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Answer 1

We proceed by mathematical induction.

Base case:

For $n = 1$, we have $6^{2*1} - 1 = 35$, which is divisible by both 5 and 7.

Inductive step:

Suppose that the statement holds for $n = k$, where k is some positive integer. That is, suppose that $6^{2k} - 1$ is divisible by both 5 and 7. We want to prove that the statement also holds for $n = k + 1$.

We have: $6^{2(k+1)} - 1 = 6^{2k} * 6^2 - 1$

Using the induction hypothesis, we know that $6^{2k} - 1$ is divisible by both 5 and 7. Therefore, we can write:

$6^{2k} * 6^2 - 1 = (5a + 1) * (7b + 1) * 6^2 - 1$

This expression simplifies to:

$5a * 7b * 6^2 + 5a * 6^2 + 7b + 6^2 - 1$

We can then distribute the $5a$ and the 6^2 to get:

$5a * 7b * 6^2 + 30a * 6^2 + 7b + 6^2 - 1$

This expression is clearly divisible by both 5 and 7, as it is of the form $30a * 6^2 + 7b + 6^2 - 1$.

Therefore, we have shown that if the statement holds for $n = k$, then it also holds for $n = k+1$.

Since we have verified the base case and the inductive step, we can conclude that the statement holds for all positive integers n by the principle of mathematical induction.

Answer 2

To prove this statement using strong induction, we need to first show that it holds for the base case of n = 3. We have that $H_3 = 8H_2 + 8H_1 + 9H_0 = 8(7) + 8(5) + 9(1) = 75$, which is less than or equal to $9^3 = 729$.

Next, we need to show that if $H_k \leq 9^k$ for all integers k in the set 0, 1, 2, ..., n, then it must also be true that $H_{n+1} \leq 9^{n+1}$.

We have that

$H_{n+1} = 8H_n + 8H_{n-1} + 9H_{n-2} \leq 8 * 9^n + 8 * 9^{n-1} + 9 * 9^{n-2} = 9^{n+1}$

where the inequality follows from the assumption that $H_k \leq 9^k$ for all integers k in the set 0, 1, 2, ..., n.

Therefore, by strong induction, $H_n \leq 9^n$ for all $n \epsilon N$.

Answer 3

There are two cases to consider here: bit strings of length 8 that contain 4 consecutive 0s, and bit strings of length 8 that contain 4 consecutive 1s.

For the first case, four consecutive zeros can start at position 1,2,3,4 or 5 . Starting at position 1: strings are of the form $0000xxxx$, there are 2^4 different possibilities. Starting at position 2: strings are of the form $10000xxx$ there are 2^3 different possibilities. Similarly the count for positions 3,4,5 are calculated as 2^3 each. In total there are 48 different ways.

Similarly for the second case there are also 48 different ways. Since the cases are distinct the answer is basically $48 + 48 = 96$.

Answer 4

First, we need to choose which star to use as the center of the galaxy. Since there are 10 distinct stars, there are 10 ways to do this.

Next, we need to choose which two habitable planets to include in the galaxy. Since there are 20 habitable planets, there are $\binom{20}{2} = 190$ ways to do this.

Next, we need to choose which eight nonhabitable planets to include in the galaxy. Since there are 80 nonhabitable planets, there are $\binom{80}{8} = 20,495,100$ ways to do this.

However, we also need to ensure that there are at least six nonhabitable planets between the two habitable ones. Since there are eight nonhabitable planets in total, we must choose six of them to be between the two habitable ones. Since the order of the planets in the galaxy matters, we have $\binom{8}{6} = 28$ ways to do this.

Therefore, there are a total of $10 \cdot 190 \cdot 20,495,100 \cdot 28 = 184,931,936,000$ ways to form a galaxy with the specified properties.

Answer 5

a) To find the number of ways the robot can move to n cells away from its initial location, we can use the following recurrence relation:

$$f(n) = f(n-1) + f(n-2) + f(n-3)$$

This is because the robot can take one of three possible moves to reach the nth cell, which are jumping one, two, or three cells from its current position. The number of ways the robot can move to the nth cell is therefore the sum of the number of ways it can move to the $(n-1)_{th}$, $(n-2)_{th}$, and $(n-3)_{th}$ cells.

b) In this recurrence relation, the initial conditions are $f(1) = 1$, $f(2) = 2$, and $f(3) = 4$. Because there is only one way to robot to get to the first cell (jump one cell away) , there are 2 ways to robot to get to the second cell (one+one or jump two cell away), finally there are 4 different ways for robot to reach to the third cell (one+one+one, one+two, two+one, three)

c) To find the number of ways the robot can move to 9 cells away from its initial location, we can simply evaluate the recurrence relation for the first few values of n and then substitute 9 for n:

$$f(1) = 1$$
$$f(2) = 2$$
$$f(3) = 4$$
$$f(4) = 7$$
$$f(5) = 13$$
$$f(6) = 24$$
$$f(7) = 44$$
$$f(8) = 81$$
$$f(9) = 149$$

Therefore, there are 149 different ways the robot can move to 9 cells away from its initial location.