IE 203 - Operations Research II

Final

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Duration: 2 hours
Important notice: 1. This is a CLOSED BOOK exam.
2. You may keep the questions.

Question I (25 pts.)

Consider the following integer programming problem

$$\begin{array}{lrll} \max & z = & 8x_1 + 5x_2 & \\ & \text{s.t.} & x_1 + x_2 & \leq 6 \\ & & 9x_1 + 5x_2 & \leq 45 \\ & & x_1, x_2 & \geq 0 \\ & & x_1, x_2 & \textit{integer} \end{array}$$

a. Show graphically the feasible solution set.

b. Show graphically the convex hull of the feasible solution set and give its algebraic description (using equalities and inequalities).

c. Solve the LP relaxation of the problem using simplex algorithm and obtain optimal tableau.

d. Generate the Gomory cut corresponding to the basic variable whose optimal value has the largest fractional part in the optimal tableau obtained in part (c).

e. Add the new cut to the obtained optimal tableau and reoptimize it using dual simplex algorithm for finding the next LP solution.

f. Show graphically the generated Gomory cut on the feasible solution graphic drawn in part (a).

Question II (25 pts.)

Show that

$$e^{x_1+x_2-2} \geq \frac{x_1^2 + x_2^2}{4}$$

for all $x_1 \geq 0$ and $x_2 \geq 0$.

Question III (30 pts.)

In order to graduate from Boğaziçi University, Zeynep needs to pass all of the three subjects she is taking this semester. She is now enrolled in French, German, and statistics. Zeynep's busy schedule of extracurricular activities allows her to spend only 2 hours per week on studying. Zeynep's probability of passing each course depends on the number of hours she spends studying for the course. These values are given in the following table. Use dynamic programming to determine how many hours per week Zeynep should spend studying each subject in order to maximize her probability of passing all of the courses. Passing courses are independent events. Clearly describe the stages, states, policy decision variables, the recursive relation, and boundary conditions.

	Probability of passing course		
Hours of study per week	French	German	Statistics
0	0.60	0.40	0.20
1	0.80	0.60	0.50
2	0.85	0.80	0.70

Please see the reverse!

Question IV (20 pts.)

Assume that the probability of rain tomorrow is 0.5 if it is raining today, and assume that the probability of its being clear (no rain) tomorrow is 0.9 if it is clear today. Also assume that these probabilities do not change if information is also provided about the weather before today.

a. Formulate the evolution of the weather as a Markov chain by defining its states and giving its (one-step) transition matrix.

b. Draw the state transition diagram of the Markov chain.

c. Knowing that the probability that it will rain today is 0.5 what is the probability that it will rain 2 days from now?

d. Knowing that the probability that it will be clear today is 0.3 what is the probability that it will be clear again 5 days from now?