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Answer 1

a)

Since t_A and t_B are independent and uniformly distributed between $[0, 100]$ milliseconds, the joint density function $f(t_A, t_B)$ can be expressed as the product of the individual density functions: $f(t_A, t_B) = f(t_A) * f(t_B) = 1/10000$, for$0 \leq t_A, t_B \leq 100$.

The joint cumulative distribution function F(t_A, t_B) is the probability that both servers complete processing and sending responses by the given times. Since the servers receive the packets simultaneously, we have: $F(t_A, t_B) = P(T_A \leq t_A \text{ and } T_B \leq t_B) = P(T_A \leq t_A) \cdot P(T_B \leq t_B) = \frac{t_A}{100} \cdot \frac{t_B}{100} = \frac{t_A t_B}{10000}$.

b)

The probability that server A processes the packet and sends a response within the first 30 milliseconds is given by: $P(t_A \leq 30) = 30/100 = 0.3$.

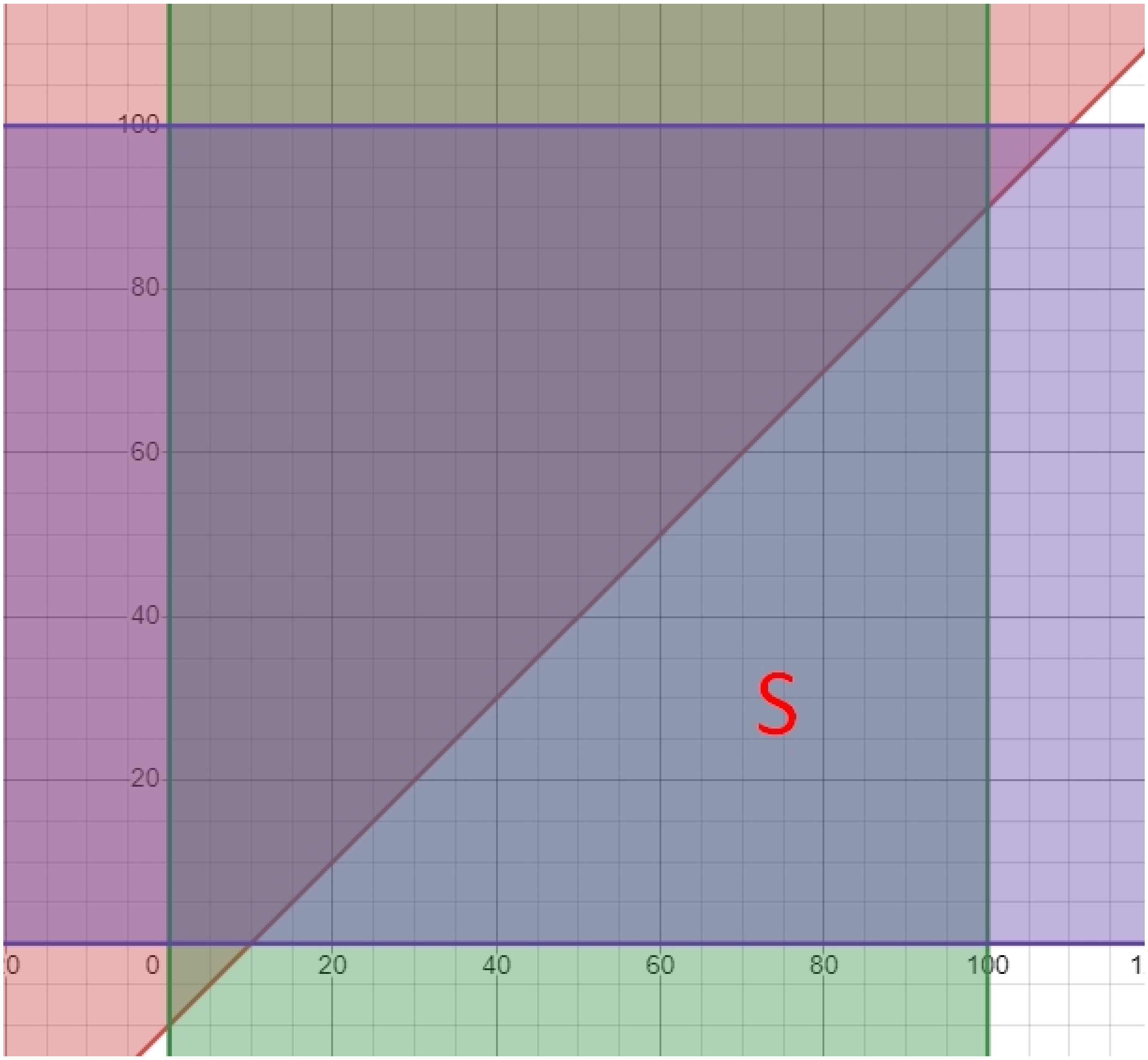
The probability that server B takes between 40 and 60 milliseconds to process the packet and send a response is given by: $P(40 \leq t_B \leq 60) = (60 - 40)/100 = 0.2$.

$$P(t_A \leq 30 \text{ and } 40 \leq t_B \leq 60) = P(t_A \leq 30) * P(40 \leq t_B \leq 60) = 0.3 * 0.2 = 0.06$$

.

c)

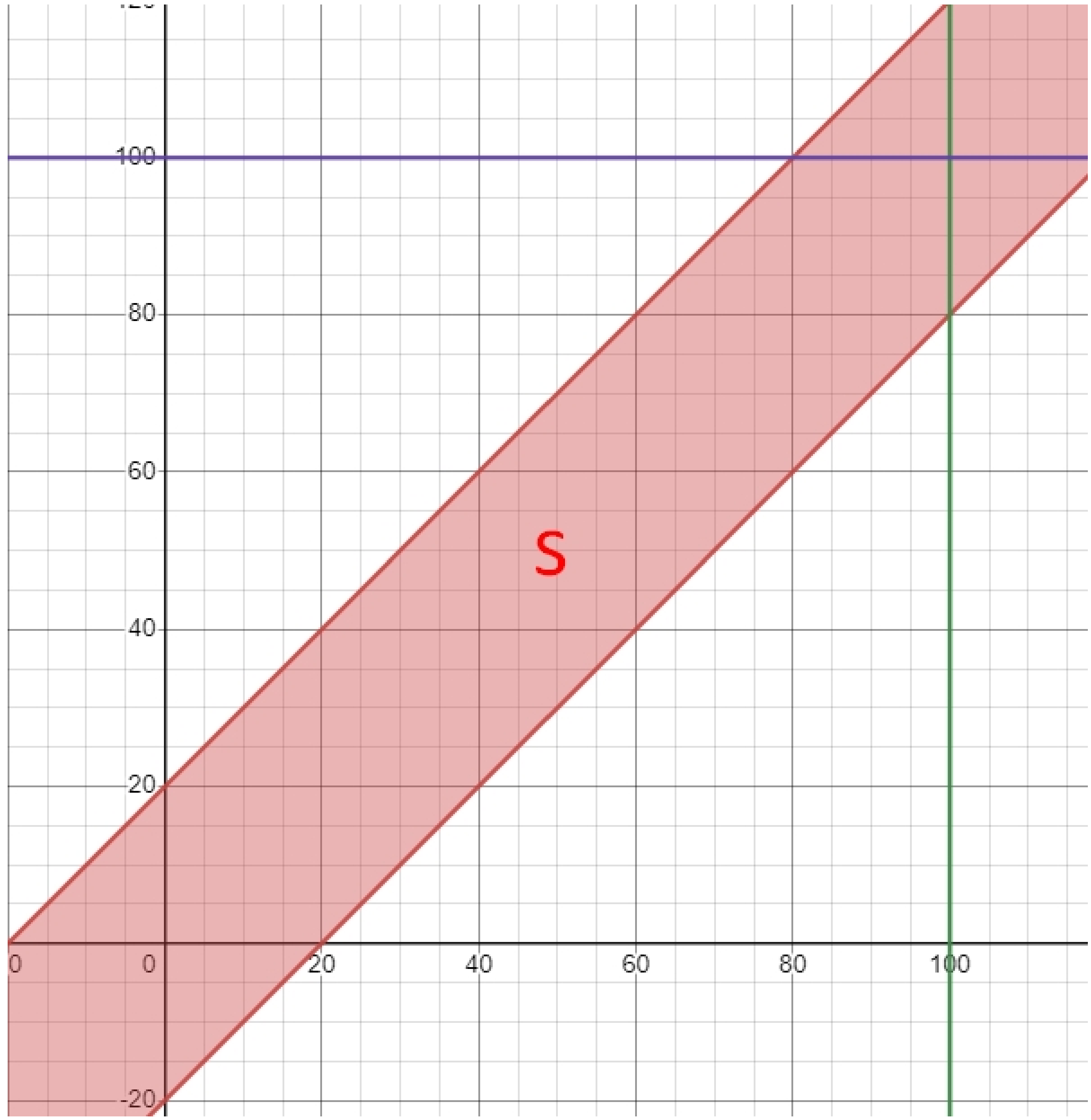
$P(t_A \leq t_B + 10)$ is asked. In other words, if we think like $t_A = x$ and $t_B = y$ the shown area over the whole are is asked.



Which is 4050/10000 So the answer is 0.405

d)

With the same logic as before this area over the whole are is asked.



Which is 3600/10000 So the answer is 0.36

Answer 2

a)

We can think the distribution of being a frequent shopper as a binomial distribution with $E(X) = 150 * 0.60$ and $Var(X) = 150 * 0.60 * 0.4$

To use in Central Limit Theorem our parameters are $\mu = E(x) = 90$ and $\delta = \sqrt{Var(X)} = 6$

So $P\{x \geq 97.5\} = P\{x > 97\}$ is asked.

$$P\{Z_n > \frac{97-90}{6}\} \approx \Phi(-1.16)$$

b)

Similar to the part a, $P\{x \leq 22.5\} = P\{x < 22\}$ is asked.

$$P\{Z_n \leq \frac{22-15}{3.67}\} \approx \Phi(1.90)$$

Answer 3

$P_{Normal}\{170 < x < 180\}$ is asked. Which is equal to:

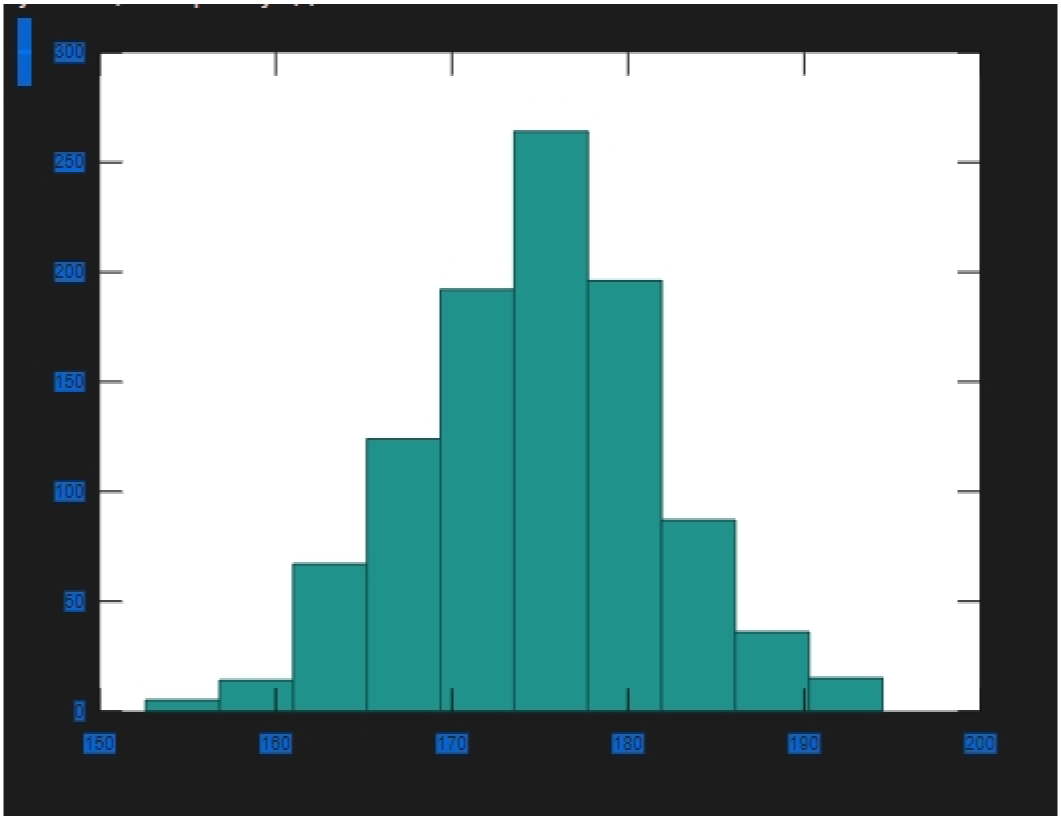
$$P\{\frac{170-\mu}{\delta} < z < \frac{180-\mu}{\delta}\}$$

For $\mu = 175$ and $\delta = 7$ the answer is $\Phi(0.71) - \Phi(-0.71)$.

Answer 4

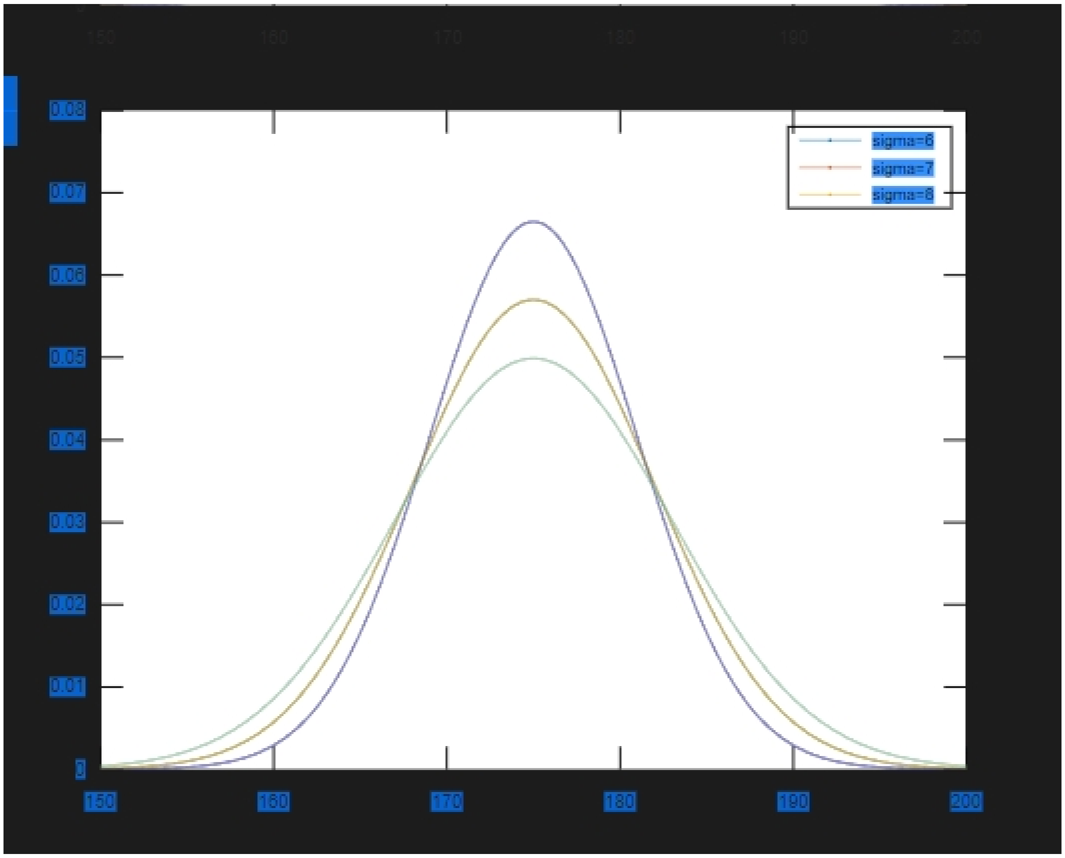
a)

```
mu = 175;
sigma = 7;
samples = normrnd(mu, sigma, [1, 1000]);
hist(samples);
```



b)

```
x = 150:0.1:200;
sigmas = [6, 7, 8];
for i = 1:length(sigmas)
    sigma = sigmas(i);
    y = normpdf(x, mu, sigma);
    plot(x, y);
    hold on;
end
legend('sigma=6', 'sigma=7', 'sigma=8');
```



c)

```
percent = [0.45, 0.50, 0.55];
probability = zeros(length(percent), 1);
for i = 1:length(percent)
    limit = percent(i);
    success = 0;
    for j = 1:1000
        heights = normrnd(175, 7, [150, 1]);
        proportion = sum(heights >= 170 & heights <= 180) / 150;
        if proportion >= limit
            success = success + 1;
        end
    end
    probability(i) = success / 1000;
end
disp(probability);
```

```
0.9640
0.7620
0.2720
```