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Section 1
Homework 4

Part 1:

In my design, I have implemented three collision strategies: Linear, Quadratic and Double hashing. I have a HashTable class, inside this class I have a constructor, destructor, insert, remove and search functions, also display, getSize, getNoOfElements and analyze functions.

In my insert function, I first take the modulo of the input value, reverse it for the case for double hashing. With these steps, my hashing is complete. The second step is, to insert the value to the corresponding place that is found via hashing. If the position is empty, then there is no problem, but if is not, I start probing. In the probing I check, if the place is occupied, if so then I look for the second place according to the strategy that is being used. The loop ends whenever it arrives at the same index (base index) that it has set off from.

In my remove function, I again take the modulo of the input value and follow the same procedures that were explained for the insert Until when probing starts, if the hash value is equal to the first index the function looks for we directly remove that value, as that is the value that we are trying to delete. But when this is not the case, the function looks for other indexes regarding the collision strategy that has been used. The probe continues while the according index of the array, which is found by linear, quadratic or double probing is not empty. Also, in the probing the function checks if the sequence repeats, as for it arrives at the same index that it has started, the function returns false.

In my search function, the initial procedure and probing procedure for probing strategies are same with the remove function, but in this function number of probes are also counted.

The display function displays the contents of the table, getSize function returns the size of the table, and analyze function counts the number of probes for successful searches which searches all of the current items in the table and the number of unsuccessful searches which simulates unsuccessful searches for every index in the table and counts the number of probes.

Part 2:

Input file:

```
I 33
I 119
I 116
I 16
I 29
I 155
I 166
I 106
R 98
I 229
I 219
I 151
I 169
I 96
I 202
I 144
I 153
I 21
R 108
I 186
I 172
I 141
S 219
S 99
I 28
I 135
I 221
I 90
R 40
I 70
I 107
I 6
I 230
I 64
I 94
I 53
I 122
S 53
S 74
I 26
I 32
R 57
I 121
I 91
I 49
I 24
I 190
I 81
I 178
I 25
```

```
I 164
I 34
I 76
S 87
S 178
I 85
I 13
I 11
R 183
I 104
I 117
I 222
I 17
I 195
I 41
I 235
I 46
R 89
S 42
S 121
I 42
I 9
I 242
I 136
I 66
I 159
I 177
I 132
I 194
I 38
I 188
I 216
R 216
R 42
R 81
R 91
```

Output:

1)

```
144 inserted. (LINEAR)
144 inserted. (QUADRATIC)
144 inserted. (DOUBLE)

153 inserted. (LINEAR)
153 inserted. (QUADRATIC)
153 inserted. (DOUBLE)

21 inserted. (LINEAR)
21 inserted. (QUADRATIC)
21 inserted. (DOUBLE)

108 not removed. (LINEAR)
108 not removed. (QUADRATIC)
108 not removed. (DOUBLE)

186 inserted. (LINEAR)
186 inserted. (QUADRATIC)
186 inserted. (DOUBLE)

172 inserted. (LINEAR)
172 inserted. (QUADRATIC)
172 inserted. (DOUBLE)

141 inserted. (LINEAR)
141 inserted. (QUADRATIC)
141 inserted. (DOUBLE)

219 found after 0 probes. (LINEAR)
219 found after 0 probes. (QUADRATIC)
219 found after 0 probes. (DOUBLE)

99 not found after 0 probes. (LINEAR)
99 not found after 0 probes. (QUADRATIC)
99 not found after 0 probes. (DOUBLE)

28 inserted. (LINEAR)
28 inserted. (QUADRATIC)
28 inserted. (DOUBLE)

135 inserted. (LINEAR)
135 inserted. (QUADRATIC)
135 inserted. (DOUBLE)

221 inserted. (LINEAR)
221 inserted. (QUADRATIC)
221 inserted. (DOUBLE)

90 inserted. (LINEAR)
90 inserted. (QUADRATIC)
90 inserted. (DOUBLE)

40 not removed. (LINEAR)
40 not removed. (QUADRATIC)
40 not removed. (DOUBLE)

70 inserted. (LINEAR)
70 inserted. (QUADRATIC)
70 inserted. (DOUBLE)
```

2)

```
144 inserted. (LINEAR)
144 inserted. (QUADRATIC)
144 inserted. (DOUBLE)

153 inserted. (LINEAR)
153 inserted. (QUADRATIC)
153 inserted. (DOUBLE)

21 inserted. (LINEAR)
21 inserted. (QUADRATIC)
21 inserted. (DOUBLE)

108 not removed. (LINEAR)
108 not removed. (QUADRATIC)
108 not removed. (DOUBLE)

186 inserted. (LINEAR)
186 inserted. (QUADRATIC)
186 inserted. (DOUBLE)

172 inserted. (LINEAR)
172 inserted. (QUADRATIC)
172 inserted. (DOUBLE)

141 inserted. (LINEAR)
141 inserted. (QUADRATIC)
141 inserted. (DOUBLE)

219 found after 0 probes. (LINEAR)
219 found after 0 probes. (QUADRATIC)
219 found after 0 probes. (DOUBLE)

99 not found after 0 probes. (LINEAR)
99 not found after 0 probes. (QUADRATIC)
99 not found after 0 probes. (DOUBLE)

28 inserted. (LINEAR)
28 inserted. (QUADRATIC)
28 inserted. (DOUBLE)

135 inserted. (LINEAR)
135 inserted. (QUADRATIC)
135 inserted. (DOUBLE)

221 inserted. (LINEAR)
221 inserted. (QUADRATIC)
221 inserted. (DOUBLE)

90 inserted. (LINEAR)
90 inserted. (QUADRATIC)
90 inserted. (DOUBLE)

40 not removed. (LINEAR)
40 not removed. (QUADRATIC)
40 not removed. (DOUBLE)

70 inserted. (LINEAR)
70 inserted. (QUADRATIC)
70 inserted. (DOUBLE)
```

3)

```
107 inserted. (LINEAR)
107 inserted. (QUADRATIC)
107 inserted. (DOUBLE)

6 inserted. (LINEAR)
6 inserted. (QUADRATIC)
6 inserted. (DOUBLE)

230 inserted. (LINEAR)
230 inserted. (QUADRATIC)
230 inserted. (DOUBLE)

64 inserted. (LINEAR)
64 inserted. (QUADRATIC)
64 inserted. (DOUBLE)

94 inserted. (LINEAR)
94 inserted. (QUADRATIC)
94 inserted. (DOUBLE)

53 inserted. (LINEAR)
53 inserted. (QUADRATIC)
53 inserted. (DOUBLE)

122 inserted. (LINEAR)
122 inserted. (QUADRATIC)
122 inserted. (DOUBLE)

53 found after 0 probes. (LINEAR)
53 found after 0 probes. (QUADRATIC)
53 found after 0 probes. (DOUBLE)

74 not found after 0 probes. (LINEAR)
74 not found after 0 probes. (QUADRATIC)
74 not found after 0 probes. (DOUBLE)

26 inserted. (LINEAR)
26 inserted. (QUADRATIC)
26 inserted. (DOUBLE)

32 inserted. (LINEAR)
32 inserted. (QUADRATIC)
32 inserted. (DOUBLE)

57 not removed. (LINEAR)
57 not removed. (QUADRATIC)
57 not removed. (DOUBLE)

121 inserted. (LINEAR)
121 inserted. (QUADRATIC)
121 inserted. (DOUBLE)

91 inserted. (LINEAR)
91 inserted. (QUADRATIC)
91 inserted. (DOUBLE)

49 inserted. (LINEAR)
49 inserted. (QUADRATIC)
49 inserted. (DOUBLE)
```

4)

```
24 inserted. (LINEAR)
24 inserted. (QUADRATIC)
24 inserted. (DOUBLE)

190 inserted. (LINEAR)
190 inserted. (QUADRATIC)
190 inserted. (DOUBLE)

81 inserted. (LINEAR)
81 inserted. (QUADRATIC)
81 inserted. (DOUBLE)

178 inserted. (LINEAR)
178 inserted. (QUADRATIC)
178 inserted. (DOUBLE)

25 inserted. (LINEAR)
25 inserted. (QUADRATIC)
25 inserted. (DOUBLE)

164 inserted. (LINEAR)
164 inserted. (QUADRATIC)
164 inserted. (DOUBLE)

34 inserted. (LINEAR)
34 inserted. (QUADRATIC)
34 inserted. (DOUBLE)

76 inserted. (LINEAR)
76 inserted. (QUADRATIC)
76 inserted. (DOUBLE)

87 not found after 0 probes. (LINEAR)
87 not found after 0 probes. (QUADRATIC)
87 not found after 0 probes. (DOUBLE)

178 found after 0 probes. (LINEAR)
178 found after 0 probes. (QUADRATIC)
178 found after 0 probes. (DOUBLE)

85 inserted. (LINEAR)
85 inserted. (QUADRATIC)
85 inserted. (DOUBLE)

13 inserted. (LINEAR)
13 inserted. (QUADRATIC)
13 inserted. (DOUBLE)

11 inserted. (LINEAR)
11 inserted. (QUADRATIC)
11 inserted. (DOUBLE)

183 not removed. (LINEAR)
183 not removed. (QUADRATIC)
183 not removed. (DOUBLE)

104 inserted. (LINEAR)
104 inserted. (QUADRATIC)
104 inserted. (DOUBLE)
```

5)

```
117 inserted. (LINEAR)
117 inserted. (QUADRATIC)
117 inserted. (DOUBLE)

222 inserted. (LINEAR)
222 inserted. (QUADRATIC)
222 inserted. (DOUBLE)

17 inserted. (LINEAR)
17 inserted. (QUADRATIC)
17 inserted. (DOUBLE)

195 inserted. (LINEAR)
195 inserted. (QUADRATIC)
195 inserted. (DOUBLE)

41 inserted. (LINEAR)
41 inserted. (QUADRATIC)
41 inserted. (DOUBLE)

235 inserted. (LINEAR)
235 inserted. (QUADRATIC)
235 inserted. (DOUBLE)

46 inserted. (LINEAR)
46 inserted. (QUADRATIC)
46 inserted. (DOUBLE)

89 not removed. (LINEAR)
89 not removed. (QUADRATIC)
89 not removed. (DOUBLE)

42 not found after 0 probes. (LINEAR)
42 not found after 3 probes. (QUADRATIC)
42 not found after 0 probes. (DOUBLE)

121 found after 0 probes. (LINEAR)
121 found after 0 probes. (QUADRATIC)
121 found after 0 probes. (DOUBLE)

42 inserted. (LINEAR)
42 inserted. (QUADRATIC)
42 inserted. (DOUBLE)

9 inserted. (LINEAR)
9 inserted. (QUADRATIC)
9 inserted. (DOUBLE)

242 inserted. (LINEAR)
242 inserted. (QUADRATIC)
242 inserted. (DOUBLE)

136 inserted. (LINEAR)
136 inserted. (QUADRATIC)
136 inserted. (DOUBLE)

66 inserted. (LINEAR)
66 inserted. (QUADRATIC)
66 inserted. (DOUBLE)
```

6)

```
159 inserted. (LINEAR)
159 inserted. (QUADRATIC)
159 inserted. (DOUBLE)

177 inserted. (LINEAR)
177 inserted. (QUADRATIC)
177 inserted. (DOUBLE)

132 inserted. (LINEAR)
132 inserted. (QUADRATIC)
132 inserted. (DOUBLE)

194 inserted. (LINEAR)
194 inserted. (QUADRATIC)
194 inserted. (DOUBLE)

38 inserted. (LINEAR)
38 inserted. (QUADRATIC)
38 inserted. (DOUBLE)

188 inserted. (LINEAR)
188 inserted. (QUADRATIC)
188 inserted. (DOUBLE)

216 inserted. (LINEAR)
216 inserted. (QUADRATIC)
216 inserted. (DOUBLE)

216 removed. (LINEAR)
216 removed. (QUADRATIC)
216 removed. (DOUBLE)

42 removed. (LINEAR)
42 removed. (QUADRATIC)
42 removed. (DOUBLE)

81 removed. (LINEAR)
81 removed. (QUADRATIC)
81 removed. (DOUBLE)

91 removed. (LINEAR)
91 removed. (QUADRATIC)
91 removed. (DOUBLE)
```

7)

```
----------------------LINEAR TABLE----------------------
Table size: 101
0:202
1:
2:
3:104
4:
5:106
6:107
7:6
8:
9:9
10:
11:11
12:
13:13
14:
15:116
16:16
17:219
18:119
19:221
20:121
21:21
22:122
23:117
24:24
25:25
26:26
27:229
28:28
29:29
30:230
31:222
32:32
33:33
34:135
35:34
36:17
37:235
38:136
39:132
40:141
41:41
42:
43:144
44:242
45:38
46:46
47:
48:
49:49
50:151
51:
52:153
53:53
54:155
55:
56:
57:
58:159
59:
```

8)

```
60:
61:
62:
63:164
64:64
65:166
66:66
67:
68:169
69:
70:70
71:172
72:
73:
74:
75:
76:76
77:178
78:177
79:
80:
81:
82:
83:
84:
85:186
86:85
87:188
88:
89:190
90:90
91:
92:
93:194
94:94
95:195
96:96
97:
98:
99:
100:
Number of elements in the table: 64

Average number of probes for successful searches: 0.712871
Average number of probes for unsuccessful searches: 6.51485
------------------------------------------------------------
```

9)

```
---------------------QUADRATIC TABLE----------------
Table size: 101
0:202
1:
2:
3:104
4:
5:106
6:107
7:6
8:
9:9
10:
11:11
12:
13:13
14:
15:116
16:16
17:219
18:119
19:221
20:121
21:21
22:122
23:
24:24
25:25
26:26
27:229
28:28
29:29
30:
31:132
32:230
33:33
34:135
35:34
36:32
37:235
38:38
39:136
40:141
41:117
42:17
43:144
44:242
45:222
46:46
47:
48:
49:49
50:151
51:
52:153
53:53
54:155
55:
56:
57:41
58:159
```

10)

```
59:
60:
61:
62:
63:164
64:64
65:166
66:66
67:
68:169
69:
70:70
71:172
72:
73:
74:
75:
76:76
77:178
78:
79:
80:177
81:
82:
83:
84:
85:186
86:85
87:188
88:
89:190
90:90
91:
92:
93:194
94:94
95:195
96:96
97:
98:
99:
100:
Number of elements in the table: 64

Average number of probes for successful searches: 1.06931
Average number of probes for unsuccessful searches: 8.82178
------------------------------------------------------
```

11)

```
----------------------DOUBLE TABLE-----------------
Table size: 101
0:202
1:
2:
3:104
4:
5:106
6:107
7:
8:
9:9
10:
11:11
12:6
13:13
14:
15:116
16:16
17:219
18:119
19:221
20:121
21:21
22:
23:
24:24
25:25
26:26
27:229
28:28
29:29
30:
31:66
32:32
33:33
34:135
35:136
36:117
37:
38:38
39:177
40:141
41:41
42:85
43:144
44:
45:
46:46
47:
48:188
49:49
50:151
51:
52:153
53:53
54:155
55:
56:
57:
```

12)

```
58:159
59:122
60:230
61:
62:34
63:164
64:64
65:166
66:
67:
68:169
69:
70:70
71:172
72:
73:
74:
75:132
76:76
77:178
78:
79:195
80:222
81:
82:
83:
84:
85:186
86:
87:235
88:17
89:190
90:90
91:
92:
93:194
94:94
95:
96:96
97:
98:
99:242
100:
Number of elements in the table: 64

Average number of probes for successful searches: 1.42574
The analysis is made only for successful searches in double hashing
------------------------------------------------------------------
```

Table Size: 101

(If a hash function uses modulo arithmetic, the table size should be prime.)

Part 3:

Load factor: α = Number of items in the table / size of the table = 64 / 101 = 0.6336633663

Theoretical average number of probes for linear probing:

$$\frac{1}{2}\left[1+\frac{1}{1-\alpha}\right] \qquad \textit{for a successful search}$$

$$\frac{1}{2}\left[1+\frac{1}{(1-\alpha)^2}\right] \qquad \textit{for an unsuccessful search}$$

- These results show only the average for one search, therefore we need to multiply the results that we have found for a successful search with the number of items in the table (64) and for an unsuccessful search we need to multiply the average found in the formula with the size of the table (101).

Theoretical number of probes for linear probing:

$$\frac{1}{2}\left(1+\frac{1}{\left(1-\frac{64}{101}\right)}\right) = 1.864864865$$

- For successful searches

$$\frac{1}{2}\left(1+\frac{1}{\left(1-\frac{64}{101}\right)^2}\right) = 4.225712199$$

- For unsuccessful searches

Empirical number of probes for linear probing:

```
Average number of probes for successful searches: 0.712871
Average number of probes for unsuccessful searches: 6.51485
```

Theoretical average number of probes for quadratic & double (excluding unsuccessful search) probing:

$$\left[\frac{1}{\alpha}(\log_e \frac{1}{1-\alpha})\right] = \frac{-\log_e(1-\alpha)}{\alpha} \qquad \textit{for a successful search}$$

$$\frac{1}{1-\alpha} \qquad \textit{for an unsuccessful search}$$

- These results show only the average for one search.

Theoretical number of probes for linear probing & double hashing:

$$-\frac{\ln\left(1-\frac{64}{101}\right)}{\frac{64}{101}} = 1.584757235$$

- For successful searches

$$\frac{1}{1-\frac{64}{101}} = 2.72972973$$

- For unsuccessful searches (excluding double hashing)

Empirical number of probes for quadratic probing:

```
Average number of probes for successful searches: 1.06931
Average number of probes for unsuccessful searches: 8.82178
```

Empirical number of probes for double hashing:

```
Average number of probes for successful searches: 1.42574
The analysis is made only for successful searches in double hashing
```

Analysis:

In linear and quadratic probing and double hashing, the cost of search depends on the length of the cluster. And if the load factor of the table increases, number of the collisions and the probability of the formation of the cluster increases. In our experiment the load factor is 64/101 which is nearly 63% which can be considered high; therefore, we have a high collision rate and rather long clusters. Whilst probing more collisions occur compared to a list which would have shorter clusters and a lower load factor. Therefore, some of the empirical results are higher than the theoretical ones, which are given previously, especially the ones for the unsuccessful searches. We see that our results for the successful searches are less than the theoretical ones, because our table is large enough to contain lots of values, so the function finds the items rather easy.