

LOG111 Hand-in 3

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Proof. By construction, the set $\Gamma_1 \cup \Gamma_2$ is unsatisfiable, so by compactness, there is a finite unsatisfiable subset $\Delta \subseteq \Gamma_1 \cup \Gamma_2$.

Consider

$$\Delta_1 := \Delta \setminus \Gamma_2 \qquad \Delta_2 := \Delta \setminus \Gamma_1.$$

We claim that Δ_1 and Δ_2 respectively axiomatize $\text{Th}(\Gamma_1)$ and $\text{Th}(\Gamma_2)$. We prove that this is the case for Δ_1 ; the argument for Δ_2 is completely analogous.

We need to prove that for any formula φ , $\Gamma_1 \vdash \varphi$ iff $\Delta_1 \vdash \varphi$. To this end, it suffices to prove their semantic counterpart by soundness and completeness.

The if direction is an immediate consequence of monotonicity. In the other direction, suppose that $\Gamma_1 \models \varphi$ and let $M \models \Delta_1$. If $M \models \Gamma_1$ then we are done. On the other hand, if $M \not\models \Gamma_1$ then it follows that $M \models \Delta_2$, but this means that $M \models \Delta_1 \cup \Delta_2$ contradicting the fact that $\Delta = \Delta_1 \cup \Delta_2$ is unsatisfiable. \square

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$$\frac{\frac{\frac{[c_x < c_y]^2 \quad [c_y < c_x]^1}{\text{T}} \quad \frac{\frac{c_x < c_x}{\text{R}} \quad \frac{\perp}{c_x \not< c_y}}{\text{RAA}_2^2}}{\perp} \quad c_x < c_y}{\frac{\perp}{c_y \not< c_x} \text{RAA}_2^1} \perp$$