Student Information

Full Name : Burak YILDIZ
Id Number : 2449049

Answer 1

a)

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \iff (\neg p \vee \neg q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

The statement is a contradiction.

b)

$$
\begin{aligned}
& p \to ((q \vee \neg q) \to (p \wedge q)) \\
& \equiv p \to ((T) \to (p \wedge q)) && \text{Negation Law} \\
& \equiv p \to ((F) \vee (p \wedge q)) && \text{Table 7 Line 1} \\
& \equiv p \to (p \wedge q) && \text{Identity Law} \\
& \equiv \neg p \vee (p \wedge q) && \text{Table 7 Line 1} \\
& \equiv (\neg p \vee p) \wedge (\neg p \vee q) && \text{Distributive Law} \\
& \equiv T \vee (\neg p \vee q) && \text{Negation Law} \\
& \equiv \neg p \vee q && \text{Identity Law}
\end{aligned}
$$

Answer 2

a) $\forall x \exists y (W(x, y))$
b) $\neg(\forall y \exists x (F(x, y)))$
c) $\forall x (W(x, P) \to A(Ali, x))$
d) $\exists y (W(Busra, y) \to F(TUBITAK, y))$
e) $\exists x \exists y \exists z (S(x, y) \wedge S(x, z) \wedge (y \neq z))$
f) $\forall x \forall y \exists z (W(x, z) \wedge W(y, z) \wedge (x = y))$
g) $\exists x \exists y \exists z (W(x, z) \wedge W(y, z) \wedge \forall t (W(t, z) \to (t = x \vee t = y)))$

Answer 3

The letter 'a' after the step numbers means that we made an assumption and still in that assumption box. 'aa' means assumption inside assumption

1.	$p \to q$	premise
2.	$(q \wedge \neg r) \to s$	premise
3.	$\neg s$	premise
4.a	$(q \wedge \neg r)$	assumed
5.a	s	$\to e, 2, 4$
6.a	$\bot$	$\neg e, 3, 5$
7.	$\neg(q \wedge \neg r)$	$\neg i, 4-6$
8.a	p	assumed
9.a	q	$\to e, 1, 8$
10.aa	$\neg r$	assumed
11.aa	$q \wedge \neg r$	$\wedge i, 9, 10$
12.aa	$\bot$	$\neg e, 7-11$
13.a	r	$\neg i, 10-12$
14.	$p \to r$	$\to i, 8-13$

Answer 4

By using the claims and the facts we can transform the sentences into the following statement to proof.

$$p, p \to (q \wedge r), r \to s \vdash \neg(s \to \neg q)$$

The letter 'a' after the step numbers means that we made an assumption and still in that assumption box.

1.	p	premise
2.	$p \to (q \wedge r)$	premise
3.	$r \to s$	premise
4.	$q \wedge r$	$\to e, 1, 2$
5.	r	$\wedge e, 4$
6.	s	$\to e, 3, 5$
7.	q	$\wedge e, 4$
8. a	$s \to \neg q$	assumed
9. a	$\neg q$	$\to e, 6, 8$
10.a	$\bot$	$\neg e, 7, 9$
11.	$\neg(s \to \neg q)$	$\neg i, 8-10$

Answer 5

The letter 'a' after the step numbers means that we made an assumption and still in that assumption box. 'aa' means assumption inside assumption

1.	$\forall x(P(x) \to (Q(x) \to R(x)))$	premise
2.	$\exists x(P(x))$	premise
3.	$\forall x(\neg R(x))$	premise
4.a	$P(c)$	assumed
5.a	$P(c) \to (Q(c) \to R(c))$	$\forall e, 1$
6.a	$Q(c) \to R(c)$	$\to e, 4, 5$
7.a	$\neg R(c)$	$\forall e, 3$
8.aa	$Q(c)$	assumed
9.aa	$R(c)$	$\to e, 6, 8$
10.aa	$\bot$	$\neg e, 7, 9$
11.a	$\neg Q(c)$	$\neg i 8, 8 - 10$
12.a	$\exists x(\neg Q(x))$	$\exists i, 11$
13.	$\exists x(\neg Q(x))$	$\exists e, 2, 4 - 12$