Firstly, I defined three functions for updating the centroids, memberships, and plotting the current state. These functions are taken from the lab.

Euclidean Distance
$$\frac{1}{2} \left( \frac{x_i}{x_i} , \frac{x_j}{x_j} \right) = \frac{1}{2} \left( \frac{x_i}{x_i} - \frac{x_j}{x_j} \right)^{2}$$

$$= \sqrt{\frac{2}{2}} \left( \frac{x_i}{x_i} - \frac{x_j}{x_j} \right)^{2}$$

I constructed the B matrix by using the Euclidean distance formula above. Then, I plot the connectivity matrix to show connected data points.

I calculated D and L matrices using the following formulas.

I applied following steps where R = 5 smallest eigenvectors and K = 5. Using these eigenvectors, I constructed Z matrix. I initalized the centroids as the 85, 129, 167, 187, and 270th rows of Z matrix.

Lastly, I run k-means clustering algorithm on Z matrix and plot the clustering result. Formulas above are taken from the lecture notes.