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SAYISAL ANALİZ
= 2. ÖDEV =
= Part 2 =

$[3,1] \to [-4,4]$
$[1,2] \to [2,2]$
$[2,1] \to [-1,4]$

$a_{11} + 2a_{12} + a_{13} = 2$
$2a_{11} + a_{12} + a_{13} = -1$
$3a_{11} + a_{12} + a_{13} = -4$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -1 \\ 3 & 1 & 1 & -4 \end{array}\right]$$

{Matrix Factorization}

$A.\vec{x} = \vec{b}$
$L.\underbrace{U.\vec{x}}_{y} = \vec{b}$
$L.\vec{y} = \vec{b}$

$E_2 - 2E_1 \to E_2$
$E_3 - 3E_1 \to E_3$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & -5 & -2 \end{bmatrix}$$

$E_3 - \frac{5}{3}E_2 \to E_3$

$$\underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -1/3 \end{bmatrix}}_{U}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix}$$

$$\underbrace{L.y = b \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}}$$

$\underline{y_1 = 2}$, $\underline{y_2 = -5}$, $\underline{y_3 = -5/3}$

$$U \cdot x = y \Rightarrow \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -1/3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -5/3 \end{bmatrix}}$$

$\underline{x_1 = -3}$, $\underline{x_2 = 0}$, $\underline{x_3 = 5}$

$[1,2] \to [2,2]$ $\quad a_{21} + 2a_{22} + a_{23} = 2$

$[2,1] \to [-1,4]$ $\quad 2a_{21} + a_{22} + a_{23} = 4$

$[3,1] \to [-4,4]$ $\quad 3a_{21} + a_{22} + a_{23} = 4$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 3 & 1 & 1 & 4 \end{array}\right]$$

{LU Matrix Factorization}

$A \cdot \vec{x} = \vec{b}$

$L \cdot U \cdot \vec{x} = \vec{b}$

$L \cdot \vec{y} = \vec{b}$

$E_2 - 2E_1 \to E_2$

$E_3 - 3E_1 \to E_3$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & -5 & -2 \end{bmatrix} \quad E_3 - 5/3\, E_2 \to E_3 \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -1/3 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix}$$

$$L \cdot y = b \Rightarrow \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$$U \cdot x = y \Rightarrow \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -1/3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}}_{}$$

$\underline{y_1 = 2}$, $\underline{y_2 = 0}$, $\underline{y_3 = -2}$

$\underline{x_1 = 0}$, $\underline{x_2 = -2}$, $\underline{x_3 = 6}$

Inverse of A matrices ;

$$A = \left[\begin{array}{ccc|ccc} -3 & 0 & 5 & 1 & 0 & 1 \\ 0 & -2 & 6 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad E_1 \Leftrightarrow -\frac{E_1}{3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5/3 & -1/3 & 0 & 0 \\ 0 & -2 & 6 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad E_2 \longleftrightarrow -\frac{1}{2}E_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5/3 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -3 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} E_1 + 5/3 E_3 \to E_1 \\ E_2 + 3E_3 \to E_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 0 & 5/3 \\ 0 & 1 & 0 & 0 & -1/2 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \longrightarrow \begin{bmatrix} -3 & 0 & 5 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1/3 & 0 & 5/3 \\ 0 & -1/2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1/3 & 0 & 5/3 \\ 0 & -1/2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$