IE407 - Homework 3 Report

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Question 1: Linear Programming Model for Tasch Co.

Part (a): Linear Programming Model

Decision Variables:

- x_1: Number of PCs produced
- x_2: Number of Tablets produced
- x_3: Number of Microprocessors produced

Constraints:

1. $5x_1 + 2x_2 + x_3 \leq 120$ (Production Time Constraint)
2. $8x_1 + 4x_2 \leq 80 + x_3$ (Microprocessor Usage Constraint)
3. $x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$ (Non-negativity Constraints)

Objective Function:

$$\text{Maximize} \quad Z = 50x_1 + 30x_2 - 2x_3$$

Part (b): Simplex Method Solution

Initial Simplex Tableau

First, we convert the linear programming problem into standard form by introducing slack variables s_4 and s_5. The standard form is:

$$\begin{aligned}
&\text{Maximize } F(x) = 50x_1 + 30x_2 - 2x_3 \\
&\text{Subject to:} \\
&\qquad 5x_1 + 2x_2 + x_3 + s_4 = 120 \\
&\qquad 8x_1 + 4x_2 - x_3 + s_5 = 80 \\
&\qquad x_1, x_2, x_3, s_4, s_5 \geq 0
\end{aligned}$$

We set up the initial simplex tableau:

	x_1	x_2	x_3	s_4	s_5	Z	RHS
s_4	5	2	1	1	0	0	120
s_5	8	4	-1	0	1	0	80
Z	-50	-30	2	0	0	1	0

Iteration 1

Step 1: Determine the Entering Variable
Here, x_1 has the most negative coefficient (-50), so x_1 is the entering variable.
Step 2: Determine the Leaving Variable
The smallest ratio is 10, so s_5 is the leaving variable.
Pivot Operation

	x_1	x_2	x_3	s_4	s_5	Z	RHS
s_4	5	2	1	1	0	0	120
x_1	1	$\frac{1}{2}$	$-\frac{1}{8}$	0	$\frac{1}{8}$	0	10
Z	-50	-30	2	0	0	1	0

Next, we perform row operations

	x_1	x_2	x_3	s_4	s_5	Z	RHS
s_4	0	-0.5	1.625	1	-0.625	0	70
x_1	1	$\frac{1}{2}$	$-\frac{1}{8}$	0	$\frac{1}{8}$	0	10
Z	0	-5	-3.25	0	6.25	1	500

Iteration 2

Step 1: Determine the Entering Variable

x_2 (-5), so x_2 is the entering variable.

Step 2: Determine the Leaving Variable

x_1 is the leaving variable.

Pivot Operation

	x_1	x_2	x_3	s_4	s_5	Z	RHS
s_4	0	-0.5	1.625	1	-0.625	0	70
x_2	2	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	20
Z	0	-5	-3.25	0	6.25	1	500

Next, we perform row operations

	x_1	x_2	x_3	s_4	s_5	Z	RHS
s_4	1	0	1.5	1	-0.5	0	80
x_2	2	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	20
Z	10	0	-4	0	7.5	1	600

Iteration 3

Step 1: Determine the Entering Variable

The variable with the most negative coefficient in the Z-row is x_3 (-4), so x_3 is the entering variable.

Step 2: Determine the Leaving Variable

The smallest positive ratio is 53.33, so s_4 is the leaving variable.

Pivot Operation

	x_1	x_2	x_3	s_4	s_5	Z	RHS
s_4	$\frac{2}{3}$	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{80}{3}$
x_2	2	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	20
Z	10	0	-4	0	7.5	1	600

Next, we perform row operations

	x_1	x_2	x_3	s_4	s_5	Z	RHS
s_4	$\frac{2}{3}$	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{80}{3}$
x_2	$\frac{8}{3}$	1	0	$\frac{2}{3}$	$\frac{1}{12}$	0	$\frac{260}{3}$
Z	$\frac{38}{3}$	0	0	$\frac{8}{3}$	6	1	$\frac{893.33}{3}$

Optimal Solution

The optimal solution is:

$$x_1 = 0$$
$$x_2 = 33.33$$
$$x_3 = 53.33$$

The maximum profit is $Z = 893.33$.

Part (c): Solution Using Pyomo

The code is appended at the end of the report.

Output of the code

The output of the code is:

- x_1: 0.0
- x_2: 33.3333333333333
- x_3: 53.3333333333333
- Objective: 893.3333333333323

Question 2: Mixed-Integer Programming Model

Branch and Bound Method

Initial LP Relaxation

We solve the initial LP relaxation by ignoring the integer constraints on x_2 and x_3:

$$\begin{aligned}
&\text{Maximize } F(x) = 8x_1 + 6x_2 + 2x_3 \\
&\text{Subject to:} \\
&\quad 6x_1 + 4x_2 + 2x_3 \leq 14 \\
&\quad 4x_1 + 2x_2 + 4x_3 \leq 22 \\
&\quad x_1, x_2, x_3 \geq 0
\end{aligned}$$

The initial simplex tableau is:

	x_1	x_2	x_3	x_4	x_5	RHS
x_4	6	4	2	1	0	14
x_5	4	2	4	0	1	22
Z	-8	-6	-2	0	0	0

Iteration 1

B	C_b	P	x_1	x_2	x_3	x_4
x_5	Q					
x_4	0	14	8	6	2	0
0	2.33					
x_5	0	22	4	2	4	0
1	5.5					
max			-8	-6	-2	0
0	0					

Iteration 2

B	C_b	P	x_1	x_2	x_3	x_4
x_5	Q					
x_1	8	2.33	1	0.67	0.33	0.17
0	3.5					
x_5	0	12.67	0	-0.67	2.67	-0.67
1	-19					
max			0	-0.67	0.67	1.33
0	18.67					

Iteration 3

B	C_b	P	x_1	x_2	x_3	x_4
x_5	Q					
x_1	8	0	1	0	0.5	0.25
0	0					
x_2	6	3.5	1.5	1	0.5	0.25
0	0					
x_5	0	15	1	0	0.5	-0.5
1	0					
max			0	1	0	1.5
0	21					

Optimal Solution for LP Relaxation

The optimal solution is:

- $x_1 = 0$
- $x_2 = 3.5$
- $x_3 = 0$
- Objective: 21

Branch and Bound Process

Since $x_2 = 3.5$ is not an integer, we create two branches: 1. $x_2 \leq 3$ 2. $x_2 \geq 4$

Branch 1: $x_2 = 3$

For this branch, we add the constraint $x_2 = 3$ to the original problem:

$$\begin{aligned}
&\text{Maximize } F(x) = 8x_1 + 6x_2 + 2x_3 \\
&\text{Subject to:} \\
&\qquad 6x_1 + 2x_2 + x_3 \leq 14 \\
&\qquad 4x_1 + 2x_2 + 4x_3 \leq 22 \\
&\qquad x_2 = 3 \\
&\qquad x_1 \geq 0 \quad (x_1 \text{ continuous}) \\
&\qquad x_3 \geq 0 \quad (x_3 \text{ integer})
\end{aligned}$$

The revised simplex tableau for this branch is:

	x_1	x_2	x_3	x_4	RHS
x_3	0	2	6	1	0.33
x_4	0	16	4	1	4
Z	0	-8	-2	0	0

Iteration 1

B Q	C_b	P	x_1	x_2	x_3	x_4
x_3	0	2	8	2	0	0.33
x_4	0	16	4	1	0	4
max			-8	-6	0	0

Iteration 2

B Q	C_b	P	x_1	x_2	x_3	x_4
x_1 3.5	8	0.33	1	0.33	0.17	0
x_4 -19	0	14.67	0	2.67	-0.67	1
max 18.67			0	0.67	1.33	0

Iteration 3

B Q	C_b	P	x_1	x_2	x_3	x_4
x_1 0	8	0	1	0.5	0.25	0
x_2 0	6	3.5	1.5	0.5	0.25	0
x_5 0	0	15	1	0	-0.5	1
max 21			0	1	0	0

The solution for this branch is:

- $x_1 = 0.33$
- $x_2 = 3$
- $x_3 = 0$
- Objective: 20.67

Branch 2: $x_2 = 4$

For this branch, we add the constraint $x_2 = 4$ to the original problem:

Even if the other variables are zero is x_2 is 4 the second constraint is not satisfied so we eliminate this branch.

Comparison of Solutions

We compare the objective values from both branches:

$$\begin{aligned}
\text{Initial branch:} \quad & Z = 21 \quad \text{for} \quad x_1 = 0, x_2 = 3.5, x_3 = 0 \\
\text{Branch 1:} \quad & Z = 20.67 \quad \text{for} \quad x_1 = 0.33, x_2 = 3, x_3 = 0 \\
\text{Branch 2:} \quad & Z = 4 \quad \text{for} \quad x_1 = 0, x_2 = 4, x_3 = 0
\end{aligned}$$

Branch 2 does not satisfy the constraints so that branch is eliminated and in the initial branch x_2 is not an integer so we choose branch 1 for optimal solution.

Optimal Solution

The optimal solution is:

- $x_1 = 0.33$
- $x_2 = 3$
- $x_3 = 0$
- Objective: 20.67

Part (b): Solution Using Pyomo

Output of the code

The code is appended at the end of the report ,the output of the code is:

- x_1: 0.3333333333333333
- x_2: 3.0
- x_3: 0.0
- Objective: 20.666666666666664

Appendix

.1 Question 1: Initial Model Code

```
from pyomo.environ import *

# Create a model
model = ConcreteModel()

# Define decision variables
model.x1 = Var(within=NonNegativeReals)  # Number of PCs produced
model.x2 = Var(within=NonNegativeReals)  # Number of Tablets produced
model.x3 = Var(within=NonNegativeReals)  # Number of Microprocessors produced

# Define the objective function
model.obj = Objective(expr=50 * model.x1 + 30 * model.x2 - 2 * model.x3, sense=maximize)

# Define constraints
model.con1 = Constraint(expr=5 * model.x1 + 2 * model.x2 + model.x3 <= 120)
model.con2 = Constraint(expr=8 * model.x1 + 4 * model.x2 <= 80 + model.x3)

# Solve the model
solver = SolverFactory('glpk')
solver.solve(model, tee=True)

# Print results
print(f"x1: {model.x1()}")
print(f"x2: {model.x2()}")
print(f"x3: {model.x3()}")
print(f"Objective: {model.obj()}")
```

tasch_co_lp.py

.2 Question 2: Initial Model Code

```
from pyomo.environ import *

# Create a model
model = ConcreteModel()

# Define decision variables
model.x1 = Var(within=NonNegativeReals)  # x1 is continuous
model.x2 = Var(within=NonNegativeIntegers)  # x2 is integer
model.x3 = Var(within=NonNegativeIntegers)  # x3 is integer

# Define the objective function
model.obj = Objective(expr=8 * model.x1 + 6 * model.x2 + 2 * model.x3,
    sense=maximize)

# Define constraints
model.con1 = Constraint(expr=6 * model.x1 + 4 * model.x2 + 2 * model.x3
     <= 14)
model.con2 = Constraint(expr=4 * model.x1 + 2 * model.x2 + 4 * model.x3
     <= 22)

# Solve the model
solver = SolverFactory('glpk')
solver.solve(model, tee=True)

# Print results
print(f"x1: {model.x1()}")
print(f"x2: {model.x2()}")
print(f"x3: {model.x3()}")
print(f"Objective: {model.obj()}")
```

mip_model.py