CENG 384 - Signals and Systems for Computer Engineers
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Homework 1

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1. (a) By multipling with the complext conjugate of the denominator we found z as $\frac{\sqrt{2}+\sqrt{6}}{8} + (\frac{\sqrt{2}-\sqrt{6}}{8})j$ so the real part of z is $\frac{\sqrt{2}+\sqrt{6}}{8}$ and the imaginary part is $\frac{\sqrt{2}-\sqrt{6}}{8}$.

   (b) To find the magnitude and phase of z, we calculate the magnitude (r) using the formula $r = \sqrt{a^2+b^2}$, where a is the real part and b is the imaginary part of z. This yields the magnitude of z as $\sqrt{1/4}$ thus 1/2.
   The phase (θ) of z is given by $\theta = \tan^{-1}\left(\frac{b}{a}\right)$, with a being the real part and b being the imaginary part of z. Thus, the phase of z is calculated as $\tan^{-1}\left(\frac{\frac{\sqrt{2}-\sqrt{6}}{8}}{\frac{\sqrt{2}+\sqrt{6}}{8}}\right)$ which then calculated as $\tan^{-1}(\frac{2-\sqrt{3}}{2})$, which is the angle z makes with the positive real axis in the complex plane.

2.
   - Time scale, expand by 2
   - Time shift , shift by 4 to right

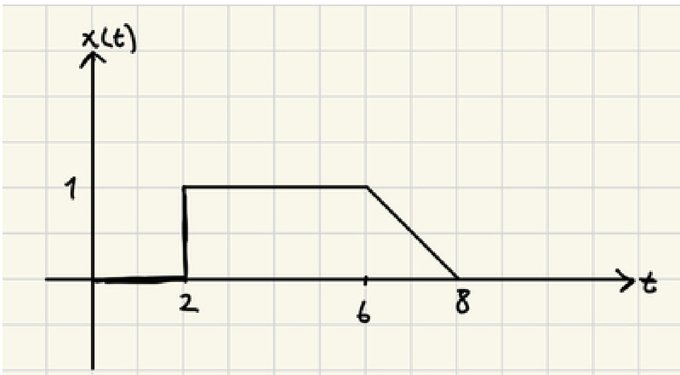

Figure 1: $y(t) = x\left(\frac{1}{2}t - 2\right)$

3. (a) $x[n] = \delta(n-1) + 2\delta(n-2) + \delta(n-3) + \delta(n+3) - \delta(n) - \delta(n+1) - \delta(n+2)$

   (b) Shown in figure 2.

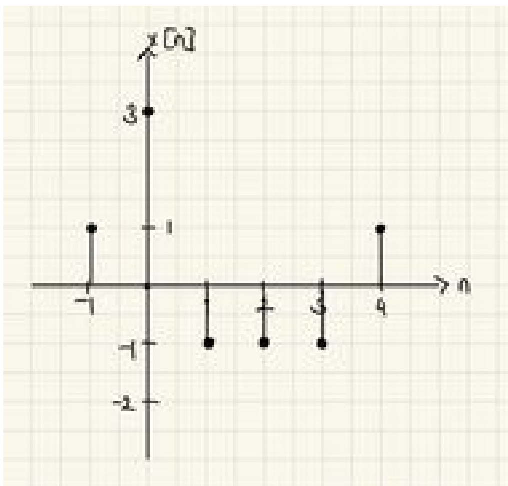

Figure 2: $y = [n] = x[2n+2] + x[1-n]$

(c) i. $x[2n+2] = \delta(2n+1) + 2\delta(2n) + \delta(2n-1) + \delta(2n+5) - \delta(2n+2) - \delta(2n+3) - \delta(2n+4)$
$= 2\delta(n) - \delta(n+1) - \delta(n+2)$

ii. $x[1-n] = \delta(-n) + 2\delta(-n-1) + \delta(-n-2) + \delta(-n+4) - \delta(-n+1) - \delta(-n+2) - \delta(-n+3)$
$= \delta(n) + 2\delta(n+1) + \delta(n+2) + \delta(n-4) - \delta(n-1) - \delta(n-2) + \delta(n-3)$

iii. So the result $x[2n+2] + x[1-n] = 3\delta[n] + \delta[n+1] + \delta(n-4) - \delta(n-1) - \delta(n-2) - \delta(n-3)$

4. (a) The signal $x_1[n] = \cos\left(\frac{5\pi}{2}n\right)$ is a periodic signal because the cosine function is inherently periodic with a period of 2π. The fundamental period N can be found by setting $\frac{5\pi}{2}N = 2\pi k$ where k is an integer. The smallest non-zero N that satisfies this equation is $N = 2$, hence the fundamental period of $x_1[n]$ is $\frac{4x\pi}{5}$.

(b) In order $\sin[5n]$ to be periodic, its continuous-time counterpart must have a rational period. However, the signal $x_2[n]\sin(5n)$ has a fundamental period of $2\pi/5$.
There is no integer t_0 such that,

$$\sin[5n] = \sin[5(n+t_0)]$$

(c) The signal $x_3(t) = 5\sin(4t + \frac{\pi}{3})$ is periodic since the sine function has a period of 2π. The fundamental period T is found by setting $4T = 2\pi k$, with k being an integer. The smallest T that satisfies this equation is $T = \frac{\pi}{2}$, making it the fundamental period of $x_3(t)$.

5. To show that $\delta(at) = \frac{1}{|a|}\delta(t)$, we will use the time scaling property of the delta function. The density of the delta function is,

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Now consider the scaled delta function $\delta(at)$ and let's apply it to the same integral using a change of variables:

$$\int_{-\infty}^{\infty} \delta(at)\, dt$$

Let $u = at$, which implies $du = a\, dt$ and thus $dt = \frac{du}{a}$. Substituting these into the integral, we get:

$$\int_{-\infty}^{\infty} \delta(u)\, \frac{du}{a}$$

Now solving the integral, since a is the constant :

$$\frac{1}{|a|}\int_{-\infty}^{\infty} \delta(u) du$$

Since the integral equates to 1 because it is the same function we used , it follows that $\delta(at) = \frac{1}{|a|}\delta(t)$ is proved.

6. (a) The difference equation for the overall system S, when subsystems $S1$ and $S2$ are connected in series, is obtained by substituting $S1$'s output into $S2$. For $S1 : y_1[n] = 4x_1[n] + 2x_1[n-1]$ and $S2 : y_2[n] = y_1[n-2]$, the overall output $y[n]$ is:

$$y[n] = 2x[n-3] + 4x[n-2]$$

(b) Reversing the order of the subsystems does not change the difference equation for the overall system. If $S2$ is applied first and then $S1$, the overall system would still be:

$$y[n] = 2x[n-3] + 4x[n-2]$$

Thus, the series connection of $S1$ and $S2$ is commutative for this particular configuration.

(c) To verify if the overall system S is linear, we check if it satisfies the superposition principle. For two inputs $x_1[n]$ and $x_2[n]$, and scalars a and b, if the output due to $ax_1[n] + bx_2[n]$ is $ay_1[n] + by_2[n]$, where $y_1[n]$ and $y_2[n]$ are outputs corresponding to $x_1[n]$ and $x_2[n]$ individually, the system is linear. The system S is linear because the difference equation consists of linear operations on the input $x[n]$.

(d) A system is time-invariant if a time shift in the input signal results in an identical time shift in the output signal. For the system S, a time shift in the input $x[n-k]$ results in the output $y[n-k]$ which is the same as shifting $y[n]$ by k samples. Thus, the system S is time-invariant as it meets this condition.

7. (a) The system $y[n] = n \cdot x[n]$ was tested for linearity using the python code provided below. The test confirmed that the system satisfies the superposition principle and is therefore a linear system.

(b) The system $y[n] = x[n]^2$ was tested for linearity using the python code provided below. The test showed that the system does not satisfy the superposition principle, indicating that the system is non-linear.

```python
from sympy import symbols, Function, simplify

n = symbols('n', integer=True)
a, b = symbols('a b', real=True)
x1 = Function('x1')(n)
x2 = Function('x2')(n)

y1a = n * x1
y2a = n * x2
y1b = x1**2
y2b = x2**2

x_combined = a*x1 + b*x2

y_combined_a = n * x_combined
y_combined_b = x_combined**2

if simplify(y_combined_a) == simplify(a*y1a + b*y2a):
    linear_a = "Linear"
else:
    linear_a = "Non-Linear"

if simplify(y_combined_b) == simplify(a*y1b + b*y2b):
    linear_b ="Linear"
else :
    linear_b = "Non-Linear"

print('Given system  is a {} system.'.format(linear_a))
print('Given system  is a {} system.'.format(linear_b))
```