Student Information

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Answer 1

We define the generating function A(x) as the following:

$$A(x) = a0 + a1x + a2x^2 + a3x^3 + ...$$

Substituting the values of the coefficients in the recurrence relation, we get:

$$A(x) = a0 + a1x + (3a0 + 4a1)x^2 + (3a1 + 4a2)x^3 + ...$$

We can simplify this to:

$$A(x) = a0 + a1x + (3 + 4x)A(x)x^2 + (3x + 4A(x))x^3 + ...$$

Collecting like terms, we get:

$$A(x) = a0 + a1x + A(x)(3x^2 + 4x^3 + ...) + (3x^2 + 4x^3 + ...)A(x)$$

This simplifies to:

$$A(x) = a0 + a1x + A(x)(3x^2 + 4x^3 + ...) + (3x^2 + 4x^3 + ...)A(x)$$

We can solve for A(x) by dividing both sides of the equation by $(3x^2 + 4x^3 + \dots)$, which gives us:

$$A(x) = \frac{a_0 + a_1 x}{1 - (3x^2 + 4x^3 + \dots)}$$

Since the recurrence relation starts with $a_0 = a_1 = 1$, we have:

$$A(x) = \frac{1 + x}{1 - (3x^2 + 4x^3 + \dots)}$$

We can simplify this further by factoring the denominator:

$$A(x) = \frac{1 + x}{1 - 3x^2 - 4x^3 - \dots}$$

This can be written as:

$$A(x) = \frac{1 + x}{1 - 3x^2 - 4x^3 - \dots}$$

This is a geometric series, so we can simplify it to:

$$A(x) = \frac{1 + x}{1 - x(3x + 4)}$$

Answer 2

a)

The recurrence relation for the given sequence is

$$a_n = 3a_{n-1} - 4$$

The generating function for the sequence is

$$G(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$

Substituting the recurrence relation into the generating function, we get

$$G(x) = a_0 + a_1x + (3a_1 - 4a_0)x^2 + (3a_2 - 4a_1)x^3 + \dots$$

Substituting the initial values of the sequence into the generating function, we get

$$G(x) = 2 + 5x + (3(5) - 4(2))x^2 + (3(11) - 4(5))x^3 + \dots$$

Simplifying the generating function, we get

$$G(x) = 2 + 5x - 4x^2 + 7x^3 + \dots$$

Writing the generating function in closed form, we get

$$G(x) = 2 + (5 - 4x^2)x + 7x^3 + \dots$$

b)

We can decompose the fraction into partial fractions by finding the roots of the denominator, $1 - 3x + 2x^2$. The roots are x = 1/2 and x = 1.

Then, we can write the partial fractions as follows:

$$G(x) = \frac{A}{x - 1/2} + \frac{B}{x - 1}$$

Substituting x = 1/2 and x = 1 into the equation and solving for A and B, we find that A = 3/4 and B = 1.

We can now write the generating function in terms of partial fractions:

$$G(x) = \frac{7 - 9x}{1 - 3x + 2x^2} = \frac{3/4}{x - 1/2} + \frac{1}{x - 1}$$

Answer 3

a)

The relation R is not an equivalence relation because it does not satisfy the reflexive property.

The reflexive property states that every element in a set must be related to itself. In other words, for any element a in the set, aRa must be true.

However, in the case of the relation R, it is not possible for an element to be related to itself because for example if we choose a as 3, then the hypotenuse will be $\sqrt{18}$ which is not an integer. Therefore, aRa is not true for some elements, and the relation R does not satisfy the reflexive property.

b)

The relation R is an equivalence relation because it satisfies the reflexive, symmetric, and transitive properties.

The reflexive property states that every element in a set must be related to itself. In other words, for any element (x, y) in the set, (x, y)R(x, y) must be true. This is satisfied because the equation 2x + y = 2x + y is always true.

The symmetric property states that if a is related to b, then b must be related to a. In other words, if (x1, y1)R(x2, y2), then (x2, y2)R(x1, y1). This is satisfied because the equation 2x1 + y1 = 2x2 + y2 is symmetric in x1 and x2, and y1 and y2.

The transitive property states that if a is related to b, and b is related to c, then a must be related to c. In other words, if (x1, y1)R(x2, y2) and (x2, y2)R(x3, y3), then (x1, y1)R(x3, y3). This is satisfied because if 2x1 + y1 = 2x2 + y2 and 2x2 + y2 = 2x3 + y3, then 2x1 + y1 = 2x3 + y3.

Since the relation R satisfies the reflexive, symmetric, and transitive properties, it is an equivalence relation.

The equivalence class of (1, -2) under the relation R consists of all pairs of integers (x, y) that satisfy the equation 2x + y = 2(1) + (-2) = 0. Some examples of pairs in this equivalence class are (0, 0), (1, -2), (2, -4), (3, -6), etc.

In the Cartesian coordinate system, the equivalence class of (1, -2) represents the line

$$2x + y = 0$$

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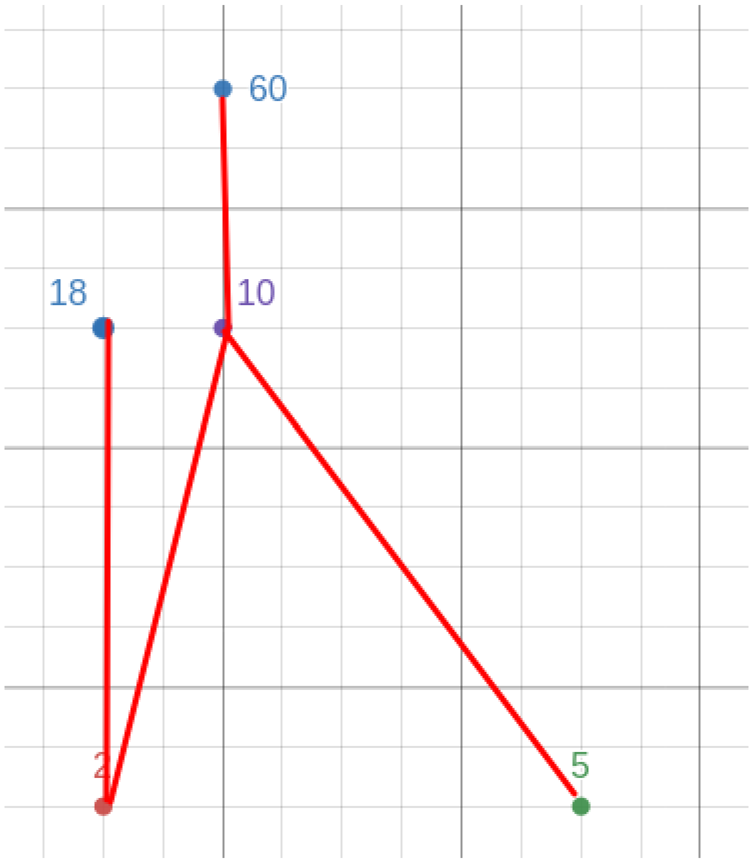
Answer 4

a)



b)

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

c)

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

The (x, y) pairs that satisfy the condition, are

$$\begin{pmatrix} (10, 2) \\ (18, 2) \\ (10, 5) \\ (60, 2) \\ (60, 10) \\ (60, 5) \end{pmatrix}$$

d)

We can not make this relation a total order by only removing one element. Because there more than one points that prevent total order.

If we are allowed to remove two elements from A and add one element, we can remove bot points that prevents, when we remove the points 5,18 than add 180. This will create a total order. (2-10-60-180)