Conjugate Interaction - Dispersal Matricies that Optimizes the Community Stability

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- **Motivation :** Complexity versus Stability question : Weak interactions stabilize food webs.
- **Aim #1:** Comparing the stability of uniform interaction / dispersal matricies with exponentialy distributed interaction / dispersal matricies, as a function of $\alpha_a = \alpha_m$.

Community (A)	Dispersal (M)
$\sim U(-1,1)$	$\sim U(-1,1)$
$\sim U(-1,1)$	$\sim \exp(\alpha_m)$
$\sim \exp(\alpha_a)$	$\sim U(-1,1)$
$\sim \exp(\alpha_a)$	$\sim \exp(\alpha_m)$

- **Aim #2:** Stability as a linear or a non-linear function of parameters : When both matricies are exponentially distributed with α_a and α_m, how does stability change as a multiplicative / additive function of the $\{\alpha_a, \alpha_m\}$ pair?
- **Aim #3:** Generalization : Find conjugate interaction-dispersal matrix pairs that optimizes the stability of a food web.
- **Aim #4:** Validation : Can we use real data sets do validate the stability of the conjugate pairs?
- **Aim #5:** Future work : Abundance vector, source and sink added to the system, perturbation in abundance.

Parameter Definitions

- N : Number of species.
- P_N : Number of patches (habitats).
- X_i^l : Abundance of species i in patch l.
- a_{ij}^l : Interaction coefficient between species i and species j in patch l.

- $m_i^{l\to k}$: Migration coefficient of species i from patch l to patch k.

  Simulation Rules

- Dispersal is symmetric, i.e., $m_i^{l\to k} = m_i^{k\to l}$
- Same species have the same dispersal rate, i.e., $m_i^{l\to k} = m_i \ \forall k, l$.
- Initial Conditions : Each patch is identical, and abundance of species are randomly drawn.
- **Governing ODE**

$$\frac{dX_i^l}{dt} = X_i^l \sum_{j=1}^{N} a_{ij}^l X_j^l - \sum_{k=1(l\neq k)}^{P_N} m_i^{l\to k} X_i^l + \sum_{k=1(k\neq l)}^{P_N} m_i^{k\to l} X_i^k \tag{1}$$

- **Jacobian Matrix :** Can be automatized via MATLAB. (Or can be symbolically hardcoded as well)