IE 407 - FUNDAMENTALS OF OR | SPRING 2023

HOMEWORK 1

Due date: 23.03.2024

GUIDELINE:

Please submit your work to corresponding submission on ODTUClass. You may complete this homework assignment as a group comprising a maximum of two students.

Your submission should include:

1- Homework report (.pdf file): Your report should include your results and comments. It must be 5 pages at most. The format must adhere to Times New Roman font, size 12, and justified text alignment. Your report can include a (scanned) hand-drawn graphical solution for question 1.
2- Excel File: Your Excel file should include your model, solution, and sensitivity analysis results.
3- Your report should include a cover page including your names, surnames, and student ID numbers.
4- Make sure to organize your findings, data, and calculations neatly for easy review. Please submit a single compressed (.zip) file consisting of your report (.pdf) and Excel.

Question 1 (50 points). Tasch Co. plans to produce hiking and sprint shoes. The weekly demand for hiking shoes is estimated to be higher than 500 pairs. The production capacity is at most 5000 pairs of shoes regardless of the shoe type. With the current labor force available, the production of a hundred pairs of hiking shoes requires 2 hours of labor force. Similarly, the production of a hundred pairs of sprint shoes requires 1 hour of labor force. Available working hours in a week is 40 hours.

Both shoes are made of cotton. The cotton required to produce a pair of hiking and sprint shoes is 0.2 and 0.5 kg, respectively. The available warehouse capacity for cotton is 500 kg.With all material and labor costs included, producing a pair of hiking and sprint shoes costs \$3 and \$1 respectively. The linear programming model for minimizing the cost of shoe production is as follows.

$Min\ z = 3x_1 + x_2$

Subject to

$x_1 \geq 500$ (1) Hiking shoe demand constraint

$x_1 + x_2 \leq 5000$ (2) total production capacity constraint

$\frac{2x_1}{100} + \frac{x_2}{100} \leq 40$ (3) labor force constraint

$0.2x_1 + 0.5x_2 \leq 500$ (4) cotton material constraint

$x_1, x_2 \geq 0$ (5) non-negativity constraint

Solve this model with graphical method by answering the following questions:

a) What is the optimal objective function value and the optimal solution (i.e., what are the optimal values of z^* (objective) and x_1^* and x_2^* (solution))? (*10 points*)

   Also show the feasible region, isocost line and the optimal solution on the graph. (*10 points*)

   Identify the binding, non-binding, and redundant constraints on the graph. (*10 points*)

b) What could happen if the cost of producing a pair of sprint shoes increases by \$1 and becomes \$2? How much the cost of sprint shoes could be increased without changing the current optimal solution? (*10 points*)

c) In the spring season, people are more likely to go hiking and the demand for hiking shoes may increase. Based on your solution in part (a), how much increase in the demand is possible without changing the current basis? (*10 points*)

Question 2 (50 points). Küpraş refinery sells two types of oil products: high emission (LE) and clean emission (CE) with prices of $40 and $70 per barrel, respectively.

Both types require carbon-saturated oil (CO) and hydrogen-saturated oil (HO) for production. The specifications for blending the LE and CE oils are as follows:

Table 1: specifications for LE and CE oils

Oil	Steam pressure upper limit	Lowest emission rating	Highest demand estimation	Minimum delivery requirement
LE	19	87	110,000	45,000
CE	20	94	90,000	6,000

The specifications and inventory capacities of CO and HO are as follows:

Table 2: Specifications and inventory capacities of CO and HO

Oil	steam pressure	emission rating	Warehouse space
CO	21	83	50,000
HO	15	97	100,000

Küpraş refinery aims to maximize its weekly revenue by satisfying steam pressure, emission rating, and demand constraints.

Formulate and solve a linear programming model to maximize the weekly revenue of Küpraş.

a) How many products should be produced from each type to maximize revenue? (*10 points*)

   What should be the material composition (amount of CO and HO) of LE and CE blends? (*10 points*) Use a linear programming software (i.e. Excel Solver) to solve the problem.

   Note: Answer parts b,c,d,e, and f separately.

b) For what values of the price of LE, the current basis remains optimal (the same variables are selected to be in the basis)? What happens to the optimal solution and optimal objective function value when the price of LE is increased by $1? (*8 points*)
c) For what values of the price of CE, the current basis remains optimal? What happens to the optimal solution and optimal objective function value when the price of CE is increased by $1 and decreased by $10? (*8 points*)
d) For what values of warehouse space of CO, the current basis remains optimal? What is the most Küpraş should be willing to pay to increase the warehouse space of CO by 1,000 barrels? What happens to the optimal solution and optimal objective function value when the warehouse space of CO is increased by 80,000 barrels? (*8 points*)

e) For what values of the minimum delivery requirement of LE, current basis remains optimal? What is the shadow price for this constraint? What happens to the optimal solution and optimal objective function value when the minimum delivery requirement of LE is increased by 3,000 barrels per week? (*6 points*)