13.02.2025

IE 203 PS 1

1- A young couple, Eve and Adam, want to divide their main household chores (marketing, cooking, dishwashing, and laundering) between them so that each has two task, and the total time they spend on household duties is kept to a minimum. Their efficiencies on these tasks differ, where the time each would need to perform the task is given by the following table. Formulate an IP model for this problem for the instance below and for the generalized, parametric version of the problem.

	Time needed per week (hours)			
	Marketing	**Cooking**	**Dishwashing**	**Laundry**
Eve	4.5	7.8	3.6	2.9
Adam	4.9	7.2	4.3	3.1

2- A company manufactures three products, whose daily labor and raw material requirements are given in the following table:

Product	**Hr/unit**	**lb/unit**
1	3	4
2	4	3
3	5	6

The profits per unit of three products are \$25, \$30, and \$22, respectively. The company has two options for locating its plant. The two locations differ primarily in the availability of labor and raw material, as shown in the following table. Formulate the problem as an MIP, maximizing the profit.

Location	**Daily hour**	**Raw material**
1	100	100
2	90	120

3- A manager is supposed to select projects to invest in with a limited budget B. Each project has a profit p_i and a cost c_i associated with it. Formulate the problem to find out which projects should be chosen to maximize profit then write the mathematical interpretation of each constraint below.

a) Project 1 and 2 cannot be selected at the same time.
b) If project 3 is selected project 4 should be selected as well.
c) If project 1 is selected, then project 3 should not be selected.
d) Exactly 5 projects should be selected in total.
e) Either project 6 and 7 are chosen together or neither of them are chosen.
f) Either project 1 or 5 are chosen, but not both.
g) Among projects $\{1, 2, \ldots, 10\}$ at least 3, at most 5 projects should be chosen.
h) Neither project 5 nor 6 can be chosen unless either 3 or 4 is chosen
i) At least one project among 1, 2, 3 or at most two projects from 4, 5, 6 are selected.
j) If both 11 and 12 are chosen, then at least one of the 13 and 14 should be selected.

4- Formulate a binary integer program for the **minimum set cover problem**. Given a universal set (U) of elements and a collection S of sets of elements, the minimum set cover problem is to identify the smallest collection of sets whose union equals the universe, or in other words covers all elements.

5- Formulate a binary integer program for the **set partition problem**. Given a universal set (U) of elements and a collection S of sets of elements, the set partition problem is to identify a collection of sets whose union equals the universe and each element is contained in exactly one of the sets in this collection.

6- Formulate a binary integer program for the **maximum set packing problem**. Given a universal set, U of elements and a collection S of sets of elements, the maximum set packing problem is to identify the biggest collection of sets whose union equals the universe and each element is contained in at most one of the sets in this collection.

7- The **Generalized Assignment Problem** (GAP) can be described using the terminology of knapsack problems. Given n items and m knapsacks, let p_{ij} denote the profit of item j and w_{ij} denote the weight of item j, if it is assigned to knapsack i. Assign items to knapsacks, maximizing the total profit and without exceeding the capacity, c_i, of any knapsack i. Formulate the problem as an integer programming model.

8- Formulate a mixed integer problem for the **capacitated facility location problem**. In this problem, there is a set of candidate locations I to open facilities, and a set of customers J. Each facility location has an associated fixed cost for opening, f_i, and a capacity of u_i. Each customer has a demand of d_j. Additionally, there is a cost of transporting a unit of product from a location to a customer, c_{ij}. The aim is to minimize to total costs.

9- A graph $G' = (V', E')$ with $V' = \{1,2,3,4\}$ and $E' = \{\{1,2\},\{2,3\},\{3,4\},\{4,1\},\{1,3\}\}$ is shown in Figure 1.

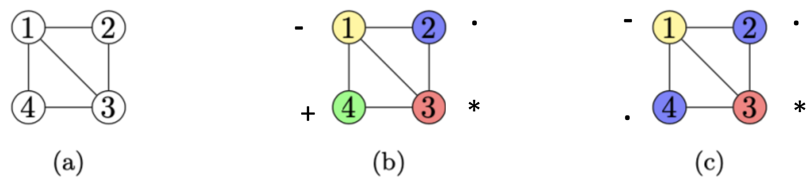

Figure 1: (a) A graph G' on 4 vertices, (b) A non-optimal coloring of G', (c) An optimal coloring of G'

Given a graph $G = (V, E)$, formulate an integer program for the **minimum graph coloring problem** which is to identify a valid coloring for a given graph with fewest possible colors. In a valid coloring, two adjacent vertices have different colors.

10- Given a graph $G = (V, E)$, formulate an integer program for the **maximum independent (stable) set problem**, where an independent set is a set of vertices such that no two vertices in it are adjacent.

11- Given a graph $G = (V, E)$, formulate an integer program for the **maximum clique problem**, where a clique is a set of vertices such that every vertex pair in it are adjacent.