Preliminary Experimentation Report: Analysis of First-Player Advantage in Random Hex Games

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1. Implementation

A Hex game was implemented in Python. The board is represented as an grid using a 2D array. Connectivity between cells is defined by the standard 6-neighbor topology of hexagonal grids.

- **Winning Condition Check:** A Depth First Search (DFS) algorithm is implemented to detect continuous paths. The game ends when Player 1 (Red) connects Top-Bottom or Player 2 (Blue) connects Left-Right.
- **Agents:** Two "Random Agents" were used. At each turn, an agent selects a move from the set of available empty cells with uniform probability.

2. Experimental Setup

Simulations were conducted on four standard board sizes: 5x5, 7x7, 9x9 and 11x11 . Player 1 always moves first. To compare, two separate batches of experiments were used:

- **Batch A:** 1000 games per board size.
- **Batch B:** 10000 games per board size.

3. Preliminary Results

The win rates for Player 1 (P1) across different board sizes are summarized below.

Table 1: Results from 1000 Simulated Games

```
--- Preliminary Experiment: Random Hex Simulation ---
Board Size   | Games      | P1 Wins    | P2 Wins    | P1 Win Rate
-------------------------------------------------------------------
5x5          | 1000       | 564        | 436        | %56.40
7x7          | 1000       | 537        | 463        | %53.70
9x9          | 1000       | 510        | 490        | %51.00
11x11         | 1000       | 534         | 466         | %53.40
```

Table 2: Results from 10,000 Simulated Games

```
--- Preliminary Experiment: Random Hex Simulation ---
Board Size   | Games      | P1 Wins    | P2 Wins    | P1 Win Rate
-------------------------------------------------------------------
5x5          | 10000      | 5781       | 4219       | %57.81
7x7          | 10000      | 5350       | 4650       | %53.50
9x9          | 10000      | 5259       | 4741       | %52.59
11x11         | 10000      | 5194        | 4806        | %51.94
```

4. Discussion

The experimental data yields two significant observations regarding the nature of the Hex game:

1. **Confirmation of First-Player Advantage:** In every single configuration, Player 1 achieved a win rate higher than 50%. Even with completely random moves, taking the initiative to place the first stone provides a measurable statistical advantage. This aligns with theoretical proofs found in the literature.
2. **Impact of Board Size:** The results from the larger dataset (Table 2) show a clear negative correlation between board size and first-player win rate.
    - On a smaller board, the first move occupies 4% of the total board area, providing a massive strategic foothold (win rate).
    - On a larger board, the impact of a single random stone on global connectivity is less significant. Consequently, the win rate approaches 50%.

Comparing Table 1 and Table 2, we observed that the 1000 game batch showed some statistical noise (e.g., the win rate spiked at). However, increasing the sample size to 10000 smoothed out this anomaly, confirming a consistent downward trend in advantage as the board scales up.

5. Conclusion and Future Work

This preliminary experimentation successfully quantified the baseline advantage of the first player in Hex. We demonstrated that while the first-move advantage is inherent, its impact decreases as the state space complexity increases.

Future Work: The current "Random Agent" serves as a baseline. For the next phase, we will implement a Heuristic Agent based on the Shortest Path (Dijkstra) algorithm. We aim to measure:

1. The performance difference between a Heuristic Agent and a Random Agent.
2. Whether the first-player advantage becomes more pronounced when two intelligent agents compete against each other.