IE 203 - Operations Research II

Midterm

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Duration: 2 hours
Important notice: 1. This is a CLOSED BOOK exam.
2. You may keep the questions.

Question I (15 pts.)

For each one of the following sets, determine whether it is a polyhedron. Explain your answers.

a. $S_1 = \{\mathbf{x} \in \mathbb{R}^2 : x_1 \cos\theta + x_2 \sin\theta \geq 1 \text{ for all } 0 \leq \theta \leq \pi/2,\ x_1 + x_2 \leq \sqrt{2},\ x_1 \geq 0,\ x_2 \geq 0\}$

b. $S_2 = \{x \in \mathbb{R} : x^2 - 3x + 2 \leq 0\}$

c. The empty set.

Question II (35 pts.)

Given four items with weights 4, 7, 5 and 3 kg. and values 40, 42, 25 and 12 TL., and a knapsack with 10 kg. capacity, which items should be put into the knapsack so that total value is maximized?

a. Give a binary integer programming formulation of the problem.

b. Solve the problem using the branch-and-bound algorithm. You can solve the LP relaxations of the subproblems using the simple ordering algorithm explained during the lectures. Show all your work in detail (i.e., the subproblems, their solutions, lower and upper bounds). Also represent clearly the branch-and-bound search tree that you construct using **depth first search**.

Question III (25 pts.)

Consider the following function

$$f(x_1, x_2) = 2x_1^4 - 5x_1^3 + 2x_2^3 - 9x_2^2 + 10x_2 - 3.$$

a. Is it a convex function over its domain, which is $\mathbb{R}^2$? Justify your answer! If it is not, can you give a range for the values of the variables where it is convex?

b. Determine its stationary points. Which ones are locally minimal? Please show all your work.

Question IV (25 pts.)

Consider the single facility location problem and suppose that the squared Euclidean distance is used to measure the distance between the unknown facility location $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and known customer locations $\mathbf{a}_j = \begin{pmatrix} a_{1j} \\ a_{2j} \end{pmatrix}$ for $j = 1, 2, \ldots, n$. Customer demands h_j for $j = 1, 2, \ldots, n$ are also given.

a. Formulate the problem so that that the sum of the weighted squared customer-to-facility distances, where the weights are the demands, is minimized.

b. Solve the formulation to find a closed form formula for an optimal facility location.

c. Use the formula you have obtained in (b) to calculate the optimal facility location for customer locations $\begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, and demands $10, 5, 40, 7$.