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Q. 1

Prove that the compound proposition

$$\neg(p \land q) \leftrightarrow (\neg q \rightarrow p)$$

is logically equivalent to

$$(p \lor q) \land (\neg p \lor \neg q)$$

using logical identities and algebraic manipulation techniques. Write down the identity used in each step.

S. 1

$\neg(p \land q) \leftrightarrow (\neg q \rightarrow p)$	*Given*
$\neg(p \land q) \leftrightarrow (q \lor p)$	*Table 7, Equation 1 in 1.3*
$(\neg p \lor \neg q) \leftrightarrow (q \lor p)$	*Table 6, De Morgan's Law*
$((\neg p \lor \neg q) \land (q \lor p)) \lor (\neg(\neg p \lor \neg q) \land \neg(q \lor p))$	*Table 8, Equation 3 in 1.3*
$((\neg p \lor \neg q) \land (q \lor p)) \lor ((p \land q) \land (\neg q \land \neg p))$	*Table 6, De Morgan's Law*
$((\neg p \lor \neg q) \land (q \lor p)) \lor F$	*By Truth Table*
$((\neg p \lor \neg q) \land (q \lor p))$	*Identity Laws*

$$\neg(p \land q) \leftrightarrow (\neg q \rightarrow p) \equiv ((\neg p \lor \neg q) \land (q \lor p))$$

Q. 2

Translate the following English sentences into compound predicate logic propositions using the predicates below.

$I(x, y)$: x is an intern in faculty y.
$E(x, y)$: x has employee id number y.
$S(x, y)$: x is supervised by y.
$A(x, y)$: x is admitted to job position y.
$J(x, y)$: x is a job position in faculty y.

Besides the indicated predicates, you are only allowed to use additional variables, existential and universal quantifiers along with logical connectives, and equals ($=$) and not equals ($\neq$) relations if necessary. Use of any other notation within your statements will cause the corresponding answers to be evaluated as 0.

a. Two different interns in the same faculty cannot have the same employee id number.

b. There are some interns in all faculties who are supervised by no one but themselves.

c. At most two interns can be admitted to each job position in the medicine faculty.

S. 2

a. $\forall x_1, x_2, y(I(x_1, y) \wedge I(x_2, y) \rightarrow \neg(E(x_1, y) \wedge E(x_2, y)))$

b. $\exists x \forall y(I(x, y) \wedge S(x, x))$

c. $\exists x_1, x_2 \forall j((x_1 \neq x_2) \wedge J(j, medicine) \wedge (A(x_1, j) \vee A(x_2, j)))$

Q. 3

Using natural deduction rules for propositional logic, prove the following statements.

a. $p \vee \neg q, p \vee r \vdash (r \to q) \to p$

b. $\vdash ((q \to p) \to q) \to q$

S. 3

a. $p \vee \neg q, p \vee r \vdash (r \to q) \to p$

1.	$p \wedge \neg q$	premise
2.	$p \wedge r$	premise
3.	p	assumption
4.	$\neg p$	$\neg i$, 3
5.	$\neg q$	Disjunctive Syllogism in page 5 1, 4
6.	$\neg\neg q$	$\neg i$ 5
7.	q	$\neg\neg e$ 6
8.	r	Disjunctive Syllogism in page 5 2, 4
9.	$p \to q$	$\to i$ 3, 7
10.	$r \to q$	$\to i$ 7, 8
11.	p	$\vee e$ 2, 9, 10
12.	$r \to q$	$\to i$ 8, 7
13.	$(r \to q) \to p$	$\to i$ 12, 11

b. $\vdash ((q \to p) \to q) \to q$

1.	$(q \to p) \to p$	Assumption
2.	$\neg q$	Assumption
3.	q	Assumption
4.	$\perp$	$\neg e$ 2, 3
5.	p	$\perp e$ 4
6.	$q \to p$	$\to i$ 3, 5
7.	q	$\to e$ 6, 1
8.	$\perp$	$\neg e$ 2, 7
9.	q	$\neg i$ 2, 8
10.	$((q \to p) \to q) \to q$	$\to i$ 1, 9

Q. 4

Using natural deduction rules for propositional and predicate logic, prove the following statements.

a. $\neg\forall(P(x) \rightarrow Q(x)) \vdash \exists x(P(x) \wedge \neg Q(x))$

b. $\forall x \forall y(P(x, y)) \rightarrow \neg P(y, x)),\ \forall x \exists y P(x, y) \vdash \neg\exists v \forall z P(z, v)$

S. 4

a. $\neg\forall(P(x) \rightarrow Q(x)) \vdash \exists x(P(x) \wedge \neg Q(x))$

1. $\neg\forall(P(x) \rightarrow Q(x))$	Premise
2. $\exists x \neg(P(x) \rightarrow Q(x))$	Proof 3 in page 5
3. $\exists x(P(x) \wedge \neg Q(x))$	Proof 4 in page 5

Proofs

Proof 1: $\neg P \wedge \neg Q \equiv \neg(P \vee Q)$

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$P \vee Q$	$\neg(P \vee Q)$
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

Proof 2: Disjunctive Syllogism

$$P \vee Q, \neg P \vdash Q$$

1. $P \vee Q$ — Premise
2. $\neg P$ — Premise
    3. $\neg Q$ — Assumption
    4. $\neg P \wedge \neg Q$ — $\wedge i$ 2,3
    5. $\neg(P \vee Q)$ — Proof 1 in page 5
    6. $\neg(P \vee Q) \wedge (P \vee Q)$ — $\wedge i$ 1,5
7. Q — RAA 3-6

Proof 3: $\exists x \neg P(x) \equiv \neg(\forall x P(x))$

$$\exists x \neg P(x) \vdash \neg \forall x P(x)$$

1. $\exists x \neg P(x)$ — Premise
    2. $\forall x P(x)$ — Assumption
        3. $\neg P(a)$ — Existential Instantiation
        4. $P(a)$ — $\forall e$ 2
        5. $\bot$ — $\neg e$ 3, 4
    6. $\bot$ — $\exists e$ 1, 3-5
7. $\neg \forall x P(x)$ — $\neg i$ 2-6

Proof 4: $\neg(P \to Q) \vdash P \wedge \neg Q$

$$\neg(P \to Q) \vdash P \wedge \neg Q$$

1. $\neg(P \to Q)$ — Premise
    2. $\neg(P \wedge \neg Q)$ — Assumption
        3. P — Assumption
            4. $\neg Q$ — Assumption
            5. $P \wedge \neg Q$ — $\wedge i$ 3, 4
            6. $\bot$ — $\bot\ i$ 2, 5
        7. Q — $\neg i$ 4, 6
    8. $P \to Q$ — $\to i$ 3, 7
    9. $\bot$ — $\bot\ i$ 1, 8
10. $P \wedge \neg Q$ — $\neg i$ 2, 9