Student Information

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Answer 1

a)

The regular expression:

$$\left((a \cup b)^* \, aa \, (a \cup b)^* \, bb \, (a \cup b)^*\right) \cup \left((a \cup b)^* \, bb \, (a \cup b)^* \, aa \, (a \cup b)^*\right)$$

This is equivalent to:

$$(a \cup b)^*((aa(a \cup b)^*bb) \cup (bb(a \cup b)^*aa))(a \cup b)^*$$

b)

Formally define and draw an NFA that recognizes the language.

Let M be a NFA:

$$M = (K, \Sigma, \Delta, s, F)$$

where,

- $K = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}\}$
- $\Sigma = \{a, b\}$
- $s = q_0$, the initial state
- $F = \{q_{11}\}$, final states
- The transition relation$\Delta = \begin{Bmatrix} (q_0, a, q_1), (q_0, b, q_1), (q_0, e, q_1), (q_1, e, q_0), (q_1, a, q_2), (q_1, b, q_6), \\ (q_2, a, q_3), (q_3, a, q_4), (q_3, b, q_4), (q_3, e, q_4), (q_4, e, q_3), (q_4, b, q_5), \\ (q_5, b, q_{10}), (q_6, b, q_7), (q_7, a, q_8), (q_7, b, q_8), (q_7, e, q_8), (q_8, e, q_7), \\ (q_8, a, q_9), (q_9, a, q_{10}), (q_{10}, a, q_{11}), (q_{10}, b, q_{11}), (q_{10}, e, q_{11}), (q_{11}, e, q_{10})\} \end{Bmatrix}$

The NFA M:

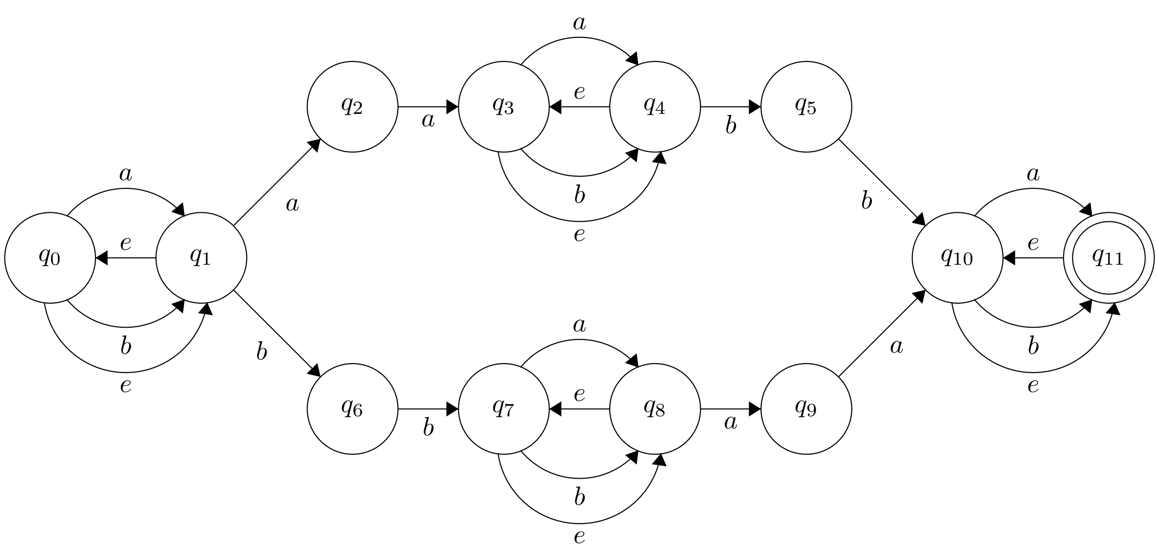

Figure 1: M

c)

We want to construct DFA. In other words we want to acquire:

$$M' = (K', \Sigma, \delta', s', F')$$

While constructing a DFA from an NFA, $E(q)$ should be computed for $q \in K$. Instead of doing this, I will compute $E(q)$ for some, or all, $q \in K$ when it is needed. Also, it can be decided that $s' = q_{100}$

We start from the initial state of DFA, $s' = q_{100} = E(q_0)$.

$$E(q_0) = \{q_0, q_1\}$$

$$(q_0, a, q_1), (q_1, a, q_2)$$

are all the transitions (q, a, p) for some $q \in E(q_0)$. It follows that

$$\delta'(q_{100}, a) = E(q_1) \cup E(q_2)$$

I need to compute $E(q_1), E(q_2)$:

- $E(q_1) = \{q_0, q_1\}$
- $E(q_2) = \{q_2\}$

$$\delta'(q_{100}, a) = \{q_0, q_1, q_2\}$$

There is a new state. Let the state be q_{101}. Similarly,

$$(q_0, b, q_1), (q_1, b, q_6)$$

are all the transitions (q, b, p) for some $q \in E(q_0)$. It follows that

$$\delta'(q_{100}, b) = E(q_1) \cup E(q_6)$$

I have $E(q_1)$. I need to compute $E(q_6)$:

- $E(q_6) = \{q_6\}$

$$\delta'(q_{100}, b) = \{q_0, q_1, q_6\}$$

There is a new state. Let the state be q_{102}.

New States and Transitions

States:

- $q_{101} = \{q_0, q_1, q_2\}$
- $q_{102} = \{q_0, q_1, q_6\}$

Transitions:

- $\delta'(q_{100}, a) = q_{101}$
- $\delta'(q_{100}, b) = q_{102}$

$$q_{101}$$

Repeating the calculation for newly introduced state q_{101}.

$$q_{101} = \{q_0, q_1, q_2\}$$

So,

$$E(q_{101}) = \{q_0, q_1, q_2\}$$

$$(q_0, a, q_1), (q_1, a, q_2), (q_2, a, q_3)$$

are all the transitions (q, a, p) for some $q \in E(q_{101})$. It follows that

$$\delta'(q_{101}, a) = E(q_1) \cup E(q_2) \cup E(q_3)$$

I have $E(q_1), E(q_2)$. I need to compute $E(q_3)$:

- $E(q_3) = \{q_3, q_4\}$

$$\delta'(q_{101}, a) = \{q_0, q_1, q_2, q_3, q_4\}$$

There is a new state. Let the state be q_{103}. Similarly,

$$(q_0, b, q_1), (q_1, b, q_6)$$

are all the transitions (q, b, p) for some $q \in E(q_{101})$. It follows that

$$\delta'(q_{101}, b) = E(q_1) \cup E(q_6)$$

I have $E(q_1), E(q_6)$. No need to compute anything.

$$\delta'(q_{101}, b) = \{q_0, q_1, q_6\}$$

There is not a new state, it is same with the q_{102}.

New States and Transitions

States:

- $q_{103} = \{q_0, q_1, q_2, q_3, q_4\}$

Transitions:

- $\delta'(q_{101}, a) = q_{103}$
- $\delta'(q_{101}, b) = q_{102}$

$$q_{102}$$

Repeating the calculation for newly introduced state q_{102}.

$$q_{102} = \{q_0, q_1, q_6\}$$

So,

$$E(q_{102}) = \{q_0, q_1, q_6\}$$

$$(q_0, a, q_1), (q_1, a, q_2)$$

are all the transitions (q, a, p) for some $q \in E(q_{102})$. It follows that

$$\delta'(q_{102}, a) = E(q_1) \cup E(q_2)$$

I have $E(q_1), E(q_2)$. No need to compute anything.

$$\delta'(q_{102}, a) = \{q_0, q_1, q_2\}$$

There is not a new state, it is same with the q_{101}. Similarly,

$$(q_0, b, q_1), (q_1, b, q_6), (q_6, b, q_7)$$

are all the transitions (q, b, p) for some $q \in E(q_{102})$. It follows that

$$\delta'(q_{102}, b) = E(q_1) \cup E(q_6) \cup E(q_7)$$

I have $E(q_1), E(q_6)$. I need to compute $E(q_7)$:

- $E(q_7) = \{q_7, q_8\}$

$$\delta'(q_{102}, b) = \{q_0, q_1, q_6, q_7, q_8\}$$

There is a new state. Let the state be q_{104}.

New States and Transitions

States:

- $q_{104} = \{q_0, q_1, q_6, q_7, q_8\}$

Transitions:

- $\delta'(q_{102}, a) = q_{101}$
- $\delta'(q_{102}, b) = q_{104}$

q_{103}

Repeating the calculation for newly introduced state q_{103}.

$$q_{103} = \{q_0, q_1, q_2, q_3, q_4\}$$

So,

$$E(q_{103}) = \{q_0, q_1, q_2, q_3, q_4\}$$

$$(q_0, a, q_1), (q_1, a, q_2), (q_2, a, q_3), (q_3, a, q_4)$$

are all the transitions (q, a, p) for some $q \in E(q_{103})$. It follows that

$$\delta'(q_{103}, a) = E(q_1) \cup E(q_2) \cup E(q_3) \cup E(q_4)$$

I have $E(q_1), E(q_2), E(q_3)$. I need to compute $E(q_4)$:

- $E(q_4) = \{q_3, q_4\}$

$$\delta'(q_{103}, a) = \{q_0, q_1, q_2, q_3, q_4\}$$

There is not a new state, it is same with the q_{103}. Similarly,

$$(q_0, b, q_1), (q_1, b, q_6), (q_3, b, q_4), (q_4, b, q_5)$$

are all the transitions (q, b, p) for some $q \in E(q_{103})$. It follows that

$$\delta'(q_{103}, b) = E(q_1) \cup E(q_4) \cup E(q_5) \cup E(q_6)$$

I have $E(q_1), E(q_4), E(q_6)$. I need to compute $E(q_5)$:

- $E(q_5) = \{q_5\}$

$$\delta'(q_{103}, b) = \{q_0, q_1, q_3, q_4, q_5, q_6\}$$

There is a new state. Let the state be q_{105}.

New States and Transitions

States:

- $q_{105} = \{q_0, q_1, q_3, q_4, q_5, q_6\}$

Transitions:

- $\delta'(q_{103}, a) = q_{103}$
- $\delta'(q_{103}, b) = q_{105}$

q_{104}

Repeating the calculation for newly introduced state q_{104}.

$$q_{104} = \{q_0, q_1, q_6, q_7, q_8\}$$

So,

$$E(q_{104}) = \{q_0, q_1, q_6, q_7, q_8\}$$

$$(q_0, a, q_1), (q_1, a, q_2), (q_7, a, q_8), (q_8, a, q_9)$$

are all the transitions (q, a, p) for some $q \in E(q_{104})$. It follows that

$$\delta'(q_{104}, a) = E(q_1) \cup E(q_2) \cup E(q_8) \cup E(q_9)$$

I have $E(q_1), E(q_2)$. I need to compute $E(q_8), E(q_9)$:

- $E(q_8) = \{q_7, q_8\}$
- $E(q_9) = \{q_9\}$

$$\delta'(q_{104}, a) = \{q_0, q_1, q_2, q_7, q_8, q_9\}$$

There is a new state. Let the state be q_{106}. Similarly,

$$(q_0, b, q_1), (q_1, b, q_6), (q_6, b, q_7), (q_7, b, q_8)$$

are all the transitions (q, b, p) for some $q \in E(q_{104})$. It follows that

$$\delta'(q_{104}, b) = E(q_1) \cup E(q_6) \cup E(q_7) \cup E(q_8)$$

I have $E(q_1), E(q_6), E(q_7), E(q_8)$. No need to compute anything.

$$\delta'(q_{104}, b) = \{q_0, q_1, q_6, q_7, q_8\}$$

There is not a new state, it is same with the q_{104}.

New States and Transitions

States:

- $q_{106} = \{q_0, q_1, q_2, q_7, q_8, q_9\}$

Transitions:

- $\delta'(q_{104}, a) = q_{106}$
- $\delta'(q_{104}, b) = q_{104}$

q_{105}

Repeating the calculation for newly introduced state q_{105}.

$$q_{105} = \{q_0, q_1, q_3, q_4, q_5, q_6\}$$

So,

$$E(q_{105}) = \{q_0, q_1, q_3, q_4, q_5, q_6\}$$

$$(q_0, a, q_1), (q_1, a, q_2), (q_3, a, q_4)$$

are all the transitions (q, a, p) for some $q \in E(q_{105})$. It follows that

$$\delta'(q_{105}, a) = E(q_1) \cup E(q_2) \cup E(q_4)$$

I have $E(q_1), E(q_2), E(q_4)$. No need to compute anything.

$$\delta'(q_{105}, a) = \{q_0, q_1, q_2, q_3, q_4\}$$

There is not a new state, it is same with the q_{103}. Similarly,

$$(q_0, b, q_1), (q_1, b, q_6), (q_3, b, q_4), (q_4, b, q_5), (q_5, b, q_{10}), (q_6, b, q_7)$$

are all the transitions (q, b, p) for some $q \in E(q_{105})$. It follows that

$$\delta'(q_{105}, b) = E(q_1) \cup E(q_4) \cup E(q_5) \cup E(q_6) \cup E(q_7) \cup E(q_{10})$$

I have $E(q_1), E(q_4), E(q_5), E(q_6), E(q_7)$. I need to compute $E(q_{10})$:

- $E(q_{10}) = \{q_{10}, q_{11}\}$

$$\delta'(q_{105}, b) = \{q_0, q_1, q_3, q_4, q_5, q_6, q_7, q_8, q_{10}, q_{11}\}$$

There is a new state. Let the state be q_{107}.

New States and Transitions

States:

- $q_{107} = \{q_0, q_1, q_3, q_4, q_5, q_6, q_7, q_8, q_{10}, q_{11}\}$

Transitions:

- $\delta'(q_{105}, a) = q_{103}$
- $\delta'(q_{105}, b) = q_{107}$

q_{106}

Repeating the calculation for newly introduced state q_{106}.

$$q_{106} = \{q_0, q_1, q_2, q_7, q_8, q_9\}$$

So,

$$E(q_{106}) = \{q_0, q_1, q_2, q_7, q_8, q_9\}$$

$$(q_0, a, q_1), (q_1, a, q_2), (q_2, a, q_3), (q_7, a, q_8), (q_8, a, q_9), (q_9, a, q_{10})$$

are all the transitions (q, a, p) for some $q \in E(q_{106})$. It follows that

$$\delta'(q_{106}, a) = E(q_1) \cup E(q_2) \cup E(q_3) \cup E(q_8) \cup E(q_9) \cup E(q_{10})$$

I have $E(q_1), E(q_2), E(q_3), E(q_8), E(q_9), E(q_{10})$. No need to compute anything.

$$\delta'(q_{106}, a) = \{q_0, q_1, q_2, q_3, q_4, q_7, q_8, q_9, q_{10}, q_{11}\}$$

There is a new state. Let the state be q_{108}. Similarly,

$$(q_0, b, q_1), (q_1, b, q_6), (q_7, b, q_8)$$

are all the transitions (q, b, p) for some $q \in E(q_{106})$. It follows that

$$\delta'(q_{106}, b) = E(q_1) \cup E(q_6) \cup E(q_8)$$

I have $E(q_1), E(q_6), E(q_8)$. No need to compute anything.

$$\delta'(q_{106}, b) = \{q_0, q_1, q_6, q_7, q_8\}$$

There is not a new state, it is same with q_{104}.

New States and Transitions

States:

- $q_{108} = \{q_0, q_1, q_2, q_3, q_4, q_7, q_8, q_9, q_{10}, q_{11}\}$

Transitions:

- $\delta'(q_{106}, a) = q_{108}$
- $\delta'(q_{106}, b) = q_{104}$

q_{107}

Repeating the calculation for newly introduced state q_{107}.

$$q_{107} = \{q_0, q_1, q_3, q_4, q_5, q_6, q_7, q_8, q_{10}, q_{11}\}$$

So,

$$E(q_{107}) = \{q_0, q_1, q_3, q_4, q_5, q_6, q_7, q_8, q_{10}, q_{11}\}$$

$$(q_0, a, q_1), (q_1, a, q_2), (q_3, a, q_4), (q_7, a, q_8), (q_8, a, q_9), (q_{10}, a, q_{11})$$

are all the transitions (q, a, p) for some $q \in E(q_{107})$. It follows that

$$\delta'(q_{107}, a) = E(q_1) \cup E(q_2) \cup E(q_4) \cup E(q_8) \cup E(q_9) \cup E(q_{11})$$

I have $E(q_1), E(q_2), E(q_4), E(q_8), E(q_9)$. I need to compute $E(q_{11})$:

- $E(q_{11}) = \{q_{10}, q_{11}\}$

$$\delta'(q_{107}, a) = \{q_0, q_1, q_2, q_3, q_4, q_7, q_8, q_9, q_{10}, q_{11}\}$$

There is not a new state, it is same with q_{108}. Similarly,

$$(q_0, b, q_1), (q_1, b, q_6), (q_3, b, q_4), (q_4, b, q_5), (q_5, b, q_6), (q_6, b, q_7), (q_7, b, q_8), (q_{10}, b, q_{11})$$

are all the transitions (q, b, p) for some $q \in E(q_{107})$. It follows that

$$\delta'(q_{107}, b) = E(q_1) \cup E(q_4) \cup E(q_5) \cup E(q_6) \cup E(q_7) \cup E(q_8) \cup E(q_{11})$$

I have $E(q_1), E(q_4), E(q_5), E(q_6), E(q_7), E(q_8), E(q_{11})$. No need to compute anything.

$$\delta'(q_{107}, b) = \{q_0, q_1, q_3, q_4, q_5, q_6, q_7, q_8, q_{10}, q_{11}\}$$

There is not a new state, it is same with q_{107}.

New States and Transitions

States:

-

Transitions:

- $\delta'(q_{107}, a) = q_{108}$
- $\delta'(q_{107}, b) = q_{107}$

$$q_{108}$$

Repeating the calculation for newly introduced state q_{108}.

$$q_{108} = \{q_0, q_1, q_2, q_3, q_4, q_7, q_8, q_9, q_{10}, q_{11}\}$$

So,

$$E(q_{108}) = \{q_0, q_1, q_2, q_3, q_4, q_7, q_8, q_9, q_{10}, q_{11}\}$$

$$(q_0, a, q_1), (q_1, a, q_2), (q_2, a, q_3), (q_3, a, q_4), (q_7, a, q_8), (q_8, a, q_9), (q_9, a, q_{10}), (q_{10}, a, q_{11})$$

are all the transitions (q, a, p) for some $q \in E(q_{108})$. It follows that

$$\delta'(q_{108}, a) = E(q_1) \cup E(q_2) \cup E(q_3) \cup E(q_4) \cup E(q_8) \cup E(q_9) \cup E(q_{10}) \cup E(q_{11})$$

I have $E(q_1), E(q_2), E(q_3), E(q_4), E(q_8), E(q_9), E(q_{10}), E(q_{11})$. No need to compute anything.

$$\delta'(q_{108}, a) = \{q_0, q_1, q_2, q_3, q_4, q_7, q_8, q_9, q_{10}, q_{11}\}$$

There is not a new state, it is same with the q_{108}. Similarly,

$$(q_0, b, q_1), (q_1, b, q_6), (q_3, b, q_4), (q_4, b, q_5), (q_7, b, q_8), (q_{10}, b, q_{11})$$

are all the transitions (q, b, p) for some $q \in E(q_{107})$. It follows that

$$\delta'(q_{108}, b) = E(q_1) \cup E(q_4) \cup E(q_5) \cup E(q_6) \cup E(q_8) \cup E(q_{11})$$

I have $E(q_1), E(q_4), E(q_5), E(q_6), E(q_8), E(q_{11})$. No need to compute anything.

$$\delta'(q_{108}, b) = \{q_0, q_1, q_3, q_4, q_5, q_6, q_7, q_8, q_{10}, q_{11}\}$$

There is not a new state, it is same with q_{107}.

New States and Transitions

States:

-

Transitions:

- $\delta'(q_{108}, a) = q_{108}$
- $\delta'(q_{108}, b) = q_{107}$

Constructing the DFA

Finally, we reached the the end and we have new states as follows:

- $q_{101} = \{q_0, q_1, q_2\}$
- $q_{102} = \{q_0, q_1, q_6\}$
- $q_{103} = \{q_0, q_1, q_2, q_3, q_4\}$
- $q_{104} = \{q_0, q_1, q_6, q_7, q_8\}$
- $q_{105} = \{q_0, q_1, q_3, q_4, q_5, q_6\}$
- $q_{106} = \{q_0, q_1, q_2, q_7, q_8, q_9\}$
- $q_{107} = \{q_0, q_1, q_3, q_4, q_5, q_6, q_7, q_8, q_{10}, q_{11}\}$
- $q_{108} = \{q_0, q_1, q_2, q_3, q_4, q_7, q_8, q_9, q_{10}, q_{11}\}$

Since, in the NFA M, q_{11} is the final states, states that include q_{11} are final states of DFA M. In this situation, they are q_{107} and q_{108}. So, we have

$$M' = (K', \Sigma, \delta', s', F')$$

where,

- $K' = \{q_{100}, q_{101}, q_{102}, q_{103}, q_{104}, q_{105}, q_{106}, q_{107}, q_{108}\}$
- $\Sigma = \{a, b\}$
- $s' = q_{100}$, the initial state
- $F' = \{q_{107}, q_{108}\}$, final states
- The transition relation δ':

q_0	σ	$\delta'(q, \sigma)$
q_{100}	a	q_{101}
q_{100}	b	q_{102}
q_{101}	a	q_{103}
q_{101}	b	q_{102}
q_{102}	a	q_{101}
q_{102}	b	q_{104}
q_{103}	a	q_{103}
q_{103}	b	q_{105}
q_{104}	a	q_{106}
q_{104}	b	q_{104}
q_{105}	a	q_{103}
q_{105}	b	q_{107}
q_{106}	a	q_{108}
q_{106}	b	q_{104}
q_{107}	a	q_{108}
q_{107}	b	q_{107}
q_{108}	a	q_{108}
q_{108}	b	q_{107}

So, we get the following DFA:

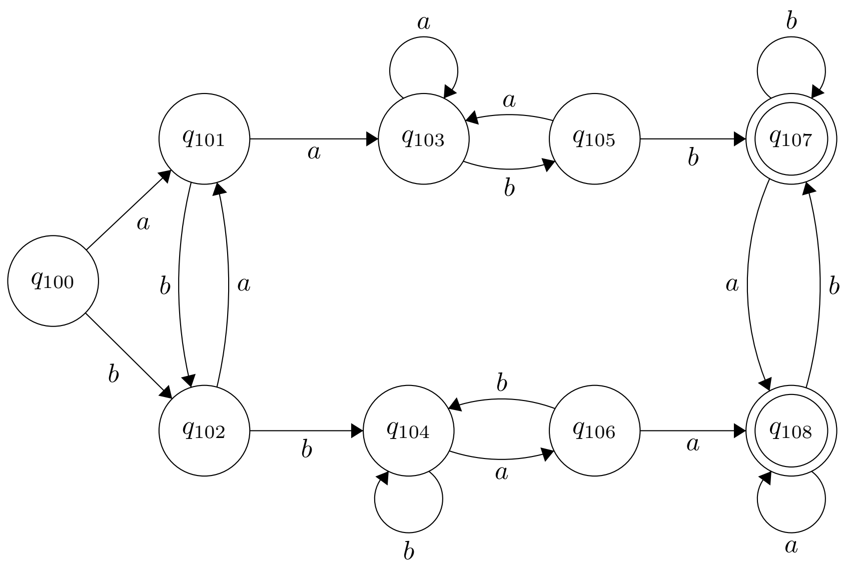

Figure 2: M'

Actually we can merge q_{107} and q_{108} if we want to acquire in order to minimize a little bit more. This will changed nothing:

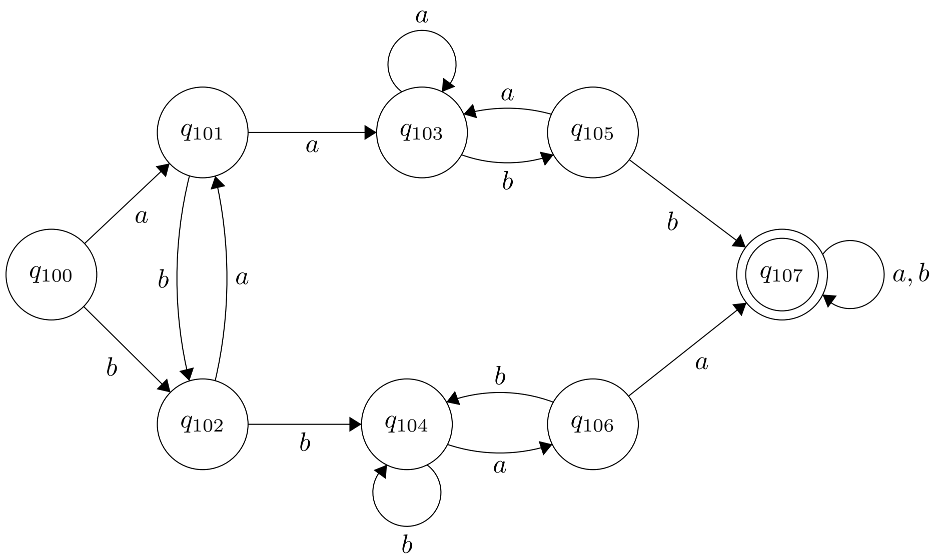

Figure 3: M_2'

d)

Tracing in NFA

$$\begin{aligned}(q_0, bbabb) &\vdash_M (q_1, babb)\\ &\vdash_M (q_6, abb)\end{aligned}$$

This way is failed.

$$\begin{aligned}(q_0, bbabb) &\vdash_M (q_1, bbabb)\\ &\vdash_M (q_6, babb)\\ &\vdash_M (q_7, abb)\\ &\vdash_M (q_8, abb)\\ &\vdash_M (q_9, bb)\end{aligned}$$

This way is failed.

$$\begin{aligned}(q_0, bbabb) &\vdash_M (q_1, babb)\\ &\vdash_M (q_0, babb)\\ &\vdash_M (q_1, abb)\\ &\vdash_M (q_0, abb)\\ &\vdash_M (q_1, bb)\\ &\vdash_M (q_0, bb)\\ &\vdash_M (q_1, b)\\ &\vdash_M (q_0, b)\\ &\vdash_M (q_1, e)\end{aligned}$$

This way is failed.

Actually there is a no way to accept this string. In other words, w' **cannot be accepted** by NFA M. It can be more clearly seen in DFA.

Tracing in DFA

$$\begin{aligned}(q_{100}, bbabb) &\vdash_M (q_{102}, babb)\\ &\vdash_M (q_{104}, abb)\\ &\vdash_M (q_{106}, bb)\\ &\vdash_M (q_{104}, b)\\ &\vdash_M (q_{104}, e)\end{aligned}$$

String w' is **not accepted** by DFA M' because the automata is not stopped at the one of the final states.

Answer 2

a)

If L_1 is a regular language, it should satisfy pumping lemma. According to pumping lemma, if L_1 is regular, there should be an integer $p \geq 1$ such that $w = a^m b^n \in L_1$ with $|w| \geq p$ can be written as $w = xyz$ such that:

- $|y| > 0$,
- $|xy| < p$, and
- $xy^i z \in L_1$ for each $i \geq 0$.

When we focus on y, there are 4 possible y, while $k, l \geq 1$:

1. $y = e$
2. $y = a^k$
3. $y = b^l$
4. $y = a^k b^l$

We can notice that there is not y such that satisfies the conditions of pumping lemma.

1. $y \neq e$, from pumping lemma itself. (Contradiction)
2. $y \neq a^k$, y cannot include *some* a: There are some situations that is valid but not all situation is valid. If the $m = n + 1$, $m \not> n$ when $i = 0$ for all $y = a^k$, while $k \geq 1$. (Contradiction)
3. $y \neq b^l$, y cannot include *some* b: $m \not> n$ when for some $i > 0$. (Contradiction)
4. $y \neq a^k b^l$, y cannot include both *some* a and *some* b: New strings out of pattern are generated (Example: $aaabbbaabb$). (Contradiction)

Since all possible y cause contradictions (at some points), which means that there is no integer p that L_1 is **not regular language** according to pumping lemma. According to *Theorem 2.3.1* in the textbook, languages accepted by finite automata are closed under complementation. So, $\overline{L_1}$ is also **not regular** language which is L_2

So, L_2 is **not regular** language.

b)

L_4 is not regular

L_4 defines a language whose strings include equal number of a's and b's where a's are located before b's. We can use pumping lemma to show that L_4 is not regular.

Assume L_4 is regular languages. Then there should be an integer $p \geq 1$ such that $w = a^n b^n \in L_4$ with $|w| \geq p$ can be written as $w = xyz$ such that:

- $|y| > 0$,
- $|xy| < p$, and
- $xy^i z \in L_4$ for each $i \geq 0$.

If $y = a^i$ for some $i > 0$, $xz = a^{n-i} b^n \notin L_4$ for $y^0 = e$, contradicting theorem. In other words, there is not any y that satisfies $xy^i z \in L_4$ for each $i \geq 0$. New strings, $xy^i z$, either are out of pattern or does not have same number of a's and b's.

So, L_4 is **not regular** language according to pumping lemma.

L_5 is regular

In L_5, it stated that $m, n \in \mathbb{N}$, which means $m, n \geq 0$. So, L_5 can be expressed as:

$$L_5 = a^* b^*$$

because of the definition of Kleene star (Kleene star basically means *zero or more occurences*).

So, it means that L_5 has a regular expression. If there is certain regular expression for a language, that language is regular language (It is also stated book in page 50: *Regular languages are all languages that can be described by regular expressions.*).

Therefore, L_5 is **regular**.

L_6 is regular

Again, since L_6 is represented by certain regular expression, L_6 is regular.

$L_4 \cup L_5 \cup L_6$ **is regular**

At that point, we know that:

- L_4 is not regular
- L_5 is regular
- L_6 is regular

If all of them was regular, we can say that the union of them is also regular because the languages are closed under union. However, L_4 is not regular.

When we focus on $L_4 \cup L_5$, we can notice that L_4 is subset of L_5, $L_4 \subset L_5$. So their union should be L_5:

$$L_4 \cup L_5 = L_5$$

So

$$L_4 \cup L_5 \cup L_6 = L_5 \cup L_6$$

So, according to *Theorem 2.3.1* in the textbook, languages accepted by finite automata are closed under union. So, $L_5 \cup L_6$ is also **regular**.

Finally, we can say that $L_4 \cup L_5 \cup L_6$ is **regular**.