

# Evolutionary Dynamics on Dispersal Networks

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## Abstract

We consider meta-community stability in the context of the degree distribution for meta-community dispersal networks. In particular we characterize meta-community stability for the case of symmetric and heavy-tailed degree distributions for intra-community interactions (May's community matrix) and the coupled dispersal network in the meta-community generalization.

## Questions

- **mutation-selection balance** How does the entropy change as a function of 1) mutation rate, 2) dispersal rate, 3) selection gradient, 4) dispersal network structure
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- **Dispersal network degree distribution** Effect of  $\exp()$  vs.  $N()$  after integrating over interaction networks
- **Is ‘few strong many weak’ robust to dispersal network structure?** Are other interaction networks more robust for particular dispersal networks?
- **Complexity versus stability relationship for different network structure?** Meta-community stability versus # of species for different degree distributions
- **Stability parameter space for particular degree distributions** e.g. stability as a function of  $\alpha_a = \alpha_m$ .
- **Mutualism vs. antagonism** Mutualism means that interaction coefficients are positive for both interacting species, and antagonism means one is positive and one is negative

# Binary sequences

$$\begin{aligned}
 x_{000} &= [0, 0, 0] \\
 x_{001} &= [0, 0, 1] \\
 &\vdots \\
 x_{111} &= [1, 1, 1]
 \end{aligned}$$

With three

We consider the following degree distributions

Community (A)	Dispersal (M)
$\sim U(-1, 1)$	$\sim U(-1, 1)$
$\sim U(-1, 1)$	$\sim \exp(\alpha_m)$
$\sim \exp(\alpha_a)$	$\sim U(-1, 1)$
$\sim \exp(\alpha_a)$	$\sim \exp(\alpha_m)$

- **Aim #2:** Stability as a linear or a non-linear function of parameters : When both matrices are exponentially distributed with  $\alpha_a$  and  $\alpha_m$ , how does stability change as a multiplicative / additive function of the  $\{\alpha_a, \alpha_m\}$  pair?
- **Aim #3:** Generalization : Find conjugate interaction-dispersal matrix pairs that optimizes the stability of a food web.
- **Aim #4:** Validation : Can we use real data sets to validate the stability of the conjugate pairs?
- **Aim #5:** Future work : Abundance vector, source and sink added to the system, perturbation in abundance.

## Parameter Definitions

- $N$  : Number of species.
- $P_N$  : Number of patches (habitats).
- $X_i^l$  : Abundance of species  $i$  in patch  $l$ .
- $a_{ij}^l$  : Interaction coefficient between species  $i$  and species  $j$  in patch  $l$ .
- $m_i^{l \rightarrow k}$  : Migration coefficient of species  $i$  from patch  $l$  to patch  $k$ .

## Simulation Rules

- Dispersal is symmetric, i.e.,  $m_i^{l \rightarrow k} = m_i^{k \rightarrow l}$
- Same species have the same dispersal rate, i.e.,  $m_i^{l \rightarrow k} = m_i \forall k, l$ .

- Initial Conditions : Each patch is identical, and abundance of species are randomly drawn.

Meta-community dynamics

The governing ODE for the Lotka-Volterra predator prey equations (i.e. *the mass action equations*) on a dispersal network is as follows

$$\frac{dX_i^l}{dt} = r_i^l X_i^l + \sum_{j=1}^N a_{ij}^l X_i^l X_j^l + \sum_{\substack{k=1 \\ k \neq l}}^P m_i^{k \rightarrow l} X_i^k - \sum_{\substack{k=1 \\ l \neq k}}^P m_i^{l \rightarrow k} X_i^l$$

For the purpose of notation, we define

$$\frac{dX_i^l}{dt} \equiv f(X_i^l).$$

The general Jacobian matrix for the meta-community dynamics is then

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f(X_1^1)}{\partial X_1^1} & \frac{\partial f(X_1^1)}{\partial X_2^1} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^1} & \frac{\partial f(X_1^1)}{\partial X_1^2} & \frac{\partial f(X_1^1)}{\partial X_2^2} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^P} \\ \frac{\partial f(X_2^1)}{\partial X_1^1} & \frac{\partial f(X_2^1)}{\partial X_2^1} & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_2^1)}{\partial X_N^P} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f(X_N^1)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f(X_1^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f(X_2^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \frac{\partial f(X_{N-1}^P)}{\partial X_N^P} \\ \frac{\partial f(X_N^P)}{\partial X_1^1} & \frac{\partial f(X_N^P)}{\partial X_1^2} & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_N^P)}{\partial X_{N-1}^P} & \frac{\partial f(X_N^P)}{\partial X_N^P} \end{bmatrix}$$

where species 1 :  $N$  in patch  $l$  are stacked as the top rows, followed by species 1 :  $N$  in patch  $l + 1$ , and so on.

For the case  $N = 2, P = 2$  the equations are

$$\begin{aligned} \frac{dX_1^1}{dt} &= r_1^1 X_1^1 - a_1^1 (X_1^1)^2 + a_{1,2}^1 X_1^1 X_2^1 + m_1^{2 \rightarrow 1} X_1^2 - m_1^{1 \rightarrow 2} X_1^1 \\ \frac{dX_2^1}{dt} &= r_2^1 X_2^1 - a_2^1 (X_2^1)^2 + a_{2,1}^1 X_2^1 X_1^1 + m_2^{2 \rightarrow 1} X_2^2 - m_2^{1 \rightarrow 2} X_2^1 \\ \frac{dX_1^2}{dt} &= r_1^2 X_1^2 - a_1^2 (X_1^2)^2 + a_{1,2}^2 X_1^2 X_2^2 + m_1^{1 \rightarrow 2} X_1^1 - m_1^{2 \rightarrow 1} X_1^2 \\ \frac{dX_2^2}{dt} &= r_2^2 X_2^2 - a_2^2 (X_2^2)^2 + a_{2,1}^2 X_2^2 X_1^2 + m_2^{1 \rightarrow 2} X_2^1 - m_2^{2 \rightarrow 1} X_2^2 \end{aligned}$$

So the Jacobian is

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f(X_1^1)}{\partial X_1^1} & \frac{\partial f(X_1^1)}{\partial X_2^1} & \frac{\partial f(X_1^1)}{\partial X_1^2} & \frac{\partial f(X_1^1)}{\partial X_2^2} \\ \frac{\partial f(X_2^1)}{\partial X_1^1} & \frac{\partial f(X_2^1)}{\partial X_2^1} & \frac{\partial f(X_2^1)}{\partial X_1^2} & \frac{\partial f(X_2^1)}{\partial X_2^2} \\ \frac{\partial f(X_1^2)}{\partial X_1^1} & \frac{\partial f(X_1^2)}{\partial X_2^1} & \frac{\partial f(X_1^2)}{\partial X_1^2} & \frac{\partial f(X_1^2)}{\partial X_2^2} \\ \frac{\partial f(X_2^2)}{\partial X_1^1} & \frac{\partial f(X_2^2)}{\partial X_2^1} & \frac{\partial f(X_2^2)}{\partial X_1^2} & \frac{\partial f(X_2^2)}{\partial X_2^2} \end{bmatrix}$$

$$= \begin{bmatrix} r_1^1 - 2a_1^1 X_1^1 + a_{1,2}^1 X_2^1 - m_1^{1 \rightarrow 2} & a_{1,2}^1 X_1^1 & m_1^{2 \rightarrow 1} & 0 \\ a_{1,2}^1 X_2^1 & r_2^1 - 2a_2^1 X_2^1 + a_{2,1}^1 X_1^1 - m_2^{1 \rightarrow 2} & 0 & m_2^{2 \rightarrow 1} \\ m_1^{1 \rightarrow 2} & 0 & r_1^2 - 2a_1^2 X_1^2 + a_{1,2}^2 X_2^2 - m_1^{2 \rightarrow 1} & a_{1,2}^2 X_1^2 \\ 0 & m_2^{1 \rightarrow 2} & a_{2,1}^2 X_2^2 & r_2^2 - 2a_2^2 X_2^2 + a_{2,1}^2 X_1^2 - m_2^{2 \rightarrow 1} \end{bmatrix}$$

We partition the Jacobian into intra-community interaction and meta-community dispersal components

$$\mathbf{J} = \mathbf{J}_A + \mathbf{J}_M$$

where

$$\mathbf{J}_A = \begin{bmatrix} r_1^1 - 2a_1^1 X_1^1 + a_{1,2}^1 X_2^1 & a_{1,2}^1 X_1^1 & 0 & 0 \\ a_{1,2}^1 X_2^1 & r_2^1 - 2a_2^1 X_2^1 + a_{2,1}^1 X_1^1 & 0 & 0 \\ 0 & 0 & r_1^2 - 2a_1^2 X_1^2 + a_{1,2}^2 X_2^2 & a_{1,2}^2 X_1^2 \\ 0 & 0 & a_{2,1}^2 X_2^2 & r_2^2 - 2a_2^2 X_2^2 + a_{2,1}^2 X_1^2 \end{bmatrix}$$

$$\mathbf{J}_M = \begin{bmatrix} -m_1^{1 \rightarrow 2} & 0 & m_1^{2 \rightarrow 1} & 0 \\ 0 & -m_2^{1 \rightarrow 2} & 0 & m_2^{2 \rightarrow 1} \\ m_1^{1 \rightarrow 2} & 0 & -m_1^{2 \rightarrow 1} & 0 \\ 0 & m_2^{1 \rightarrow 2} & 0 & -m_2^{2 \rightarrow 1} \end{bmatrix}$$

We can further decompose  $\mathbf{J}_M$  into emigration and immigration networks

$$\mathbf{J}_M = \mathbf{J}_{M_e} + \mathbf{J}_{M_i}$$

where the emigration matrix is the diagonal matrix  $\mathbf{J}_{M_e}$  and the immigration matrix  $\mathbf{J}_{M_i}$  forms the off-diagonal terms.

## General community dynamics

For a general community dynamics model  $g$  in detailed balance,

$$\tilde{a}_{i,j}^l = \frac{\partial g(X_i^l)}{\partial X_j^l} \Big|_{\substack{X_i^l = X_i^{l*} \\ X_j^l = X_j^{l*}}}$$

## Degree distributions

Here we consider meta-community stability as a function of the probability distribution for the underlying network components. We are particularly interested in how dispersal network degree distribution models modulate the meta-community stability in the context of different intra-community interactions networks.

We make a distinction between the graphical structure of the interaction and dispersal networks, versus the structure of the resulting Jacobian at detailed balance.

## Dispersal networks

## Interactions networks