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Answer 1

a)

The sample mean ($\bar{x}$) of the 16 measurements is 6.81 liters and the sample standard deviation (s) is 1.06 liters.

The formula to calculate the confidence interval is:

$$\text{CI} = \bar{x} \pm \left(\text{critical value} \times \frac{s}{\sqrt{n}}\right)$$

With a 98% confidence level, the critical value is approximately 2.602 (obtained from a software).

Substituting the values:

$$\text{CI} = 6.81 \pm \left(2.602 \times \frac{1.06}{\sqrt{16}}\right)$$

Simplifying the calculation:

$$\text{CI} = 6.81 \pm (2.602 \times 0.265)$$

This results in the confidence interval for the mean gasoline consumption after the improvement:

$$\text{CI} = [6.14, 7.48]$$

Therefore, we can be 98% confident that the true mean per 100 km gasoline consumption of the car after the improvement falls within the range of 6.14 to 7.48 liters.

b)

Null hypothesis (H_0): There is no significant reduction in the gasoline consumption after the improvement.
Alternative hypothesis (H_1): There is a significant reduction in the gasoline consumption after the improvement.

We can conduct a one-sample t-test using the sample data provided. The mean of the 16 measurements is 6.81 liters with a sample standard deviation of 1.06 liters. The population mean before the improvement is given to be 7.5 liters.

Using the t-test, we calculate the t-statistic as follows:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where $\bar{x}$ is the sample mean, μ is the population mean, s is the sample standard deviation, and n is the sample size.

Substituting the values:

$$t = \frac{6.81 - 7.5}{\frac{1.06}{\sqrt{16}}}$$

Simplifying the calculation:

$$t = \frac{-0.69}{0.265}$$

The degrees of freedom for this test is $n - 1 = 16 - 1 = 15$.

Next, we compare the calculated t-statistic with the critical value at a 5% significance level. For a one-tailed test, the critical value is approximately -1.753.

Performing the calculations, we find:

$$t \approx \frac{-0.69}{0.265} \approx -2.60$$

Since -2.60 is smaller (in the left tail) than -1.753, we reject the null hypothesis.

Therefore, at a 5% level of significance, we can claim that the improvement is effective, indicating a significant reduction in the gasoline consumption.

c)

Given that before the improvement, the car was consuming 6.5 liters of gasoline per 100 km, we can compare this value with the null hypothesis.

Null hypothesis (H_0): There is no significant reduction in the gasoline consumption after the improvement.

Since the null hypothesis assumes no change in the gasoline consumption, if the value of 6.5 liters per 100 km falls within the confidence interval or the range of plausible values for the mean gasoline consumption after the improvement, we would fail to reject the null hypothesis.

However, if the value of 6.5 liters per 100 km falls outside the confidence interval, we would have evidence to reject the null hypothesis and conclude that there is a significant reduction in the gasoline consumption after the improvement.

Without any calculations or additional information about the confidence interval, we cannot immediately accept or reject H_0 based solely on the given value of 6.5 liters per 100 km. We need the confidence interval or additional statistical analysis to make a conclusive determination.

Answer 2

a)

Since the null hypothesis assumes no change or similarity to the previous year's rent prices, Ali's claim that the prices are similar should be considered as the null hypothesis (H_0). Ahmet's claim of an increase in prices will be the alternative hypothesis (H_1).

Therefore:
Null Hypothesis (H_0): The average rent price in Ankara is similar to the previous year ($\mu =$ 5000TL).
Alternative Hypothesis (H_1): There is an increase in the average rent price in Ankara ($\mu >$ 5000TL).

b)

Performing the t-test, Ahmet would calculate the t-statistic using the formula:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where $\bar{x}$ is 5500, μ is 5000, s is 2000, and n is 100. Calculating $t = 2.5$
For a one-tailed test at a 5% level of significance, the critical value is approximately 1.645.
Since $t >$ critical value, we can reject the null hypothesis.

c)

The P-value for $t = 2.5$ is approximately 0.0062.

The P-value is less than the significance level (0.05), indicating strong evidence to reject the null hypothesis. Therefore, we can conclude that there is a significant increase in rent prices compared to last year based on the given data.

d)

To compare the rent prices in Ankara and Istanbul, they conducted a two-sample t-test with the following information:

For Ankara: Sample size (n_1) = 100, Sample mean ($\bar{x_1}$) = 5500TL, Standard deviation (σ_1) = 2000TL
For Istanbul: Sample size (n_2) = 60, Sample mean ($\bar{x_2}$) = 6500TL, Standard deviation (σ_2) = 3000TL

The hypotheses are:
Null hypothesis (H_0): The rent prices in Ankara are the same as in Istanbul ($\mu_1 = \mu_2$)
Alternative hypothesis (H_1): The rent prices in Ankara are lower than in Istanbul ($\mu_1 < \mu_2$)

Calculating Z: ($Z = \frac{5500-6500}{\sqrt{(\frac{2000^2}{100})+(\frac{3000^2}{60})}} \approx -2.29$)
Critical value: For a one-tailed test at a 1% level of significance, the critical value is approximately -2.33.
Since the calculated t-statistic is greater than the critical value, we fail to reject the null hypothesis.

Therefore, at a 1% level of significance, we can claim that the rent prices in Ankara are not lower than in Istanbul based on the provided evidence.

Answer 3

To test the dependency of the number of rainy days on the season in Ankara, we conducted a chi-square test. The null hypothesis (H_0) assumes independence, while the alternative hypothesis (H_1) suggests dependence.

The observed and expected values are as follows:

	Rainy	Non Rainy	Total
Winter	34	56	90
Spring	32	58	90
Summer	15	75	90
Autumn	19	71	90
Total	100	260	360

Using these values, we calculated the chi-square statistic:

$$\chi^2 = 14.7323$$

With degrees of freedom (df) equal to 3. Comparing this value to the chi-square distribution table at a significance level of 0.05, the critical value is 7.8147. Since the calculated chi-square statistic is greater than the critical value, we reject the null hypothesis, indicating a dependence between the number of rainy days and the seasons.

Answer 4

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observed = [34 56; 32 58; 15 75; 19 71];

row_totals = sum(observed, 2);
col_totals = sum(observed);
total = sum(row_totals);
expected = (row_totals * col_totals) / total;

chi2 = sum(sum((observed - expected).^2 ./ expected));

df = (size(observed, 1) - 1) * (size(observed, 2) - 1);

p_value = 1 - chi2cdf(chi2, df);

fprintf('Chi-square statistic: %.2f\n', chi2);
fprintf('P-value: %.4f\n', p_value);
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Chi-square statistic: 14.73
P-value: 0.0021
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