CENG 384 - Signals and Systems for Computer Engineers
Spring 2024
Homework 4

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1.

Question 1

a) $y(t) = \iint x(t) - 6y(t)\,dt + 4x(t) - 5y(t)\,dt$

$y'(t) = \int x(t) - 6y(t)\,dt + 4x(t) - 5y(t)$

$\int x(t) - 6y(t)\,dt = y'(t) + 5y(t) - 4x(t)$

$x(t) - 6y(t) = y''(t) + 5y'(t) - 4x'(t)$

$$\boxed{x(t) + 4x'(t) = y''(t) + 5y'(t) + 6y(t)}$$

b) $H(jw) = \dfrac{Y(jw)}{X(jw)} = \dfrac{4jw + 1}{jw^2 + 5jw + 6} = \dfrac{A}{(jw+3)} + \dfrac{B}{(jw+2)}$

partial fraction

for $(jw = -3)$ → A will be 11

similary B is -7

so $\boxed{H(jw) = \dfrac{11}{(jw+3)} - \dfrac{7}{(jw+2)}}$

c) To find $h(t)$ we will apply inverse FT to $H(t)$

since $e^{-at}u(t) \leftrightarrow \dfrac{1}{a+jw}$ from the lookup table

$$\boxed{h(t) = \left(11 \cdot e^{-3t} - 7 \cdot e^{-2t}\right) u(t)}$$

d) $Y(jw) = H(jw) \cdot X(jw)$

$= \dfrac{1}{(4jw+1)} \cdot \dfrac{(4jw+1)}{(jw+3)(jw+2)} = \dfrac{-1}{jw+3} + \dfrac{1}{jw+2}$

applying inverse FT to $Y(jw)$

$$\boxed{y(t) = \left(-e^{-3t} + e^{-2t}\right) u(t)}$$

2.

Question 2

a) $H(j\omega) = \dfrac{(j\omega+4)}{-\omega^2+5j\omega+6} = \dfrac{Y(j\omega)}{X(j\omega)} = \dfrac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k}$

$$\boxed{= y'' + 5y' + 6y = x'(t) + 4x(t)}$$

b) To find $h(t)$ we will apply inverse FT to $H(j\omega)$

$$\boxed{F^{-1}\left(\frac{-1}{j\omega+3} + \frac{2}{j\omega+2}\right) = -e^{-3t} + 2e^{-2t} = h(t)}$$

$\dfrac{(j\omega+4)}{(j\omega+3)\cdot(j\omega+2)}$ after finding the partial eq.

c)

$Y(j\omega) = H(j\omega).X(j\omega) = \dfrac{\cancel{(j\omega+4)}}{(j\omega+3)(j\omega+2)} \cdot \left[\dfrac{1}{j\omega+4} - \cancel{\dfrac{1}{(j\omega+4)^2}}\right]$

$= \dfrac{-1}{(j\omega+4)\cdot(j\omega+2)} = \dfrac{A}{j\omega+4} + \dfrac{B}{j\omega+2}$

$B = 1/2 \qquad A = -1/2$

$$\boxed{Y(j\omega) = \frac{-1}{2j\omega+8} + \frac{1}{2j\omega+4}}$$

d) Applyn inverse ft to $Y(j\omega)$

$$\boxed{Y(t) = \left(\frac{-1}{2}\cdot e^{-4t} + \frac{1}{2}\cdot e^{-2t}\right)u(t)}$$

Q3) DT LTI

from lookup table

a) $h(n) \leftrightarrow H(e^{j\omega})$

$a^n u(n)\ |a|<1 \leftrightarrow \dfrac{1}{1-ae^{-j\omega}}$

$$x[n] = \left(\frac{2}{3}\right)^n u(n)$$

$$y(n) = n\left(\frac{2}{3}\right)^{n+1} u(n) \Rightarrow y(n) = \frac{2}{3}\cdot n\cdot\left(\frac{2}{3}\right)^n u(n)$$

F response $H(e^{j\omega})$

$n\cdot x(n) \leftrightarrow j\cdot \dfrac{dX(e^{j\omega})}{d\omega}$

$(n+1)a^n u(n)\ |a|<1 \xleftrightarrow{FT} \dfrac{1}{(1-ae^{-j\omega})^2}$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \qquad X(e^{j\omega}) = \frac{1}{1-\frac{2}{3}e^{-j\omega}} = \frac{3}{3-2e^{-j\omega}}$$

$$y(n) = \frac{2}{3}\cdot n\left(\frac{2}{3}\right)^n\cdot u(n)$$

$$= \frac{2}{3}\left((n+1)\left(\frac{2}{3}\right)^n u(n) - \left(\frac{2}{3}\right)^n u(n)\right)$$ to ease the FT

$$Y(e^{j\omega}) = \frac{2}{3}\left(\frac{1}{(1-\frac{2}{3}e^{-j\omega})^2} - \frac{1}{(1-\frac{2}{3}e^{-j\omega})}\right)$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\frac{2}{3}\left(\frac{1}{(1-\frac{2}{3}e^{-j\omega})^2} - \frac{1}{1-\frac{2}{3}e^{-j\omega}}\right)}{\frac{1}{1-\frac{2}{3}e^{-j\omega}}}$$

$$= \frac{2}{3}\left(\frac{1}{1-\frac{2}{3}e^{-j\omega}} - 1\right) = \frac{2}{3}\,\frac{2e^{-j\omega}}{3-2e^{-j\omega}}$$

3. (a)

b) $h(n) \leftrightarrow H(e^{jw})$

from part a) $H(e^{jw}) = \frac{2}{3} \cdot \frac{2e^{-jw}}{3-2e^{-jw}}$

$\left(\frac{1}{3}\right)$

$= \frac{2}{3} \cdot \frac{2}{3} \cdot e^{-jw} \frac{1}{1-\frac{2}{3}e^{-jw}}$

time shift

$h(n) = \left(\frac{2}{3}\right)^{n} \cdot \left(\frac{2}{3}\right)^{n-1} \cdot u(n-1)$

$h(n) = \left(\frac{2}{3}\right)^{n+1} \cdot u(n-1)$

c)

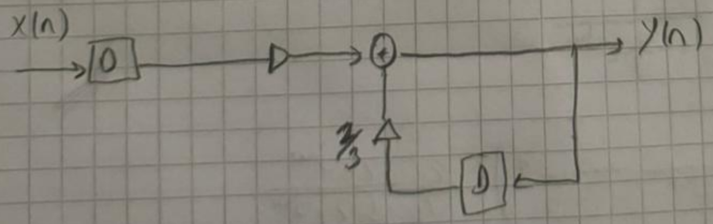
from part a) $H(e^{jw}) = \frac{4e^{-jw}}{9-6e^{-jw}} = \frac{Y(e^{jw})}{X(e^{jw})}$

$4e^{-jw} \cdot X(e^{jw}) = 9Y(e^{jw}) - 6e^{-jw} \cdot Y(e^{jw})$

inverse froier trnsform $4 \cdot x(n-1) = 9y(n) - 6y(n-1)$

d)

$\left\{ y(n) = \frac{4}{9} x(n-1) + \frac{2}{3} y(n-1) \right\}$



(b)

4.

4) a) $y[n] = \frac{-1}{8} y[n-2] + 2x[n] + \frac{3}{4} y[n-1]$

$y[n] + \frac{1}{8} y[n-2] - \frac{3}{4} y[n-1] = 2x[n]$

b)

$Y(e^{jw}) + \frac{1}{8} e^{-2jw} \cdot Y(e^{jw}) - \frac{3}{4} e^{-jw} Y(e^{jw}) = 2 \cdot X(e^{jw})$

$$H(jw) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{2}{\frac{e^{-2jw}}{8} - \frac{3}{4} e^{-jw} + 1} = \frac{16}{e^{-2jw} - 6e^{-jw} + 8} = \frac{8}{e^{-jw}-4} - \frac{8}{e^{-jw}-2}$$

$$= \frac{-2}{1-\frac{1}{4} e^{-jw}} + \frac{4}{1-\frac{1}{2} e^{-jw}} = H(jw)$$

c) $h(n) \to H(jw)$

$$H(e^{jw}) = \frac{2}{1-\frac{3}{4}e^{-jw} + \frac{1}{8} e^{-2jw}} = \frac{A}{(1-\frac{1}{4}e^{-jw})} + \frac{B}{(1-\frac{1}{2}e^{-jw})}$$

$A+B=2$

$e^{-jw}\left(\frac{A}{2}+\frac{B}{4}\right)=0$

$2A+B=0$

$A=-2$

$B=4$

$$= \frac{-2}{1-\frac{1}{4}e^{-jw}} + \frac{4}{1-\frac{1}{2}e^{-jw}} = H(e^{jw})$$

$$\left((-2)\cdot\left(\frac{1}{4}\right)^n + 4\left(\frac{1}{2}\right)^n\right) u(n) = h(n)$$

d) $F(x[n]) = \frac{1}{1-\frac{1}{4}e^{-jw}} = X(e^{jw})$

$$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw}) = \frac{4}{4-e^{-jw}}\left(\frac{8}{e^{-jw}-4} - \frac{8}{e^{-jw}-2}\right)$$

$$= \frac{-32}{(4-e^{-jw})^2} + \frac{16}{4-e^{-jw}} - \frac{16}{-2+e^{-jw}} = \frac{-2}{(1-\frac{1}{4}e^{-jw})^2} + \frac{4}{1-\frac{1}{4}e^{-jw}} + \frac{8}{1-\frac{1}{2}e^{-jw}}$$

$$= -2\left((n+1)\left(\frac{1}{4}\right)^n u(n)\right) + 4\left(\frac{1}{4}\right)^n u(n) + 8\cdot\left(\frac{1}{2}\right)^n u(n)$$

5. 5) $X(e^{j\omega})\left(H_1(e^{j\omega}) + H_2(e^{j\omega})\right) = Y(e^{j\omega})$

from ii) $h(n) = \left(\frac{1}{3}\right)^n u(n)$

$$H_1(e^{j\omega}) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$$

from i) $$H_1(e^{j\omega}) + H_2(e^{j\omega}) = \frac{5e^{-j\omega} - 12}{e^{-2j\omega} - 7e^{-j\omega} + 12}$$

$$H_2(e^{j\omega}) = \frac{5e^{-j\omega} - 12}{(e^{-j\omega}-4)\cdot(e^{-j\omega}-3)} + \frac{-1}{1-\frac{1}{3}e^{-j\omega}} = \frac{A}{e^{-j\omega}-4} + \frac{B}{e^{-j\omega}-3} + \frac{-1}{1-\frac{1}{3}e^{-j\omega}}$$

$A + B = 5$

$-3A - 4B = -12$ $\quad A = 8$

$B = -3$

$$H_2(e^{j\omega}) = \frac{-8}{4\left(1-\frac{1}{4}e^{-j\omega}\right)} + \frac{3}{3\left(1-\frac{1}{3}e^{-j\omega}\right)} + \frac{-1}{\left(1-\frac{1}{3}e^{-j\omega}\right)}$$

$$H_2(e^{j\omega}) = \frac{-2}{\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

inv FT.

$$h_2[n] = -2\left(\frac{1}{4}\right)^n u(n)$$

6. import numpy as np

```
import matplotlib.pyplot as plt

# Define the range for n
n_min = -50
n_max = 50
n = np.arange(n_min, n_max + 1)

# Define the signal x[n]
x_n = (1/2)**np.abs(n)

# Define the frequency range for omega
omega = np.linspace(-np.pi, np.pi, 1000)

# Compute the DTFT
X_omega = np.array([np.sum(x_n * np.exp(-1j * w * n)) for w in omega])

# Plot the magnitude of the DTFT
plt.figure(figsize=(10, 6))
plt.plot(omega, np.abs(X_omega), label='|X(e^{j})|')
plt.title('DTFT of the signal x[n] = (1/2)^|n|')
plt.xlabel('Frequency ()')
plt.ylabel('Magnitude |X(e^{j})|')
plt.grid(True)
plt.legend()
plt.show()
```