20.03.2025

IE 203 PS 5

1- Determine whether each of the following functions are convex, concave, neither or both in the given domains.

a) $f(x) = x^3$ in $[0, \infty)$
b) $f(x) = x^3$ in $\mathbb{R}$
c) $f(x) = \frac{1}{x}$ in $(0, \infty)$
d) $f(x) = x^a$ in $(0, \infty)$ where $0 \leq a \leq 1$
e) $f(x) = \ln x$ in $(0, \infty)$
f) $f(x) = ax + b$ in $\mathbb{R}$

2- Find the stationary points of the following functions and classify them as local minimum, local maximum, or inflection point. check that whether the extremums are global maximum or global minimum on $\mathbb{R}$.

a) $f(x) = x.e^{-x}$
b) $f(x) = 6x^5 - 4x^3 + 10$
c) $f(x) = (3x-2)^2(2x-3)^2$

3- Suppose $g: I \to R$ and $h: C \to R$ are two convex functions.

a) Show that $f = max\{g, h\}$ is a convex function.
b) If g is non-decreasing, show that $g \circ h$ is a convex function.

4- Determine whether the following functions are convex, concave, or neither.

a) $f(x_1, x_2) = x_1^3 + 2x_2^2$
b) $f(x_1, x_2) = -x_1^2 - 5x_2^2 + 2x_1x_2 + 10x_1 - 10\, x_2$
c) $f(x_1, x_2, x_3) = x_1x_2 + 2x_1^2 + x_2^2 + 2x_3^2 - 6x_1x_3$
d) $f(x_1, x_2) = x_1 e^{-(x_1 + x_2)}$

5- Determine the stationary points of the following functions and indicate whether they are local minimum, local maximum, or saddle points.

a) $f(x_1, x_2) = -x_1^2 - 5x_2^2 + 2x_1x_2 + 10x_1 - 10x_2$
b) $f(x_1, x_2) = x_1^4 + x_2^2 + x_1x_2$
c) $f(x_1, x_2) = 2x_1^3 - 3x_1^2 - 6x_1x_2(x_1 - x_2 - 1)$
d) $f(x_1, x_2) = x_1^3 + x_2^3 - 3x_1x_2$