Answer Key

Other (130 Questions)

Q1. (A)	Q2. (B)	Q3. (B)	Q4. (A)	Q5. (D)
Q6. (B)	Q7. (A)	Q8. (D)	Q9. (B)	Q10. (C)
Q11. (B)	Q12. (C)	Q13. (C)	Q14. (D)	Q15. (A)
Q16. (B)	Q17. (A)	Q18. (B)	Q19. (C)	Q20. (A)
Q21. (A)	Q22. (D)	Q23. (B)	Q24. (A)	Q25. (A)
Q26. (A)	Q27. (C)	Q28. (A)	Q29. (B)	Q30. (C)
Q31. (C)	Q32. (A)	Q33. (C)	Q34. (A)	Q35. (A)
Q36. (C)	Q37. (D)	Q38. (D)	Q39. (D)	Q40. (A)
Q41. (A)	Q42. (A)	Q43. (A)	Q44. (C)	Q45. (B)
Q46. (D)	Q47. (C)	Q48. (A)	Q49. (D)	Q50. (B)
Q51. (D)	Q52. (A)	Q53. (B)	Q54. (B)	Q55. (C)
Q56. (B)	Q57. (D)	Q58. (A)	Q59. (C)	Q60. (B)
Q61. (C)	Q62. (A)	Q63. (A)	Q64. (A)	Q65. (D)
Q66. (A)	Q67. (D)	Q68. (D)	Q69. (D)	Q70. (A)
Q71. (A)	Q72. (B)	Q73. (C)	Q74. (C)	Q75. (D)
Q76. (C)	Q77. (B)	Q78. (A)	Q79. (B)	Q80. (C)
Q81. (C)	Q82. (A)	Q83. (C)	Q84. (D)	Q85. (A)
Q86. (A)	Q87. (D)	Q88. (B)	Q89. (A)	Q90. (A)
Q91. (B)	Q92. (A)	Q93. (D)	Q94. (B)	Q95. (B)
Q96. (A)	Q97. (D)	Q98. (B)	Q99. (D)	Q100.(C)
Q101.(B)	Q102.(C)	Q103.(C)	Q104.(A)	Q105.(D)

Q106.(A)	**Q107.(D)**	**Q108.(D)**	**Q109.(B)**	**Q110.(D)**
Q111.(B)	**Q112.(B)**	**Q113.(D)**	**Q114.(A)**	**Q115.(A)**
Q116.(A)	**Q117.(C)**	**Q118.(B)**	**Q119.(D)**	**Q120.(A)**
Q121.(A)	**Q122.(D)**	**Q123.(A)**	**Q124.(A)**	**Q125.(A)**
Q126.(A)	**Q127.(A)**	**Q128.(C)**	**Q129.(D)**	**Q130.(C)**

Solutions

Q1. Solution

Correct Answer: (A)

Speed of electron in n^{th} orbit

$= 2.18 \times 10^6 \times \frac{Z}{n}$

For hydrogen atom, $Z = 1$

Speed of electron in n^{th} orbit for hydrogen atom

$= \frac{2.18 \times 10^6}{n}\ \mathrm{m\ s^{-1}}$

So, as the value of n increases, orbiting speed of electron decreases.

Hence, I is correct.

Radii of an electron in n^{th} orbit

$= 0.529 \frac{n^2}{Z}\ \mathring{\mathrm{A}}$

Radii of an electron for hydrogen atom in n^{th} orbit

$= 0.529 \frac{n^2}{1}\ \mathring{\mathrm{A}}$

Radii are directly proportional to n^2 not to n.

Hence, II is incorrect.

Time taken by an electron to complete one revolution

$= 1.534 \times 10^{-10} \times \frac{n^3}{Z^2}\ \mathrm{s}$

T is proportional to $\frac{n^3}{Z^2}$

We know, frequency with which electrons orbit in nucleus $f = \frac{1}{T}$

So, f is inversely proportional to n^3 for hydrogen atom.

Hence, III is correct.

Binding force with which electrons

is bound to nucleus$= \frac{KZe^2}{r^2}$

In hydrogen atom, binding force

$= \frac{Ke^2}{r^2}$

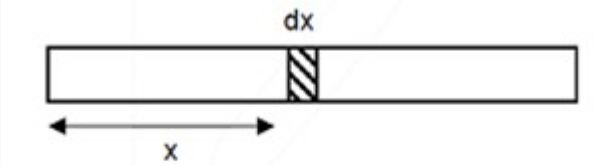
And, we know, as we shift to outer orbit, r increases.

So, binding force decreases.

Hence, IV is incorrect.

Q2. Solution

Correct Answer: (B)



$X_{cm} = \frac{1}{M}\int_0^{\iota} x.\, dM$

$dM = \rho.\, dx = \left(a + b\left(\frac{x}{\iota}\right)^2\right).\, dx$

$x_{cm} = \frac{\int x dM}{\int dm} = \frac{\int x\rho dx}{\int \rho dx} = \frac{\int_0^{\iota} x\left(a+\frac{bx^2}{\iota^2}\right)dx}{\int_0^{\iota}\left(a+\frac{bx^2}{\iota^2}\right)dx}$

$= \frac{a\left(\frac{x^2}{2}\right)_0^{\iota}+\frac{b}{\iota^2}\left(\frac{x^4}{4}\right)_0^{\iota}}{a(x)_0^{\iota}+\frac{b}{\iota^2}\left(\frac{x^3}{3}\right)_0^{\iota}}$

$= \frac{a\frac{\iota^2}{2}+b\frac{\iota^2}{4}}{a\iota+\frac{b\iota}{3}} = \frac{(2a+b)}{(3a+b)}\frac{\iota}{4} \times 3$

$= \frac{3\iota}{4}\left(\frac{2a+b}{3a+b}\right)$

Q3. Solution

Correct Answer: (B)

Given, $S = 125\ J/kg - K$

$\theta = 25^oC,\ \theta = 300^oC,\ m = 2.5 \times 10^4 J/kg$, then

$\frac{1}{2}mv^2 \times \frac{1}{2} = mS\Delta\theta + mL$

$\Rightarrow \quad \frac{v^2}{4} = 125\ (300 - 25) + 2.5 \times 10^4$

$\Rightarrow \quad \frac{v^2}{4} = 59375$

$\Rightarrow \quad v^2 = 59375 \times 4$

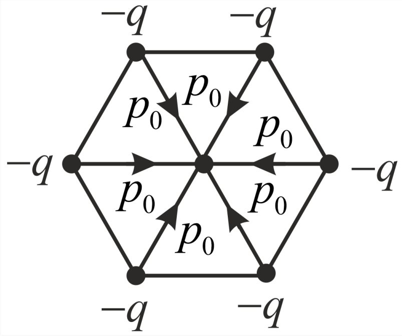
$\Rightarrow \quad v = 237500 \quad \Rightarrow \quad v = 487.3\ m/s$

$\Rightarrow \quad \approx 490\ m/s$

Q4. Solution

Correct Answer: (A)

This system is a combination of six equal dipoles each of dipole moment $p_0 = qa$



From the polygon law of vector,

$$\Sigma \overrightarrow{\mathrm{p}} = 0$$

Q5. Solution

Correct Answer: (D)

According to the equation of continuity, speed of the fluid is maximum at the minimum cross-sectional area of a horizontal tube and so will be the kinetic energy. And according to Bernoulli's theorem, the pressure will be minimum, where kinetic energy is maximum. So Assertion is false.
The reason is true, as excess pressure due to fluid is ρgh. So, for fluids of different densities, pressure can be the same at different depths.

Q6. Solution

Correct Answer: (B)

According to Newton's law of gravitation,

Gravitational force $F = G\frac{m_1 m_2}{r^2}$

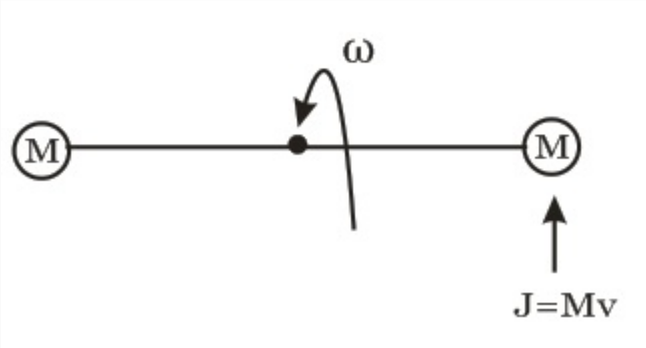
Where, $G = 6.67 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$ = Gravitational constant.

$$\Rightarrow\ G = \frac{Fr^2}{m_1 m_2} = \frac{[M^1L^1T^{-2}][L^2]}{[M^2]} \Rightarrow\ G = [M^{-1}L^3T^{-2}]$$

Q7. Solution

Correct Answer: (A)

Let ω be the angular velocity of the rod. Applying, angular impulse = change in angular momentum about centre of mass of the system



$$(M\mathrm{v})\left(\frac{L}{2}\right)=(2)\left(\frac{ML^2}{4}\right)\omega$$

$$\Rightarrow\ \omega=\frac{\mathrm{v}}{\mathrm{L}}$$

Q8. Solution

Correct Answer: (D)

Sample X is undoped. Sample Y is pentavalent as energy level is close to conduction band. Sample Z is trivalent as energy level is close to valence band.

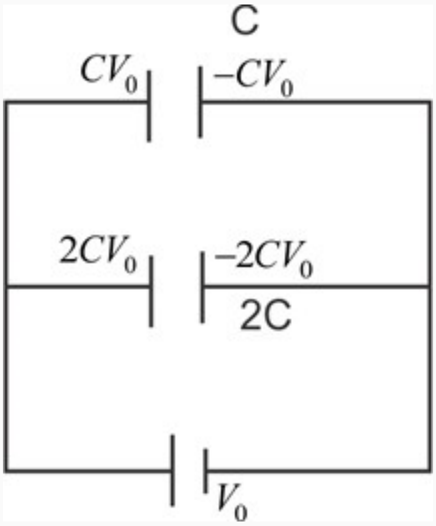
Q9. Solution

Correct Answer: (B)

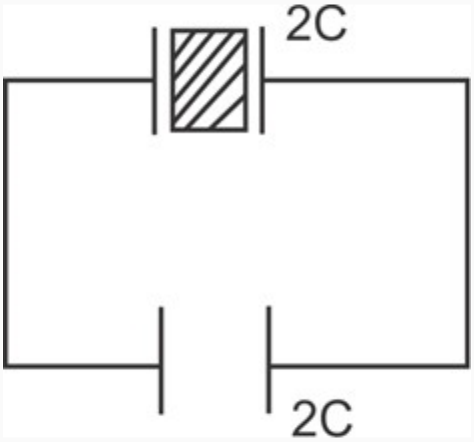
A water drop in air behaves as converging lens.

Q10. Solution

Correct Answer: (C)



Initial state



As both capacitors have the same capacitance total charge $= CV_0 + 2CV_0$
$= 3CV_0$

It will be divided equaly so $q_1 = q_2 = \frac{3CV_0}{2}$ & potential $= \frac{q}{C} = \frac{\left(\frac{3CV_0}{2}\right)}{2C} = \frac{3V_0}{4}$,

Q11. Solution

Correct Answer: (B)

$\phi = \mathrm{B\pi r}^{2} \quad \varepsilon = \frac{\mathrm{d}\phi}{\mathrm{dt}} = \mathrm{N\pi r^2}\ \frac{\mathrm{dB}}{\mathrm{dt}}$
$= \mathrm{N\pi r^2 \mu_0 n} \frac{\mathrm{di}}{\mathrm{dt}}$
$\mathrm{I} = \frac{\varepsilon}{\mathrm{R}}$ and $\Delta\mathrm{Q} = \mathrm{I}\Delta\mathrm{t} = \frac{\mathrm{N\pi r^2 \mu_0 n}}{\mathrm{R}} \Delta\mathrm{t}$
$\Delta\mathrm{Q} = \frac{100\times\pi\times\left(2\times10^{-2}\right)^2\times10^4\times4\pi\times10^{-7}\times10}{20}$
$= 8 \times 10^{-4}\mathrm{c} = 800\mu\mathrm{C}$

Q12. Solution
Correct Answer: (C)

Displacement current is given as rate of flow of charge during charging or discharging of the capacitor. It is given by

$i_d = \frac{dq}{dt}$

Given here $q = q_0 \sin(2\pi f t)$

Putting the value of charge in displacement current equation,

$i_d = \frac{d}{dt}(q_0 \sin 2\pi f t) = q_0 2\pi f \cos 2\pi f t$

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Q13. Solution
Correct Answer: (C)

For given condition we can apply direct formula $l_1 = \left(\frac{\mu}{\mu+1}\right) l$ ~

Q14. Solution
Correct Answer: (D)

Here, $\omega_1 = 3\pi$ rad s^{-1}, $I_1 = I$
$I_2 = \frac{75}{100} I_1 = \frac{3}{4} I$, $\omega_2 = ?$
As $I_2 \omega_2 = I_1 \omega_1$
$\therefore \omega_2 = \frac{I_1}{I_2} \times \omega_1 = \frac{4}{3} \times 3\pi = 4\pi$ rad s^{-1} .

Q15. Solution
Correct Answer: (A)

$F = 105$ dyne $= 105 \times 10^{-5}$ N, $T = 7 \times 10^{-2}$ Nm^{-1}
Force due to surface tension, $(2\pi r)T = F$
Circumference, $2\pi r = \frac{F}{T} = \frac{105 \times 10^{-5}}{7 \times 10^{-2}}$
$= 15 \times 10^{-3}$ m
$= 1.5 \times 10^{-2}$ m
$= 1.5$ cm ,

Q16. Solution
Correct Answer: (B)

The time period of mass,
$T=2\pi\sqrt{\frac{m}{k}}$
Given, $m=3\ \mathrm{kg}$
$k=48\ \mathrm{N\ m}^{-1}$
$T=2\pi\sqrt{\frac{3}{48}}=2\pi\sqrt{\frac{1}{16}}$
$=2\pi\times\frac{1}{4}=\frac{\pi}{2}$ ^

Q17. Solution
Correct Answer: (A)

Refractive index of certain glass $=\mu=1.5$, Wavelength of light in vacuum$=\ 6000\ \overset{o}{\mathrm{A}}$.

Wavelength of light when pass through glass is decreased due to light is travel through rare medium to denser medium.

$\lambda'=\frac{\lambda}{\mu}=\frac{6000}{1.5}=4000\ \mathring{\mathrm{A}}$

^

Q18. Solution
Correct Answer: (B)

$\mathrm{MeV}-$ sec is not a unit of energy while others are the unit of energy.

volt is a work per unit charge and coulomb is a unit of charge so, the $\mathrm{volt}-\mathrm{coulomb}$ is a unit of energy.
$\mathrm{Henry}-(\mathrm{ampere})^2$ and $\mathrm{Farad}-(\mathrm{volt})^2$ are also the units of energy.

^

Q19. Solution

Correct Answer: (C)

Here, $A_2 = 2A_1$

$\because$ Intensity α (Amplitude)^2

$\therefore \frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{2A_1}{A_1}\right)^2 = 4$

Maximum intensity, $I_m = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$= \left(\sqrt{I_1} + \sqrt{4I_1}\right)^2 = \left(3\sqrt{I_1}\right)^2 = 9I_1$

or $I_1 = \frac{I_m}{9}$ ......(i)

Resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$= I_1 + 4I_1 + 2\sqrt{I_1(4I_1)}\cos\phi$

$= 5I_1 + 4I_1\cos\phi = I_1 + 4I_1 + 4I_1\cos\phi$

$= I_1 + 4I_1(1 + \cos\phi)$

$= I_1 + 8I_1\cos^2\frac{\phi}{2}$ $\left(\because 1 + \cos\phi = 2\cos^2\frac{\phi}{2}\right)$

$= I_1\left(1 + 8\cos^2\frac{\phi}{2}\right)$

Putting the value of I_1 from eqn. (i), we get

$I = \frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$!

Q20. Solution

Correct Answer: (A)

$$\frac{\Delta p}{p} = \frac{\Delta m}{m} + \frac{\Delta V}{V}$$

$$= \frac{0.05}{5} \times 100 + \frac{0.05}{1} \times 100$$

$$= 6$$

$$\frac{\Delta p}{p} = 6\%$$

Q21. Solution

Correct Answer: (A)

Given,

The maximum wavelength of light, which increases the conductivity, $\lambda = 600$ nm

Hence the energy of this photon must be equal to band gap (E_g).

$\therefore E_g = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{600\times10^{-9}}$ J

$= \frac{6.6\times3\times10^{-17}}{600\times1.6\times10^{-19}}$ eV $= 2.06$ eV

Q22. Solution

Correct Answer: (D)

According to Kepler's law $T^2 \propto R^3$ If N is the frequencs then $N^2 \propto (R)^{-3}$ or

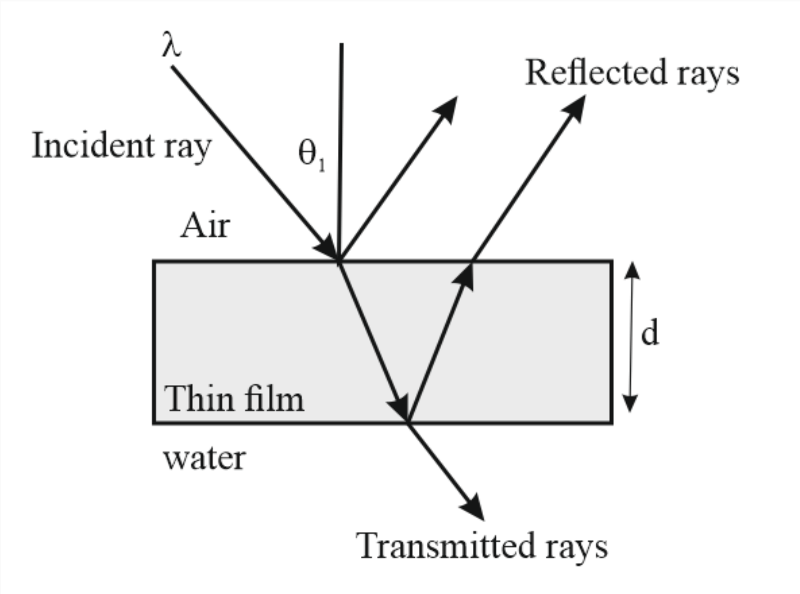
$\frac{N_2}{N_1} = \left(\frac{R_2}{R_1}\right)^{-3/2} \Rightarrow \frac{R_1}{R_2} = \left(\frac{N_2}{N_1}\right)^{2/3}$,

Q23. Solution
Correct Answer: (B)

Given,

Refractive index of the oil, $\mu_{\text{oil}} = \frac{4}{3}$

The wavelength of the light, $\lambda = 9.6 \times 10^{-7}$ m



When the light will fall on the film, some part of the light will be reflected from the upper surface and some part from the lower surface and rest will be transmitted.

The two reflected parts of light will interfere according to their path difference created by the oil film.

For normal incidence, the extra path difference created will be given by,

$\Delta x = (2d)\mu_{\text{oil}}$, where d is the thickness of the oil film.

For destructive interference(dark), $\Delta x = (2n-1)\frac{\lambda}{2}$.

For constructive interference(bright), $\Delta x = n\lambda$.

For minimum thickness increment for changing dark to bright will be, $2\mu\,\Delta\,d = \frac{\lambda}{2}$.

$\Delta t = \frac{\lambda}{4\mu_{\text{oil}}} = \frac{9.6\times10^{-7}}{4\times1.2} = 2\times10^{-7}\text{m}$

Q24. Solution
Correct Answer: (A)

$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{(127+273)}{(227+273)} = \frac{1}{5}$
$W = \eta Q_1 = \frac{1}{5} \times 10^4 \text{ J} = 2000 \text{ J}$

Q25. Solution

Correct Answer: (A)

Since the rope has a mass, the tension along its length is variable. At the top end of its tension is (3 + 1)g N or 4g N. At the bottom end, its tension is 1g N.

Now, speed of the transverse wave on the string, $c = \sqrt{\frac{T}{\mu}}$

$$v\lambda = \sqrt{\frac{T}{\mu}} \qquad (\because \quad c = v\lambda)$$

As frequency v and μ, the linear density are constant, so $\frac{\sqrt{T}}{\lambda}$ is a constant.

$$\frac{\sqrt{T_{Top}}}{\lambda_{Top}} = \frac{\sqrt{T_{Bottom}}}{\lambda_{Bottom}}; \quad \frac{\sqrt{4g}}{\lambda_{top}} = \frac{\sqrt{1g}}{0.05 \text{ m}}$$

$$\Rightarrow \quad \lambda_{top} = 0.05 \times 2 \text{ m} = 0.1 \text{ m}.$$

Q26. Solution

Correct Answer: (A)

$$\overrightarrow{DA} = -2\cos 30^{o}\hat{i} - 2\sin 30^{o}\hat{k} = \left(-\sqrt{3}\hat{i} - \hat{k}\right)$$

$$\overrightarrow{AB} = 2\hat{j}$$

$$\therefore \vec{M} = i\left(\overrightarrow{DA} \times \overrightarrow{AB}\right)$$

$$= \frac{1}{2}\left[\left(-\sqrt{3}\hat{i} - \hat{k}\right) \times \left(2\hat{j}\right)\right]$$

$$= -\sqrt{3}\hat{k} + \hat{i}$$

$$= \left(\hat{i} - \sqrt{3}\hat{k}\right) \text{A-m}^2$$

Q27. Solution

Correct Answer: (C)

We have,

$\overrightarrow{\text{B}}$ is the resultant magnetic field,

$\overrightarrow{\text{H}}$ is magnetic intensity,

$\overrightarrow{\text{I}}$ is the intensity of magnetization.

We know that,

$B = \mu_0(H + I)$
$\because \ I = \chi_m H$
$\Rightarrow B = \mu_0 H(1 + \chi_m) \ \ \ldots (1)$
also $B = \mu H \ \ldots (2)$

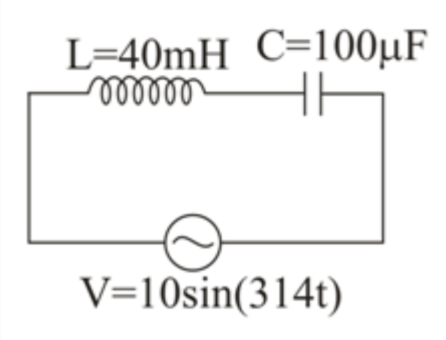
From Eqs. (1) and (2),
$\mu = \mu_0(1 + \chi_m) \ \ \ldots (3)$
Also,

$\mu = \mu_0 \mu_r \ \ldots (4)$

From Eqs. (3) and (4),
$\mu_r = 1 + \chi_m$
So, the only wrong relation is $\mu_0 = \mu(1 + \chi_m)$.

Q28. Solution

Correct Answer: (A)



$z = \sqrt{R^2 + (X_C - X_L)^2}$

$R = 0$

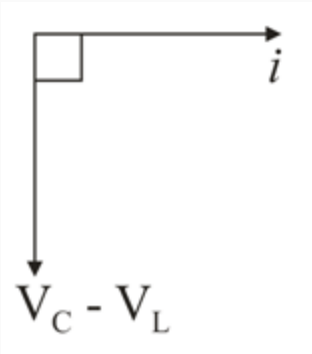
$Z = X_C - X_L$

$= \frac{1}{\omega C} - \omega L$

$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$

$= 31.84 - 12.56$

$= 19.28\ \Omega$



$i = \frac{V_0}{Z} \sin\left(314t + \frac{\pi}{2}\right)$

$\therefore i = \frac{V_0}{Z} \cos(314t) \Rightarrow i = \frac{10}{19.28} \cos(314t)$

$\Rightarrow i = 0.52 \cos(314t)$

Q29. Solution

Correct Answer: (B)

According to activity law, $R = R_0 e^{-\lambda t}$

$\therefore$ $R_1 = R_0 e^{-\lambda t_1}$ and $R_2 = R_0 e^{-\lambda t_2}$

$\therefore \quad \frac{R_1}{R_2} = \frac{R_0 e^{-\lambda t_1}}{R_0 e^{-\lambda t_2}} = e^{-\lambda t_1}\ e^{\lambda t_2} = e^{-\lambda(t_1 - t_2)}$

or $\mathrm{R_1 = R_2 e^{-\lambda(t_1 - t_2)}}$

Q30. Solution
Correct Answer: (C)

Here, $E_1 = E_2$
$n_1 h v_1 = n_2 h v_2$
So, $\frac{n_1}{n_2} = \frac{v_2}{v_1}$

Q31. Solution
Correct Answer: (C)

After addition of Cd it oxidises into Cd^{2+}
$\mathrm{NO_3^-(aq)} \quad + \quad \mathrm{4H^+(aq)} \quad + \quad \mathrm{3e^-} \quad \rightarrow \quad \mathrm{NO(g)} \quad + \quad \mathrm{2H_2O}(l)$
$0.1 - x \qquad\qquad 0.4 - 4x$
X = 0.06
$[\mathrm{NO_3^-}]$ remaining$= 0.1 - 0.06 \approx 0.04$ M
$[\mathrm{H^+}]$ remaining$= 0.4 - 4 \times 0.06 = 0.4 - 0.24 = 0.16$ M
$\mathrm{E_{NO_3^-/NO}} = \mathrm{E^\circ_{NO_3^-/NO}} - \frac{0.591}{3} \log \frac{1}{[\mathrm{NO_3^-}][\mathrm{H^+}]^4}$
$= 0.95 - \frac{0.0591}{3} \log \frac{1}{(0.04)(0.16)^4} = 0.95 - \frac{0.0591}{3} \log \frac{1}{4 \times 10^{-2} \times 10^{-8} \times 16^4}$
$= 0.95 - \frac{0.0591}{3} \log \frac{1}{4 \times 10^{-2} \times 10^{-8} \times 2^{+16}}$
$= 0.95 - \frac{0.0591}{3} \log \frac{10^{10}}{2^2 \times 2^{16}}$
$= 0.95 - \frac{0.0591}{3} [10 - 18 \times 0.3010] = 0.95 - \frac{0.0591}{3} \times 4.582$
$= 0.95 - 0.09 = 0.86$ V

Q32. Solution
Correct Answer: (A)

$\mathrm{A} \rightarrow \mathrm{IV}, \mathrm{B} \rightarrow \mathrm{II}, \mathrm{C} \rightarrow \mathrm{V}, \mathrm{D} \rightarrow \mathrm{I}$ Methenaglobinemia is a condition caused by elevated levels of methenoglobin in the blood is due to the presence of 2000ppm or above nitrate in drinking water. Kidney damage is due to the presence of 1000ppm of lead in drinking water. Lead get deposit in the kidney and damages it. Bones and teeth damages due to presence of 50ppm or above fluoride in drinking water. Less than 1 ppm of dissolved oxygen in water can stop the growth of fish due to less supply of oxygen.

Q33. Solution
Correct Answer: (C)

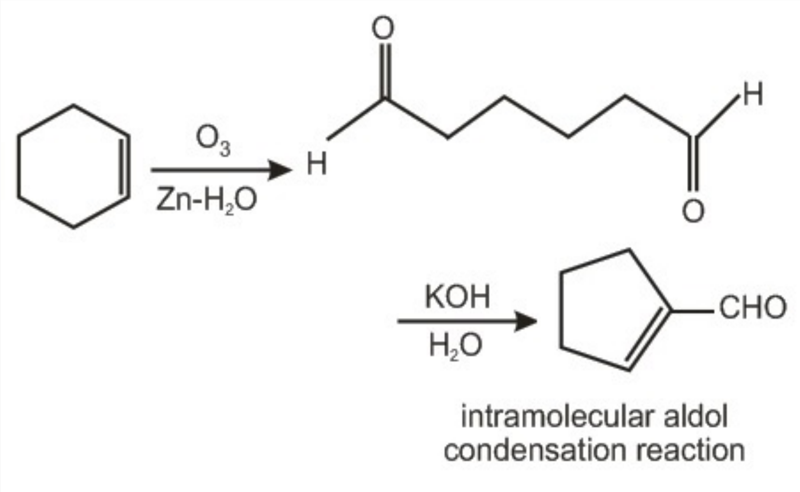
Chromatography method is used to separate sugars.They are separated by the chromatographic adsorption of their coloured esters.

Q34. Solution
Correct Answer: (A)

Hints: $\frac{1}{\lambda} = \mathrm{z}^2 \cdot \mathrm{R_H} \left[1/\mathrm{n}_1^2 - 1/\mathrm{n}_2^2\right]$ $\Rightarrow \frac{1}{\lambda} = (\mathrm{z})^2 \cdot \mathrm{R_H} \left\{\frac{1}{1} - \frac{1}{4}\right\} = \frac{3}{4} \mathrm{R_H z^2}$ $\therefore \lambda \propto 1/\mathrm{z}^2$ Hence, for shortest λ, z must be maximum, which is for Li^{+2}.

Q35. Solution

Correct Answer: (A)



Q36. Solution

Correct Answer: (C)

$$C_6H_5NH_2 \xrightarrow{NaNO_2/HCl} \underset{(X)}{C_6H_5N_2^+Cl^-} \xrightarrow{HNO_2} \underset{(Y)}{C_6H_5NO_2} + N_2 + HCl$$

Q37. Solution

Correct Answer: (D)

The value of lattice energy depends on the charges present on the two ions and distance between them. It shall be high if charges are high and ionic radii are small.

Q38. Solution

Correct Answer: (D)

$t_{1/2} \propto \frac{1}{a^{n-1}}$

$\frac{t_2}{t_1} = \left(\frac{a_1}{a_2}\right)^{n-1}$

$\frac{160}{320} = \left(\frac{58}{29}\right)^{n-1}$

n = 0.

Q39. Solution

Correct Answer: (D)

$\Delta G^o = \Delta H^o - T\Delta S^o = -54.07 \times 1000 - 298 \times 10 = -57050\ Jmol^{-1}$

$\Delta G^\circ = -2.303RT\log_{10}K$

$-57050 = -5705\log_{10}K$

$\log_{10}K = 10$

Q40. Solution

Correct Answer: (A)

Wilkinson's catalyst is $[(Ph_3P)_3RhCl]$. Here Rh assumes dsp^2 hybrid state and the geometry is square planar.

Q41. Solution

Correct Answer: (A)

$$\mathrm{Li\ (g) \rightarrow Li^{+3} + 3e^-\ ;\quad IP_I + IP_{II} + IP_{III} = 203 \cdot 1\ ev/species}$$
$$\mathrm{Li \rightarrow Li^+(g) + 1e^-;\quad 5 \cdot 4eV/species\ (IP_I)}$$
$$\mathrm{Li^+(g) \rightarrow Li^{2+} + 1e^-;\quad IP_{II} = ??}$$
$$\mathrm{Li^{+2} \rightarrow Li^{+3}\ (g);\quad IP_{III} = IE_{Li^{2+}} = 13 \cdot 6 \times (3)^2 = 122.4\ eV/species}$$

Q42. Solution

Correct Answer: (A)

NH_4NO_3 will evolve NH_3 pungent smelling gas with NaOH. - NO_2 brown gas with con. H_2SO_4 - No reaction with water Thus, NH_4NO_3 can be used to label all three beakers. (c) $(NH_4)_2CO_3$ will evolve pungent smelling gas NH_3 with NaOH - Effervescence of CO_2 gas with con. H_2SO_4 - No reaction with water. Thus, $(NH_4)_2CO_3$ can be used to label all three beakers. Thus, options (a, c) are correct.

Q43. Solution

Correct Answer: (A)

$$\underset{(X)}{C_2H_5OH} + [O] \xrightarrow[\text{in } CH_2Cl_2]{PCC} \underset{(Y)}{CH_3CHO}$$

$$\underset{(Y)}{CH_3CHO} + 4NaOH + 3I_2 \longrightarrow CHI_3 + HCOONa + 3H_2O + 3NaI$$

Q44. Solution

Correct Answer: (C)

The Si, Ge and Sn belong to third, fourth and fifth period of group 14 of periodic table. Moving down the group, size of atoms increases, so the bond length order is $Si - Si < Ge - Ge < Sn - Sn$ Bond enthalpy is inversely proportional to bond length, so bond enthalpy order is $\begin{matrix} Si - Si < Ge - Ge < Sn - Sn \\ 297 > 260 > 240. \left(kJmol^{-1}\right) \end{matrix}$

Q45. Solution

Correct Answer: (B)

$$NaCN \quad + \quad ZnS \quad \rightarrow \quad \underset{\text{Soluble complex}}{Na_2[Zn(CN)_4]}$$

A layer of this zinc complex is formed on the surface of ZnS and due to this ZnS is prevented from the froth formation while PbS form froth. (i.e., NaCN is added as depressant for ZnS).

Q46. Solution

Correct Answer: (D)

In XeF_2, XeF_4 and XeF_6(g), the number of lone pairs on Xe respectively are 3,2 and 1 . The number of bond pairs are 2,4 and 6 respectively. The total number of valence electrons on Xe are 8 .

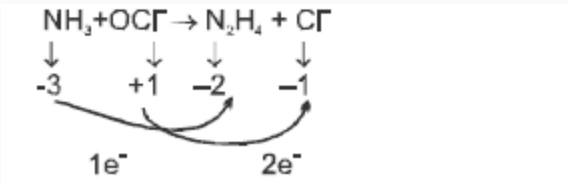
Q47. Solution

Correct Answer: (C)

The most unreactive metal that the ion which has the most positive reduction potential will be released to first. Further Li^+ has a very negative reduction potential hence it will not be released in aqueous medium. so after Cu^{+2}, H_2 will evolve at cathode

Q48. Solution

Correct Answer: (A)



$2\,NH_3 + OCl^- \longrightarrow N_2H_4 + Cl^- + H_2O$

Coefficient of $N_2H_4 = 1$

Q49. Solution

Correct Answer: (D)

Tyndal effect is due to Scattering of light and not due to charge.

Q50. Solution

Correct Answer: (B)

Lowering in freezing point is a colligative property, colligative property depends on the number of molecule atom or ions and molality. In 0.1 m NaCl. Molality is maximum out of four and NaCl dissociates into Na^+, Cl^- (i = 2).

Colligative property $\propto$ m $\times$ i

So, 0.1 m NaCl > 0.1 m C_2H_5OH > 0.01 m NaCl > 0.01 m C_2H_5OH

Q51. Solution

Correct Answer: (D)

Fe^{2+} due to presence of 4 unpaired electrons.

Q52. Solution
Correct Answer: (A)

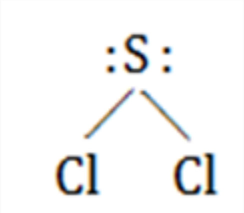
the organic substance being more soluble in organic solvent in transferred from aqueous layer to the organic layer.

Q53. Solution
Correct Answer: (B)

$H_2 + Cl_2 \longrightarrow 2HCl, \quad \Delta H = -194$ kJ. Here, 2 molecules of HCl are formed and the value of ΔH is -194 kJ. Hence, the change in enthalpy (ΔH) for the formation of one molecule of HCl $= \frac{-194}{2} = -97$ kJ

Q54. Solution
Correct Answer: (B)

In SCl_2 there are two lone pair and two bond pairs around so SCl_2 has bent shape and hybridisation state of sulphur is sp^3 .



Q55. Solution
Correct Answer: (C)

Concentration will fall from $0.1M$ concentration to $0.025M$ concentration within 2 half lives.
$2 \times T_{1/2} = 40$ min
$\therefore T_{1/2} = 20$ min
Rate of reaction $= K.c = \frac{0.693}{T_{1/2}}.c = \frac{0.693}{20} \times 10^{-2}M/\min = 3.47 \times 10^{-4}M/\min^{-1}$.

Q56. Solution
Correct Answer: (B)

Redius $=$ Na $>$ Mg $>$ Al $>$ Si
Moving through period, atomic radii decreases due to more Z_{eff} across period from left to right.

Q57. Solution
Correct Answer: (D)

The stability of carbocation is decided by the following order,

$+\mathrm{M}-\text{effect} \; > \text{Hyperconjugation} \;\; \text{effect} \; > +\mathrm{I}-\text{effect} > -\mathrm{I}\text{ effect} > -\mathrm{M}-\text{effect}$

I. $\underset{+\mathrm{I}-\text{effect}}{\mathrm{CH_3}-\overset{+}{\mathrm{C}}\mathrm{H}-\mathrm{CH_3}}$ II. $\underset{+\mathrm{M}-\text{effect}}{\mathrm{CH_3}-\overset{+}{\mathrm{C}}\mathrm{H}-\mathrm{OCH_3}}$ III. $\underset{-\mathrm{M}-\text{effect}}{\mathrm{CH_3}-\overset{+}{\mathrm{C}}\mathrm{H}-\mathrm{COCH_3}}$

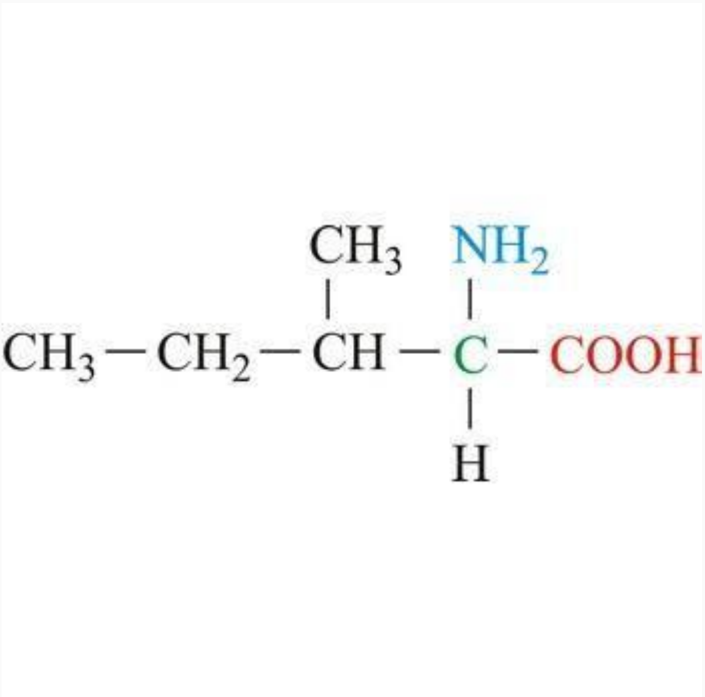
Therefore, the decreasing order of stability of the ions is $\mathrm{II} > \mathrm{I} > \mathrm{III}$.

Q58. Solution
Correct Answer: (A)

For the given crystal,
$\mathrm{a} = \mathrm{b} \neq \mathrm{c}, \alpha = \beta = 90^{\circ}, \gamma = 120^{\circ}$
These are the characteristic of a hexagonal system.

Q59. Solution
Correct Answer: (C)

$\mathrm{C_6H_{13}NO_2}$



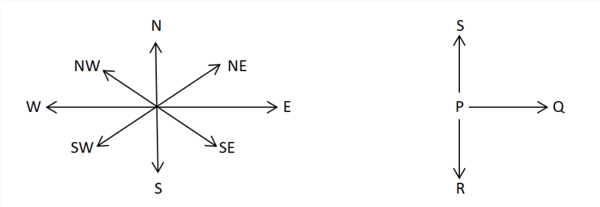
Q60. Solution
Correct Answer: (B)

The hydration enthalpies of alkali metals decreases with increase in ionic radii. Since Li has low ionic radius it has the highest hydration enthalpy.

Ionic radius increases as we go down the group due to the addition of new energy shell with each corresponding element. Since Rb has the highest number of energy shells it has the highest ionic radius.

Q61. Solution
Correct Answer: (C)

As it is given that P is standing in the middle of all the players such as Q is to the East of P, R is to the south of P and S is to the North of P, i.e.



Therefore, the direction of S with respect to Q is North-West.

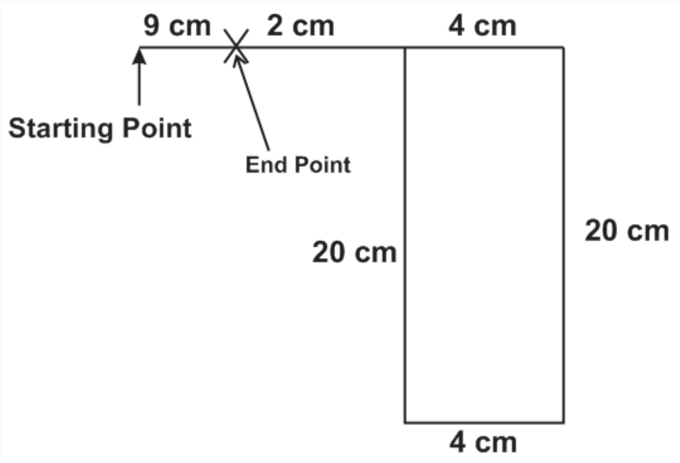
Hence, the correct answer is North-West.

Q62. Solution

Correct Answer: (A)

According to the given information, An insect walks 15 cm East, then it turns to its right and walks for another 20 cm in the South direction, then it turns right and walks 4 cm in the West direction, then it turns North and walks 20 cm, then finally it turns to its left and walks 2 cm in the West direction.

The following direction diagram can be made,

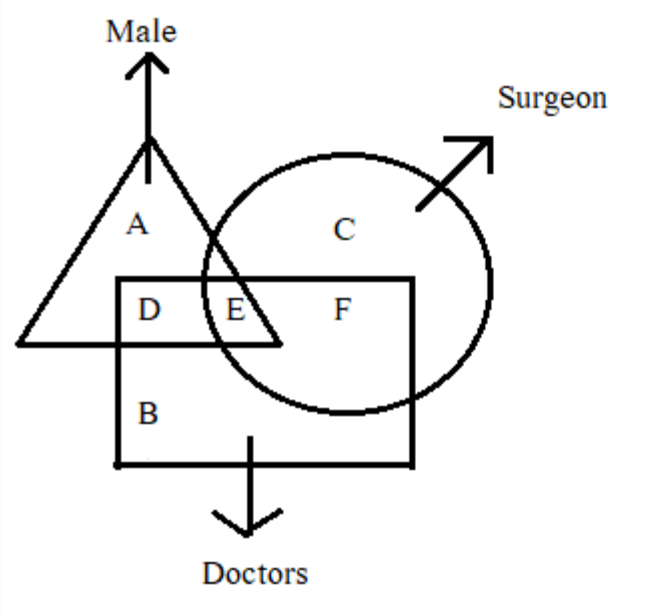


So, the endpoint is towards the East of the starting point.

Hence, the correct answer is 9 cm East.

Q63. Solution
Correct Answer: (A)

According to the given question, the Venn diagrams have to be interpreted to find the common part. From the given figure we can very well observe that:



A is the area representing the male population who are neither doctors nor surgeons. B is the area depicting the doctors who are neither male nor surgeons.

E is the area that is common between circle, triangle and square, therefore, the males who are doctors, as well as surgeons, are represented by E.

Hence, the correct answer is E.

Q64. Solution
Correct Answer: (A)

Diabetes is a hormonal disease.

Smallpox is an infection caused by the variola virus.

Conjunctivitis is a viral or bacterial infection.

Chickenpox is caused by a virus called varicella zoster.

Plague is caused by the bacterium, Yersinia pestis.

Hence, all the diseases are either viral or bacterial except Diabetes.

Hence, Diabetes is correct.

Q65. Solution
Correct Answer: (D)

$$100, \xrightarrow{\div(-5)} -20, \xrightarrow{\div(-5)} 4, \xrightarrow{\div(-5)} -0.8, \xrightarrow{\div(-5)} 0.16, \xrightarrow{\div(-5)} -0.032$$

Q66. Solution
Correct Answer: (A)

$(8 \times 9) - 3 = 69; (7 \times 5) - 6 = 29$
$(4 \times 7) - 9 = 19; (9 \times 8) - 4 = 68$

Q67. Solution
Correct Answer: (D)

1. M is to the immediate right of $R : RM\_\_\_\_$. 2. O is between M and $P : RMOPP\_\_$. 3. N is third to the left of $P : RN\_PQM$. So, the full arrangement becomes: R N Q P O M. Q is sitting opposite to P.

Q68. Solution
Correct Answer: (D)

We have to calculate the number representing the word BODY,

B$= 32$

O$= 65$ or 66 or 85 or 86

D$= 30$

Y$= 87$

So, $32,\ 65,\ 30,\ 87$ is correct answer

Q69. Solution
Correct Answer: (D)

After merging the first and second column, we can see that the circles which are common are carried forward to the third column.
Now, in the D option, we can see the same logic which is after merging the first and second column, we can see that only one circle is common and that is carried forward in the third column. So, option D is the right answer.

Q70. Solution

Correct Answer: (A)

The fig. (X) is similar to Form I. So, when the sheet shown in fig. (X) is folded to form a box (cuboid), then the two rectangular-shaded faces lie opposite to each other, two rectangular white faces lie opposite to each other and the two square shaped faces (one shaded and one white) lie opposite to each other. Clearly, the cuboids shown in figures (2) and (4) cannot be formed as in each of the two cuboids the two shaded rectangular faces appear adjacent to each other. So, only the cuboids in figures (1) and (3) can be formed.

Q71. Solution

Correct Answer: (A)

Total number of Q type cars produced in 2001 $= (60-40)\%$ of $4,40,000 = 88,000$. Total number of Q type cars produced in 2000 $= (45-30)\%$ of $3,50,000 = 52,500$. $\therefore$ Required difference $= (88000 - 52500) = 35,500$.

Q72. Solution

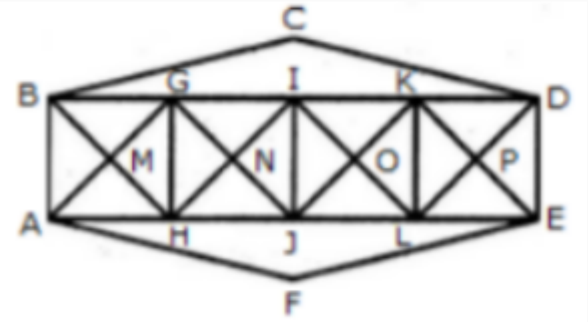
Correct Answer: (B)

If the percentage production of P type cars in 2001 $\begin{aligned} &= \text{ Percentage production of } P \text{ type cars in 2000} \\ &= 30\%. \end{aligned}$ then,

number of P type cars produced in 2001 $\begin{aligned} &= 30\% \text{ of } 4,40,000 \\ &= 1,32,000 \end{aligned}$

Q73. Solution

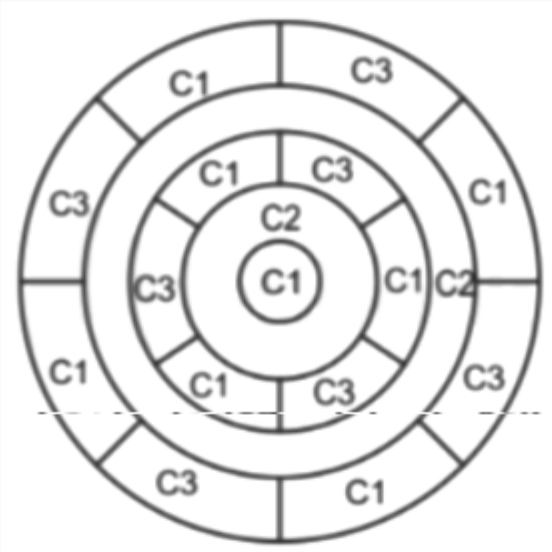
Correct Answer: (C)

The figure may be labelled as shown. 

Triangles: The simplest triangles are BGM, GHM, HAM, ABM, GIN, UN, JHN, HGN, IKO, KLO, LJO, JIO, KDP, DEP, ELP, LKP, BCD and AFE i.e. 18 in number. The triangles composed of two components each are ABG, BGH, GHA, HAB, HGI, GIJ, IJH, JHG, JIK, IKL, KLJ, LJI, LKD, KDE, DEL and ELK i.e. 16 in number. The triangles composed of four components each are BHI, GJK, ILD, AGJ, HIL and JKE i.e. 6 in number. Total number of triangles in the figure $= 18 + 16 + 6 = 40$. Squares : ☐ The squares composed of two components each are MGNH, NIOJ and OKPL i.e. 3 in number. The squares composed of four components each are BGHA, GIJH, IKLJ and KDEL i.e. 4 in number. Total number of squares in the figure $= 3 + 4 = 7$.

Q74. Solution

Correct Answer: (C)



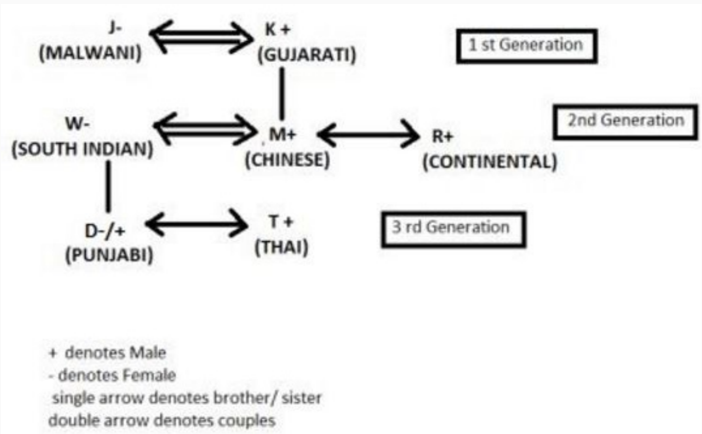
AS shown in the figure above minimum three colours C1, C2 and C3 are required to paint the figure given above such that no two adjacent regions have the same colour. Hence, 3 is the correct answer.

Q75. Solution

Correct Answer: (D)

Members: M, K, J, T, R, D, W
Cuisine: Chinese, Continental, Thai, Punjabi, South India, Gujarati, Malwani

The given diagram below illustrates the relation among the family members. It shows the members of the first generation, second generation, and third generation respectively. The one which has '+' sign are the male members and the one which has '-' sign are the female members.



As per the family tree, K, M, and R are the male members, but D can either be male or female since it cannot be determined.

So, the data is inadequate to count the total male members in the family.

Hence, this is the correct answer.

Q76. Solution
Correct Answer: (C)

According to the given information, A and B are brothers. C and D are sisters. The son of A is the brother of D. A is the brother of B. Both A and B are male. C and D are sisters. Both C and D are female.

Since the son of A is the brother of D. So, A is the father of C and D. Now, since B is the brother of A. Therefore, B is the uncle of C.
Hence, this is the correct answer.

Q77. Solution
Correct Answer: (B)

According to the given statement,

Laughter is the best medicine.

Except for the second conclusion, all others are irrelevant because the statement implies that laughter is something that is beneficial from a health perspective. Laughing triggers healthy physical and emotional changes in the body.

Hence, laughter is good for health, is the correct answer.

Q78. Solution
Correct Answer: (A)

Given series:

$6, 12, 48, 264, ?$

By observing closely, we find the following pattern:

$6 + 6 = 12$

$12 + (6)^2 = 48$

$48 + (6)^3 = 264$

$264 + (6)^4 = 1560$

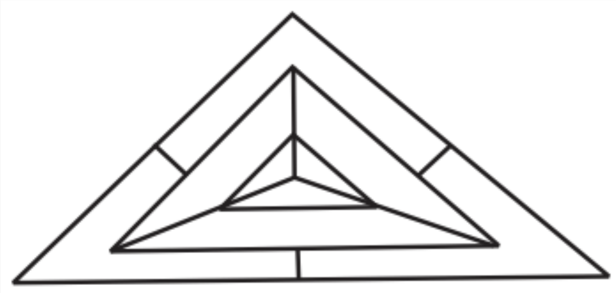
The missing number in the series is 1560.

Hence, option A is the correct answer

Q79. Solution

Correct Answer: (B)

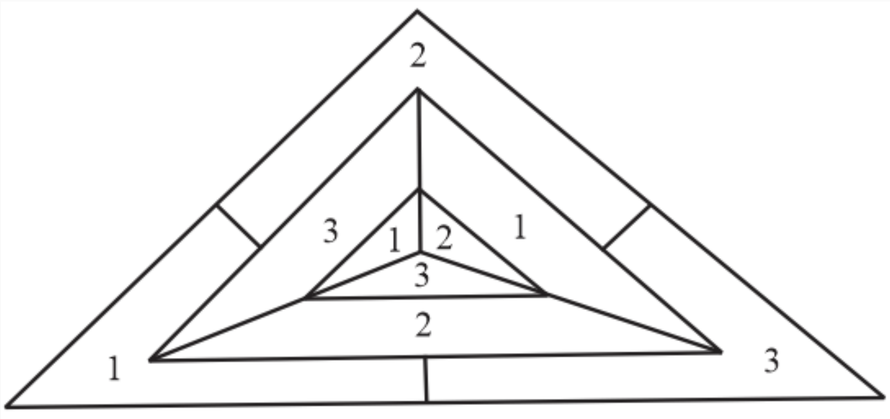
Given figure,



Here, we have to find out the minimum number of colors required to color the entire figure if no adjacent faces have the same color.

Here, we can see that each section to the given triangle is divided into three parts.

Now, if we pick three colors 1, 2, and 3 to fill the triangle the following figure will be obtained,



Hence, the correct answer is 3.

Q80. Solution
Correct Answer: (C)

According to the question,

Given information is, $11,\ 33,\ 30,\ 90,\ 87,\ ?$

Now we find the next term of given series,

So, The pattern used in this series is, $\times 3,\ \ -3,\ \ \times 3,\ \ -3\ldots\ldots$

$33\ =\ 11\ \times\ 3, 30\ =\ 33 - 3, 90\ =\ 30 \times 3,\ 87\ =\ 90 - 3,\ ?\ =\ 87\ \times\ 3\ =\ 261$

So the next term of this series is 261.

Hence, the answer is 261.

Q81. Solution
Correct Answer: (C)

Correct answer: good person who helps all

The idiom 'good Samaritan' means "a compassionate person who helps everyone without seeing the limits and egos". Any person who is selfless and treats everyone in same kind is called a 'Good Samaritan'. In other words, we call them generous.

Example - Sonu Sood worked as a Good Samaritan during covid -19.

Q82. Solution
Correct Answer: (A)

Correct answer: mischief, trickery

The phrase 'hanky-panky' means 'activities done in a secret, concealed and in an enclosed manner.' It means to behave in a dishonest or unacceptable way. Generally, a negative feeling word as this phrase hides the wrong activities done secretly like cheating and carries the similar words like 'trickery and mischief'.

Example - This group is famous in a negative way for its hanky-panky behaviour.

Q83. Solution
Correct Answer: (C)

In the given question, the given statement is 'An annually conducted nationwide survey by a leading health research organisation shows a continuing marked decline in the use of illegal drugs like hashish and charas by high school seniors over the last five years.'

It is required to answer In using the results of the survey described above. In order to make conclusions about illegal drug use in the teenage population as a whole, which of the following, if true, casts most doubt on the relevance of the survey results.

To find the required solution, read the statement carefully. Identify the reason and find the conclusions that can be made from this.

The statement is in present tense and the answer is 'Illegal drug use by teenagers is highest in those areas of the country where teenagers are least likely to stay in high school for their senior year.'

Q84. Solution
Correct Answer: (D)

This question depends on understanding the passage as a whole. The passage begins by describing a long-held belief regarding humans' circadian rhythms: that the SCNs control them. It then goes on to explain that new findings have led scientists to believe that other organs and tissues may be involved in regulating the body's circadian rhythms as well. A The passage does not challenge the morerecent findings. Furthermore, the recent findings that the passage recounts do not contradict earlier findings; rather, when placed alongside those earlier findings, they have led scientists to reach additional conclusions. B The passage does not discuss a two-sided debate; no findings or conclusions are disputed by any figures in the passages. C There is only one question at issue in the passage: whether the SCN alone control human circadian rhythms. Furthermore, nothing in the passage suggests that researchers have been puzzled for a long time about this. D Correct. The new evidence regarding circadian rhythm-related gene activity in all the body's tissue has led scientists to revise their long-standing belief that the SCN alone control circadian rhythms.

Q85. Solution
Correct Answer: (A)

Rave refers to talking incoherently as if one were mad or out of his mind. 'Talk wildly' is correct as talk wildly means conversing in an aggressive and impolite manner.

'Influence' is incorrect because influence is to cause-effect over other's thinking and doing things. 'Talk mildly' is incorrect because talk mildly is to talk softly in a low voice/tone. 'Encourage' is incorrect because encourage means to motivate positively.

Hence, 'Talk wildly' is correct.

Q86. Solution
Correct Answer: (A)

‘Effete’ refers to something which is weak and is no longer capable of effective action. ‘Weak’ means lack of strength and capacity.

‘Strong’ is incorrect as strong means being capable of doing something. ‘Result’ is incorrect as a result denotes the outcome of an activity. ‘Effect’ is incorrect as effect means the change caused by activity.

Therefore, 'Weak' is the most suitable option for the given the word 'Effete'.

Q87. Solution
Correct Answer: (D)

This question asks about what is NOT specifically mentioned in the passage with regard to functions regulated by the SCN. Those functions, as identified in the passage, are blood pressure, body temperature, activity level, alertness, and the release of melatonin. A The passage includes activity level in its list of functions regulated by the SCN. B The passage includes blood pressure in its list of functions regulated by the SCN. C The passage includes alertness in its list of functions regulated by the SCN. D Correct. While the passage does say that cells in the human retina transmit information to the SCN, there is no suggestion that the SCN reciprocally control vision.

Q88. Solution
Correct Answer: (B)

The word 'calumny' indicates to the producing of false statements that damages someone's else's reputation.
From the given options:
The word 'Apology' indicates to something that is a very poor example of a particular object or matter.
The word 'Eulogy' indicates to the showing of praise and acclamation.
The word 'enjoyment' indicates to the moment full of joy.
The word 'reservation' indicates to the prebooking of a particular thing.
Hence, the required antonym is 'Eulogy'.

Q89. Solution
Correct Answer: (A)

The simple future is expressed as:
‘will’ + first form of verb
The doer is ‘we’ (plural), therefore ‘execute’ (plural) fits in the sentence.

Hence, the correct answer is 'execute' which is the first form of the verb.

Q90. Solution
Correct Answer: (A)

$2\cos^2\theta+\cos\theta+1=2\left(\cos^2\theta+\frac{\cos\theta}{2}+\frac{1}{2}\right)$
$2\left\{\left(\cos\theta+\frac{1}{4}\right)^2+\frac{7}{16}\right\}$
Given expression is maximum when $\cos\theta=1$ and minimum when $\cos\theta=-\frac{1}{4}$
$\Rightarrow M=2\left(\left(\frac{5}{4}\right)^2+\frac{7}{16}\right)=2\left(\frac{32}{16}\right)=4$

and $m=2\left(\frac{7}{16}\right)=\frac{7}{8}$

Hence, $\left[\frac{M}{m}\right]=\left[\frac{32}{7}\right]=4$

Q91. Solution
Correct Answer: (B)

Use 'a' with consonant sound.
We use articles as per the sound of words. The words beginning with the sound of A, E, I, O, U, i.e., vowel sound take article "an" before them, while those beginning with consonant sounds take article "a" before them. ‘European’ is pronounced as ‘youropean’, as ‘y’ is not a vowel; therefore, the article to be used is ‘A’.

Q92. Solution
Correct Answer: (A)

Equation of plane containing the line of intersection of planes is, $(2x-y)+\lambda\,(y-3z)=0$ ......(i)

Also, plane (i) is perpendicular to $4x+5y-3z-8=0$

$\therefore\ 4\,(2)+5\,(\lambda-1)-3\,(-3\lambda)=0$

$\Rightarrow 14\lambda=-3\Rightarrow\lambda=-\frac{3}{14}$

Put the value of λ in (i), we get $28x-17y+9z=0$, which is the required plane.

Q93. Solution
Correct Answer: (D)

If matrix has no inverse it means the value of determinant should be zero.

$$\therefore\qquad \begin{vmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{vmatrix}=0$$

As we know if any two rows or columns are identical then the value of determinant is zero.
So, if we put $x=1$, then column Ist and IIIrd are identical

Q94. Solution

Correct Answer: (B)

Since, $\frac{\pi}{4} < 1$

$\therefore \left[\frac{\pi}{4}\right] = 0$

$\therefore \lim_{x \to \frac{\pi}{2}} \frac{\left[\frac{x}{2}\right]}{\ln(\sin x)} = \lim_{x \to \frac{\pi}{2}} \frac{0}{\ln(\sin x)} = 0$

Q95. Solution

Correct Answer: (B)

Given, the length of latus rectum$=\frac{1}{3}\times$length of major axis

$\therefore \ \frac{2b^2}{a} = \frac{1}{3}(2a)$

$\Rightarrow 6b^2 = 2a^2$

$\Rightarrow \ 3a^2\left(1 - e^2\right) = a^2, \ \ \left(\because \ b^2 = a^2\left(1 - e^2\right)\right)$

$\Rightarrow 3e^2 = 2$

$\Rightarrow \ e = \sqrt{\left(\frac{2}{3}\right)}$, eccentricity is always positive of an ellipse.

Q96. Solution

Correct Answer: (A)

Let $f\left(x\right) = 1 - \sqrt{1+x^2} + x \log_e\left(x + \sqrt{x^2+1}\right)$

Differentiate both sides w.r.t. x,

$\Rightarrow f'\left(x\right) = -\frac{x}{\sqrt{1-x^2}} + \log_e\left(x + \sqrt{x^2+1}\right) + \frac{x\left(1+\frac{x}{\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}} \Rightarrow f'\left(x\right) = \frac{-x}{\sqrt{1-x^2}} + \log_e\left(x + \sqrt{x^2+1}\right)$

$\Rightarrow f'\left(x\right) = \log_e\left(x + \sqrt{x^2+1}\right)$

For $x \geq 0$, $f'(x) \geq 0$

$\Rightarrow f(x)$ is an increasing function

Now, $x \geq 0$

$\Rightarrow f(x) \geq f(0)$

$\Rightarrow 1 - \sqrt{1+x^2} + x \log_e\left(x + \sqrt{x^2+1}\right) \geq 0$

$\Rightarrow 1 + x \log_e\ \left(x + \sqrt{1+x^2}\right) \geq \sqrt{1+x^2}$

Q97. Solution

Correct Answer: (D)

Given, $y^2 + 4y + 4x + 2 = 0$

$\Rightarrow (y+2)^2 + 4x - 2 = 0$

$\Rightarrow (y+2)^2 = -4\ \left(x - \frac{1}{2}\right)$

Replace $y + 2 = Y, x - \frac{1}{2} = X$

We have, $Y^2 = -4X$

This is a parabola with directrix at $X = 1$

$\Rightarrow x - \frac{1}{2} = 1 \Rightarrow x = \frac{3}{2}$

Q98. Solution

Correct Answer: (B)

$\sum_{k=1}^{10}\left[\frac{(-1)^{k-1}}{k}\left({}^{10}C_k\right)\right]$

$= \frac{{}^{10}C_1}{1} - \frac{{}^{10}C_2}{2} + \frac{{}^{10}C_3}{3} - \ldots. - \frac{{}^{10}C_{10}}{10}$

Consider the expansion $(1 - x)^{10} = {}^{10}C_0 - {}^{10}C_1 x + {}^{10}C_2 x^2 - \ldots. + {}^{10}C_{10} x^{10}$

$\Rightarrow \frac{(1-x)^{10} - {}^{10}C_0}{x} = -\left[{}^{10}C_1 - {}^{10}C_2 x + \ldots. + {}^{10}C_{10} x^9\right]$

Integrating both sides from x = 0 to x = 1, we get

$\int_0^1 \frac{1-(1-x)^{10}}{x} dx = \int_0^1 \left[{}^{10}C_1 - {}^{10}C_2 x + \ldots - {}^{10}C_{10} x^9\right] dx$

$= {}^{10}C_1 - \frac{{}^{10}C_2}{2} + \frac{{}^{10}C_3}{3} - \ldots - \frac{{}^{10}C_{10}}{10}$

To find LHS, consider $I_n = \int_0^1 \frac{1-(1-x)^n}{x} dx$

$\Rightarrow \quad I_{n+1} - I_n = \int_0^1 (1-x)^n dx = \frac{1}{n+1}$

$\therefore \quad I_{n+1} = \frac{1}{n+1} + I_n$

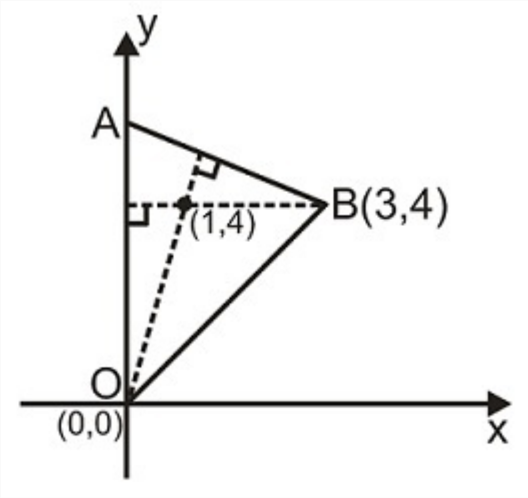
$I_{10} = \int_0^1 \frac{1-(1-x)^{10}}{x} dx$

$= \frac{1}{10} + I_9 = \frac{1}{10} + \frac{1}{9} + I_8 = \ldots.$

$= \frac{1}{10} + \frac{1}{9} + \frac{1}{8} + \ldots. + 1$

Q99. Solution

Correct Answer: (D)



Let A be (x, y)

We know that product of slopes of perpendicular lines are -1.

Let A be (x, y)

Slope AH $\times$slope $OB = -1$

$3x + 4y = 19$

Slope AB $\times$slope $OH = -1$

$x + 4y = 19$

On solving both the equations, we get,

$\Rightarrow \mathrm{y} - 4 = -\frac{3}{4}(\mathrm{x} - 1)$

Put $x = 0$

$\Rightarrow \mathrm{y} = 4 + \frac{3}{4} = \frac{19}{4}$

$\Rightarrow \mathrm{A} \equiv \left(0, \frac{19}{4}\right)$

Hence the correct answer is(d)

Q100. Solution

Correct Answer: (C)

We have statements $p, q \rightarrow T$ and $r, s \rightarrow F$
Option (a) $(q \wedge r) \vee (\sim p \wedge s) \equiv (T \wedge F) \vee (F \wedge F)$
$\equiv F \vee F \equiv F$
Option (b) $(\sim p \rightarrow q) \leftrightarrow (r \wedge s) \equiv (p \vee q) \rightarrow (r \vee s)$
$(\because p \rightarrow q \equiv \sim p \vee q)$
$\equiv (\sim pvq) \vee (r \wedge s) \wedge ((p \vee q) \vee (\sim r \wedge s))$
$(\because p \leftrightarrow q \equiv (\sim p \vee q) \wedge (pv - q))$
$\equiv (\sim (T \vee T) \vee (F \wedge F) \wedge ((T \vee F) \vee (F \wedge F)))$
$\equiv (F \vee F) \wedge (T \vee T) \equiv F \wedge T \equiv F$
Option (c) $(p \rightarrow q) \vee (r \leftrightarrow s)$
$\equiv (\sim p \vee q) \vee ((\sim r \vee s) \wedge (r \vee \sim s))(\because p \rightarrow q \equiv \sim p \vee q)$
$\equiv (F \vee T) \vee ((T \vee F) \wedge (F \vee T))$
$\equiv T \vee (T \wedge T) \equiv T \vee T \equiv T$
Option (d) do similar as option (a).

Q101. Solution

Correct Answer: (B)

Total possible outcomes
When a card is drawn = 52
Favourable outcomes card
drawn is either black or non - Black card of king "=28
$\therefore P = \frac{28}{52} = \frac{7}{13} \Rightarrow 13P = 6 + 1$
$13P - 6 = 1$

Q102. Solution

Correct Answer: (C)

Let $z = x + iy$
Given, $\left|\frac{z+2i}{2z+i}\right| < 1$
$\Rightarrow \frac{\sqrt{(x)^2+(y+2)^2}}{\sqrt{(2x)^2+(2y+1)^2}} < 1$
$\Rightarrow \quad x^2 + y^2 + 4 + 4y < 4x^2 + 4y^2 + 1 + 4y$
$\Rightarrow \quad 3x^2 + 3y^2 > 3$
$\Rightarrow \quad x^2 + y^2 > 1$

Q103. Solution

Correct Answer: (C)

We have,

$$\vec{a} \times \left[\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$$

$$= \vec{a} \times \left[\left(\vec{a} \cdot \vec{b}\right)\vec{a} - \left(\vec{a} \cdot \vec{a}\right)\vec{b}\right]$$

$$= \left(\vec{a} \cdot \vec{b}\right)\left(\vec{a} \times \vec{a}\right) - \left(\vec{a} \cdot \vec{a}\right)\left(\vec{a} \times \vec{b}\right)$$

$$= \left(\vec{a} \cdot \vec{b}\right)\vec{0} - \left(\vec{a} \cdot \vec{a}\right)\left(\vec{a} \times \vec{b}\right)$$

$$= \left(\vec{a} \cdot \vec{a}\right)\left(\vec{b} \times \vec{a}\right).$$

Q104. Solution

Correct Answer: (A)

Given that,

$$y = Ae^{x} + Be^{2x} + Ce^{3x} \qquad \ldots\text{(i)}$$

On differentiating both sides w.r.t. x, we get

$$y' = Ae^{x} + 2Be^{2x} + 3Ce^{3x}$$

From Eq. (i)

$$Ae^{x} = y - Be^{2x} - Ce^{3x}$$

$$\therefore \quad y' = y + Be^{2x} + 2Ce^{3x} \qquad \ldots\text{(ii)}$$

Again differentiating w.r.t. x, we get $y'' = y' + 2Be^{2x} + 6Ce^{3x}$ From Eq. (ii)

$Be^{2x} \quad = y' - y - 2Ce^{3x}$

$\therefore \quad y'' = y' + 2y' - 2y - 4Ce^{3x} + 6Ce^{3x}$

$$y'' = 3y' - 2y + 2Ce^{3x} \qquad \ldots\text{(iii)}$$

Again differentiating w.r.t. x, we get $y''' = 3y'' - 2y' + 6Ce^{3x}$ From Eq. (iii)

$2Ce^{3x} = y'' - 3y' + 2y$

$\therefore \quad y''' = 3y'' - 2y' + 3\left(y'' - 3y' + 2y\right)$

$\Rightarrow \quad y''' = 6y'' - 11y' + 6y$

$\Rightarrow \quad y''' - 6y'' + 11y' - 6y = 0$

Q105. Solution
Correct Answer: (D)

Given equation,
$4\left(x^2+\frac{1}{x^2}\right)+16\left(x+\frac{1}{x}\right)-57=0$

Let, $x+\frac{1}{x}=y; x^2+\frac{1}{x^2}=y^2-2$

$\Rightarrow 4y^2+16y-65=0$

$\Rightarrow y=-\frac{13}{2}$ or $\frac{5}{2}$

When, $y=\frac{5}{2}$

$x+\frac{1}{x}=\frac{5}{2}\Rightarrow x=2$ or $\frac{1}{2}$

When, $y=-\frac{13}{2}$

$\Rightarrow x+\frac{1}{x}=-13/2$

$\Rightarrow 2x^2+13x+2=0$

$\Rightarrow x=\frac{-13\pm\sqrt{153}}{4}$

Since x is rational, $x=2$ or $\frac{1}{2}$

Hence, their product is 1.

Q106. Solution
Correct Answer: (A)

Let the boys arrange themselves in $5!$ ways. Girls can sit in between or besides them such that maximum one girl is between any 2 boys.

1 B 2 B 3 B 4 B 5 B 6

Clearly, there are 1 to 6 places i.e. total 6 places for 5 girls.

Required number of ways $=$ $5!\times 6!$

Q107. Solution

Correct Answer: (D)

Slope $M=\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$

$\therefore\ M=\frac{3t^2-3}{2t}$ at $t=2\Rightarrow M=\frac{9}{4}$

Equation of tangent at $(4,2)$ is $M=\frac{9}{4}$

$\therefore\ (y-2)=\frac{9}{4}(x-4)\Rightarrow y=\frac{9}{4}x-7.$

Q108. Solution

Correct Answer: (D)

Let $\mathrm{I}=\int\frac{\cos 4\mathrm{x}-1}{\cot\mathrm{x}-\tan\mathrm{x}}\mathrm{dx}$

$$\left\{\begin{array}{c}\because\ \cos 2x=1-2\sin^2 x\\ -2\sin^2 x=\cos 2x-1\\ \therefore -2\sin^2 2x=\cos 4x-1\end{array}\right\}$$

$\Rightarrow\mathrm{I}=\int\frac{-2\sin^2 2x}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}dx$

$\Rightarrow\mathrm{I}=\int\frac{-2\sin^2 2x}{\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}}dx$

$\Rightarrow\mathrm{I}=\int\frac{-\sin^2 2x(2\sin x\cos x)}{\cos 2x}dx\quad\{\because\cos^2 x-\sin^2 x=\cos 2x\}$

$\Rightarrow\mathrm{I}=\int\frac{-\sin^2 2x(\sin 2x)}{\cos 2x}dx\ \{\because 2\sin x\cos x=\sin 2x\}$

$\Rightarrow\mathrm{I}=-\int\frac{\sin 2\mathrm{x}\left(1-\cos^2 2\mathrm{x}\right)}{\cos 2\mathrm{x}}\mathrm{dx}\quad\{\because\sin^2 2x+\cos^2 2x=1\}$

On using the substitution method

Put $\cos 2\mathrm{x}=t\ \ .....(1)$

Now, differentiate both the sides,

$\Rightarrow\quad-\sin 2\mathrm{xdx}=\frac{\mathrm{dt}}{2}$

$\Rightarrow\mathrm{I}=\frac{1}{2}\int\frac{1-\mathrm{t}^2}{\mathrm{t}}\mathrm{dt}$

$\Rightarrow\mathrm{I}=\frac{1}{2}\int\left(\frac{1}{t}-t\right)dt$

$\Rightarrow\mathrm{I}=\frac{1}{2}\log_e t-\frac{\mathrm{t}^2}{4}+\mathrm{c}$

From equation(1)

$\Rightarrow\mathrm{I}=\frac{1}{2}\log_e\cos 2\mathrm{x}-\frac{\cos^2 2\mathrm{x}}{4}+\mathrm{c}\ .$

Q109. Solution
Correct Answer: (B)

We have

$2f(x)+3f\left(\frac{1}{x}\right)=x^2-1 \qquad \ldots.(1)$

Putting $\frac{1}{x}$ in place of $x,$ in the above equation

$3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x^2}-1 \quad \ldots\ldots(2)$
Using $2\times(1)-3\times(2),$ we get

$f(x)=\frac{\left(2x^2+3\right)\left(1-x^2\right)}{5x^2}$

Since $f(-x)=f(x)$,

$\Rightarrow$ It is an even function.

Q110. Solution
Correct Answer: (D)

Three determinant of second order negative value are possible $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$

No. of possible determinant with element 0,1 are $2^4 = 16$

so determinant with non negative value is 13

Q111. Solution
Correct Answer: (B)

Number of arrangements = $5!3!/2=360$

Q112. Solution

Correct Answer: (B)

Let

$\lambda = \frac{x^2+y^2}{x^2+xy+4y^2}$

$\Rightarrow \lambda = \frac{1+\left(\frac{y}{x}\right)^2}{1+\frac{y}{x}+4\left(\frac{y}{x}\right)^2}$

Put $\frac{y}{x} = z$, then

$\Rightarrow \lambda = \frac{1+z^2}{1+z+4z^2}$

$\Rightarrow \lambda + \lambda z + 4\lambda z^2 = 1 + z^2$

$\Rightarrow z^2(4\lambda - 1) + z(\lambda) + \lambda - 1 = 0$

Since z is real,

$D \geq 0$

We know that for a quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ the discriminant is given by $D = b^2 - 4ac$.

$\Rightarrow\ \lambda^2 - 4(4\lambda - 1)(\lambda - 1) \geq 0$

$\Rightarrow\ \lambda^2 - 4\left(4\lambda^2 - 5\lambda + 1\right) \geq 0$

$\Rightarrow 15\lambda^2 - 20\lambda + 4 \leq 0$

$\Rightarrow \lambda \in \left(\frac{10-4\sqrt{5}}{15}, \frac{10+4\sqrt{5}}{15}\right)$.

Q113. Solution

Correct Answer: (D)

Let the equation of tangent to the parabola $y^2 = x$, having slope m is, $y = mx + \frac{1}{4m}$ For common tangent to the circle $x^2 + y^2 - 6y + 4 = 0$ $\therefore$ Radius $\sqrt{9-4} = \frac{3-\frac{1}{4m}}{\sqrt{1+m^2}}$ $\Rightarrow\ 5\left(1+m^2\right) = 9 + \frac{1}{16m^2} - \frac{3}{2m}$ $\Rightarrow\ m = \frac{1}{2}$ So, equation of common tangent is

$$y = \frac{1}{2}x + \frac{1}{2}$$

$$\Rightarrow\ x - 2y + 1 = 0.$$

Q114. Solution

Correct Answer: (A)

We can reduce the function by substituting

$x = \cos\theta \Rightarrow \quad \theta = \cos^{-1} x$

$y = \cos\left(2\tan^{-1}\left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)\right) - 2\cos^{-1}\left(\sqrt{\frac{1-\cos\theta}{2}}\right)$

$y = \cos\left(2\tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right)\right) - 2\cos^{-1}\left(\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2}}\right)$

$= \cos\left(2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right) - 2\cos^{-1}\left(\sin\frac{\theta}{2}\right)$

As $\theta = \cos^{-1} x$

$\theta \in [0, \pi) \Rightarrow \frac{\theta}{2} \in \left[0, \frac{\pi}{2}\right)$

In the first quadrant $\tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$

$= \cos\left(2 \times \frac{\theta}{2}\right) - 2\cos^{-1}\left(\sin\frac{\theta}{2}\right)$

As $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$y = \cos\theta - 2\left(\frac{\pi}{2} - \sin^{-1}\left(\sin\frac{\theta}{2}\right)\right)$

As $\sin^{-1}\left(\sin\frac{\theta}{2}\right) = \frac{\theta}{2}$ in the first quadrant

$y = \cos\theta - \pi + \theta$

$y = x - \pi + \cos^{-1} x$

$\frac{dy}{dx} = 1 - \frac{1}{\sqrt{1-x^2}}$

Q115. Solution

Correct Answer: (A)

Given points $A(k,\ 2k),\ B(3k,\ 3k)$ and $C(3,\ 1)$ are collinear.

$\therefore$ Slope of AB = Slope of BC

$\therefore \frac{2k-2k}{3k-k} = \frac{1-3k}{3-3k}$

$\Rightarrow \frac{k}{2k} = \frac{1-3k}{3-3k} \Rightarrow 3-3k = 2(1-3k)$

$\Rightarrow 1 = \ -3k \ \Rightarrow \ \ k = \ -\frac{1}{3}$

$\therefore$ Given points become $A\left(-\frac{1}{3},\ \ -\frac{2}{3}\right),\ B\ (-1,\ \ -1)$ and $C(3,\ 1)$.

$\therefore$ Equation of line passing through B and C is

$y+1 = \frac{1+1}{3+1}\ (x+1)$

$\Rightarrow\ \ y+1 = \frac{2}{4}(x+1)$

$\Rightarrow 2(y+1) = (x+1)$

$\Rightarrow 2y-x+1 = 0$

Now, the distance from (0, 0) to the above line is

$d = \frac{|2(0)-0+1|}{\sqrt{2^2+1^2}}\ \Rightarrow\ \ = \frac{1}{\sqrt{4+1}} = \frac{1}{\sqrt{5}}$

Alternate Solution

Equation of line passing through $(k,\ 2k)$ and $(3k,\ k)$ is

$(y-2k) = \frac{3k-2k}{3k-4}\ (x-k)$

$\Rightarrow y-2k = \frac{k}{2k}(x-k)$

$\Rightarrow y-2k = \frac{1}{2}(x-k)$ ....(i)

Since, above line is passing through (1, 1).

$\therefore 1-2k = \frac{1}{2}(1-k)$

$\Rightarrow 2-4k = \ \ -1-k$

$\Rightarrow 1-3k\ \Rightarrow k = \frac{1}{3}$

On putting $k = \frac{1}{3}$ in equation (i), we get

$y-\frac{2}{3} = \frac{1}{2}\left(x-\frac{1}{3}\right)$

$\Rightarrow 3y-2 = \frac{1}{2}(3x-1)$

$\Rightarrow 6y-4 = 3x-1$

$\Rightarrow 6y-3x-3 = 0$

Or $2y-x-1 = 0$

$\therefore$ Perpendicular distance from (0, 0) to the above line is

$d = \frac{|2(0)-0-1|}{\sqrt{2^2+1^2}} = \frac{1}{\sqrt{4+1}}$

$= \frac{1}{\sqrt{5}}$

Q116. Solution

Correct Answer: (A)

As we know that $\tan^{-1}x_1+\tan^{-1}x_2+\cdots+\tan^{-1}x_n = \tan^{-1}\left(\frac{S_1-S_3+S_5-S_7+\cdots}{1-S_2+S_4-S_6+\cdots}\right)$, where S_k denotes the sum of the products of $x_1, x_2, \cdots, x_n$ taken k at a time.

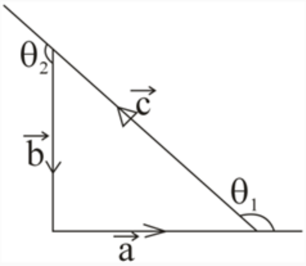
$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z = \tan^{-1}\left\{\frac{x+y+z-xyz}{1-xy-yz-zx}\right\}$

$= \tan^{-1}0 = \pi$ or 0

Q117. Solution

Correct Answer: (C)

$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 = \left|\vec{c}\right|^2$ (property of triangles)



From the diagram,

$\pi - \theta_1 + \pi - \theta_2 = \frac{\pi}{2}$

$\Rightarrow \theta_1 + \theta_2 = \frac{3\pi}{2}$

Q118. Solution

Correct Answer: (B)

Given that, $\cos\frac{x}{2}\cos\frac{x}{2^2}\cos\frac{x}{2^3}\ldots\cos\frac{x}{2^n}$

$= \frac{\sin x}{2^n \sin\left(\frac{x}{2^n}\right)} \quad \ldots(\text{i})$

Taking logarithm to the base on both sides of Eq. (i) and then differentiating w.r.t. x, we get

$\sum_{n=1}^{n} \frac{1}{2^n}\tan\frac{x}{2^n} = \left(\frac{1}{2^n}\cot\frac{x}{2^n} - \cot x\right)$

$\therefore \lim_{n\to\infty}\sum_{n=1}^{n} \frac{1}{2^n}\tan\frac{x}{2^n} = \lim_{n\to\infty}\left(\frac{1}{x}\times\frac{\frac{x}{2^n}}{\tan\frac{x}{2^n}} - \cot x\right)$

$= \left(\frac{1}{x} - \cot x\right)$

We have, $f(x) = \begin{cases} \frac{1}{x} - \cot x, & x \in (0,\pi) - \left\{\frac{\pi}{2}\right\} \\ \frac{2}{\pi}, & x = \frac{\pi}{2} \end{cases}$

Clearly, $\lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}}\left(\frac{1}{x} - \cot x\right)$

$= \frac{2}{\pi} = f\left(\frac{\pi}{2}\right)$

Hence, $f(x)$ is continuous at $x = \frac{\pi}{2}$.

Q119. Solution

Correct Answer: (D)

$\int_{\sqrt{3}}^{\sqrt{18}}[x]dx = \int_{\sqrt{3}}^{\sqrt{4}}[x]dx + \int_{2}^{3}[x]dx + \int_{3}^{4}[x]dx + \int_{4}^{\sqrt{18}}[x]dx = \int_{\sqrt{3}}^{\sqrt{4}} 1dx + \int_{2}^{3} 2dx + \int_{3}^{4} 3dx + \int_{4}^{\sqrt{18}} 4dx$
$= (2-\sqrt{3}) + (2) + (3) + 4(\sqrt{18}-4) = 2-\sqrt{3}+2+3+4\sqrt{18}-16 = -9-\sqrt{3}+12\sqrt{2}$

$= 12\sqrt{2}-\sqrt{3}-9$
$= b\sqrt{2}+c\sqrt{3}+9$ Where $a=-9, b=12, c=-1$ $a+b+c=2$

Q120. Solution

Correct Answer: (A)

This question can be easily done by using the concept of characteristic equation of a matrix.
Consider the equation det $(\mathrm{A}\text{ - }\mathrm{kI}) = 0$, where k is a scalar.
Now, when we solve the equation $|\mathrm{A}-\mathrm{kI}| = 0$, then we get a polynomial in k.
Here, if we put k = A(matrix), then the equation obtained is called as the characteristic equation of square matrix A.
The characteristic polynomial of A is $|\mathrm{A}-\mathrm{kI}| = 0$.

$\therefore \left|\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix} - \mathrm{k}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right| = 0$

$\Rightarrow \left|\begin{bmatrix}3-\mathrm{k} & 2\\ 7 & 5-\mathrm{k}\end{bmatrix}\right| = 0$

$\Rightarrow \begin{vmatrix}3-\mathrm{k} & 2\\ 7 & 5-\mathrm{k}\end{vmatrix} = 0$

$\Rightarrow \ (3-\mathrm{k})(5-\mathrm{k}) - 14 = 0$

$\Rightarrow \ \mathrm{k}^2 - 8\mathrm{k} + 1 = 0$

$\therefore \ \mathrm{A}^2 - 8\mathrm{A} + \mathrm{I} = 0$

On comparing the above equation with the given equation $\mathrm{A}^2 + \lambda\mathrm{A} + \mu\mathrm{I} = \mathrm{O}$, we get
$\lambda = -8, \mu = 1 \Rightarrow \frac{\mu-\lambda}{3} = 3$

Q121. Solution
Correct Answer: (A)

Here the angles of a polygon of n sides form an $\mathrm{A.P.}$ whose first term is 120^o and common difference is 5^o.
Now, the sum of interior angles
$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2 \times 120 + (n-1)5]$
$S_n = \frac{n}{2}[240 + 5n - 5] = \frac{n}{2}(235 + 5n)$ ...(1)
Also the sum of interior angles $= 180\,(n-2)$
From equation (1),
$\therefore \frac{n}{2}(235 + 5n) = 180n - 360$
$\Rightarrow \frac{5n}{2}(47 + n) = 180\,(n-2)$
$\Rightarrow n(47 + n) = 72(n-2)$
$\Rightarrow n^2 + 47n = 72n - 144$
$\Rightarrow n^2 - 25n + 144 = 0$
$\Rightarrow (n-16)(n-9) = 0$

$\Rightarrow n \neq 16 \therefore\ n = 9.$
$\therefore\ n \neq 16$, because for $n = 16$, one of the interior angle will become 180°, which is not possible.
For $n = 16 \Rightarrow 120^{\circ},\ 125^{\circ}, \ldots\ldots., 180^{\circ}, \ldots\ldots, 195^{\circ}$ and for$n = 9 \Rightarrow 120^{\circ},\ 125^{\circ}, \ldots\ldots, 160^{\circ}$.

Q122. Solution
Correct Answer: (D)

Reflexive
since $m^2\ \geq 0\, \forall\, \text{real}\, m$
hence mRm
Symmetric
if $nm \geq 0 \Rightarrow mn \geq 0\, \forall\, \text{real}\, m\, \&\, n.$
hence, $nRm \Rightarrow mRn.$
Transitive
if $nm \geq 0\, \&\, mp \geq 0$
$\Rightarrow\ nm^2p \geq 0 \Rightarrow np \geq 0$
hence $nRm.\ \&\ mRp \Rightarrow nRm$
So, the relation is equivalent relation.

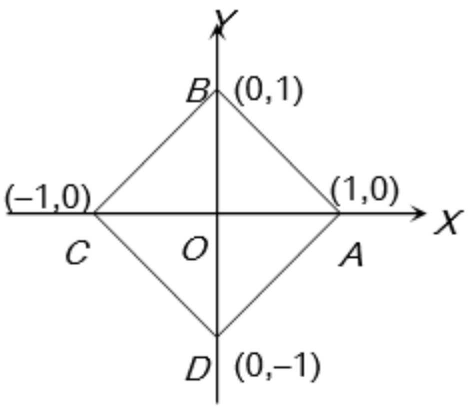
Q123. Solution
Correct Answer: (A)

$\alpha =\ -\omega,\ \beta = -\omega^2$
$\alpha^{2015} =\ -\omega^2,\ \beta^{2015} =\ -\omega$

Q124. Solution

Correct Answer: (A)

Required locus of the point (x, y) is the curve $|x| + |y| = 1$. If the point lies in the first quadrant, then $x > 0, y > 0$ and so $|x| + |y| = 1 \Rightarrow x + y = 1$, which is straight line AB. If the point (x, y) lies in second quadrant then $x < 0, y > 0$ and so $|x| + |y| = 1 \Rightarrow -x + y = 1$



Similarly for third and fourth quadrant, the equations are $-x - y = 1$ and $x - y = 1$. Hence the required locus is the curve consisting of the sides of the square $ABCD$.

Q125. Solution

Correct Answer: (A)

Let co-ordinates of P and Q be $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$, respectively.

Since, PQ subtends a right angle at the vertex $(0,0)$, then $(m_{OP})(m_{OQ}) = -1$,

$\Rightarrow \left(\frac{2at_1-0}{at_1^2-0}\right)\left(\frac{2at_2-0}{at_2^2-0}\right) = -1,$

$\Rightarrow \left(\frac{2}{t_1}\right)\left(\frac{2}{t_2}\right) = -1, \Rightarrow t_1t_2 = -4 \quad \ldots(1)$

If (h,k) is the point of intersection of normals at P and Q, then, we know that, the point of intersection of the normals at $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$ is

$\left(2a + a\left(t_1^2 + t_2^2 + t_1t_2\right),\ -at_1t_2(t_1+t_2)\right),$

$\Rightarrow h = 2a + a\left(t_1^2 + t_2^2 + t_1t_2\right) \quad \ldots(2)$

and $k = -at_1t_2(t_1+t_2) \quad \ldots(3)$

In order to find the locus, we have to eliminate t_1 and t_2 from equations (2) and (3).

On putting, $t_1t_2 = -4$ in equation (3), we get $k = 4a(t_1+t_2)$,

$\Rightarrow (t_1+t_2) = \frac{k}{4a} \quad \ldots(4)$

Now, from equation (1), we have,

$h = 2a + a\left(t_1^2 + t_2^2 + t_1t_2\right),$

Using $p^2 + q^2 = (p+q)^2 - 2pq$, we get

$h - 2a = a\left\{(t_1+t_2)^2 - 2t_1t_2 + t_1t_2\right\},$

$\Rightarrow h - 2a = a\left\{(t_1+t_2)^2 - t_1t_2\right\},$

On putting the values of t_1+t_2 and t_1t_2, we get

$h - 2a = a\left\{\left(\frac{k}{4a}\right)^2 - (-4)\right\},$

$\Rightarrow h - 2a = a\left[\frac{k^2}{16a^2} + 4\right],$

$\Rightarrow h - 6a = \frac{k^2}{16a}, \Rightarrow 16a(h-6a) = k^2.$

Replacing (h,k) by (x,y), we get the required locus as

$y^2 = 16a\left(x - 6a\right)$, which is a parabola.

Q126. Solution

Correct Answer: (A)

$A^2 = A \cdot A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} = \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix} = O \Rightarrow A^3 = A \cdot A^2 = 0$ and $A^n = 0$, for all $n \geq 2$.

Q127. Solution

Correct Answer: (A)

Let $P(2, -1,4)$

Now, any point Q on line $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1} = \lambda$ is

$Q = (10\lambda - 3, \ -7\lambda + 2\ , \lambda)$

Direction Ratio of PQ

$= (10\lambda - 3 - 2, \ -7\lambda + 2 + 1, \ \lambda - 4)$

$= (10\lambda - 5, \ -7\lambda + 3, \ \lambda - 4)$

$\because PQ$ is perpendicular to given line

$\therefore 10(10\lambda - 5) - 7(-7\lambda + 3) + \ (\lambda - 4) = 0$

$\Rightarrow 150\ \lambda = 75 \ \Rightarrow \lambda = \frac{1}{2}$

$\therefore Q \equiv \left(2, -\frac{3}{2}, \frac{1}{2}\right)$

Distance $PQ = \sqrt{\frac{1}{4} + \frac{49}{4}} = \frac{5}{\sqrt{2}} \approx 3.53$ which lies in $(3,4)$.

Q128. Solution

Correct Answer: (C)

$\lim_{n\to\infty} \left[\frac{n(n+1)(2n+1)}{6n^3}\right] = \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6} = \frac{1}{3}$. Note : Students should remember that $\lim_{n\to\infty} \frac{\sum n}{n^2} = \frac{1}{2}, \lim_{n\to\infty} \frac{\sum n^2}{n^3} = \frac{1}{3}$ and $\lim_{n\to\infty} \frac{\sum n^3}{n^4} = \frac{1}{3}$.

Q129. Solution
Correct Answer: (D)

From the given condition

$^nC_r :^n C_{r+1} :^n C_{r+2} = 2 : 15 : 70$

$\Rightarrow \frac{^nC_r}{^nC_{r+1}} = \frac{2}{15}$ and $\frac{^nC_{r+1}}{^nC_{r+2}} = \frac{15}{70}$

$\Rightarrow \frac{\left(\frac{n!}{(n-r)!\cdot r!}\right)}{\left(\frac{n!}{(n-r-1)!\cdot(r+1)!}\right)} = \frac{2}{15}$ and $\frac{\left(\frac{n!}{(n-r-1)!\cdot(r+1)!}\right)}{\left(\frac{n!}{(n-r-2)!\cdot(r+2)!}\right)} = \frac{15}{70}$

$\Rightarrow \frac{(n-r-1)!\cdot(r+1)!}{(n-r)!\cdot r!} = \frac{2}{15}$ and $\frac{(n-r-2)!\cdot(r+2)!}{(n-r-1)!\cdot(r+1)!} = \frac{3}{14}$

$\Rightarrow \frac{(n-r-1)!\cdot(r+1)\cdot r!}{(n-r)\cdot(n-r-1)!\cdot r!} = \frac{2}{15}$ and $\frac{(n-r-2)!\cdot(r+2)\cdot(r+1)!}{(n-r-1)\cdot(n-r-2)!\cdot(r+1)!} = \frac{3}{14}$

$\Rightarrow \frac{r+1}{n-r} = \frac{2}{15}$ and $\frac{r+2}{n-r-1} = \frac{3}{14}$

$\Rightarrow\ 17r = 2n - 15$ and $17r = 3n - 31$

$\Rightarrow 3n - 31 = 2n - 15,\ \Rightarrow n = 16$ and $r = 1$

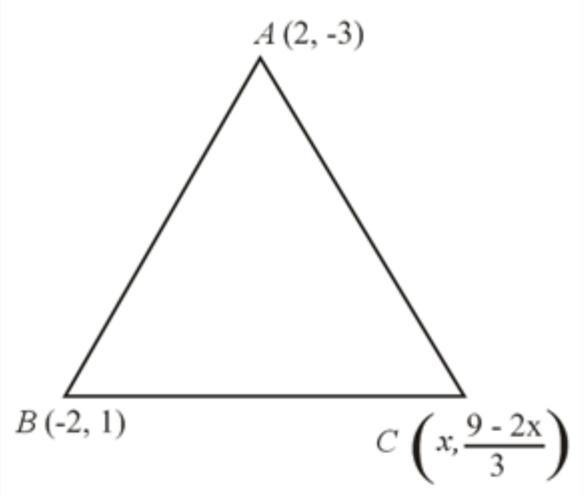
Hence, average $= \frac{^nC_r + ^nC_{r+1} + ^nC_{r+2}}{3}$

$= \frac{^{16}C_1 + ^{16}C_2 + ^{16}C_3}{3}$

$= 232.$

Q130. Solution
Correct Answer: (C)

Given $A(2, -3)$ $B(-2, 1)$ The third vertex lies on $2\ x + 3\ y = 9$ i.e. $C\left(x, \frac{9-2x}{3}\right)$



$\therefore$ Let $P(h, k)$ be any point on the required locus i.e P is the centroid of the triangle ABC
$\Rightarrow \left(\frac{2-2+x}{3}, \frac{-3+1+\frac{9-2x}{3}}{3}\right) = (h, k) \therefore\ h = \frac{x}{3},\ k = \frac{3-2x}{9}$ Eliminating x from the above equations
$\Rightarrow 9\ k = 3 - 2(3\ h) \Rightarrow 9\text{k} = 3 - 6h \Rightarrow 2h + 3k = 1$ Hence, the locus of $P(h, k)$ is $2\ x + 3\ y = 1$