20.02.2025

IE 203 PS 2

1- Prove or Disprove: The following is set convex.

a) $S = \{x \in \mathbb{R}^3_+ : x_1 + x_2 - x_3 \geq 2, x_2 + x_3 = 5\}$

b) $S = \{x \in \mathbb{Z}^3_+ : x_1 + x_2 - x_3 \geq 2, x_2 + x_3 = 5\}$

c) $S = \{x \in \mathbb{Z}^2_+ : x_1 + x_2 \leq 2, x_1 - x_2 \geq 1, x_1 \geq \frac{3}{2}\}$

2- Given the set $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4,\ x \geq 1\}$, is S a convex set? Give an algebraic answer. (Quiz question from last year)

3- Solve the following IP using branch and bound algorithm. Use Simplex Algorithm to solve the linear relaxation at each node of the tree.

$$\begin{aligned} \max \quad & 2x_1 + x_2 \\ s.t. \quad & x_1 + x_2 \leq 5 \\ & 3x_1 - x_2 \leq 6 \\ & x, y \in \mathbb{Z}_{0,+} \end{aligned}$$

4- Solve the following IP using branch and bound algorithm by BFS strategy, clearly demonstrate the B&B tree including upper bounds and lower bounds at each node of the branch and bound tree. For every fathomed branch, state the fathoming type. (Quiz question from last year)

$$\begin{aligned} \max \quad & x_1 + 3x_2 \\ s.t. \quad & 2x_1 + 2x_2 \leq 11 \\ & -2x_1 + x_2 \leq 1 \\ & x_1 + 2x_2 \geq 1 \\ & x, y \in \mathbb{Z}_{0,+} \end{aligned}$$

5- Solve the following IP using branch and bound algorithm.

$$\begin{aligned} \max \quad & x + 2y \\ s.t. \quad & x - y \leq \frac{3}{4} \\ & x - y \geq \frac{1}{2} \\ & \mathrm{x} + \mathrm{y} \leq 3 \\ & x, y \in \mathbb{Z}_{0,+} \end{aligned}$$

6- Four projects are available for investment. Their costs and revenues are given below. The budget for investment is 6 units. Develop a knapsack BIP formulation that maximizes the profit and solve using branch and bound algorithm.

Projects	*Revenue*	*Cost*
1	8	3
2	13	5
3	5	2
4	11	4