

ASSIGNMENT 2: CARDINALITY

LOG121: Set theory

Semester H24

INSTRUCTIONS The following exercises are not mandatory but should help you understand the course material better. All of these exercises are taken from Goldrei. Please *submit your solutions on Canvas*; both *legibly* handwritten and typeset solutions are accepted. The deadline is on the evening of the *15th October*.

EXERCISE (6.13) Provide bijections witnessing the following:

(b) $A \times (B \times C) \approx (A \times B) \times C$

(f) $C^{A \times B} \approx (C^B)^A$

EXERCISE (6.33) Let A and B be countable. Then $A \cup B$ is countable.

EXERCISE (6.37) Assume AC to prove that if X is infinite then $\mathbb{N} \preceq X$. (*Hint*: Recall that under AC, \preceq is total.)

EXERCISE (6.55) Let X be such that $X \times X \approx X$. Prove that

(a) if $X \neq \emptyset$ then X is infinite.

(b) $2^X \approx X^X$.

(c) $X + X := (X \times \{0\}) \cup (X \times \{1\}) \approx X$.