AIRFLOW ESTIMATION FROM RESPIRATORY SOUNDS

by

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1. INTRODUCTION

2. EXPERIMENTAL SETUP AND DATA

3. AIRFLOW CURVE ESTIMATION

3.1. Autoregressive Models

Autoregressive modeling is a widely used method in spectral signal analysis. It is also used in respiratory signal analysis and since the respiratory sounds have spectral characteristics changing over time, either we divide the sound into small pieces and assume the small pieces are stationary or we model the respiratory sounds as time varying autoregressive processes. It is reported that when a respiratory sound signal is modeled as TVAR process, the first AR coefficient is highly correlated with the airflow for the sounds recorded at trachea and have less but still correlation for the sounds recorded at chest wall. We will test this for the sounds recorded at chest wall and try to find best settings for most correlation in this subsection. We will try to extract the AR coefficients with three methods:

- Windowing Based Autoregressive Modeling: Divide the signal into overlapping windows assume each window is independent from others.
- Time Varying Autoregressive Modeling with Basis Functions: Assume the AR parameters are combination of sinusoidal signals.
- Time Varying Autoregressive Modeling with Particle Filters: Assume the AR parameters are changing slowly.

3.1.1. Univariate Autoregressive Model

Univariate AR model is a difference equation where the current measurement of the signal is a linear combination of past measured values and a white Gaussian noise. The model is equivalent to a filter with infinite impulse response where the input is white Gaussian noise. The equation of an AR model is given in (3.1). In this equation a_i is the i^{th} order AR coefficient, N is the order of model and e is the noise term where

$\mu_e = 0$ and σ_e^2 is constant.

$$x(n) = \sum_{i=1}^{N} a_i x(n-i) + e(n) \tag{3.1}$$

Before looking at mean, variance, autocorrelation and spectral content of a general AR model, we must set some conditions on a_i's to make the model stationary. The stationarity requires the mean, variance and autocorrelation function is constant. If the mean is constant then (3.2) must hold. We know that μ_e is 0, then either $\sum_{i=1}^{N} a_i = 1$ or $\mu_x = 0$ is true.

$$\mu_x = \sum_{i=1}^{N} a_i \mu_x + \mu_e \tag{3.2}$$

We require $E[X(0)] = 0$ for initial conditions for AR processes and it ensures $\mu_x = 0$. Then for variance we can write the equations in (3.3), (3.4) and (3.5).

$$\sigma_X^2 = E[X(n)X(n)] - \mu_X^2 \tag{3.3}$$

$$\sigma_X^2 = \sum_{i=1}^{N} a_i E[X_n X_{n-i}] + E[X_n e_n] - \mu_X^2 \tag{3.4}$$

$$\sigma_X^2 = \sum_{i=1}^{N} a_i R_{XX}(i) + \sigma_e^2 \tag{3.5}$$

We know that Autocorrelation coefficients are very important in analysis of an AR model. The AR coefficients can be directly calculated in presence of autocorrelation coefficients.

$$r_{xx}(i) = R_{xx}(k)/R_{xx}(0) \tag{3.6}$$

$$r_{xx}(i) = E[x(n)x(n-i)]/R_{xx}(0) \tag{3.7}$$

$$r_{xx}(i) = \frac{\sum_{j=1}^{N} E[a_j x(n-j)x(n-i)]}{R_{xx}(0)} \tag{3.8}$$

$$r_{xx}(i) = \sum_{j=1}^{N} a_j r_{xx}(i-j) \tag{3.9}$$

The equation in (3.9) is very useful since we can rewrite it as in (3.10), this equation is called Yule-Walker equation. So, given a signal generated with an AR model, we can find the coefficients of that model by solving the equation in (3.10). Let's call this equation as $r = Ra$, the the coefficients can easily be found with $a = R^{-1}r$.

$$\begin{pmatrix} r_{XX}(1) \\ r_{XX}(2) \\ \vdots \\ r_{XX}(N-1) \\ r_{XX}(N) \end{pmatrix} = \begin{pmatrix} 1 & r_{XX}(1) & ... & r_{XX}(N-2) & r_{XX}(N-1) \\ r_{XX}(1) & 1 & ... & ... & r_{XX}(N-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{XX}(N-2) & ... & ... & 1 & r_{XX}(1) \\ r_{XX}(N-1) & r_{XX}(N-2) & ... & r_{XX}(1) & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{N-1} \\ a_N \end{pmatrix} \tag{3.10}$$

Although solving the equation (3.10) seems to be very straightforward, it requires exact knowledge on correlation coefficients. However, when we are given a signal with finite length we can just estimate the correlation coefficients, and the error in calculated AR coefficients will depend on the condition number of R matrix. To solve this equation, there are several methods in literature, throughout this thesis we will use Burg's Method to find the AR coefficients, since it is more stable than the others. [Delft'teki makale]

Algorithm 1 AR Coefficients Estimation With Overlapping Windows

procedure WINDOWEDAR(signal, order, winLen, overlap)
    $windowStart \leftarrow 1$; $L \leftarrow Length(signal)$;
    while $windowStart \leq L - winLen$ **do**
        $temp \leftarrow signal(windowStart : (windowStart + windowLength))$;
        $AR(:, i) \leftarrow EstimateAR(temp, N)$;
        $windowStart \leftarrow windowStart + windowLength$;
    end while
    return AR;
end procedure

3.1.2. Time Varying Autoregressive Model

While univariate autoregressive model is very useful for many signals, the method doesn't use the continuity of the signal and treats each segment independently. Assuming the coefficients at different time instants are correlated with each other and building some structured models is another method which is widely applied for nonstationary signals. TVAR modeling were applied to respiratory signals and satisfactory results have been obtained in many papers. The equation describing a TVAR process is given in (3.11).

$$x(n) = \sum_{i=1}^{N} a_i(n)x(n-i) + e(n) \tag{3.11}$$

We will use two methods for solving this equation for a_is. One of them is modeling a_is as combination of sinusoids, the other one is modeling a_is as slowly changing parameters.

<u>3.1.2.1. TVAR Coefficients Estimation With Fourier Basis Functions.</u> In this method, the AR coefficients are assumed to be combination of sinusoidal functions. Then equations in (3.12) and (3.13) can be written.

$$x(n) = \sum_{i=1}^{N} x(n-i) \sum_{j=1}^{M} c_{ij}u_j(n-i) + e(n) \tag{3.12}$$

$$a_i(n) = \sum_{j=1}^{M} c_{ij}u_j(n-i) \tag{3.13}$$

Equation (3.12) can be rewritten in vector form as in (3.14) where X_i is the diagonal matrix with the diagonal elements are adjacent elements of x starting from i, U is the matrix whose columns are basis vectors, and the c_i's are the unknown parameters in this equation.

$$x = \sum_{i=1}^{N} X_i U c_i + e \tag{3.14}$$

Let $Y_i = X_i U$, the equation can be written in matrix form as in

$$x = \begin{pmatrix} Y_1 | Y_2 \dots Y_N \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{pmatrix} + e \tag{3.15}$$

The equation in (3.15) describes an overdetermined set of equations, we will use least squares approach to solve this. After finding **c** vector, which has NxM elements, calculating $a_i(n)$, TVAR coefficients, will be straightforward.

$$\mathbf{c} = (Y^T Y)^{-1} Y^T x \tag{3.16}$$

We will try to find the optimum values for AR order, number of basis vectors, and the range of frequencies spanned by basis vectors to get the best correlation.

Algorithm 2 TVAR Coefficients Estimation With Fourier Basis Functions

```
procedure TVARFourierBasis(signal, order, freqRange, numBasis)
    δ_f = freqRange/numBasis;
    for i=1:numBasis do
        U(:, 2 * i − 1) = sin(i * δ_f);
        U(:, 2 * i) = cos(i * δ_f);
    end for
    Generate Y: Project signal onto U;
    c = (Y^T Y)^{−1} Y^T x
    for i=1:N do
        AR(:, i) = U c_i;
    end for
    return AR;
end procedure
```

<u>3.1.2.2. TVAR Coefficients Estimation With Kalman Filter.</u> Kalman filter is a very old but still popular algorithm in field of information processing. It is the best estimator for one linear systems with Gaussian noise. It is used everywhere from tracking applications to computer games.

Before explaining how Kalman filter works, we must first give the required equations to describe the model where it is applied Kalman filter is invented to work on dynamic systems where we can record the noisy observations of transformations of the process in interest. So we may give two equations, for both measurement (3.17) and process (3.18) model.

$$y_n = H_n x_n + v_n \tag{3.17}$$

$$x_n = F_n x_{n-1} + B_n u_n + w_n \tag{3.18}$$

In (3.18) x is the state vector, u is control input to the system, B is the control matrix, F is the state transition matrix and w is process noise. In (3.17) the H is the transform matrix and v is the measurement noise. When a model desribed in (3.17) and (3.18) is present, we can observe y and we are after estimatin x one can apply Kalman filter. When the process and measurement noises are Gaussian then Kalman is the optimal estimator.

In our problem, there is not any control input to system and equations convert to (3.19) and (3.20). In these equations y is the recorded sound amplitude, x is the AR coefficients, H is equal to raw vector containing previous N values of y where N is the AR order.

$$y_n = H_n x_n + v_n \tag{3.19}$$

$$x_n = F_n x_{n-1} + w_n \tag{3.20}$$

Kalman filter includes two stages, prediction and measurement update. The prediction equations are given in (3.21) and (3.22). The measurement update equations are given in (3.23), (3.24) and (3.25). In these equations $\hat{x}_{n|m} = E[x_n|y_{1:m}]$ and $P_{n|m} = E[(x_n - \hat{x}_{n|m})(x_n - \hat{x}_{n|m})^T|y_{1:m}]$.

Prediction:

$$\hat{x}_{n|n-1} = F\hat{x}_{n-1|n-1} \tag{3.21}$$

$$P_{n|n-1} = FP_{n-1|n-1}F^T + \sigma_w^2 \tag{3.22}$$

Measurement Update:

$$K_n = P_{n|n-1}H(H_n^T P_{n|n-1}H_n + \sigma_v^2)^{-1} \tag{3.23}$$

$$\hat{x}_{n|n} = \hat{x}_{n|n-1} + K_n(y_n - H_n^T \hat{x}_{n|n-1}) \tag{3.24}$$

$$P_{n|n} = (I - K_n H_n^T)P_{n|n-1} \tag{3.25}$$

In the equations above, the filter uses only measurement values before N to estimate a value of x_N. In order to add the information from complete signal, the estimations can be smoothed with Rauch-Tung-Striebel backward recursions given in (3.26), (3.27) and (3.28).

$$J_n = P_{n|n}F^T P_{n+1|n}^{-1} \tag{3.26}$$

$$\hat{x}_{n|N} = \hat{x}_{n|n} + J_n(x_{n+1|N} - x_{n+1|n}) \tag{3.27}$$

$$P_{n|N} = P_{n|n} + J_n(P_{n+1|N} - P_{n+1|n})J_n^T \tag{3.28}$$

There process noise, state transition matrix and measurement noise are not being updated, and we must tune them. The tuning process and results will be given in Experiments & Results section.

3.2. Time Frequency Analysis

Short Time Fourier Trasnform is a technique which is widely used and very useful for analysis of signals with a time varying spectral characteristics. STFT is used for analysis of respiratory signals since the have a nonstationary nature.

$$X_m(f) = \sum_{n=-\infty}^{\infty} x(n)w(n-mR)e^{-j2\pi fn} \tag{3.29}$$

STFT is defined in (3.29). It can be seen as a sliding Fourier Transform (FT). In this equation, R is hop size and w is the window function which zero outside of a predefined range and is used to pick the part of signal which will be input of FT and smooth the signal to make it stationary. The window is an important parameter for STFT, it is the effective parameter to adjust time-frequency resolution. First of all, the window length must be so small that it must ensure that the selected portion is stationary. There is also a trade-off between resolution in time and resolution in frequency, this trade-off is also controlled with window length. As the window length increases the frequency resolution increases and the time resolution decreases. So, for wideband signals one can use smaller window lengths whereas for narrowband signals greater window lengths may be used. The output of STFT operation is a two dimensional complex matrix, which describes the magnitude and phase of frequency band component. For most of the time and for respiratory sounds too, we need the magnitude information and we convert the complex matrix to a real matrix which will give information about energy directly. The resulting matrix is usually visualized by a heatmap as in 3.1. When we look at figure 3.1, respiratory sound, magnitude plot of its STFT and corresponding airflow we can see a trend seems related to airflow in evolution of some horizontal lines. They correspond to the energy at a frequency band over the time. We will run experiments to test the correlation between the evolution of energy at each frequency band with the airflow. We will tune STFT by tweaking the window type, window length and number of FFT bins in Experiment & Results section.

3.3. Unifying Estimations

When we have an estimation problem with desired output d and observations in x vector if each observation in x and d are jointly wide sense stationary (wss) the optimum estimator for mean square error (MSE) is the Wiener filter. We can write

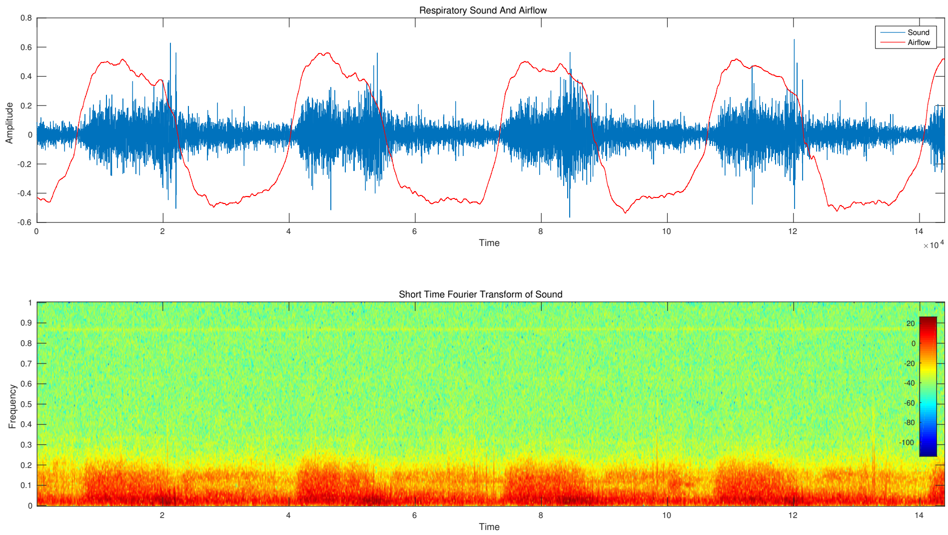

Figure 3.1. A respiratory sound, corresponding airflow and STFT magnitude plot

down the equations and derive the optimum filter. The estimation equation is given in (3.30), $\hat{d}$ is the estimation in this equation. Estimation error is given in (3.31) and the error measure we are interested in is given in (3.33)

$$\hat{d} = \sum_{i=1}^{N} w_i x_i = \mathbf{w^T x} \tag{3.30}$$

$$e = d - \hat{d} = d - \mathbf{w^T x} \tag{3.31}$$

$$E[e^2] = E[|d - \hat{d}|^2] \tag{3.32}$$

$$E[e^2] = E[d^2 - 2d\mathbf{x^T w} + \mathbf{w^T x x^T w}] \tag{3.33}$$

In order to minimize the error we can differentiate with respect to $\mathbf{w}$ and find the value where derivative is zero. For error to be at its minimum the expectation in (3.34) must be zero, this requires $\mathbf{x}$ and e to be uncorrelated.

$$\frac{\partial E[e^2]}{\partial \boldsymbol{w}} = 2E[e\frac{\partial e}{\partial \boldsymbol{w}}] = -2E[ex_i] \tag{3.34}$$

If we rewrite error as difference between d_n and $\mathbf{w_{opt}^T x}$, then we can state the equations in (3.35) and (3.36).

$$E[(d_n - \mathbf{w_{opt}^T x})\mathbf{x}] = 0 \tag{3.35}$$

$$E[\mathbf{x}\mathbf{x}^\mathbf{T}]\mathbf{w_{opt}} = E[\mathbf{x}d_n] \tag{3.36}$$

The first expectation in (3.36) is autocovariance matrix of $\mathbf{x}$ and the second expectation is crosscovariance between x and d. The filter w_{opt} is the called as Wiener filter.
In previous sections we end up with vectors which are estimations of airflow. We will use a weighted sum of these estimations as our observations and weights will be decided by using this Wiener filter approach.

3.4. Experiments & Results

In this section, the experiments to reach the best tuning parameters and the results will be given for the methods explained in this chapter.

3.4.1. Univariate Autoregressive Model

There are 4 parameters to be optimized in univariate autoregressive modeling method. These parameters are the coefficient degree, AR order, window length and overlap.
Firstly, we will try to answer the question about which AR order and which coefficient order is best. For this purpose, experiments will be done for AR orders from 1 to 15 with three different window length 500, and 50% overlap.
The inference from figures is that the correlation is decreasing with increasing coefficient order and the correlation with absolute value of flow is significantly greater. We will continue our analysis with absolute value of airflow for AR estimators.

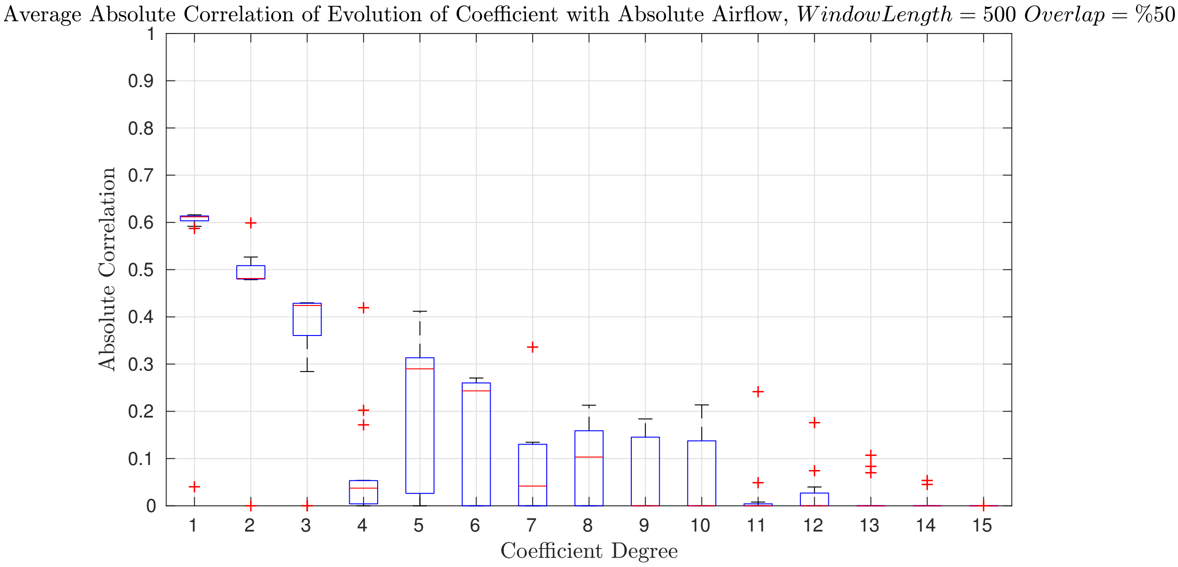

Figure 3.2. Boxplot for correlation coefficient of AR coefficient evolution with absolute value of airflow for different coefficient orders

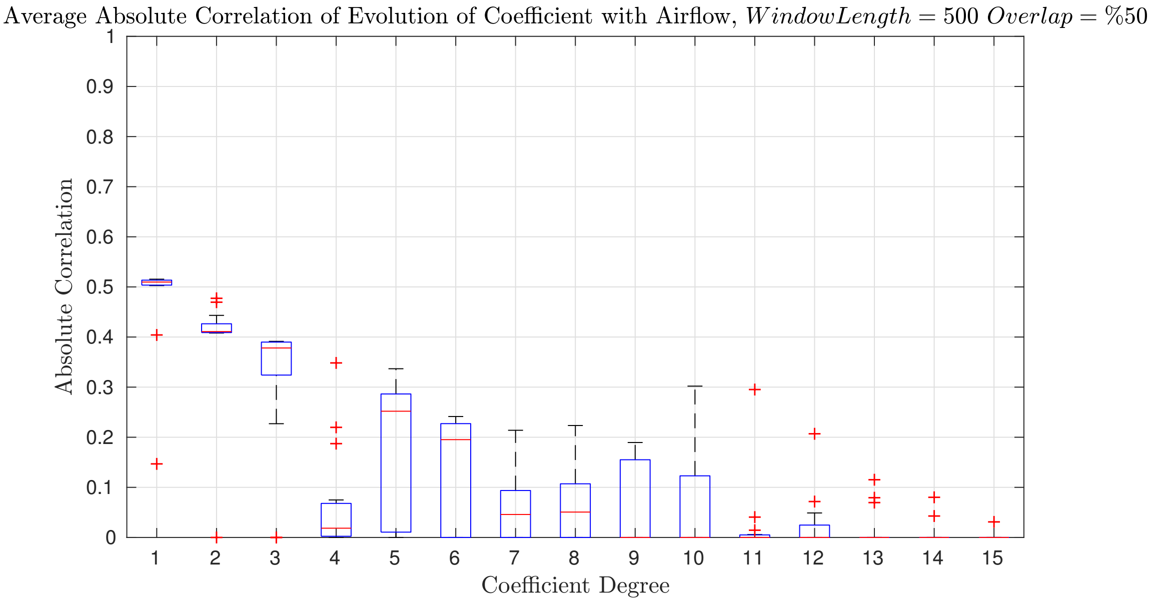

Figure 3.3. Boxplot for correlation coefficient of AR coefficient evolution with airflow for different coefficient orders