Meta-Community Stability on Dispersal Networks

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Abstract

We consider meta-community stability in the context of the meta-community dispersal network degree distribution. In particular we characterize meta-community stability for the case of symmetric and heavy-tailed degree distributions for intra-community interactions (May's community matrix) and the coupled dispersal network in the meta-community generalization.

Questions

- **Dispersal network degree distribution** Effect of exp() vs. N() after integrating over interaction networks
- **Is 'few strong many weak' robust to dispersal network structure?** Are other interaction networks more robust for particular dispersal networks?
- **Complexity versus stability relationship for different network structure?** Meta-community stability versus # of species for different degree distributions
- **Stability parameter space for particular degree distributions** e.g. stability as a function of $\alpha_a = \alpha_m$.

We consider the following degree distributions

Community (A)	Dispersal (M)
$\sim U(-1,1)$	$\sim U(-1,1)$
$\sim U(-1,1)$	$\sim \exp(\alpha_m)$
$\sim \exp(\alpha_a)$	$\sim U(-1,1)$
$\sim \exp(\alpha_a)$	$\sim \exp(\alpha_m)$

- **Aim #2:** Stability as a linear or a non-linear function of parameters : When both matricies are exponentially distributed with α_a and α_m, how does stability change as a multiplicative / additive function of the $\{\alpha_a, \alpha_m\}$ pair?

- **Aim #3:** Generalization : Find conjugate interaction-dispersal matrix pairs that optimizes the stability of a food web.
- **Aim #4:** Validation : Can we use real data sets do validate the stability of the conjugate pairs?
- **Aim #5:** Future work : Abundance vector, source and sink added to the system, perturbation in abundance.

  Parameter Definitions

- N : Number of species.
- P_N : Number of patches (habitats).
- X_i^l : Abundance of species i in patch l.
- a_{ij}^l : Interaction coefficient between species i and species j in patch l.
- $m_i^{l\to k}$: Migration coefficient of species i from patch l to patch k.

  Simulation Rules

- Dispersal is symmetric, i.e., $m_i^{l\to k} = m_i^{k\to l}$
- Same species have the same dispersal rate, i.e., $m_i^{l\to k} = m_i \ \forall k, l$.
- Initial Conditions : Each patch is identical, and abundance of species are randomly drawn.

Meta-community dynamics

The governing ODE for the Lotka-Volterra predator prey equations (i.e. *the mass action equations*) on a dispersal network is as follows

$$\frac{dX_i^l}{dt} = r_i^l X_i^l + \sum_{j=1}^{N} a_{ij}^l X_i^l X_j^l + \sum_{\substack{k=1\\k\neq l}}^{P} m_i^{k\to l} X_i^k - \sum_{\substack{k=1\\l\neq k}}^{P} m_i^{l\to k} X_i^l$$

For the purpose of notation, we define

$$\frac{dX_i^l}{dt} \equiv f(X_i^l).$$

The general Jacobian matrix for the meta-community dynamics is then

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f(X_1^1)}{\partial X_1^1} & \frac{\partial f(X_1^1)}{\partial X_2^1} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^1} & \frac{\partial f(X_1^1)}{\partial X_1^2} & \frac{\partial f(X_1^1)}{\partial X_2^2} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^P} \\ \frac{\partial f(X_2^1)}{\partial X_1^1} & \frac{\partial f(X_2^1)}{\partial X_2^1} & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_2^1)}{\partial X_N^P} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f(X_N^1)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f(X_1^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f(X_2^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \frac{\partial f(X_{N-1}^P)}{\partial X_N^P} \\ \frac{\partial f(X_N^P)}{\partial X_1^1} & \frac{\partial f(X_N^P)}{\partial X_1^2} & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_N^P)}{\partial X_{N-1}^P} & \frac{\partial f(X_N^P)}{\partial X_N^P} \end{bmatrix}$$

where species $1:N$ in patch l are stacked as the top rows, followed by species $1:N$ in patch $l+1$, and so on.

For the case $N=2, P=2$ the equations are

$$\begin{aligned} \frac{dX_1^1}{dt} &= r_1^1 X_1^1 - a_1^1 (X_1^1)^2 + a_{1,2}^1 X_1^1 X_2^1 + m_1^{2\to1} X_1^2 - m_1^{1\to2} X_1^1 \\ \frac{dX_2^1}{dt} &= r_2^1 X_2^1 - a_2^1 (X_2^1)^2 + a_{2,1}^1 X_2^1 X_1^1 + m_2^{2\to1} X_2^2 - m_2^{1\to2} X_2^1 \\ \frac{dX_1^2}{dt} &= r_1^2 X_1^2 - a_1^2 (X_1^2)^2 + a_{1,2}^2 X_1^2 X_2^2 + m_1^{1\to2} X_1^1 - m_1^{2\to1} X_1^2 \\ \frac{dX_2^2}{dt} &= r_2^2 X_2^2 - a_2^2 (X_2^2)^2 + a_{2,1}^2 X_2^2 X_1^2 + m_2^{1\to2} X_2^1 - m_2^{2\to1} X_2^2 \end{aligned}$$

So the Jacobian is

$$\begin{aligned} \mathbf{J} &= \begin{bmatrix} \frac{\partial f(X_1^1)}{\partial X_1^1} & \frac{\partial f(X_1^1)}{\partial X_2^1} & \frac{\partial f(X_1^1)}{\partial X_1^2} & \frac{\partial f(X_1^1)}{\partial X_2^2} \\ \frac{\partial f(X_2^1)}{\partial X_1^1} & \frac{\partial f(X_2^1)}{\partial X_2^1} & \frac{\partial f(X_2^1)}{\partial X_1^2} & \frac{\partial f(X_2^1)}{\partial X_2^2} \\ \frac{\partial f(X_1^2)}{\partial X_1^1} & \frac{\partial f(X_1^2)}{\partial X_2^1} & \frac{\partial f(X_1^2)}{\partial X_1^2} & \frac{\partial f(X_1^2)}{\partial X_2^2} \\ \frac{\partial f(X_2^2)}{\partial X_1^1} & \frac{\partial f(X_2^2)}{\partial X_2^1} & \frac{\partial f(X_2^2)}{\partial X_1^2} & \frac{\partial f(X_2^2)}{\partial X_2^2} \end{bmatrix} \\ &= \begin{bmatrix} r_1^1 - 2a_1^1 X_1^1 + a_{1,2}^1 X_2^1 - m_1^{1\to2} & a_{1,2}^1 X_1^1 & m_1^{2\to1} & 0 \\ a_{1,2}^1 X_2^1 & r_2^1 - 2a_2^1 X_2^1 + a_{2,1} X_1^1 - m_2^{1\to2} & 0 & m_2^{2\to1} \\ m_1^{1\to2} & 0 & r_1^2 - 2a_1^2 X_1^2 + a_{1,2}^2 X_2^2 - m_1^{2\to1} & a_{1,2}^2 X_1^2 \\ 0 & m_2^{1\to2} & a_{2,1}^2 X_2^2 & r_2^2 - 2a_2^2 X_2^2 + a_{2,1}^2 X_1^2 - m_2^{2\to1} \end{bmatrix} \end{aligned}$$

We partition the Jacobian into intra-community interaction and meta-community dispersal components

$$\mathbf{J} = \mathbf{J_A} + \mathbf{J_M}$$

where

$$\mathbf{J_A} = \begin{bmatrix} r_1^1 - 2a_1^1X_1^1 + a_{1,2}^1X_2^1 & a_{1,2}^1X_1^1 & 0 & 0 \\ a_{1,2}^1X_2^1 & r_2^1 - 2a_2^1X_2^1 + a_{2,1}X_1^1 & 0 & 0 \\ 0 & 0 & r_1^2 - 2a_1^2X_1^2 + a_{1,2}^2X_2^2 & a1,2^2X_1^2 \\ 0 & 0 & a_{2,1}^2X_2^2 & r_2^2 - 2a_2^2X_2^2 + a_{2,1}^2X_1^2 \end{bmatrix}$$

$$\mathbf{J_M} = \begin{bmatrix} -m_1^{1\to2} & 0 & m_1^{2\to1} & 0 \\ 0 & -m_2^{1\to2} & 0 & m_2^{2\to1} \\ m_1^{1\to2} & 0 & -m_1^{2\to1} & 0 \\ 0 & m_2^{1\to2} & 0 & -m_2^{2\to1} \end{bmatrix}$$

We can further decompose $\mathbf{J_M}$ into emigration and immigration networks

$$\mathbf{J_M} = \mathbf{J_{M_e}} + \mathbf{J_{M_i}}$$

where the emigration matrix is the diagonal matrix $\mathbf{J_{M_e}}$ and the immigration matrix $\mathbf{J_{M_i}}$ forms the off-diagonal terms.

General community dynamics

For a general community dynamics model g in detailed balance,

$$\tilde{a}_{i,j}^l = \frac{\partial g(X_i^l)}{\partial X_j^l}\bigg|_{\substack{X_i^l = X_i^{l*} \\ X_j^l = X_j^{l*}}}$$

Degree distributions

Here we consider meta-community stability as a function of the probability distribution for the underlying network components. We are particularly interested in how dispersal network degree distribution models modulate the meta-community stability in the context of different intra-community interactions networks.

We make a distinction between the graphical structure of the interaction and dispersal networks, versus the structure of the resulting Jacobian at detailed balance.

Dispersal networks

Interactions networks