Student Information

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Answer 1

(a)

Since it is given that Normal approximation can be used, the following formula can be used to determine the size of the Monte Carlo study:

$$\mathrm{N} \geq 0.25\left(\frac{z_{\alpha/2}}{\varepsilon}\right)^2$$

In question the followings are given,

$$\alpha = 0.01 \quad \varepsilon = 0.02$$

So,

$$\begin{aligned}
\mathrm{N} &\geq 0.25\left(\frac{z_{0.005}}{0.02}\right)^2 \\
&\geq 0.25\left(\frac{2.575}{0.02}\right)^2 \\
&\geq 4144.140625
\end{aligned}$$

Since N should be a integer, **4145** is the size of Monte Carlo study.

(b)

Expected Value for the Weight of an Automobile

The weight of an automobile has Gamma distribution with $\alpha_a = 190$, $\lambda_a = 0.15$. Since expected value of Gamma distribution is $\frac{\alpha}{\lambda}$,

$$\mathbf{E}\left(w_a\right) = \frac{\alpha_a}{\lambda_a} = \frac{190}{0.15} = 1266.\bar{6} \approx 1267$$

where $w_a = expected\_weight\_of\_automobile$.

Expected Value for the Weight of an Truck

The weight of an automobile has Gamma distribution with $\alpha_t = 110$, $\lambda_t = 0.01$. Since expected value of Gamma distribution is $\frac{\alpha}{\lambda}$,

$$\mathbf{E}\left(w_t\right) = \frac{\alpha_t}{\lambda_t} = \frac{110}{0.01} = 11000$$

where $w_t = expected\_weight\_of\_truck$.

Expected Total Weight of all Automobiles

The total weight of all automobiles that pass over the bridge on a day can be calculated as follows:

$$\mathbf{E}\left(\textit{expected\_number\_of\_automobiles} \times \textit{expected\_weight\_of\_automobile}\right)$$

For the convenience, let $n_a = \textit{expected\_number\_of\_automobiles}$ and $w_a = \textit{expected\_weight\_of\_automobile}$. Since these are independent events,

$$\mathbf{E}\left(n_a \times w_a\right) = \mathbf{E}\left(n_a\right) \times \mathbf{E}\left(w_a\right)$$

$\mathbf{E}\left(w_a\right)$ was calculated and $\mathbf{E}\left(w_a\right) \approx 1267$. $\mathbf{E}\left(n_a\right)$ should be calculated. Since the expected number of automobiles that pass over the bridge on a day has Poisson distribution with $\lambda = 50$ and the expected value of Poisson distribution is λ,

$$\mathbf{E}\left(n_a\right) = \lambda = 50$$

So,

$$\begin{aligned}\mathbf{E}\left(n_a \times w_a\right) &= \mathbf{E}\left(n_a\right) \times \mathbf{E}\left(w_a\right)\\ &= 50 \times \frac{190}{0.15} = 63333.\bar{3} \approx 63333\end{aligned}$$

Expected Total Weight of all Trucks

The total weight of all trucks that pass over the bridge on a day can be calculated as follows:

$$\mathbf{E}\left(\textit{expected\_number\_of\_trucks} \times \textit{expected\_weight\_of\_truck}\right)$$

For the convenience, let $n_t = \textit{expected\_number\_of\_trucks}$ and $w_t = \textit{expected\_weight\_of\_truck}$. Since these are independent events,

$$\mathbf{E}\left(n_t \times w_t\right) = \mathbf{E}\left(n_t\right) \times \mathbf{E}\left(w_t\right)$$

$\mathbf{E}\left(w_t\right)$ was calculated and $\mathbf{E}\left(w_t\right) = 11000$. $\mathbf{E}\left(n_t\right)$ should be calculated. Since the expected number of automobiles that pass over the bridge on a day has Poisson distribution with $\lambda = 10$ and the expected value of Poisson distribution is λ,

$$\mathbf{E}\left(n_t\right) = \lambda = 10$$

So,

$$\begin{aligned}\mathbf{E}\left(n_t \times w_t\right) &= \mathbf{E}\left(n_t\right) \times \mathbf{E}\left(w_t\right)\\ &= 10 \times 11000 = 110000\end{aligned}$$

Answer 2

Algorithm 5.1 of the textbook can be used to generate the weights of automobiles and trucks because the weights of them have Poisson distribution. The sample MATLAB code for automobiles:

```
% automobiles
lambda = 50;       % parameter
U = rand;          % generated uniform variable
i = 0;             % initial value
F = exp(-lambda); % initial value of F(0)
while (U>=F)
  i = i + 1;
  F = F + exp(-lambda) * lambda^i / gamma(i+1);
end
n_a = i;           % total number of automobiles
```

Since the weight of each automobile and truck has a Gamma random variable, the formula from example 5.11 can be used. This value may be used to estimate the probability that the total weight of all the vehicles that pass over the bridge on a day is more than 200 tons.

```
X = sum(-1/0.15 * log(rand(190,1))); % formula from example 5.11
```

This variable is valid for only 1 occurence. Therefore, it is necessary to calculate new variable j times (number of automobiles or trucks) and sum them up.

By using the matlab code that is attached and provided (hw4.m);

- The estimated probability of having the total weight of all the vehicles that pass over the bridge in a day more than 200 tons is **0.222437**
- The estimated total weight of all the vehicles that pass over the bridge in a day is **173632.748586** kilograms which is approximately **173.6** tons.
- The estimated Standard deviation is **35781.459393**. In this simulation $\alpha = 0.01$ and $\varepsilon = 0.02$ are used, so the study yields to accurate results within the error margin of 0.02 and 0.99% of the time.

  Since the theoretical Standard deviation is $\text{Std}(X) = \dfrac{\sigma}{\sqrt{N}}$, Std($X$) can be reduced by using the larger study sizes.

The screenshot of the output of `hw4.m`,

```
>> hw4
Estimated probability = 0.222437
Expected weight = 173632.748586
Standard deviation = 35781.459393
```

The source code (hw4.m) that is used for simulation.

```
N = 4145; % size of Monte Carlo Simulation
total_weight = zeros(N,1); % vector to keep the total weight of vehicles for each monte carlo run

for k=1:N
    weight = 0; % the total weight for this monte carlo run

    % automobiles
    lambda = 50;      % parameter
    U = rand;         % generated uniform variable
    i = 0;            % initial value
    F = exp(-lambda); % initial value of F(0)
    while (U>=F)
        i = i + 1;
        F = F + exp(-lambda) * lambda^i / gamma(i+1);
    end

    n_a = i;        % total number of automobiles
    for j=1:n_a     % summing total weight of automobiles
        X = sum(-1/0.15 * log(rand(190,1))); % formula from example 5.11
        weight = weight + X;
    end

    % trucks
    lambda = 10;      % parameter
    U = rand;         % generated uniform variable
    i = 0;            % initial value
    F = exp(-lambda); % initial value of F(0)
    while (U>=F)
        i = i + 1;
        F = F + exp(-lambda) * lambda^i / gamma(i+1);
    end

    n_t = i;        % total number of trucks
    for j=1:n_t     % summing total weight of trucks
        X = sum(-1/0.01 * log(rand(110,1))); % formula from example 5.11
        weight = weight + X;
    end

    total_weight(k) = weight;
end

prob_over_200 = mean(total_weight > 200000);
expected_total_weight = mean(total_weight);
std_weight = std(total_weight);

fprintf('Estimated probability = %f\n', prob_over_200);
fprintf('Expected weight = %f\n', expected_total_weight);
fprintf('Standard deviation = %f\n', std_weight);
```