Evolutionary Dynamics on Dispersal Networks

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Abstract

We consider meta-community stability in the context of the degree distribution for meta-community dispersal networks. In particular we characterize meta-community stability for the case of symmetric and heavy-tailed degree distributions for intra-community interactions (May's community matrix) and the coupled dispersal network in the meta-community generalization.

Questions

- **mutation-selection balance** How does the entropy distribution change as a function of 1) mutation rate, 2) dispersal rate, 3) selection gradient ($\alpha\delta r$), 4) dispersal network structure (degree distribution),
- **Species interactions** Is 'few strong many weak' robust to dispersal network structure? Are other interaction networks more robust for particular dispersal networks? Mutualism vs. antagonism; conjugate interaction-dispersal matrix pairs
- **Complexity versus stability**

Binary sequences

$$
\begin{aligned}
X_{000} &= [0,0,0] \\
X_{001} &= [0,0,1] \\
&\vdots \\
X_{111} &= [1,1,1]
\end{aligned}
$$

With three bits we have 2^n sequences.

Governing ordinary differential equation

The governing ODE for the logistic quasispecies on a dispersal network is as follows

$$\frac{dx_i^l}{dt} = r_i^l x_i^l \left(1 - \Sigma x^l\right) + \sum_{\substack{j \\ j \neq i}}^{N} q_0^{h(i,j)} x_j^l - \sum_{\substack{j \\ j \neq i}}^{N} q_0^{h(i,j)} x_i^l + \sum_{\substack{k \\ k \neq l}}^{P} m_i^{k \to l} x_i^k - \sum_{\substack{k \\ l \neq k}}^{P} m_i^{l \to k} x_i^l$$

with parameters and state variable symbols defined in Table X.

Table 1: Parameter and state variance definitions for the governing ODE

Symbol	**Definition**
r_i^l	intrinsic growth rate
x_i^l	concentration of sequence i in patch l
Σx^l	total sequence abundance in path i
q_0	mutation probability per bit
$h(i,j)$	Hamming distance between sequences i and j
N	Number of unique sequences per patch
$m_i^{h \to l}$	dispersal rate of species i from patch k to l
P	number of patches

We write the equation in matrix-vector notation

$$\frac{d\mathbf{x}}{dt} = \mathbf{R}\Sigma\mathbf{x} + \mathbf{Q}\mathbf{x} + \mathbf{M}\mathbf{x}$$

where

$$\mathbf{R} = \begin{bmatrix} r_1^1 & 0 & \dots & \dots & \dots & 0 \\ 0 & r_1^2 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & r_2^1 & \ddots & \vdots \\ \vdots 0 & 0 & \dots & \dots & r_N^P & \end{bmatrix}$$

The Jacobian matrix

For the purpose of notation, we define

$$\frac{dX_i^l}{dt} \equiv f(X_i^l).$$

The general Jacobian matrix for the meta-community dynamics is then

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f(X_1^1)}{\partial X_1^1} & \frac{\partial f(X_1^1)}{\partial X_2^1} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^1} & \frac{\partial f(X_1^1)}{\partial X_1^2} & \frac{\partial f(X_1^1)}{\partial X_2^2} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^P} \\ \frac{\partial f(X_2^1)}{\partial X_1^1} & \frac{\partial f(X_2^1)}{\partial X_2^1} & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_2^1)}{\partial X_N^P} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f(X_N^1)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f(X_1^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f(X_2^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \frac{\partial f(X_{N-1}^P)}{\partial X_N^P} \\ \frac{\partial f(X_N^P)}{\partial X_1^1} & \frac{\partial f(X_N^P)}{\partial X_1^2} & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_N^P)}{\partial X_{N-1}^P} & \frac{\partial f(X_N^P)}{\partial X_N^P} \end{bmatrix}$$

where species $1:N$ in patch l are stacked as the top rows, followed by species $1:N$ in patch $l+1$, and so on.

For the case $N=2, P=2$ the equations are

$$\begin{aligned} \frac{dX_1^1}{dt} &= r_1^1 X_1^1 - a_1^1 (X_1^1)^2 + a_{1,2}^1 X_1^1 X_2^1 + m_1^{2\to1} X_1^2 - m_1^{1\to2} X_1^1 \\ \frac{dX_2^1}{dt} &= r_2^1 X_2^1 - a_2^1 (X_2^1)^2 + a_{2,1}^1 X_2^1 X_1^1 + m_2^{2\to1} X_2^2 - m_2^{1\to2} X_2^1 \\ \frac{dX_1^2}{dt} &= r_1^2 X_1^2 - a_1^2 (X_1^2)^2 + a_{1,2}^2 X_1^2 X_2^2 + m_1^{1\to2} X_1^1 - m_1^{2\to1} X_1^2 \\ \frac{dX_2^2}{dt} &= r_2^2 X_2^2 - a_2^2 (X_2^2)^2 + a_{2,1}^2 X_2^2 X_1^2 + m_2^{1\to2} X_2^1 - m_2^{2\to1} X_2^2 \end{aligned}$$

So the Jacobian is

$$\begin{aligned} \mathbf{J} &= \begin{bmatrix} \frac{\partial f(X_1^1)}{\partial X_1^1} & \frac{\partial f(X_1^1)}{\partial X_2^1} & \frac{\partial f(X_1^1)}{\partial X_1^2} & \frac{\partial f(X_1^1)}{\partial X_2^2} \\ \frac{\partial f(X_2^1)}{\partial X_1^1} & \frac{\partial f(X_2^1)}{\partial X_2^1} & \frac{\partial f(X_2^1)}{\partial X_1^2} & \frac{\partial f(X_2^1)}{\partial X_2^2} \\ \frac{\partial f(X_1^2)}{\partial X_1^1} & \frac{\partial f(X_1^2)}{\partial X_2^1} & \frac{\partial f(X_1^2)}{\partial X_1^2} & \frac{\partial f(X_1^2)}{\partial X_2^2} \\ \frac{\partial f(X_2^2)}{\partial X_1^1} & \frac{\partial f(X_2^2)}{\partial X_2^1} & \frac{\partial f(X_2^2)}{\partial X_1^2} & \frac{\partial f(X_2^2)}{\partial X_2^2} \end{bmatrix} \\ &= \begin{bmatrix} r_1^1 - 2a_1^1 X_1^1 + a_{1,2}^1 X_2^1 - m_1^{1\to2} & a_{1,2}^1 X_1^1 & m_1^{2\to1} & 0 \\ a_{1,2}^1 X_2^1 & r_2^1 - 2a_2^1 X_2^1 + a_{2,1} X_1^1 - m_2^{1\to2} & 0 & m_2^{2\to1} \\ m_1^{1\to2} & 0 & r_1^2 - 2a_1^2 X_1^2 + a_{1,2}^2 X_2^2 - m_1^{2\to1} & a_{1,2}^2 X_1^2 \\ 0 & m_2^{1\to2} & a_{2,1}^2 X_2^2 & r_2^2 - 2a_2^2 X_2^2 + a_{2,1}^2 X_1^2 - m_2^{2\to1} \end{bmatrix} \end{aligned}$$

We partition the Jacobian into intra-community interaction and meta-community dispersal components

$$\mathbf{J} = \mathbf{J_A} + \mathbf{J_M}$$

where

$$\mathbf{J_A} = \begin{bmatrix} r_1^1 - 2a_1^1 X_1^1 + a_{1,2}^1 X_2^1 & a_{1,2}^1 X_1^1 & 0 & 0 \\ a_{1,2}^1 X_2^1 & r_2^1 - 2a_2^1 X_2^1 + a_{2,1} X_1^1 & 0 & 0 \\ 0 & 0 & r_1^2 - 2a_1^2 X_1^2 + a_{1,2}^2 X_2^2 & a1,2^2 X_1^2 \\ 0 & 0 & a_{2,1}^2 X_2^2 & r_2^2 - 2a_2^2 X_2^2 + a_{2,1}^2 X_1^2 \end{bmatrix}$$

$$\mathbf{J_M} = \begin{bmatrix} -m_1^{1\to2} & 0 & m_1^{2\to1} & 0 \\ 0 & -m_2^{1\to2} & 0 & m_2^{2\to1} \\ m_1^{1\to2} & 0 & -m_1^{2\to1} & 0 \\ 0 & m_2^{1\to2} & 0 & -m_2^{2\to1} \end{bmatrix}$$

We can further decompose $\mathbf{J_M}$ into emigration and immigration networks

$$\mathbf{J_M} = \mathbf{J_{M_e}} + \mathbf{J_{M_i}}$$

where the emigration matrix is the diagonal matrix $\mathbf{J_{M_e}}$ and the immigration matrix $\mathbf{J_{M_i}}$ forms the off-diagonal terms.

General community dynamics

For a general community dynamics model g in detailed balance,

$$\tilde{a}_{i,j}^l = \frac{\partial g(X_i^l)}{\partial X_j^l}\bigg|_{\substack{X_i^l = X_i^{l*} \\ X_j^l = X_j^{l*}}}$$

Degree distributions

Here we consider meta-community stability as a function of the probability distribution for the underlying network components. We are particularly interested in how dispersal network degree distribution models modulate the meta-community stability in the context of different intra-community interactions networks.

We make a distinction between the graphical structure of the interaction and dispersal networks, versus the structure of the resulting Jacobian at detailed balance.

Dispersal networks

Interactions networks