EE 464 Homework 3

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1 Derivation of Input to Output TF

To derive TF, first I will find small signal circuit equivalent. Forward converter can be treated as a buck converter with $V_{in} = n \cdot V_{in}$ where n is transformer ratio.

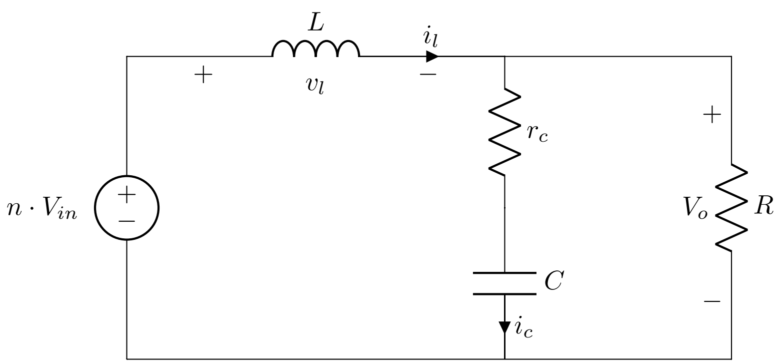

Figure 1: Simplified Forward Converter when switch is on

KVL at the outer path,

$$-nV_{in} + L\frac{di}{dt} + i_o R = 0 \tag{1}$$

Since,

$$i_o = i_l - i_c \tag{2}$$

$$L\frac{di}{dt} = nV_{in} - Ri_l + Ri_c \tag{3}$$

Find i_c by KVL on inductor and capacitor

$$-nV_{in} + L\frac{di}{dt} + i_c r_c + V_c = 0 \tag{4}$$

$$i_c = \frac{nV_{in} - L\frac{di}{dt} - V_c}{r_c} \tag{5}$$

Insert i_c into equation 3,

$$\hat{i}_l = \frac{di}{dt} = \frac{nV_{in}}{L} - \frac{r_c R}{(r_c + R)L}\frac{R}{(r_c + R)L}V_c \tag{6}$$

We know that $R >> r_c$ so we can simplify equation as,

$$\hat{i}_l = \frac{nV_{in}}{L} - \frac{r_c}{L}i_l - \frac{1}{L}v_c \tag{7}$$

Start derive V_c by KCL at output node,

$$i_c = i_l - \frac{V_o}{R} \tag{8}$$

Since,

$$v_o = v_c + r_c i_c \tag{9}$$

$$i_c = \frac{R}{R + r_c} i_l - \frac{1}{R + r_c} v_l \tag{10}$$

Since,

$$\hat{v}_c = \frac{i_c}{C} \tag{11}$$

$$\hat{v}_c = \frac{R}{C(R + r_c)} i_l - \frac{1}{C(R + r_c)} v_l \tag{12}$$

We know that $R >> r_c$ so we can simplify equation as,

$$\hat{V}_c = \frac{1}{C} i_l - \frac{1}{RC} v_l \tag{13}$$

According to state space equations,

$$\begin{bmatrix} \hat{i}_l \\ \hat{v}_c \end{bmatrix} = \begin{bmatrix} -\frac{r_c}{L} & -\frac{1}{v_c} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_l \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{n}{L} \\ 0 \end{bmatrix} V_{in} \tag{14}$$

For switch off position A matrix hasn't changed.But B matrix multiplied with duty ratio so,

$$\begin{bmatrix} \hat{i}_l \\ \hat{v}_c \end{bmatrix} = \begin{bmatrix} -\frac{r_c}{L} & -\frac{1}{v_c} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_l \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{dn}{L} \\ 0 \end{bmatrix} V_{in} \tag{15}$$

Find v_o as state space variables,

$$v_o = R i_o = R(i_l - i_c) = R(i_l - (\frac{v_o - v_c}{r_c})) \tag{16}$$

$$v_o = \frac{r_c R}{r_c + 1} i_l + \frac{1}{(r_c + 1)} v_c \tag{17}$$

Since $r_c << 1$,

$$v_o = r_c R i_l + v_c \tag{18}$$

$$C = \begin{bmatrix} R r_c \\ 1 \end{bmatrix} \tag{19}$$

$$\frac{v_o(s)}{d(s)} = C^T [SI - A]^{-1} B V_{in} \tag{20}$$

After a lengthy evaluation control to output TF is,

$$G_{vd} = \frac{\widetilde{v_o}}{\widetilde{d}}\bigg|_{\widetilde{v_i}=0} = \frac{nV_{in}}{RLC} \left[\frac{sR^2 C r_c + R + 1}{s^2 + s(\frac{r_c}{L} + \frac{1}{RC}) + \frac{1}{LC}} \right] \tag{21}$$