**IE 203 - Operations Research II**
**Quiz I**

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**Duration: 30 minutes**
**This is a CLOSED BOOK exam.**

**Question I (40 pts.)**

Consider the integer programming problem

$$\begin{array}{lrll} \max & z = & 8x_1 + 5x_2 & \\ & \text{s.t.} & x_1 + x_2 & \leq 8 \\ & & 9x_1 + 5x_2 & \leq 45 \\ & & x_1 & \leq 15 \\ & & x_2 & \leq 7 \\ & & x_1, x_2 & \geq 0 \\ & & x_1, x_2 & \textit{integer} \end{array}$$

**a.** Give the equivalent "binary" integer programming formulation.

**b.** What values do the new binary variables take (i.e., $0, 1$ assignment of the binary variables) when the original IP's optimum solution is $x_1^* = 5$ and $x_2^* = 0$?

**Question II (60 pts.)**

Consider the integer programming problem

$$\begin{array}{lrll} \max & z = & x_1 + 2x_2 & \\ & \text{s.t.} & -3x_1 + 4x_2 & \leq 4 \\ & & 3x_1 + 2x_2 & \leq 11 \\ & & 2x_1 - x_2 & \leq 5 \\ & & x_1, x_2 & \geq 0 \\ & & x_1, x_2 & \textit{integer} \end{array}$$

**a.** Show graphically the feasible solution set.

**b.** Show graphically the convex hull of the feasible solution set and give its algebraic description (using equalities and inequalities).

**c.** Find the extreme points and directions of the convex hull of the feasible solution set.

**d.** Solve the problem using the branch-and-bound algorithm. You can solve the LP relaxations of the subproblems graphically. Show all your work in detail (i.e. the subproblems, their solutions, lower and upper bounds). Also represent clearly the branch-and-bound tree.