tag{5}$$

Insert i_c into equation 3,

$$\hat{i}_l = \frac{di}{dt} = \frac{nV_{in}}{L} - \frac{r_c R}{(r_c + R)L} \frac{R}{(r_c + R)L} V_c \tag{6}$$

We know that $R >> r_c$ so we can simplify equation as,

$$\hat{i}_l = \frac{nV_{in}}{L} - \frac{r_c}{L} i_l - \frac{1}{L} v_c \tag{7}$$

Start derive V_c by KCL at output node,

$$i_c = i_l - \frac{V_o}{R} \tag{8}$$

Since,

$$v_o = v_c + r_c i_c \tag{9}$$

$$i_c = \frac{R}{R + r_c} i_l - \frac{1}{R + r_c} v_l \tag{10}$$

Since,

$$\hat{v}_c = \frac{i_c}{C} \tag{11}$$

$$\hat{v}_c = \frac{R}{C(R + r_c)} i_l - \frac{1}{C(R + r_c)} v_l \tag{12}$$

We know that $R >> r_c$ so we can simplify equation as,

$$\hat{V}_c = \frac{1}{C} i_l - \frac{1}{RC} v_l \tag{13}$$

According to state space equations,

$$\begin{bmatrix} \hat{i}_l \\ \hat{v}_c \end{bmatrix} = \begin{bmatrix} -\frac{r_c}{L} & -\frac{1}{v_c} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_l \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{n}{L} \\ 0 \end{bmatrix} V_{in} \tag{14}$$

For switch off position A matrix hasn't changed.But B matrix multiplied with duty ratio so,

$$\begin{bmatrix} \hat{i}_l \\ \hat{v}_c \end{bmatrix} = \begin{bmatrix} -\frac{r_c}{L} & -\frac{1}{v_c} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_l \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{dn}{L} \\ 0 \end{bmatrix} V_{in} \tag{15}$$

Find v_o as state space variables,

$$v_o = v_c + r_c i_c \tag{16}$$

Since,

$$i_c = i_l - \frac{v - o}{R} \tag{17}$$

$$v_o = v_c + r_c i_l - \frac{v_o r_c}{R} \tag{18}$$

$$v_o = \frac{r_c R}{r_c + R} i_l + \frac{R}{(r_c + R)} v_c \tag{19}$$

Since $r_c << 1$,

$$v_o = r_c i_l + v_c \tag{20}$$

$$C = \begin{bmatrix} r_c \\ 1 \end{bmatrix} \tag{21}$$

$$\frac{v_o(s)}{d(s)} = C^T [SI - A]^{-1} B V_{in} \tag{22}$$

After a lengthy evaluation control to output TF is,

$$G_{vd} = \frac{\widetilde{v_o}}{\widetilde{d}} \bigg|_{\widetilde{v_i}=0} = \frac{nV_{in}}{LC} \left[\frac{s r_c C + 1}{s^2 + s(\frac{r_c}{L} + \frac{1}{RC}) + \frac{1}{LC}} \right] \tag{23}$$

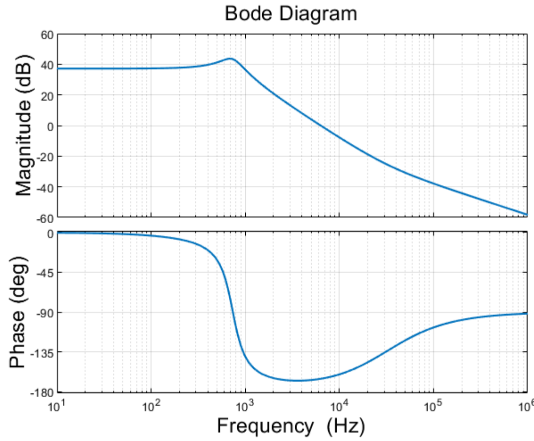

Figure 2: Bode plot of control to output TF

According to **erickson** Crossover Frequency is defined as the frequency where the magnitude of loop gain is unity.Limit for f_c is pole frequency since we want to operate unity gain at f_c.For power application switching frequency selected as one in ten or five.For this application I select as,

$$f_c = \frac{f_{sw}}{5} = 20kHz \tag{24}$$

Since $f_{ESR} = 32kHz$ and $f_{LC} = f734Hz$,

$$f_{LC} < f_c < f_{ESR} < f_{sw}/2 \tag{25}$$

Compensator Type	Relative location of the crossover and power-stage frequencies	Typical Output Capacitor
Type II (PI)	$F_{LC} < F_{ESR} < F_0 < F_S/2$	Electrolytic, POS-Cap, SP-Cap
Type III-A (PID)	$F_{LC} < F_0 < F_{ESR} < F_S/2$	POS-Cap, SP-Cap
Type III-B (PID)	$F_{LC} < F_0 < F_S/2 < F_{ESR}$	Ceramic

Figure 3: Compensator Selection table

According to table 3 I select Type 3-A compensator for this application.Type 3 compensator supplies phase boost for this converter to operate stable region.

2 Compensator Parameters

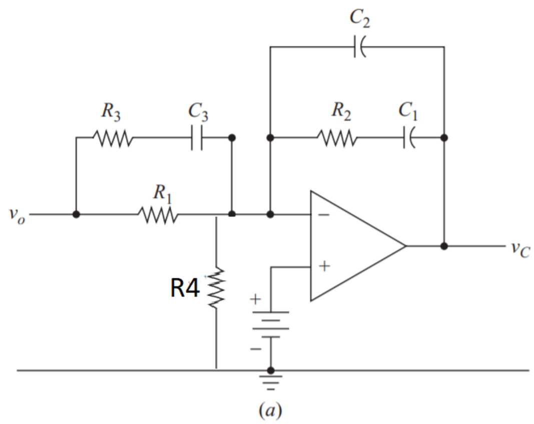

Figure 4: Type 3 Compensator

This compensator is referred from **erickson** book except one extension.R_4 added to adjust input voltage range. When $f_c = 20kHz$, from bode plot phase angle is -146 and gain is -20 db. $V_{ref} = 3V$, which corresponds -9.5db.

$$|G(jw_o)| = 29.5dB = 29.5Gain \tag{26}$$

$$\theta_{comp} = \theta_{phase-margin} - \theta_{converter} = 45 - (-146) = 191 \tag{27}$$

$$K = tan\Big(\frac{191 + 90}{2}\Big)^2 = 7.75 \tag{28}$$

$$\boxed{\begin{aligned} R_2 &= \frac{|G(j\omega_{co})|R_1}{\sqrt{K}} \\ C_1 &= \frac{\sqrt{K}}{\omega_{co}R_2} = \frac{\sqrt{K}}{2\pi f_{co}R_2} \\ C_2 &= \frac{1}{\omega_{co}R_2\sqrt{K}} = \frac{1}{2\pi f_{co}R_2\sqrt{K}} \\ C_3 &= \frac{\sqrt{K}}{\omega_{co}R_1} = \frac{\sqrt{K}}{2\pi f_{co}R_1} \\ R_3 &= \frac{1}{\omega_{co}\sqrt{K}C_3} = \frac{1}{2\pi f_{co}\sqrt{K}C_3} \end{aligned}}$$

Figure 5: Type 3 Compensator equations

To start lets choose $R_1 = 1k\Omega$, Afterwards by using formulas at figure 5, other parameters found as,

$$R_2 = 10.77k\Omega \quad , \quad C_1 = 2nF \quad , \quad C_2 = 0.26nF \quad , \quad C_3 = 22nF \quad , \quad R_3 = 130\Omega \tag{29}$$

It is suitable to measure output voltage with $\frac{v_o}{3}$,

$$v_{ref} = \frac{v_o}{3}\frac{R_4}{R_4 + R_1} \implies R_4 = \frac{v_{ref}R_1}{\frac{v_o}{3} - v_{ref}} = 1.5k\Omega \tag{30}$$

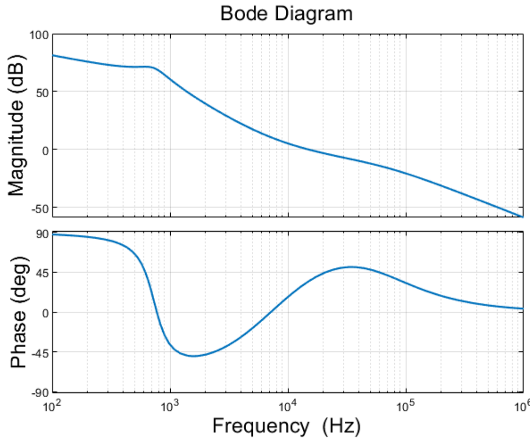

Figure 6: Bode plot of converter with compensator