Student Information

Name : Burak Metehan Tunçel

ID : 2468726

Answer 1

a) The followings are given:

- **Measurements:**

$$20.1, 12.8, 18.9, 16.4, 20.3, 10.1, 15.4, 12.4, 24.7, 18.5$$

- $\sigma = 3$

Construct 90% confidence interval Since we know what σ is, we can use the following formula

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

1. **We need to calculate the $\bar{X}$.** $measurement\_size = 10$ and $measurement\_sum = 169.6$. So,

$$\begin{aligned}\bar{X} &= measurement\_sum/measurement\_size\\ &= 169.6/10 = 16.96\end{aligned}$$

2. **We need to calculate the α for 90% confidence interval**

$$1 - \alpha = 0.90 \Rightarrow \alpha = 0.10 \text{ and } \alpha = 0.05$$

Hence, we are looking for quantiles

$$q_{0.05} = -z_{0.05} \qquad \text{and} \qquad q_{0.950} = z_{0.05}$$

$z_{0.05} = q_{0.950} = 1.6449$.

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 16.96 \pm 1.6449 \cdot \frac{3}{\sqrt{10}} = 16.96 \pm 1.5604891569632902 \approx 16.96 \pm 1.56$$

So, the interval is,

$$\begin{aligned}&[16.96 - 1.5604891569632902,\ 16.96 + 1.5604891569632902]\\ =&[15.39951084303671,\ 18.52048915696329]\\ \approx&[15.40,\ 18.52]\end{aligned}$$

Construct 99% confidence interval Since we know what σ is, we can use the following formula

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

1. **We know the $\bar{X}$.**

$$\begin{aligned}\bar{X} &= measurement\_sum/measurement\_size\\ &= 169.6/10 = 16.96\end{aligned}$$

2. **We need to calculate the α for 99% confidence interval**

$$1-\alpha = 0.99 \Rightarrow \alpha = 0.01 \text{ and } \alpha = 0.005$$

Hence, we are looking for quantiles

$$q_{0.005} = -z_{0.005} \qquad \text{and} \qquad q_{0.995} = z_{0.005}$$

$z_{0.005} = q_{0.995} = 2.5758$.

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 16.96 \pm 2.5758 \cdot \frac{3}{\sqrt{10}} = 16.96 \pm 2.4436184391185134 \approx 16.96 \pm 2.44$$

So, the interval is,

$$\begin{aligned}&[16.96 - 2.4436184391185134,\ 16.96 + 2.4436184391185134]\\ =&[14.516381560881488,\ 19.403618439118514]\\ \approx&[14.52,\ 19.40]\end{aligned}$$

b) In the textbook, is stated that in order to attain a margin of error Δ for estimating a population mean with a confidence level $(1-\alpha)$, a sample of size n should be satisfied the following inequality

$$n \geq \left(\frac{z_{\alpha/2}\cdot\sigma}{\Delta}\right)^2$$

The followings are given:

- Δ, error or margin $= 1.55$.
- $(1-\alpha)$, confidence level $= 0.98$

So, we have $\Delta = 1.55$, $\alpha = 0.02$, and $\sigma = 3$. We need to look for quantiles:

$$q_{0.01} = -z_{0.01} \qquad \text{and} \qquad q_{0.99} = z_{0.01}$$

So, we $z_{0.01} = q_{0.99} = 2.3263$.

$$n \geq \left(\frac{z_{\alpha/2}\cdot\sigma}{\Delta}\right)^2 = \left(\frac{z_{0.01}\cdot\sigma}{\Delta}\right)^2 = \left(\frac{2.3263\cdot 3}{1.55}\right)^2 = 20.272651492195628 \approx 20.27$$

So, we have the following inequality:

$$n \geq 20.27$$

So smallest required sample size that satisfies the condition is $n = 21$.

Answer 2

a) ***No***, the mean and sample size alone are not sufficient as the statistics for the restaurant. To make some measurements on spread of the data, we need at least one another parameter: standard deviation, in addition to mean and sample.

b)

- Null hypothesis H_0 (the restaurant has a greater than or equal rating of 7.5): $\mu_0 \geq 7.5$.
- Alternate hypothesis H_A (the restaurant has lower rating of 7.5): $\mu < 7.5$
- Mean is 7.4: $\bar{X} = 7.4$
- Standard deviation is 0.8: $s = 0.8$
- Sample size is 256: $n = 256$.

Since sample size, n is sufficiently large, Z and T test will give similar results. (Since their critical value is equal due to n=256).
(Also, in the text of the question, the given standard deviation 0.8 can be understood as population standard deviation (although "sample standard deviation" is stated). Therefore, in here, my assumption is that the given standard deviation 0.8 is population standard deviation.)

In here, I will use Z-test. The formula is,

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

When the given variables are inserted,

$$Z = \frac{7.4 - 7.5}{0.8/\sqrt{256}} = -2$$

Now, we need to calculate the critical value where $\alpha = 0.05$ (because our significance level is 5%).

$$z_\alpha = 1.6449$$

The acceptance region for left tail Z-test is: $[-z_\alpha,\ \infty)$. So our acceptance region is $[-1.6449,\ \infty)$. Since $-2 < -1.6449$, sufficient evidence against H_0 is provided (and sufficient evidence in favor of H_A is also provided.)

Thus, restaurant A ***should not*** be in my list of candidate restaurants to order food from.

c) Only standard deviation is changed to 1.0 from 0.8. Other data is same with the part a. The formula is,

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

When the given variables are inserted,

$$Z = \frac{7.4 - 7.5}{1.0/\sqrt{256}} = -1.6$$

We know critical value from part a.

$$z_\alpha = 1.6449$$

So our acceptance region is $[-1.6449,\ \infty)$. Since $-1.6 > -1.6449$ "$-1.6 \in [-1.6449,\ \infty)$", sufficient evidence in favor of H_0 is provided (and sufficient evidence against H_A is also provided.)

Thus, restaurant A ***should*** be in my list of candidate restaurants to order food from.

d) The mean of the user rating for restaurant is greater than the mean 7.5 which the value we desire. So, there is no need to bother about deviation.

Therefore, we ***can add*** the restaurant in out candidate list.

Answer 3

In question the followings are given:

For Computer A

Mean Run-time	$\bar{X_A} = 211$
Sample Standard Deviation	$s_A = 5.2$
Run Size	$n_A = 20$

For Computer B

Mean Run-time	$\bar{X_B} = 133$
Sample Standard Deviation	$s_B = 22.8$
Run Size	$n_B = 32$

a) In here, T-test can be used with the following hypotheses:

- Null hypothesis H_0: $\mu_A - \mu_B \geq 90$.
- Alternate hypothesis H_A: $\mu_A - \mu_B < 90$

The formula of T-test for unknown but equal standard deviations is, (where $D = \mu_A - \mu_B$).

$$t = \frac{\bar{X_A} - \bar{X_B} - D}{s_p\sqrt{\dfrac{1}{n_A} + \dfrac{1}{n_B}}}$$

When we insert data,

$$t = \frac{211 - 133 - 90}{18.24\sqrt{\dfrac{1}{20} + \dfrac{1}{32}}} = -2.3084693271786647 \approx -2.31$$

where

$$s_p = \sqrt{\frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2}} = \sqrt{\frac{(19)(5.2)^2 + (31)(22.8)^2}{20 + 32 - 2}} = 18.236666362030096 \approx 18.24$$

We need to calculate the critical value. Degrees of freedom is $d.f. = n_A + n_B - 2 = 50$. So,

$$t_\alpha = 2.403$$

The acceptance region for left tail T-test is: $[-t_\alpha,\ \infty)$. So our acceptance region is $[-2.403,\ \infty)$. Since $-2.31 > -2.403$, sufficient evidence in favor of H_0 is provided (and sufficient evidence against H_A is also provided.)

Thus, the researcher ***can claim*** that the computer B provides a 90-minutes or better improvement.

b) In here, T-test can be used with the following hypotheses:

- Null hypothesis H_0: $\mu_A - \mu_B \geq 90$.
- Alternate hypothesis H_A: $\mu_A - \mu_B < 90$

The formula of T-test for unknown and unequal standard deviations is,

$$t = \frac{\bar{X_A} - \bar{X_B} - D}{\sqrt{\dfrac{s_A^2}{n_A} + \dfrac{s_B^2}{n_B}}}$$

where $D = \mu_A - \mu_B$.

When we insert data,

$$t = \frac{211 - 133 - 90}{\sqrt{\dfrac{(5.2)^2}{20} + \dfrac{(22.8)^2}{32}}} = -2.8606315819550376 \approx -2.86$$

We need to calculate the critical value. Degrees of freedom is coming from Satterthwaite approximation:

$$d.f. = \frac{\left(\dfrac{s_A^2}{n_A} + \dfrac{s_B^2}{n_B}\right)^2}{\dfrac{s_A^4}{n_A^2(n_A - 1)} + \dfrac{s_B^4}{n_B^2(n_B - 1)}}$$

When we insert data,

$$d.f. = \frac{\left(\frac{(5.2)^2}{20} + \frac{(22.8)^2}{32}\right)^2}{\frac{(5.2)^4}{(20^2)(19)} + \frac{(22.8)^4}{(32^2)(31)}} = 35.96822746722723 \approx 36$$

So,

$$t_\alpha = 2.434$$

The acceptance region for left tail T-test is: $[-t_\alpha,\ \infty)$. So our acceptance region is $[-2.434,\ \infty)$. Since $-2.86 < -2.434$, sufficient evidence against H_0 is provided (and sufficient evidence in favor of H_A is also provided.)

Thus, the researcher ***cannot claim*** that the computer B provides a 90-minutes or better improvement.