Assignment 2 Report

1 Sudoku Assignment

1.1 Part a

Describe the representation of a solution (answer) of the problem as explained during the course.

1.1.1 Variables :

$X_{i,j}$ represents the variable which is a cell corresponding to the i-th row and j-th column in the Sudoku grid. Sudoku grid is 9×9 so $0 < i < 10$ and $0 < j < 10$, there are 81 variables in total.

1.1.2 Domain :

The domain for each variable $X_{i,j}$ can be an integer between 1 to 9, inclusive

$$D(X_{i,j}) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

1.1.3 Constraints :

There are three constraints for this problem. 1. Row Constraint: Each row must contain all the numbers from 1 to 9 once.

$$\forall i, j, k \text{ where } k \neq j : X_{i,j} \neq X_{i,k} \text{ for } i, j, k \in \{1, 2, ..., 9\}$$

2. Column Constraint: Each column must contain all the numbers from 1 to once.

$$\forall j, i, k \text{ where } k \neq i : X_{i,j} \neq X_{k,j} \text{ for } i, j, k \in \{1, 2, ..., 9\}$$

3. Subgrid Constraint: Each 3x3 subgrid must contain all the numbers from 1 to 9 once

$$\forall m, n, i, j, k, l \text{ where } k, l \neq i, j : X_{3m+i,3n+j} \neq X_{3m+k,3n+l}$$

$$\text{for } m, n, i, j, k, l \in \{0, 1, 2\}.$$

So the answer is grid X = $[x_{0,0}, x_{0,1} ... x_{9,9}]$ which satisfies all three constraints with the given domain.

1.2 Part b

Give the equation for the restriction of the problem.

The restrictions are the solving constraints of this problem with a given domain. which are :

1. Row Constraint: Each row must contain all the numbers from 1 to 9 once.

$$\forall i, j, k \text{ where } k \neq j : X_{i,j} \neq X_{i,k} \text{ for } i, j, k \in \{1, 2, ..., 9\}$$

2. Column Constraint: Each column must contain all the numbers from 1 to once.

$$\forall j, i, k \text{ where } k \neq i : X_{i,j} \neq X_{k,j} \text{ for } i, j, k \in \{1, 2, ..., 9\}$$

3. Subgrid Constraint: Each 3x3 subgrid must contain all the numbers from 1 to 9 once

$$\forall m, n, i, j, k, l \text{ where } k, l \neq i, j : X_{3m+i,3n+j} \neq X_{3m+k,3n+l}$$

$$\text{for } m, n, i, j, k, l \in \{0, 1, 2\}.$$

1.3 Part c

What is considered as a state? In addition, explain why.

The state is a configuration of the elements within the domain at a specific point in time while solving the problem. In this problem, the state is a 9x9 grid with numbers arranged within the domain. The states include the given initial numbers and the numbers added in the process of solving the problem.

1.4 Part d

What is the initial state? In addition, explain why

The initial state of Sudoku consists of a partially filled 9x9 grid, serving as the starting point for solving the puzzle. The solution is sought from this starting configuration, making it the initial state.

1.5 Part e

Which is/are the possible action(s)? In addition, explain why.

In this problem, there are two possible actions:

1. Adding a new number to the grid that meets the conditions and has not been previously placed.
2. Removing a previously placed number because the whole grid is not filled and makes adding a new number impossible which is a backtracking to previous state.

These actions help to solve the problem. New numbers are recursively attempted to be added until no further progress can be made, then alternative paths are explored by backtracking.

1.6 Part f

What is the maximum branching factor of the tree (b)? In addition, explain why.

The branching factor is the number of actions available to move to another state. In this problem, it is necessary to add a new number to any cell in the grid to move next state. If there are no restrictions for the next move, there can be up to 9 numbers to be used to create the next state, so the maximum branching factor(b) is 9.

1.7 Part g

What is the maximum dept of the search tree(m)? In addition, explain why.

The depth of the search tree corresponds to the number of moves made until the conditions are met without returning to a previous state. In this problem, each cell in the grid needs to be filled, with one number added per action starting from the initial state. Therefore, the depth of the search tree is the maximum number of cells that can be filled without backtracking, which is the total number of cells in the problem, 9x9 = 81 if all the cells are empty in the initial state.