

Chapter 2: Exercise Set

Exercise 2.1

Consider the following matrices,

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 0 & 8 & 2 \end{bmatrix} \quad (1)$$

$$\mathbf{B} = \begin{bmatrix} 7 & 2 \\ 1 & 5 \\ 9 & 4 \end{bmatrix} \quad (2)$$

Calculate the following values/matrices:

- (a) $A_{2,3}$
- (b) \mathbf{A}^T
- (c) \mathbf{B}^T
- (d) $\mathbf{A} + \mathbf{A}$
- (e) $2\mathbf{B} + 1$
- (f) $\mathbf{A}\mathbf{A}$
- (g) $\mathbf{A}\mathbf{B}$
- (h) $\mathbf{A} \odot \mathbf{A}$
- (i) $(\mathbf{I}_3\mathbf{B})\mathbf{I}_2$

Exercise 2.2

Write the following set of equations into the matrix form $\mathbf{A}\mathbf{x} = \mathbf{b}$.

$$\begin{aligned} 2x_1 + 3x_2 + x_3 + 8x_4 &= 5 \\ x_1 - x_2 + x_3 - x_4 &= 2 \\ 4x_1 + 5x_3 - 2x_4 &= -4 \\ 6x_1 - 5x_2 + 3x_3 - 9x_4 &= 0 \end{aligned} \quad (3)$$

Exercise 2.3

Let \mathbb{V} be the set of vectors $\{\mathbf{v}^{(1)}, \mathbf{v}^{(2)}\}$,

$$\mathbf{v}^{(1)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}^{(2)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (4)$$

Find the values of the coefficients c_i such that:

$$\begin{bmatrix} 1/2 \\ 4 \end{bmatrix} = \sum_i c_i \mathbf{v}^{(i)} \quad (5)$$

Exercise 2.4

Let \mathbb{V} be the set of vectors $\{\mathbf{v}^{(1)}, \mathbf{v}^{(2)}\}$,

$$\mathbf{v}^{(1)} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \mathbf{v}^{(2)} = \begin{bmatrix} -4 \\ 2 \end{bmatrix} \quad (6)$$

(a) Which of the following vectors are in the span of \mathbb{V} ?

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -10 \\ -5 \end{bmatrix}, \begin{bmatrix} -10 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (7)$$

(b) Are the vectors in the set \mathbb{V} linearly independent?

Exercise 2.5

Consider the matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 42 \\ 0 \\ 12 \end{bmatrix} \quad (8)$$

(a) Is \mathbf{b} in the range of \mathbf{A} ?

(b) If so, solve $\mathbf{Ax} = \mathbf{b}$ for \mathbf{x} . If not, describe why.

Exercise 2.6

Consider the matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 \\ 4 & 5 & 3 \\ 4 & 5 & 3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 42 \\ 0 \\ 12 \end{bmatrix} \quad (9)$$

(a) Is \mathbf{b} in the range of \mathbf{A} ?

(b) If so, solve $\mathbf{Ax} = \mathbf{b}$ for \mathbf{x} . If not, describe why.

Exercise 2.7

Consider the two vectors

$$\mathbf{v}_1 = \begin{bmatrix} 5 \\ -8 \\ 1 \\ 100 \\ 4 \\ 7 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \\ 0 \\ 0 \end{bmatrix} \quad (10)$$

(a) Calculate the L^1 norm of \mathbf{v}_1 and \mathbf{v}_2 .

(b) Calculate the L^2 norm of \mathbf{v}_1 and \mathbf{v}_2 .

Exercise 2.8

Consider the two vectors

$$\mathbf{v}_1 = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad (11)$$

- (a) Calculate the L^1 norm of \mathbf{v}_1 and \mathbf{v}_2 .
- (b) Calculate the L^2 norm of \mathbf{v}_1 and \mathbf{v}_2 .