

SAYGUSAL ANALİZ
= 2. ÖDEV =
= Part 2 =

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$$[3, 1] \rightarrow [-4, 4]$$

$$[1, 2] \rightarrow [2, 2]$$

$$[2, 1] \rightarrow [-1, 4]$$

$$a_{11} + 2a_{12} + a_{13} = 2$$

$$2a_{11} + a_{12} + a_{13} = -1$$

$$3a_{11} + a_{12} + a_{13} = -4$$

$$\begin{bmatrix} 1 & 2 & 1 & | & 2 \\ 2 & 1 & 1 & | & -1 \\ 3 & 1 & 1 & | & -4 \end{bmatrix}$$

Matris
{Matris Faktörizasyonu}

$$A \cdot \vec{x} = \vec{b}$$

$$L \cdot U \cdot \vec{x} = \vec{b}$$

$$L \cdot \vec{y} = \vec{b}$$

$$\begin{array}{l} E_2 - 2E_1 \rightarrow E_2 \\ E_3 - 3E_1 \rightarrow E_3 \end{array} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & -5 & -2 \end{bmatrix}$$

$$E_3 - \frac{5}{3}E_2 \rightarrow E_3 \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -1/3 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix}$$

$$L \cdot y = b \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$$

$$U \cdot x = y \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -1/3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 5/3 \end{bmatrix}$$

$\underline{x_1 = -3}, \underline{x_2 = 0}, \underline{x_3 = 5}$

$\underline{y_1 = 2}, \underline{y_2 = -5}, \underline{y_3 = -5/3}$

$$\begin{aligned} [1, 2] &\rightarrow [2, 2] & a_{21} + 2a_{22} + a_{23} &= 2 \\ [2, 1] &\rightarrow [-1, 4] & 2a_{21} + a_{22} + a_{23} &= 4 \\ [3, 1] &\rightarrow [-4, 4] & 3a_{21} + a_{22} + a_{23} &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 3 & 1 & 1 & 4 \end{array} \right] \quad \left\{ \text{Matrix Factorization} \right\} \quad \begin{aligned} A \cdot \vec{x} &= \vec{b} \\ L \cdot U \cdot \vec{x} &= \vec{b} \\ L \cdot \vec{y} &= \vec{b} \end{aligned}$$

$$\begin{aligned} E_2 - 2E_1 &\rightarrow E_2 \\ E_3 - 3E_1 &\rightarrow E_3 \end{aligned} \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -3 & -1 & 0 \\ 0 & -5 & -2 & -4 \end{array} \right] \quad E_3 - \frac{5}{3}E_2 \rightarrow E_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -3 & -1 & 0 \\ 0 & 0 & -1/3 & -10/3 \end{array} \right] = U$$

$$L = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 5/3 & 1 & 0 \end{array} \right] \quad L \cdot y = b \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 4 \\ 3 & 5/3 & 1 & 4 \end{array} \right] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$$U \cdot x = y \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -3 & -1 & 0 \\ 0 & 0 & -1/3 & -10/3 \end{array} \right] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \quad \begin{aligned} y_1 &= 2, y_2 = 0, y_3 = -2 \\ x_1 &= 0, x_2 = 2, x_3 = 6 \end{aligned}$$

2

Inverse of A matrices;

$$A = \left[\begin{array}{ccc|ccc} -3 & 0 & 5 & 1 & 0 & 1 \\ 0 & -2 & 6 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \quad E_1 \Leftrightarrow -\frac{E_1}{3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5/3 & -1/3 & 0 & 0 \\ 0 & -2 & 6 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \quad E_2 \Leftrightarrow -\frac{1}{2}E_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5/3 & -1/3 & 0 & 0 \\ 0 & 1 & -3 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \quad \begin{array}{l} E_1 + 5/3 E_3 \rightarrow E_1 \\ E_2 + 3 E_3 \rightarrow E_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 0 & 5/3 \\ 0 & 1 & 0 & 0 & -1/2 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc} -3 & 0 & 5 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{array} \right] \cdot \left[\begin{array}{ccc} -1/3 & 0 & 5/3 \\ 0 & -1/2 & 3 \\ 0 & 0 & 1 \end{array} \right]$$

$$A^{-1} = \left[\begin{array}{ccc} -1/3 & 0 & 5/3 \\ 0 & -1/2 & 3 \\ 0 & 0 & 1 \end{array} \right]$$
