Meta-Community Stability on Dispersal Networks

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Abstract

We consider meta-community stability in the context of the degree distribution for meta-community dispersal networks. In particular we characterize meta-community stability for the case of symmetric and heavy-tailed degree distributions for intra-community interactions (May's community matrix) and the coupled dispersal network in the meta-community generalization.

Questions

- **Dispersal network degree distribution** Effect of exp() vs. N() after integrating over interaction networks
- **Is 'few strong many weak' robust to dispersal network structure?** Are other interaction networks more robust for particular dispersal networks?
- **Complexity versus stability relationship for different network structure?** Meta-community stability versus # of species for different degree distributions
- **Stability parameter space for particular degree distributions** e.g. stability as a function of $\alpha_a = \alpha_m$.
- **Mutualism vs. antagonism** Mutualism means that interaction coefficients are positive for both interacting species, and antagonism means one is positive and one is negative

We consider the following degree distributions

Community (A)	Dispersal (M)
$\sim U(-1,1)$	$\sim U(-1,1)$
$\sim U(-1,1)$	$\sim \exp(\alpha_m)$
$\sim \exp(\alpha_a)$	$\sim U(-1,1)$
$\sim \exp(\alpha_a)$	$\sim \exp(\alpha_m)$

- **Aim #2:** Stability as a linear or a non-linear function of parameters : When both matricies are exponentially distributed with α_a and α_m, how does stability change as a multiplicative / additive function of the $\{\alpha_a, \alpha_m\}$ pair?
- **Aim #3:** Generalization : Find conjugate interaction-dispersal matrix pairs that optimizes the stability of a food web.
- **Aim #4:** Validation : Can we use real data sets do validate the stability of the conjugate pairs?
- **Aim #5:** Future work : Abundance vector, source and sink added to the system, perturbation in abundance.

Parameter Definitions

- N : Number of species.
- P_N : Number of patches (habitats).
- X_i^l : Abundance of species i in patch l.
- a_{ij}^l : Interaction coefficient between species i and species j in patch l.
- $m_i^{l\to k}$: Migration coefficient of species i from patch l to patch k.

Simulation Rules

- Dispersal is symmetric, i.e., $m_i^{l\to k} = m_i^{k\to l}$
- Same species have the same dispersal rate, i.e., $m_i^{l\to k} = m_i \ \forall k, l$.
- Initial Conditions : Each patch is identical, and abundance of species are randomly drawn.

Meta-community dynamics

The governing ODE for the Lotka-Volterra predator prey equations (i.e. *the mass action equations*) on a dispersal network is as follows

$$\frac{dX_i^l}{dt} = r_i^l X_i^l + \sum_{j=1}^{N} a_{ij}^l X_i^l X_j^l + \sum_{\substack{k=1\\k\neq l}}^{P} m_i^{k\to l} X_i^k - \sum_{\substack{k=1\\l\neq k}}^{P} m_i^{l\to k} X_i^l$$

For the purpose of notation, we define

$$\frac{dX_i^l}{dt} \equiv f(X_i^l).$$

The general Jacobian matrix for the meta-community dynamics is then

$$
\mathbf{J} = \begin{bmatrix}
\frac{\partial f(X_1^1)}{\partial X_1^1} & \frac{\partial f(X_1^1)}{\partial X_2^1} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^1} & \frac{\partial f(X_1^1)}{\partial X_1^2} & \frac{\partial f(X_1^1)}{\partial X_2^2} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^P} \\
\frac{\partial f(X_2^1)}{\partial X_1^1} & \frac{\partial f(X_2^1)}{\partial X_2^1} & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_2^1)}{\partial X_N^P} \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\frac{\partial f(X_N^1)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\frac{\partial f(X_1^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\frac{\partial f(X_2^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \frac{\partial f(X_{N-1}^P)}{\partial X_N^P} \\
\frac{\partial f(X_N^P)}{\partial X_1^1} & \frac{\partial f(X_N^P)}{\partial X_1^2} & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_N^P)}{\partial X_{N-1}^P} & \frac{\partial f(X_N^P)}{\partial X_N^P}
\end{bmatrix}
$$

where species $1:N$ in patch l are stacked as the top rows, followed by species $1:N$ in patch $l+1$, and so on.

For the case $N=2, P=2$ the equations are

$$
\begin{aligned}
\frac{dX_1^1}{dt} &= r_1^1 X_1^1 - a_1^1 (X_1^1)^2 + a_{1,2}^1 X_1^1 X_2^1 + m_1^{2\to1} X_1^2 - m_1^{1\to2} X_1^1 \\
\frac{dX_2^1}{dt} &= r_2^1 X_2^1 - a_2^1 (X_2^1)^2 + a_{2,1}^1 X_2^1 X_1^1 + m_2^{2\to1} X_2^2 - m_2^{1\to2} X_2^1 \\
\frac{dX_1^2}{dt} &= r_1^2 X_1^2 - a_1^2 (X_1^2)^2 + a_{1,2}^2 X_1^2 X_2^2 + m_1^{1\to2} X_1^1 - m_1^{2\to1} X_1^2 \\
\frac{dX_2^2}{dt} &= r_2^2 X_2^2 - a_2^2 (X_2^2)^2 + a_{2,1}^2 X_2^2 X_1^2 + m_2^{1\to2} X_2^1 - m_2^{2\to1} X_2^2
\end{aligned}
$$

So the Jacobian is

$$
\begin{aligned}
\mathbf{J} &= \begin{bmatrix}
\frac{\partial f(X_1^1)}{\partial X_1^1} & \frac{\partial f(X_1^1)}{\partial X_2^1} & \frac{\partial f(X_1^1)}{\partial X_1^2} & \frac{\partial f(X_1^1)}{\partial X_2^2} \\
\frac{\partial f(X_2^1)}{\partial X_1^1} & \frac{\partial f(X_2^1)}{\partial X_2^1} & \frac{\partial f(X_2^1)}{\partial X_1^2} & \frac{\partial f(X_2^1)}{\partial X_2^2} \\
\frac{\partial f(X_1^2)}{\partial X_1^1} & \frac{\partial f(X_1^2)}{\partial X_2^1} & \frac{\partial f(X_1^2)}{\partial X_1^2} & \frac{\partial f(X_1^2)}{\partial X_2^2} \\
\frac{\partial f(X_2^2)}{\partial X_1^1} & \frac{\partial f(X_2^2)}{\partial X_2^1} & \frac{\partial f(X_2^2)}{\partial X_1^2} & \frac{\partial f(X_2^2)}{\partial X_2^2}
\end{bmatrix} \\
&= \begin{bmatrix}
r_1^1 - 2a_1^1 X_1^1 + a_{1,2}^1 X_2^1 - m_1^{1\to2} & a_{1,2}^1 X_1^1 & m_1^{2\to1} & 0 \\
a_{1,2}^1 X_2^1 & r_2^1 - 2a_2^1 X_2^1 + a_{2,1} X_1^1 - m_2^{1\to2} & 0 & m_2^{2\to1} \\
m_1^{1\to2} & 0 & r_1^2 - 2a_1^2 X_1^2 + a_{1,2}^2 X_2^2 - m_1^{2\to1} & a_{1,2}^2 X_1^2 \\
0 & m_2^{1\to2} & a_{2,1}^2 X_2^2 & r_2^2 - 2a_2^2 X_2^2 + a_{2,1}^2 X_1^2 - m_2^{2\to1}
\end{bmatrix}
\end{aligned}
$$

We partition the Jacobian into intra-community interaction and meta-community dispersal components

$$
\mathbf{J} = \mathbf{J_A} + \mathbf{J_M}
$$

where

$$\mathbf{J_A} = \begin{bmatrix} r_1^1 - 2a_1^1 X_1^1 + a_{1,2}^1 X_2^1 & a_{1,2}^1 X_1^1 & 0 & 0 \\ a_{1,2}^1 X_2^1 & r_2^1 - 2a_2^1 X_2^1 + a_{2,1} X_1^1 & 0 & 0 \\ 0 & 0 & r_1^2 - 2a_1^2 X_1^2 + a_{1,2}^2 X_2^2 & a1,2^2 X_1^2 \\ 0 & 0 & a_{2,1}^2 X_2^2 & r_2^2 - 2a_2^2 X_2^2 + a_{2,1}^2 X_1^2 \end{bmatrix}$$

$$\mathbf{J_M} = \begin{bmatrix} -m_1^{1\to 2} & 0 & m_1^{2\to 1} & 0 \\ 0 & -m_2^{1\to 2} & 0 & m_2^{2\to 1} \\ m_1^{1\to 2} & 0 & -m_1^{2\to 1} & 0 \\ 0 & m_2^{1\to 2} & 0 & -m_2^{2\to 1} \end{bmatrix}$$

We can further decompose $\mathbf{J_M}$ into emigration and immigration networks

$$\mathbf{J_M} = \mathbf{J_{M_e}} + \mathbf{J_{M_i}}$$

where the emigration matrix is the diagonal matrix $\mathbf{J_{M_e}}$ and the immigration matrix $\mathbf{J_{M_i}}$ forms the off-diagonal terms.

General community dynamics

For a general community dynamics model g in detailed balance,

$$\tilde{a}_{i,j}^l = \frac{\partial g(X_i^l)}{\partial X_j^l}\bigg|_{\substack{X_i^l = X_i^{l*} \\ X_j^l = X_j^{l*}}}$$

Degree distributions

Here we consider meta-community stability as a function of the probability distribution for the underlying network components. We are particularly interested in how dispersal network degree distribution models modulate the meta-community stability in the context of different intra-community interactions networks.

We make a distinction between the graphical structure of the interaction and dispersal networks, versus the structure of the resulting Jacobian at detailed balance.

Dispersal networks

Interactions networks