AIRFLOW ESTIMATION FROM RESPIRATORY SOUNDS

by

İlhan Yıldırım

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APPROVED BY:

Prof. Yasemin Z. Kahya (Thesis Supervisor)	....
Assist. Prof. İpek Şen (Thesis Co-supervisor)	....
Prof. Emin Anarım	....
Assoc. Prof. Ali Emre Pusane	....

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0.1. Autoregressive Models

Autoregressive models are widely used in signal analysis and also in respiratory sound signal analysis. However since the respiratory sounds are not stationary, we are modeling the respirator sounds as time varying autoregressive processes. It is reported that when a respiratory sound signal is modeled as TVAR process, the first AR coefficient is highly correlated with the airflow for the sounds recorded at trachea. We will test this for the sounds recorded at chest wall and try to find best settings for most correlation in this subsection. We will try to extract the AR coefficients with three methods:

- Windowing Based Autoregressive Modeling: Divide the signal into overlapping windows assume each window is independent from others.
- Time Varying Autoregressive Modeling with Basis Functions: Assume the AR parameters are combination of sinusoidal signals.
- Time Varying Autoregressive Modeling with Particle Filters: Assume the AR parameters are changing slowly.

0.1.1. Univariate Autoregressive Model

Univariate AR model is a difference equation where the current measurement of the signal is a linear combination of past measured values and a white Gaussian noise. The model is equivalent to a filter with infinite impulse response where the input is white Gaussian noise. The equation of an AR model is given in (1). In this equation a_i is the i^{th} order AR coefficient, N is the order of model and e is the noise term where $\mu_e = 0$ and σ_e^2 is constant.

$$x(n) = \sum_{i=1}^{N} a_i x(n-i) + e(n) \tag{1}$$

Before looking at mean, variance, autocorrelation and spectral content of a general AR model, we must set some conditions on a_i's to make the model stationary. The stationarity requires the mean, variance and autocorrelation function is constant. If the mean is constant then (2) must hold. We know that μ_e is 0, then either $\sum_{i=1}^{N} a_i = 1$ or $\mu_x = 0$ is true.

$$\mu_x = \sum_{i=1}^{N} a_i \mu_x + \mu_e \tag{2}$$

For variance we can write (3). We know that the covariance between white Gaussian noise and any other signal is zero, so we can eliminate the last term in (3). Then, $\sigma_x^2(1 - \sum_{i=1}^{N} a_i) = \sigma_e^2$. If we combine this result with the previous inference we can say that $\mu_x = 0$ and $\sum_{i=1}^{N} a_i < 1$.

$$\sigma_x^2 = \sum_{i=1}^{N} a_i^2 \sigma_x^2 + \sigma_e^2 + 2\sigma_{xe} \tag{3}$$

Another way of looking at variance is rewriting the variance in terms of autocorrelation function. To do this, we can write the equations in (4), (5) and (6).

$$\sigma_X^2 = E[X(n)X(n)] - \mu_X^2 \tag{4}$$

$$\sigma_X^2 = \sum_{i=1}^{N} a_i E[X_n X_{n-i}] + E[X_n e_n] - \mu_X^2 \tag{5}$$

$$\sigma_X^2 = \sum_{i=1}^{N} a_i R_{XX}(i) + \sigma_e^2 \tag{6}$$

Autocorrelation coefficients are very important in analysis of an AR model. The AR coefficients can be directly calculated in presence of autocorrelation coefficients.

$$r_{xx}(i) = R_{xx}(k)/R_{xx}(0) \tag{7}$$

$$r_{xx}(i) = E[x(n)x(n-i)]/R_{xx}(0) \tag{8}$$

$$r_{xx}(i) = \frac{\sum_{j=1}^{N} E[a_j x(n-j)x(n-i)]}{R_{xx}(0)} \tag{9}$$

$$r_{xx}(i) = \sum_{j=1}^{N} a_j r_{xx}(i-j) \tag{10}$$

The equation in (10) is very useful since we can rewrite it as in (11), this equation is called Yule-Walker equation. So, given a signal generated with an AR model, we can find the coefficients of that model by solving the equation in (11). Let's call this equation as $r = Ra$, the the coefficients can easily be found with $a = R^{-1}r$.

$$\begin{pmatrix} r_{XX}(1) \\ r_{XX}(2) \\ \vdots \\ r_{XX}(N-1) \\ r_{XX}(N) \end{pmatrix} = \begin{pmatrix} 1 & r_{XX}(1) & ... & r_{XX}(N-2) & r_{XX}(N-1) \\ r_{XX}(1) & 1 & ... & ... & r_{XX}(N-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{XX}(N-2) & ... & ... & 1 & r_{XX}(1) \\ r_{XX}(N-1) & r_{XX}(N-2) & ... & r_{XX}(1) & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{N-1} \\ a_N \end{pmatrix} \tag{11}$$

Although solving the equation (11) seems to be very straightforward, it requires exact knowledge on correlation coefficients. However, when we are given a signal with finite length we can just estimate the correlation coefficients, and the error in calculated AR coefficients will depend on the condition number of R matrix. To solve this equation, there are several methods in literature, throughout this thesis we will use Burg's Method to find the AR coefficients, since it is more stable than the others. [Delft'teki makale]

0.1.2. Time Varying Autoregressive Model

While univariate autoregressive model is very useful for many signals, modeling the AR coefficients as they are varying in time is a method which is widely applied for nonstationary signals. TVAR modeling were applied to respiratory signals and satisfactory results have been obtained in many papers. The equation describing a TVAR process is given in (12).

$$x(n) = \sum_{i=1}^{N} a_i(n)x(n-i) + e(n) \tag{12}$$

We will use two methods for solving this equation for a_is. One of them is modeling a_is as combination of sinusoids, the other one is modeling a_is as slowly changing parameters.

<u>0.1.2.1. TVAR Modeling With Basis Functions.</u> In this method, the AR coefficients are assumed to be combination of sinusoidal functions. Then equations in (13) and (14) can be written.

$$x(n) = \sum_{i=1}^{N} x(n-i) \sum_{j=1}^{M} c_{ij} u_j(n-i) + e(n) \tag{13}$$

$$a_i(n) = \sum_{j=1}^{M} c_{ij} u_j(n-i) \tag{14}$$