GEBZE TEKNİK ÜNİVERSİTESİ

BİLGİSAYAR MÜHENDİSLİĞİ

MAT 214

SAYISAL ANALİZ

HOMEWORK 1

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Answer of Section 2.1

Exercise 6.a

My Output

$(3x - e^x) = 0 \quad [1,2]$ for $\varepsilon < 10^{-5}$

n	DISTANCE_TO_ROOT	ABSOLUTE_ERROR	RELATIVE_ERROR
1	0,01831092966	1,50000000000	1,00000000000
2	0,50460267601	0,25000000000	0,14285714286
3	0,20341903718	0,12500000000	0,07692307692
4	0,08323318197	0,06250000000	0,04000000000
5	0,03020315278	0,03125000000	0,02040816327
6	0,00539040379	0,01562500000	0,01030927835
7	0,00659810663	0,00781250000	0,00518134715
8	0,00063844709	0,00390625000	0,00258397933
9	0,00236731253	0,00195312500	0,00129032258
10	0,00086226838	0,00097656250	0,00064557779
11	0,00011136983	0,00048828125	0,00032289312
12	0,00026367380	0,00024414063	0,00016147263
13	0,00007618579	0,00012207031	0,00008072980
14	0,00001758357	0,00006103516	0,00004036327
15	0,00002930322	0,00003051758	0,00002018204
16	0,00000586035	0,00001525879	0,00001009092
17	0,00000586148	0,00000762939	0,00000504543

Approximation root is 1,512138366699.
Process finished with exit code 0

Other questions contains in source code.

Answer of Section 2.2

Exercise 5

$$x^4 - 3x^2 - 3 = 0$$
$$with \quad g(x) = (3x^2+3)^{1/4} \quad P_0 = 1$$

$$g'(x) = \frac{3x}{2(3+3x^2)^{3/4}}$$

$$g'(1) = 0{,}3917 \approx 0{,}4 = k$$
$$\xi = \frac{k^n}{1-k} . |P_1 - P_0|$$
$$\frac{0.4^n}{(1-0.4)} . |1{,}793572 - 1{,}565084|$$
$$n = 7.26 \approx 7 \quad has \quad step \quad theorically .$$

$$\begin{aligned}
p_1 &= g(p_0) = 1{,}5650 && \ldots\ldots\ldots\ldots\ldots\ldots \\
p_2 &= g(p_1) = 1{,}79357 && \epsilon = 0{,}12764 \\
p_3 &= g(p_2) = 1{,}88594 && \epsilon = 0{,}04856 \\
p_4 &= g(p_3) = 1{,}92284 && \epsilon = 0{,}01983 \\
p_5 &= g(p_4) = 1{,}93750 && \epsilon = 0{,}00621 \\
p_6 &= g(p_5) = 1{,}94331 && \epsilon = 0{,}000989
\end{aligned}$$

$$\epsilon < 0.001 \quad and \quad p_6 = 1.94331 \quad is \quad approximation \quad value .$$

Answer of Section 2.3

Exercise 4

Let f (x) = −x ^{3} − cos x. With p 0 = −1 and p $_1$ = 0, find p $_3$.
a) Use the Secant method.

$$f(x) = -x^3 - \cos(x) \quad P_0 = -1 \quad P_1 = 0$$
$$p = p_1 - f\frac{(p_1)*(p_1 - p_0)}{(f(p_1) - f(p_0))}$$
$$a$$
$$p_0 = -1.0000 \quad p_1 = 0{,}0000$$
$$p_2 = 0{,}6850$$
$$p_3 = -1{,}2520$$

$$Approximation \quad is \quad -1.2520$$

b) Use the method of False Position.

$$f(x) = -x^3 - \cos(x) \quad P_0 = -1 \quad P_1 = 0$$
$$f(p_0) * f(p_1) < 0 \; then,$$
$$p_n * p_{n-1}$$

$$p_0 \;=\; 0{,}0000 \quad p_1 \;=\; -0{,}685073$$
$$p_2 \;=\; -0{,}841355$$
$$p_3 \;=\; -0{,}862547$$

$$Approximation \;\; value \;\; is \;\; -0{,}862547$$

Exercise 5

Use Newton's method to find solutions accurate to within 10 $^{-4}$ for the following problems.
a. x ^{3} − 2x ^{2} − 5 = 0, [1, 4]
b. x ^{3} + 3x ^{2} − 1 = 0, [−3, −2]

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

a) $x^3 - 2x^2 - 5 = 0 \qquad ,[1,4]$

$$For \;\; x_0 = 2$$
$$p_5 = 2.690$$
$$Approximation \;\; value is \;\; 2.690$$

$$x^3 + 3x^2 - 1 = 0 \qquad ,[-3,-2]$$

b) $For \; x_0 = -3$

$$p_3 = -2.879$$
$$Approximation \;\; value is \;\; -2.879$$