Chapter 2: Exercise Set

Exercise 2.1

Consider the following matrices,

$$\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 0 & 8 & 2 \end{bmatrix} \tag{1}$$

$$\boldsymbol{B} = \begin{bmatrix} 7 & 2 \\ 1 & 5 \\ 9 & 4 \end{bmatrix} \tag{2}$$

Calculate the following values/matrices:

(a) $A_{2,3}$

(b) $\boldsymbol{A}^T$

(c) $\boldsymbol{B}^T$

(d) $\boldsymbol{A} + \boldsymbol{A}$

(e) $2\boldsymbol{B} + 1$

(f) $\boldsymbol{A}\boldsymbol{A}$

(g) $\boldsymbol{A}\boldsymbol{B}$

(h) $\boldsymbol{A} \odot \boldsymbol{A}$

(i) $(\boldsymbol{I}_3\boldsymbol{B})\boldsymbol{I}_2$

Exercise 2.2

Write the following set of equations into the matrix form $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$.

$$\begin{aligned} 2x_1 + 3x_2 + x_3 + 8x_4 &= 5 \\ x_1 - x_2 + x_3 - x_4 &= 2 \\ 4x_1 + 5x_3 - 2x_4 &= -4 \\ 6x_1 - 5x_2 + 3x_3 - 9x_4 &= 0 \end{aligned} \tag{3}$$

Exercise 2.3

Let $\mathbb{V}$ be the set of vectors $\{\boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)}\}$,

$$\boldsymbol{v}^{(1)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \boldsymbol{v}^{(2)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{4}$$

Find the values of the coefficients c_i such that:

$$\begin{bmatrix} 1/2 \\ 4 \end{bmatrix} = \sum_i c_i \boldsymbol{v}^{(i)} \tag{5}$$

Exercise 2.4

Let $\mathbb{V}$ be the set of vectors $\{\boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)}\}$,

$$\boldsymbol{v}^{(1)} = \begin{bmatrix} 2 \\ -1 \end{bmatrix},\ \boldsymbol{v}^{(2)} = \begin{bmatrix} -4 \\ 2 \end{bmatrix} \tag{6}$$

(a) Which of the following vectors are in the span of $\mathbb{V}$?

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -10 \\ -5 \end{bmatrix}, \begin{bmatrix} -10 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{7}$$

(b) Are the vectors in the set $\mathbb{V}$ linearly independent?

Exercise 2.5

Consider the matrices:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 42 \\ 0 \\ 12 \end{bmatrix} \tag{8}$$

(a) Is $\boldsymbol{b}$ in the range of $\boldsymbol{A}$?

(b) If so, solve $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$ for $\boldsymbol{x}$. If not, describe why.

Exercise 2.6

Consider the matrices:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 2 \\ 4 & 5 & 3 \\ 4 & 5 & 3 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 42 \\ 0 \\ 12 \end{bmatrix} \tag{9}$$

(a) Is $\boldsymbol{b}$ in the range of $\boldsymbol{A}$?

(b) If so, solve $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$ for $\boldsymbol{x}$. If not, describe why.

Exercise 2.7

Consider the two vectors

$$\boldsymbol{v}_1 = \begin{bmatrix} 5 \\ -8 \\ 1 \\ 100 \\ 4 \\ 7 \end{bmatrix},\ \boldsymbol{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

(a) Calculate the L^1 norm of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

(b) Calculate the L^2 norm of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

Exercise 2.8

Consider the two vectors

$$\boldsymbol{v}_1 = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \ \boldsymbol{v}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \tag{11}$$

(a) Calculate the L^1 norm of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

(b) Calculate the L^2 norm of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

Exercise 2.9

Imagine you are developping an image compression algorithm. You expect that after a certain image transformation (e.g. Wavelet), the resulting matrix can be well aproximated by a sparse matrix. Which type of norm (L^1 vs. L^2) do you think will be the best choice to quantify sparsity in your algorithm? Explain.

Exercise 2.10

Which type of norm (L^1 vs. L^2) would provide the best estimate of the minimal distance needed for:

(a) a boat to sail to a nearby harbour?

(b) a New York taxicab to drive the customer to her/his destination?

Exercise 2.11

Consider the following matrix,

$$\boldsymbol{A} = \begin{bmatrix} 11 & -3 \\ 18 & -4 \end{bmatrix} \tag{12}$$

Which of the following vectors are eigenvectors of $\boldsymbol{A}$? What are their eigenvalues?

(a)

$$\begin{bmatrix} 1 & -1 \end{bmatrix}^T \tag{13}$$

(b)

$$\begin{bmatrix} 1 & 3 \end{bmatrix}^T \tag{14}$$

(c)

$$\begin{bmatrix} -1 & 3 \end{bmatrix}^T \tag{15}$$

(d)

$$\begin{bmatrix} 2 & 4 \end{bmatrix}^T \tag{16}$$