Conjugate Interaction - Dispersal Matricies that Optimizes the Community Stability

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- **Motivation :** Complexity versus Stability question : Weak interactions stabilize food webs.
- **Aim #1:** Comparing the stability of uniform interaction / dispersal matricies with exponentialy distributed interaction / dispersal matricies, as a function of $\alpha_a = \alpha_m$.

Community (A)	Dispersal (M)
$\sim U(-1, 1)$	$\sim U(-1, 1)$
$\sim U(-1, 1)$	$\sim \exp(\alpha_m)$
$\sim \exp(\alpha_a)$	$\sim U(-1, 1)$
$\sim \exp(\alpha_a)$	$\sim \exp(\alpha_m)$

- **Aim #2:** Stability as a linear or a non-linear function of parameters : When both matricies are exponentially distributed with α_a and α_m, how does stability change as a multiplicative / additive function of the $\{\alpha_a, \alpha_m\}$ pair?
- **Aim #3:** Generalization : Find conjugate interaction-dispersal matrix pairs that optimizes the stability of a food web.
- **Aim #4:** Validation : Can we use real data sets do validate the stability of the conjugate pairs?
- **Aim #5:** Future work : Abundance vector, source and sink added to the system, perturbation in abundance.

Parameter Definitions

- N : Number of species.
- P_N : Number of patches (habitats).
- X_i^l : Abundance of species i in patch l.
- a_{ij}^l : Interaction coefficient between species i and species j in patch l.

- $m_i^{l \to k}$: Migration coefficient of species i from patch l to patch k.

  Simulation Rules

- Dispersal is symmetric, i.e., $m_i^{l \to k} = m_i^{k \to l}$
- Same species have the same dispersal rate, i.e., $m_i^{l \to k} = m_i \ \forall k, l$.
- Initial Conditions : Each patch is identical, and abundance of species are randomly drawn.
- **Governing ODE**

$$\frac{dX_i^l}{dt} = r_i^l X_i^l + \sum_{j=1}^{N} a_{ij}^l X_i^l X_j^l - \sum_{k=1(l \neq k)}^{P_N} m_i^{l \to k} X_i^l + \sum_{k=1(k \neq l)}^{P_N} m_i^{k \to l} X_i^k \tag{1}$$

- **Jacobian Matrix :** Can be automatized via MATLAB. (Or can be symbolically hardcoded as well)

$$\frac{dX_i^l}{dt} \equiv f(X_i^l)$$

$$\mathbf{J} = \begin{bmatrix}
\frac{\partial f(X_1^1)}{\partial X_1^1} & \frac{\partial f(X_1^1)}{\partial X_2^1} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^1} & \frac{\partial f(X_1^1)}{\partial X_1^2} & \frac{\partial f(X_1^1)}{\partial X_2^2} & \cdots & \frac{\partial f(X_1^1)}{\partial X_N^P} \\
\frac{\partial f(X_2^1)}{\partial X_1^1} & \frac{\partial f(X_2^1)}{\partial X_2^1} & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_2^1)}{\partial X_N^P} \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\frac{\partial f(X_N^1)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\frac{\partial f(X_1^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\frac{\partial f(X_2^2)}{\partial X_1^1} & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \frac{\partial f(X_{N-1}^P)}{\partial X_N^P} \\
\frac{\partial f(X_N^P)}{\partial X_1^1} & \frac{\partial f(X_N^P)}{\partial X_1^2} & \cdots & \cdots & \cdots & \cdots & \frac{\partial f(X_N^P)}{\partial X_{N-1}^P} & \frac{\partial f(X_N^P)}{\partial X_N^P}
\end{bmatrix}$$

For the case $N = 2, P = 2$:

$$\frac{dX_1^1}{dt} = r_1^1 X_1^1 + a_{1,1}(X_1^1)^2 + a_{1,2} X_2^1 + m^{2 \to 1} X_1^2 - m_1^{1 \to 2} X_1^1$$

$$\frac{dX_2^1}{dt} = r_2^1 X_2^1 + a_{2,2}(X_1^2)^2 + a_{2,1} X_1^2 + m_2^{2 \to 1}$$

$$\frac{dX_1^2}{dt} = r$$

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