CENG 384 - Signals and Systems for Computer Engineers
Spring 2024
Homework 2

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1)

Given the functions $x(t)$ and $h(t)$ defined as:

$$x(t) = u(t+3) * u(7-t)$$
$$h(t) = u(t-1) * u(15-t)$$

Their convolution $y(t)$ is calculated as:

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \\ &= \int_{-\infty}^{\infty} (u(\tau+3) - u(7-\tau))(u(t-\tau-1) - u(15-t+\tau))\,d\tau \end{aligned}$$

The intervals of integration are determined by the overlapping regions of $x(\tau)$ and $h(t-\tau)$. Thus, we have:
For $-2 \leq t < 8$:

$$y(t) = \int_{-3}^{t-1} d\tau = t+2$$

For $8 \leq t < 12$:

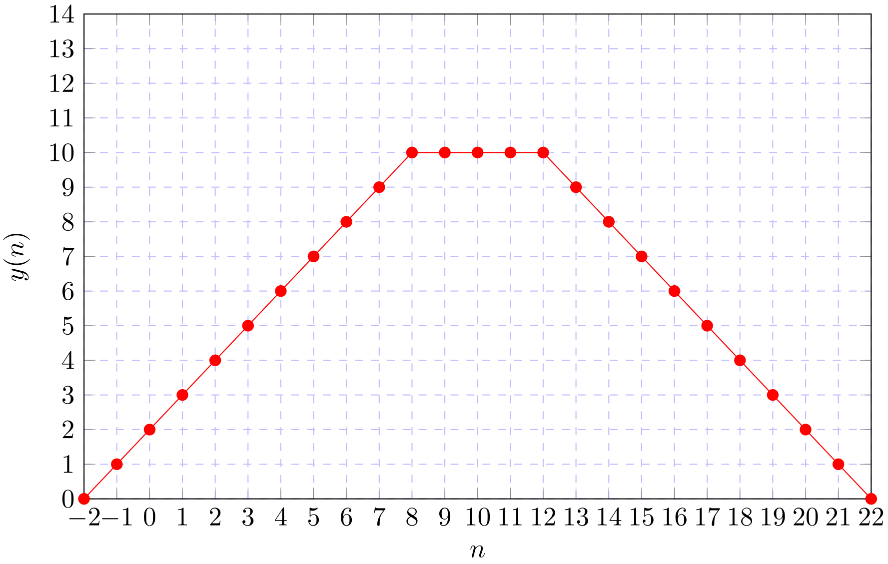
$$y(t) = \int_{-3}^{7} d\tau = 10$$

For $12 \leq t < 22$:

$$y(t) = \int_{t-15}^{7} d\tau = 22-t$$

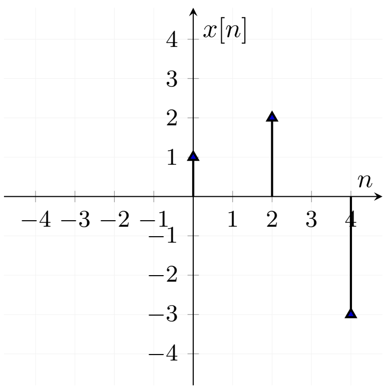
Finally, the complete piecewise function for $y(t)$ is given by:

$$y(t) = \begin{cases} t+2 & -2 \leq t < 8 \\ 10 & 8 \leq t \leq 12 \\ 22-t & 12 < t \leq 22 \\ 0 & \text{otherwise} \end{cases}$$

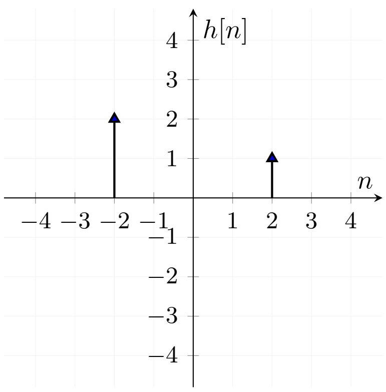


2)

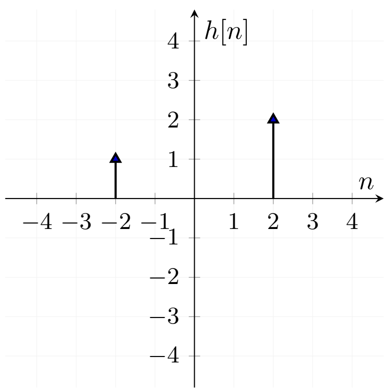
Input Signal $x[n]$



Impulse Response $h[n]$



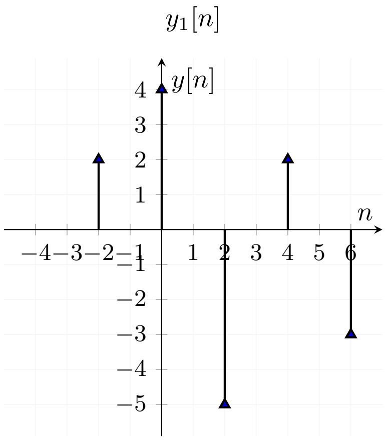
Flipped Impulse Response about the y-axis $h[n]$



a)

n	$y_1[n]$
-2	2
0	4
2	-5
4	2
6	-3

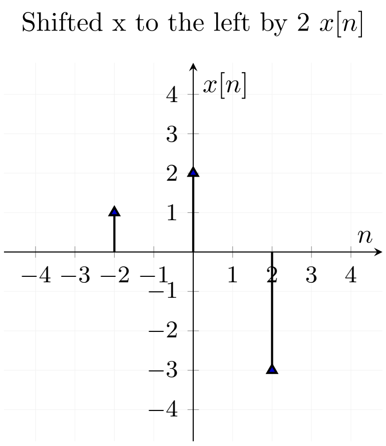
The resulting plot is $y_1[n]$:



b)

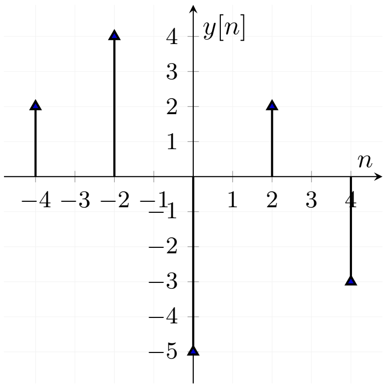
we should shift x to the left by 2

n	$y_2[n]$
-4	2
-2	4
0	-5
2	2
4	-3



The resulting plot is $y_2[n]$:

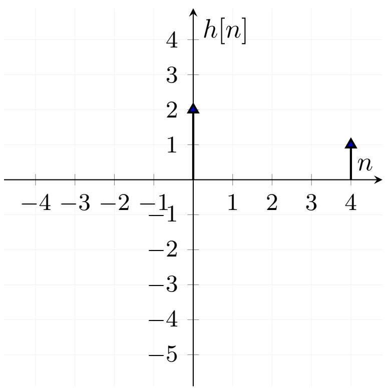
$y_2[n]$



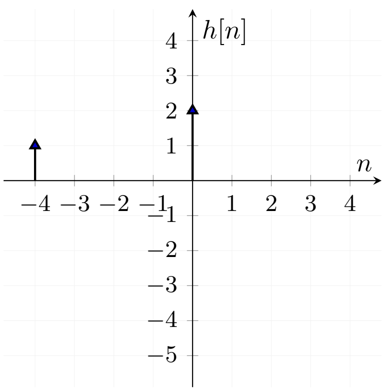
c)

n	$y_3[n]$
-2	2
0	4
2	-5
4	2
6	-3

$h[n]$ Shifted right by 2

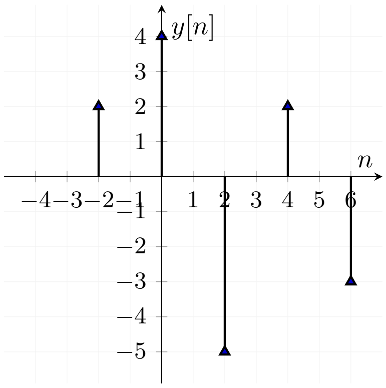


Flipped Impulse Response about the y-axis $h[n-2]$



The resulting plot is $y_3[n]$:

$y_3[n-2]$



3)

a)

The impulse response of a linear time-invariant (LTI) system is determined by applying the delta function, symbolized by $\delta[n]$, which is 1 at $n = 0$ and 0 for all other n values. When this is inserted into the system's difference equation, the result is:

$$h[n] = \frac{1}{5}\delta[n-1] + \delta[n].$$

Consequently, the impulse response can be expressed as:

$$h[n] = \begin{cases} 1 & \text{for } n = 0, \\ \frac{1}{5} & \text{for } n = 1, \\ 0 & \text{otherwise.} \end{cases}$$

b)

To analyze the given system's response, we input the shifted delta function $\delta[n-2]$. Using this input, the system's equation becomes:

$$y[n] = \frac{1}{5}\delta[n-3] + \delta[n-2].$$

Hence, for the input $x[n] = \delta[n-2]$, the output $y[n]$ is described by:

$$y[n] = \begin{cases} \frac{1}{5} & \text{for } n = 3, \\ 1 & \text{for } n = 2, \\ 0 & \text{otherwise.} \end{cases}$$

c)

A system achieves BIBO (Bounded Input, Bounded Output) stability when all bounded inputs lead to bounded outputs. This is verified when the impulse response $h[n]$ is nonzero over only a limited range of n and the total sum of $h[n]$'s absolute values remains finite, indicating that the system maintains BIBO stability.

d)

A system possesses memory when its current output is influenced by previous or subsequent input values. This characteristic is evident when the output $y[n]$ is affected by $x[n-1]$, demonstrating that the system have memory.

e)

Assume we are given two distinct input signals, $x_1[n]$ and $x_2[n]$, defined as $x_1[n] = a_1\left(-\frac{1}{5}\right)^n$ and $x_2[n] = b_1\left(-\frac{1}{5}\right)^n$, with a_1 and b_1 being non-zero constants. The system's response to these inputs can be described as:

$$y[n] = \frac{1}{5}x_1[n-1] + x_1[n] = -a_1\left(-\frac{1}{5}\right)^n + a_1\left(-\frac{1}{5}\right)^n = 0$$

$$y[n] = \frac{1}{5}x_2[n-1] + x_2[n] = -b_1\left(-\frac{1}{5}\right)^n + b_1\left(-\frac{1}{5}\right)^n = 0$$

The system produces the same output for both distinct inputs, which implies that the system's response is not unique to each input. Since we cannot uniquely determine the input from the output, the system is not invertible.

4)

a)

To obtain the system's differential equation, it is acknowledged that the system's transfer function is equivalent to the Laplace transform of its impulse response. Therefore, we establish the following relationship:

$$\frac{Y(s)}{X(s)} = \frac{2s}{s^2 - 2s + 1}$$

Consequently, this leads to the differential equation:

$$y''(t) - 2y'(t) + y(t) = 2x'(t)$$

b)

When the input $x(t)$ is zero, we obtain a homogeneous differential equation. This leads to the equation:

$$y''(t) - 2y'(t) + y(t) = 0$$

The characteristic equation for this differential equation is:

$$r^2 - 2r + 1 = 0$$

This characteristic equation has a single repeated root, $r = 1$. Thus, the general solution of the differential equation can be written as:

$$y(t) = c_1e^t + c_2te^t$$

The constants c_1 and c_2 are defined by initial conditions. Considering the system starts from rest, which means that both $y(0)$ and $y'(0)$ are zero, we deduce $c_1 = 0$ and $c_2 = 0$. As a result, the solution simplifies to:

$$y(t) = 0$$

c)

The corresponding differential equation is:

$$y''(t) - 2y'(t) + y(t) = 2x'(t)$$

Given that the input signal derivative for $t > 0$ is:

$$x'(t) = 2u(t)$$

and using the initial conditions of the system being at rest, which implies $y(0) = y'(0) = 0$, the differential equation simplifies to:

$$y''(t) - 2y'(t) + y(t) = 4$$

The homogeneous solution to this equation is:

$$y_h(t) = (c_1 + c_2t)e^t$$

Considering a particular solution $y_p(t) = A$ and substituting into the differential equation, we find:

$$A = 4$$

Using the initial conditions, we find the constants:

$$c_1 = -4, \quad c_2 = 4$$

Thus, the solution to the differential equation is:

$$y(t) = ((4t - 4)e^t + 4)u(t)$$

for $t > 0$, considering the unit step function $u(t) = 1$ for $t > 0$.

5)

a)

To determine the impulse response $h[n]$ of the system, an impulse input $x[n] = \delta[n]$ is applied. Given that the system starts without any initial output, we have $y[0]$ and $y[1]$ as zero. At $n = 2$, the input is $x[n-2] = \delta[0]$, leading to $y[2]$ being 2. At $n = 3$, the output $y[3]$ is computed as $\frac{2}{5}$. Noticing the pattern that $x[n-2] = 0$ for $n > 2$ and the difference equation's recursive characteristic, the output $y[n]$ for $n > 2$ decays by a factor of $\frac{1}{5}$ from the previous value. Hence, the output for $n > 2$ is $\frac{2}{5^{n-2}}$. Therefore, the impulse response of the system can be characterized as:

$$h[n] = \begin{cases} 0, & \text{for } n < 2, \\ 2, & \text{for } n = 2, \\ \frac{2}{5^{n-2}}, & \text{for } n > 2. \end{cases}$$

b)

Considering the given difference equation and taking its Z-transform results in the following expression:

$$y(z)\left(1 - \frac{1}{5z}\right) = 2x(z)z^{-2}$$

To find the system's transfer function, denoted as $H(z) = \frac{Y(z)}{X(z)}$, the equation is manipulated thus:

$$H(z) = \frac{2z^{-2}}{1 - \frac{1}{5z}}$$

Simplification of this expression yields the system's transfer function in the Z-domain, which elucidates the relationship between the output $Y(z)$ and the input $X(z)$ as follows:

$$H(z) = \frac{10}{5z^2 - z}$$

This represents the system's transfer function in the Z-domain, describing how the output $Y(z)$ relates to the input $X(z)$.

c)

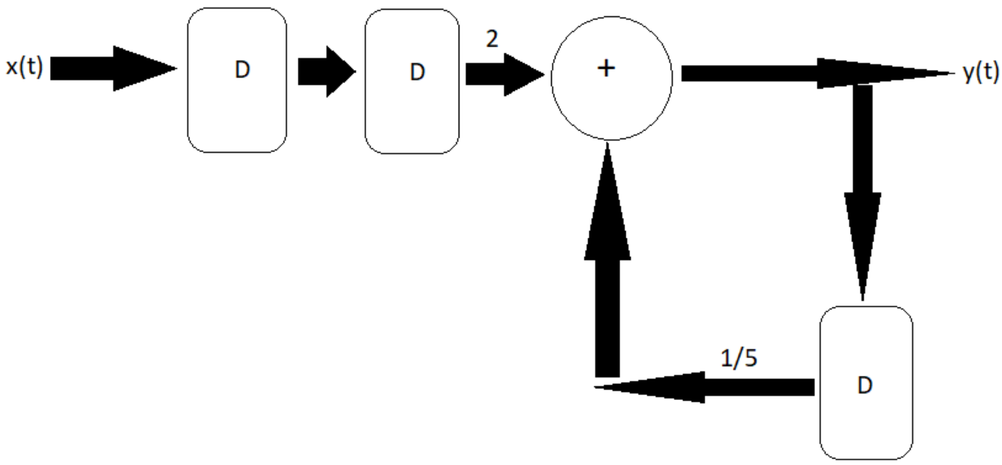


Figure 1: Block diagram representation of this system with using the adders and unit delay operators

6)

a)

The given equation can be manipulated to express $y(t)$. By doing so, we obtain:

$$y(t) = -\frac{1}{a}y'(t) - \frac{b}{a}x(t)$$

With specific values for a and b, such as $a = -2$ and $b = 8$, the equation simplifies to:

$$y(t) = \frac{1}{2}y'(t) - 4x(t)$$

Below is the corresponding block diagram:

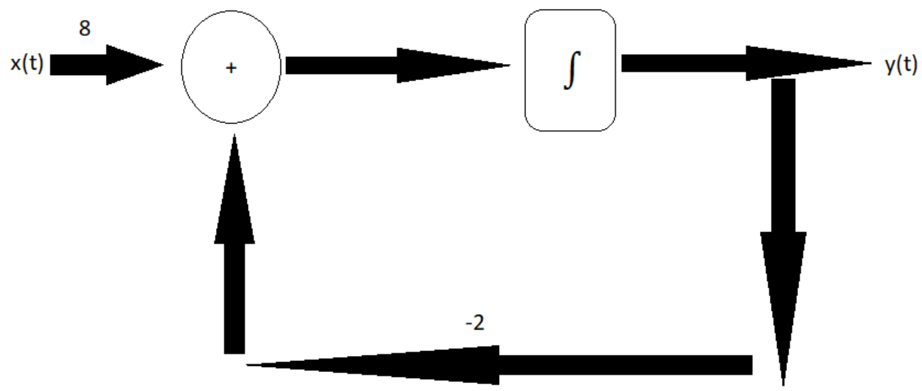


Figure 2: Block diagram representation of this system using integrators and adders

b)

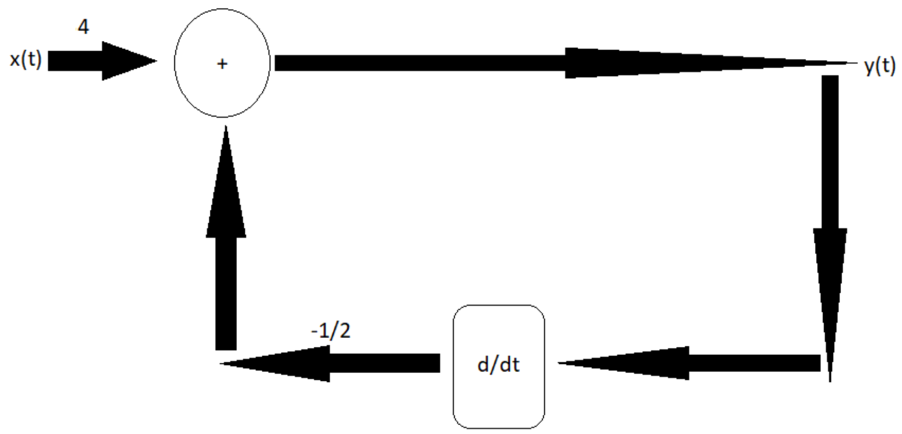


Figure 3: Block diagram representation of this system using differentiators and adderss

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7)

```
    import matplotlib.pyplot as plt

# Define the length of the signals
N = 5

# Define the input signal x[n] = delta[n-1]
x = [0] * N
x[1] = 1  # delta[n-1]

# Initialize the output signal y[n]
y = [0] * N

# Implement the  difference equation y[n] = 1/4 * y[n-1] + x[n]
for n in range(1, N):
    y[n] = 1/4 * y[n-1] + x[n]

# Plot the output signal y[n]
plt.stem(range(N), y)
plt.xlabel('n')
plt.ylabel('y[n]')
plt.grid(True)
plt.show()
```