Student Information

Name: Burak Metehan Tunçel

ID: 2468726

Answer 1

a)

$$G_1 = (V_1, \Sigma_1, R_1, S_1)$$

with

$$V_1 = \{S_1, a, b\} \qquad \Sigma_1 = \{a, b\} \qquad R_1 = \begin{Bmatrix} S_1 \to S_1bS_1bS_1aS_1, \\ S_1 \to S_1bS_1aS_1bS_1, \\ S_1 \to S_1aS_1bS_1bS_1, \\ S_1 \to e \end{Bmatrix}$$

b)

$$G_2 = (V_2, \Sigma_2, R_2, S_2)$$

with

$$V_2 = \{S_2, a, b\} \qquad \Sigma_2 = \{a, b\} \qquad R_2 = \begin{Bmatrix} S_2 \to aS_2b, \\ S_2 \to aaS_2b, \\ S_2 \to e \end{Bmatrix}$$

c)

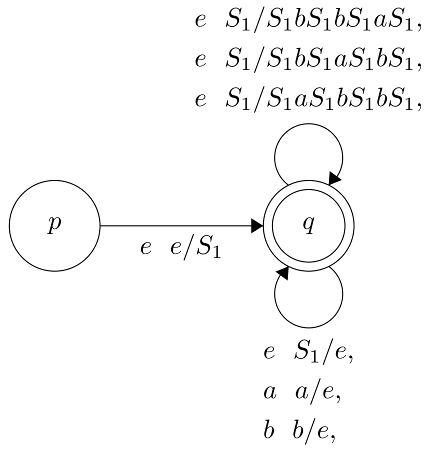
Let

$$M_1 = (\{p, q\}, \Sigma_1, V_1, \Delta, p, \{q\})$$

where, Δ contains the following transitions

$$\Delta = \begin{Bmatrix} ((p, e, e), (q, S_1)), \\ ((q, e, S_1), (q, S_1bS_1bS_1aS_1)), \\ ((q, e, S_1), (q, S_1bS_1aS_1bS_1)), \\ ((q, e, S_1), (q, S_1aS_1bS_1bS_1)), \\ ((q, e, S_1), (q, e)), \\ ((q, a, a), (q, e)), \\ ((q, b, b), (q, e)), \end{Bmatrix}$$

PDA that accepts the L_1:



d)

Let S be a new symbol and let $G_3 = (V_3, \Sigma_3, R_3, S)$ where

- $V_3 = V_1 \cup V_2 \cup \{S\} = \{S_1, S_2, S, a, b\}$
- $\Sigma_3 = \Sigma_1 \cup \Sigma_2 = \{a, b\}$
- $R_3 = R_1 \cup R_2 \cup \{S \to S_1, S \to S_2\} = \left\{\begin{array}{ll} S \to S_1, & S \to S_2, \\ S_1 \to S_1bS_1bS_1aS_1, & S_1 \to S_1bS_1aS_1bS_1, \\ S_1 \to S_1aS_1bS_1bS_1, & S_1 \to e, \\ S_2 \to aS_2b, & S_2 \to aaS_2b, \\ S_2 \to e & \end{array}\right\}$

The only rules involving S are $S \to S_1$ and $S \to S_2$, so $S \Rightarrow_G^* w$ if and only if either $S_1 \Rightarrow_{G_1}^* w$ or $S_2 \Rightarrow_{G_2}^* w$. Also, since G_1 and G_2 have disjoint sets of nonterminals, the last disjunction is equivalent to saying that $w \in L(G_1) \cup L(G_2)$.

Answer 2

a)

One way to show G_1 i ambiguous is showing that there are more than 1 derivations or parse tree for string w such that $w \in L(G_1)$. For example, there are more than 1 derivations for $w = 00111$:

$$\begin{array}{ccccccccccc} S & \Rightarrow & AS & \Rightarrow & 0A1S & \Rightarrow & 0A11S & \Rightarrow & 00111S & \Rightarrow & 00111 \\ S & \Rightarrow & AS & \Rightarrow & A1S & \Rightarrow & 0A11S & \Rightarrow & 00111S & \Rightarrow & 00111 \end{array}$$

Therefore, G_1 is ambiguous.

b)

The following grammer is unambiguous grammar for $L(G_1)$:

$$G_1' = \{V', \Sigma, R', S\}$$

where

- $V' = \{0, 1, S, A, B\}$
- $\Sigma = \{0, 1\}$
- $R' = \left\{\begin{array}{l} S \to AS \mid e, \\ A \to 0A1 \mid 0B1, \\ B \to B1 \mid e \end{array}\right\}$

c)

The left-most derivation:

$$S \Rightarrow AS \Rightarrow 0A1S \Rightarrow 00B11S \Rightarrow 00B111S \Rightarrow 00111$$

The left-most derivation parse tree: