Advanced Physics Lab I

Lab Report #5

Group 1
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We hereby declare that we (Luka Burduli and Noah Horne) are the sole authors of this lab report and have not used any sources other than those listed in the bibliography and identified as references throughout the report.

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Contents

1 Abstract

The following investigation focused on the effects of an incident magnetic field on two distinct solid materials: Copper and p-Ge. Throughout the measurement process, the Hall effect was analyzed and specific results for the hall mobility and charge carrier density were calculated. To begin, the relationship between the current in the electromagnetic coils and the resultant magnetic field was determined. For p-Ge, the Hall coefficient was calculated as $(0.007556 \pm 0.000306)[VmAT^1]$ with a corresponding mobility of $(0.2731 \pm 0.0112)[m^2V^1s^1]$ and a carrier density of $(8.26 \pm 0.335)10^{20}[m^3]$.For Copper, the Hall coefficient was found to be $(-2.08 \pm 0.08)10^{-11}[VmAT^{-1}]$, leading to a mobility of $(1.34 \pm 0.05)10^{-3}[m^2V^{-1}s^{-1}]$ and a charge carrier density of $(-2.99 \pm 0.11)10^{29}m^{-3}$. Additionally, the temperature dependence of the electrical conductivity was investigated, revealing an energy band gap for p-Ge of approximately $(0.67 \pm 0.02)eV$. These results align with theoretical expectations, demonstrating the of the Hall Effect in characterizing material properties.

2 Introduction & Theory

This investigation primarily focuses on the quantitative analysis of the Hall Effect, and the subsequent calculation of the material properties for two different solid materials: P-doped Germanium, and Copper. In solid materials, the behavior of charge carriers and their nature can vary dramatically depending on the electrical properties of the material. In cases where the electrical properties of a material are of interest, the energy bands of a medium are investigated, specifically, the valence and conduction band. The valence band of an electrical solid material refers to the highest energy range where charge carriers are still present without being subject to ionisation. Contrarily, the conduction band refers to the lowest energy level vacant to charge carriers at the absolute zero $(0K)$ temperature. In metals, which are deemed as conductors, the charge carriers (electrons) move freely with a large free path, and therefore cause the valence and conduction bands to overlap. However, in semiconducting media these energy bands remain separated by a gap width of approximately 1-2eV, a very small amount. This energy gap is small enough such that a shift in temperature (T) in a semiconductor can cause the charge carriers to gain enough energy to cross between the valence and conduction bands, closing the energy gap between the conduction and valence band and allowing the doped semiconductor to function as an intrinsic semiconductor. The energy gap is defined as the difference in energy between the valence and conduction band, e.g.

$$E_g = E_C - E_V \tag{1}$$

Semiconductors can be doped with impurities, altering the behavior of the semiconductor by moderating the movement of the charge carriers in the material. For the purposes of this investigation, the behavior a p-doped semiconductor (p-Ge) will be used.

Due to the temperature dependence of the energy gap in semiconductors, there is an associated relationship between the temperature of the semiconductor and the charge carrier concentration n that is observed. For lower temperatures, the energy gap between the valence and conduction bands is much larger, leading to less mobility μ of the charge carriers, as there is much lesser probability for the charges to move across the energy gap boundary. This temperature range is referred to as the freeze-out range, for a low number of doped impurities in the semiconductors are ionized, causing the charge carrier concentration to drop. For moderate temperatures of the semiconductors, these doped semiconductor impurities all become ionized, but not enough energy is present to allow the significant transfer of elecFigure 9: Plot of conductivity against temperature of the p-Ge sampletrons across the energy gap–this temperature range is called the extrinsic range. Introducing more energy into the medium by increasing the temperature gives enough energy to the present electrons to make the jump between the valence and conduction bands, and the semiconductor behaves as a fully non doped (intrinsic) semiconductor. From the theory developed by the Advanced Physics Lab I Manual, the electrical conductivity σ of a medium can be defined as (Wagner & Söcker, 2024):

$$\sigma = e \cdot n \cdot \mu \tag{2}$$

where n, μ and e are the charge carrier density, charge carrier mobility and the elementary charge respectively. More traditionally, it is also expressed as the reciprocal of the resistivity ρ, or:

$$\sigma = \frac{1}{\rho} = \frac{l}{R \cdot A} = \frac{I \cdot l}{U \cdot A} \tag{3}$$

where l is the length of the material sample with current I passing through and A is the cross-sectional area of the sample. Due to the direct correlation between the electrical conductivity and the charge carrier density n exhibited in equation 2, the electrical conductivity σ can be used as a measure for the rate of change of the charge carrier density, taking into account the relevant proportionality constants of the mobility μ and the elementary charge. In the intrinsic range of temperatures where a p-doped semiconductor behaves as a normal semiconductor, the rate of change of the electrical conductivity can be analyzed to determine the exact band gap present in the material being analyzed. As of such, the relationship between electrical conductivity σ and temperature T will be analyzed, and the energy gap E_g for the semiconducting solid material (p-Ge) will be found using the following formula developed from the theory presented in the Advanced Physics Lab I Manual (Wagner & Söcker, 2024):

$$(\log \sigma) \cdot T = \frac{-E_g}{2k_B} \implies E_g = -2k_B T \log \sigma \tag{4}$$

Where k_B is the Boltzmann constant. In other words, the slope of the graph of the logarithm of the conductivity ($\log \sigma$) against the reciprocal of temperature ($\frac{1}{T}$) will be analyzed on the domain of temperatures such that it appears linear (the intrinsic domain). By extracting this relevant quantity, one can simply multiply by the slope of this graph by the factor $(-2k_B)$

to find the energy gap in joules (J) which can easily be converted to electron volts: the typical unit of band gap (eV).

It is important to note that although the pre-developed theory allows for the calculation of the energy band gap, it does not allows for the calculation of important quantities in the medium such as the mobility μ and density n of the charge carriers. However, methods still exist which exploit a specific phenomenon observed in solid materials when exposed to a magnetic field: the Hall Effect.

In this investigation, a magnetic field is produced around the material sample through the use of two large conducting electromagnetic coils. the coils lay below the sample and are wrapped around a metal core. As a separate current passes through the coils a magnetic flux is induced through the iron core, which has a small air gap of width δ where the samples can be placed (see figure 1). From Ampères law, it is clear that:

$$\oint \vec{H}_l dl = H_{Fe} l_{Fe} + H_{air}\delta = \sum I_{coil} = N_{tot} I_{coil} \tag{5}$$

and by the symmetry of the coils and the iron core:

$$B_{air} = \mu_0 \mu_{r,air} H_{air} \tag{6}$$

Using these two equations, one can calculate the theoretic relationship between the current in the wires and the magnetic field strength through the sample. Substituting equation 5 into 6, one achieves the following derivation of the relationship between these two quantities.

$$B_{air} = \mu_0 \mu_{r,air} \cdot \frac{N_{tot} I_{coil} - H_{fe} l_{fe}}{\delta}$$

$$\implies H_{air} = \frac{N_{tot} I_{coil} - H_{fe} l_{fe}}{\delta}$$

$$\because \mu_{r,air} \approx 1, \mu_{r,fe} \gg \mu_{r,air} \implies B_{air} = \frac{\mu_0 N_{tot}}{\delta} I_{coil}$$

$$B_{air} = \frac{\mu_0 N_{tot}}{\delta} I_{coil} \tag{7}$$

As briefly aforementioned, the Hall Effect refers to a phenomenon used primarily in the identification and classification of solid materials. By applying a potential difference V to a solid material, a current I is induced along the axis established by the potential difference. If a magnetic field $\vec{B}$ is introduced perpendicular to the current I in the solid material, then the charge carriers in the solid material become moving charges in a magnetic field. As the Lorentz Force dictates, any moving charge q passing through a B-Field $\vec{B}$ will experience an 'Lorentz' force orthogonal to the current direction $\hat{j}$ and the B-field ($\vec{B}$), given by:

$$\vec{F}_L = q \cdot (\vec{v} \times \vec{B}) \tag{8}$$

As a force is experience by the moving charge carriers in the materials, they are deflected from their original path of motion, inducing a new electric field $\vec{E}_H$. This electric field E_H directly arises due to the Hall Effect, and is given by:

$$\vec{E}_H = \frac{1}{qn} \cdot \left(\vec{j} \times \vec{B}\right) \tag{9}$$

where $\vec{j}$ is the current density in the material, and $\vec{B}$ is the B-field at the material surface. The coefficient $\frac{1}{qn}$ present in the electric field $\vec{E}_H$ equation is given a special name, and is called the Hall Coefficient:

$$R_H = \frac{1}{qn} \tag{10}$$

Further simplifying equation 9 by computing the cross product and substituting the Hall coefficent R_H given in equation 10, one achieves an alternative expression for the electric field generated by the offset charge carriers in the material:

$$E_{H,y} = R_H \cdot j_x \cdot B_z \tag{11}$$

Where the z component of the B-Field B_z is assumed to be the component normal to the surface of the material, and the current density component j_x is along the component of original current travel. Following the right hand rule, one can easily determine which direction the charges are expected to deflect, given the direction of the current and the B-field. This electric field gives rise to an electric potential, which appears as a voltage across the surface perpendicular to the direction of original current propagation. While, small, it is still present when a B-Field is positioned in such a manner orthogonal to the direction of the current. One can denote this voltage as the Hall Voltage U_H. And it is defined by the relation to the Hall Electric Field E_H as,

$$E_{H,y} = \frac{U_H}{w} \tag{12}$$

where w is the width of the solid material through which the current flows. By combining equations 12 and 11, the Hall Coefficient R_H establishes a direct relationship between the orthogonal component of the B-field (parallel to the surface normal of the material) and the current through the material. As derived in the Advanced Physics Lab I Manual, the Hall Coefficient R_H can be expressed as,

$$R_H = \frac{U_H \cdot d}{I \cdot B_z} \tag{13}$$

where d is the thickness of the material (Wagner & Söcker, 2024). It is this formula 13 that will be used to empirically determine the Hall Coefficient of the two materials being analyzed. As the hall voltage can be measured similarly to how the voltage across a normal load can be measured, and the current is easily determined through an ammeter, it is the clear choice.

As aforementioned, the Hall Coefficient is useful as it easily allows for many intrinsic properties of its corresponding material to be determined. By combining equations 10 and 2, one can establish the following useful relation:

$$\mu = |R_H|\sigma \tag{14}$$

Now, once the Hall Coefficient R_H and Electrical conductivity σ of each medium (p-Ge and Copper) are calculated using formulas 13 and 3, the mobility can easily be found. Likewise, formula 10 can be used, given that the sign of the charge carrier is known, (based on direction of hall voltage) to find the charge carrier density n.

So, with the theory established, one can effectively calculate the hall coefficient of each medium through the empirical observations of the current and hall voltage, and from this coefficient calculate all relevant intrinsic properties of each medium. Furthermore, the theory on temperature dependent carrier density was established, and will be empirically verified through the analysis of the conductivity-temperature graphs to determine the energy band gap in a p-doped semiconductor (p-Ge).

3 Setup & Experimental Procedure

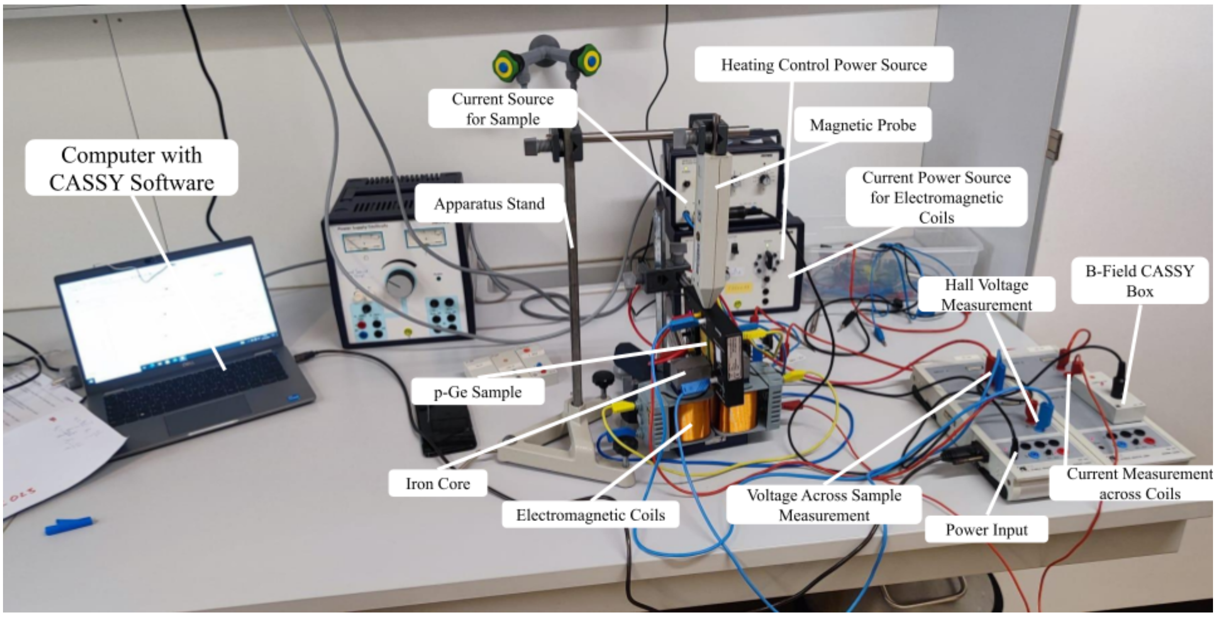

Figure 1: Setup for the Quantitative Hall Effect Investigation of p-Ge

This experiment focuses on the investigation of two distinct solid materials, p-Ge and Copper, and the categorization of their charge carriers. As the Hall Effect is directly dependent on the incident magnetic field ($\vec{B}$) across the sample, it is crucial to conduct an investigation which determines the relationship between the current through the coils and the resultant magnetic field measured at the sample. To measure the dependence of the magnetic field strength across the sample, a magnetic probe was positioned in the air gap between the iron core, and the B-Field was measured through the use of a B-Field CASSY box, as seen in figure 1. From the theory established during the introduction, it is clear that the relationship between the magnetic field density B and the current I shown in equation 7 should hold. This is verified by taking measurements of the magnetic field with no sample present inside the air gap, and varying the current through the coils from -2A to 2A with a step size of 0.5A. From this data, the values which most appropriately fit linear behavior were extracted, and the slope of the graph was taken and compared to the proportionality constant shown in equation 7.

After establishing this proportionality and ensuring its literary accuracy to a sufficient degree, the first p-doped Germanium p-Ge sample is introduced. The sample is placed between the air gap in the iron core, and is wired such that the voltage V and current I across the sample can be measured alongside the hall voltage U_H in the orthogonal direction (see Figure 1). With the wiring set up and the magnetic probe stationed next to the sample inside of the iron cores air gap, measurements of the voltage across the sample were taken with respect to a varied current I through the sample (between 0-25mA). The slope of the resultant linear plot of voltage against current was used to determine the total resistance of the sample, and the resistance was substituted into formula 3 alongside the known literary values.

The investigation then shifts its objective to focus on the nature of the Hall Voltage in p-doped Germanium. First, the Hall Voltage U_H of the p-Ge sample is determined with dependence on the magnetic field strength B across the sample for a fixed current of 0.025A. The magnetic field B is measured using the magnetic probe placed between the air gap in the iron core, and the current I is measured via the CASSY placed in series. The coil current was varied between -1.5A to 1.5A, which resulted in a range of magnetic field strengths from -0.313T to 0.308T. From this data a linear graph was created and the slope extracted was applied to equation 13 to find the Hall Coefficient R_H.

Similarly, the magnetic field was then fixed at 0.3T, and the current was varied from -0.025A to 0.025A. The same data was then applied again to equation 13 to find the hall coefficient R_H, but this time the B-field was treated as a constant.

Using the two values of the hall coefficient R_H obtained through these two measurements, formulas 10 and 14 were applied to calculate the resultant charge carrier density n and Hall mobility μ_H of the charge carriers in p-Ge.

Lastly, the temperature dependence of the hall voltage U_H and conductivity σ was measured for a temperature range between 298.15K to 413.15K. The resultant Hall Voltage U_H and conductivity σ were plotted against temperature, and the slope of the graph of the logarithm of the conductivity against the reciprocal of the temperature was used in the intrinsic range as a means of calculating the Energy band gap of the p-doped semiconductor.

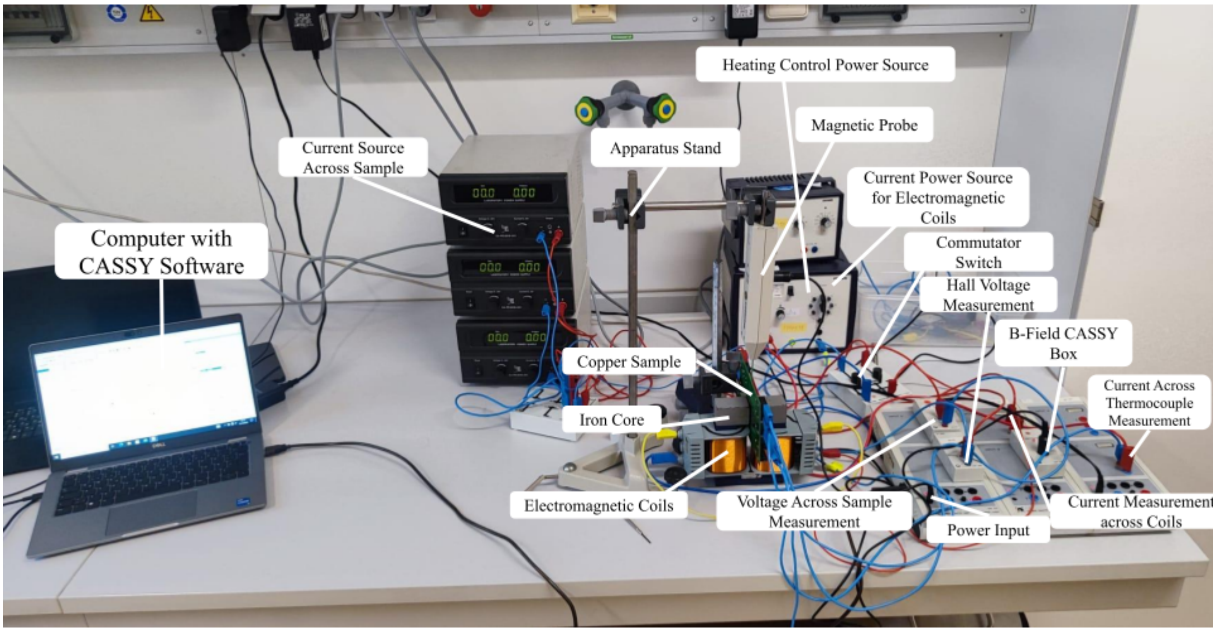

Figure 2: Setup for the Quantitative Hall Effect Analysis a of a Copper Sample

For the Copper sample, a very similar experiment was carried out, only with slightly varied current I values and more precise measurements of the hall voltage due to its very large conductivity σ. As Copper is a metal, it is a conductor, which means that its valence and conduction bands overlap, allowing for a large free mean path of the electrons in the material. To compensate for this in the experimental method, a larger current I ($\lessapprox 15A$) was passed through the sample for all measurements, and a more precise voltage V measurement instrument was used (on the order of μV) as seen in figure 2.

The conductivity was measured by taking current values of 0.5A to 15A and measuring the voltage. From this linear plot the slope was extracted as the resistance and was substituted appropriately into formula 3 to find the conductivity.

Then, the current across the copper sample was fixed at exactly 12A, and the magnetic field was varied to make observations of the hall voltage U_H. The coil current was varied again from 0A to 1.5A, resulting in an incident magnetic field strength on the sample ranging from -0.326T to 0.331T. After this measurement series, the magnetic field B was fixed at a constant value–which was assigned to a larger value of 0.412T to observe more significant results–and the current was varied from 0A to 15A.

Despite how small the scale of the measurements were, meaningful results were still found and the linear trend between the B-Field B, Current I, and Hall Voltage U_H were analyzed and substituted into equation 13 to calculate the hall coefficients of Copper. Using these two values–one for each fixation–the values of the hall mobility μ and charge carrier density n were calculated.

Lastly, in a similar fashion to the way that temperature dependence was measured for p-

Ge, the Cu sample was electrically heated in a temperature range from 305K to 415K. During the variation of temperature of the Copper sample, measurements of the Hall Voltage U_H, Voltage across the sample, and Current across the sample were measured. Using the Voltage and Current measurements, the electrical conductivity was calculated, and was resultantly plotted against temperature to observe the consequent behavior. It was expected due to the nature of conductors that after the Debye temperature the conductivity would decrease with respect to temperature. This hypothesis was tested with the empirical results of the experiment.

4 Results and Data Analysis

The first aspect of the investigation focused on the quantitative analysis of the relationship between the current through the electromagnetic coils and the magnetic field strength B experienced in the air gap between the two iron core segments. Before any measurements were taken, the theoretical constant of proportionality between this current and the magnetic field strength was calculated using formula 7. By substituting the known literature values for the number of loops in the magnetic coils $N_{tot} = 1200$, the permeability of free space $\mu_0 = 4\pi \cdot 10^{-7} VsA^{-1}m^{-1}$ and the width of the air gap of the iron core $\delta = 0.6 \cdot 10^{-2}m$, one achieves a theoretical estimate of this proportionality constant. Below is the calculation which was performed to find said theoretical value:

$$B_{air} = \frac{\mu_0 N_{tot}}{\delta} I_{coil}$$

$$\implies \frac{\mu_0 N_{tot}}{\delta} = \frac{4\pi \cdot 10^{-7}[TmA^{-1}](1200)}{0.6 \cdot 10^{-2}[m]} \approx 0.2513[TA^{-1}] \tag{15}$$

For a constant temperature, formula 7 implies a linear relationship between the magnetic field strength B and the current I through the electromagnetic coils. This relationship was examined by measuring the magnetic field density in the air gap against the current through the coils, which was varied from -2A to 2A. Below is the table of the measured values of the Magnetic Field Density, alongside the theoretical magnetic field density, which is written the last column to be compared with the measured magnetic field density (see Table 4)

Table 1: Measurements of Magnetic Field Density against Current through Coils

Current Through Coils (+- 0.001A)	Magnetic Field Density (+-0.001T)	Theoretical Magnetic Field Density (+-0.00025T)
-2.000	-0.437	-0.50265
-1.500	-0.342	-0.37699
-1.000	-0.233	-0.25133
-0.500	-0.115	-0.12566
0.000	0.000	0.00000
0.500	0.115	0.12566
1.000	0.234	0.25133
1.500	0.343	0.37699
2.000	0.447	0.50265

Observing the values of the magnetic field density B from the table above, the directly proportional positive relationship is evident. For a current of 0A through the coils, a magnetic field density of 0T is observed, which is expected. To better gauge the correlation between the empirical measurements and theoretical values, a plot of the magnetic field density against the current through the coils was plotted (in Black) in comparison to the theoretical magnetic field density (in Red). Below is the plot detailing this comparison.

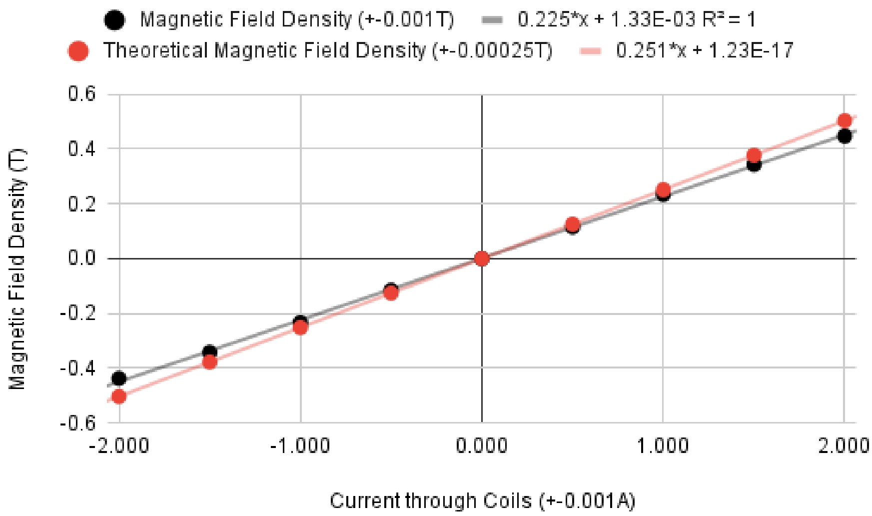

Figure 3: Plot of Magnetic Field Density against Current Through Coils

As one can see, a clearly positive proportional relationship is present between the magnetic field density B and the current I through the coils. It can be observed however that the measured magnetic field density differs from the theoretical value as it devolves into slightly non-linear behavior for the currents of larger magnitude. To compensate for this behavior to determine the final experimental value of the constant of proportionality, only the values which appear linear in form are taken. The values chosen ranged from currents of -1A to 1A. Below is the plot of the restricted domain to ensure the linearity of the results. Using the slope of this graph, the true measured constant of proportionality is calculated.

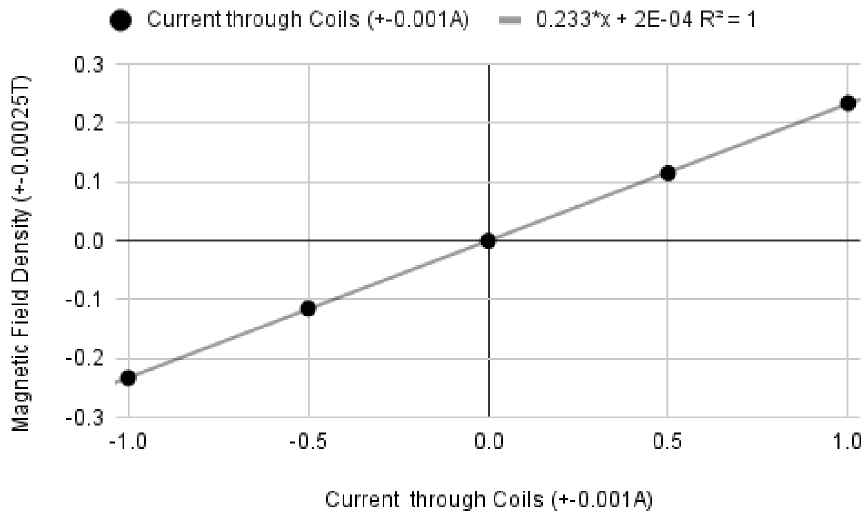

Figure 4: Plot of Magnetic Field Density against Current (Restricted to Linear Domain)

As one can see from the plot with the restricted current domain above, the plot behaves even more linearly. Extracting the slope from the graph above, the constant of proportionality is given by:

$$\text{Slope} = 0.2328 \pm 0.0008[TA^{-1}] \tag{16}$$

Comparing this value with the theoretical value calculated from equation 7, it is clear that a strong correlation exists between the empirical measurements and the theoretical value, $\frac{\mu_0 N_{tot}}{\delta} \approx 0.2513[TA^{-1}$. While there is a slight deviation of $\approx 0.0185[TA^{-1}]$, it is clear that the two values coincide to a commensurable degree. Note also however that that the theoretical value does not have any uncertainty, as it is calculated from fundamental physical constants or measurements extracted from the equipment given by the manufacturer. This disallows the two values to be considered statistically consistent, as they fall outside of each other's absolute uncertainty ranges, even though they can be clearly categorized as agreeable in nature.

For the next aspect of the investigation, the intrinsic properties of a p-doped Germanium semiconductor (p-Ge) sample were verified. To begin, the conductivity of the sample was determined through the variation of the current across the sample and the measurement of the resultant voltage which lay across the sample. For this measurement, the effect of a current range across the sample of 0-25 mA was examined in relation to the voltage across the sample. Below is the table of results for this measurement.

Table 3: Measured Voltage Across the Sample against Current		
Current Across Sample ($\pm 1mA$)	Current Across Sample ($\pm 0.001A$)	Voltage Across Sample ($\pm 0.01V$)
0	0	-0.05
1	0.001	-0.01
2	0.002	0.05
3	0.003	0.09
4	0.004	0.17
5	0.005	0.19
6	0.006	0.28
7	0.007	0.35
8	0.008	0.4
9	0.009	0.44
10	0.01	0.51
11	0.011	0.55
12	0.012	0.61
13	0.013	0.64
14	0.014	0.71
15	0.015	0.77
16	0.016	0.8
17	0.017	0.89
18	0.018	0.93
19	0.019	0.98
20	0.02	1.02
21	0.021	1.09
22	0.022	1.16
23	0.023	1.22
24	0.024	1.29
25	0.025	1.34

The table above demonstrates the relationship between the current across the sample and the resultant potential difference. As the conductivity σ of p-doped Germanium (p-Ge) is given by the ratio of these two associated quantities, one can apply formula 3 to determine it. Additionally, the resistance of the sample is defined as the voltage over the sample divided by the current over the sample–by Ohm's law–which incentivizes the calculation of the slope of the voltage against current to additionally determine the resistance of the sample. Once the resistance of the sample is calculated, the intermediate formula shown in equation 3 involving the resistance can be used, substituting in the appropriate values for the length of the sample and its cross-sectional area. The length of the sample and its cross-sectional area are given and calculated as shown below (Wagner & Söcker, 2024):

$$l_{sample} = 20 \cdot 10^{-3}[m] \tag{17}$$

$$A_{sample} = w_{sample} \cdot d_{sample} = (10 \cdot 10^{-3}[m]) \cdot (10^{-3}[m]) = 10^{-5}[m^2] \tag{18}$$

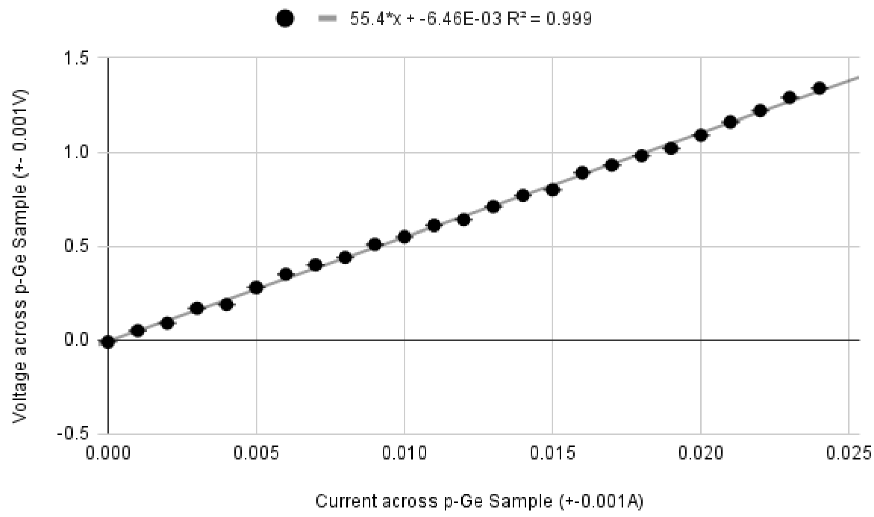

Figure 5: Plot of Voltage (V) against Current (I) for p-Ge Germanium Sample

Extracting the slope from the graph above, the resistance of the sample is determined directly.

$$\text{Slope} = R = (55.34 \pm 0.39)\Omega \tag{19}$$

Using the literature values of the length and cross-sectional area of the sample, the conductivity can be determined by applying formula 3.

$$\sigma = \frac{l}{R \cdot A} = \frac{20 \cdot 10^{-3}[m]}{55.34[\Omega] \cdot 10^{-5}[m^2]} = (36.14 \pm 0.25)[\Omega^{-1}m^{-1}] \tag{20}$$

As one can see, the conductivity σ of p-Ge is relatively small. This implies that at room temperature, p-Ge does not conduct electricity as well. However, from pre-established theory it is known that as the temperature of the semiconductor increases passed a specific threshold and into the intrinsic semiconductor range, the conductivity of the material will begin to increase. As the conductivity is low at the moment, it is safe to assume that the semiconductor exists in its saturation state at room temperature, where the doped impurities are ionized, but not enough energy is present to allow the charge carriers to efficiently jump between the conduction and valence band.

Moving on, the relationship of the Hall Voltage across the semiconductor with respect to a variable B-Field was measured. The current across the semiconductor was fixed at 0.025A to permit the flow of charge carriers in a variable magnetic field density B. From equation 13, it is evident that when subject to a fixed current, the Hall voltage U_H will exhibit linear behavior with respect to the magnetic field B across the sample. The data and graph on the next page exhibit this linear trend dictated by equation 13 in a clear manner.

Table 4: Hall Voltage U_H against measured Magnetic Field B		
Current through Wire Loops ($\pm 0.001A$)	Magnetic Field At Sample ($\pm 0.001T$)	Measured Hall Voltage Across Sample ($\pm 0.001V$)
-1.5	-0.313	-0.054
-1.4	-0.284	-0.052
-1.3	-0.27	-0.049
-1.2	-0.249	-0.046
-1.1	-0.228	-0.042
-1	-0.207	-0.039
-0.9	-0.188	-0.036
-0.8	-0.169	-0.032
-0.7	-0.148	-0.028
-0.6	-0.131	-0.024
-0.5	-0.106	-0.019
-0.4	-0.086	-0.015
-0.3	-0.069	-0.012
-0.2	-0.045	-0.007
-0.1	-0.023	-0.003
0	0	0.003
0.1	0.022	0.006
0.2	0.042	0.01
0.3	0.065	0.014
0.4	0.081	0.018
0.5	0.101	0.022
0.6	0.123	0.025
0.7	0.143	0.03
0.8	0.164	0.033
0.9	0.182	0.036
1	0.205	0.041
1.1	0.224	0.045
1.2	0.244	0.048
1.3	0.265	0.051
1.4	0.286	0.054
1.5	0.308	0.057

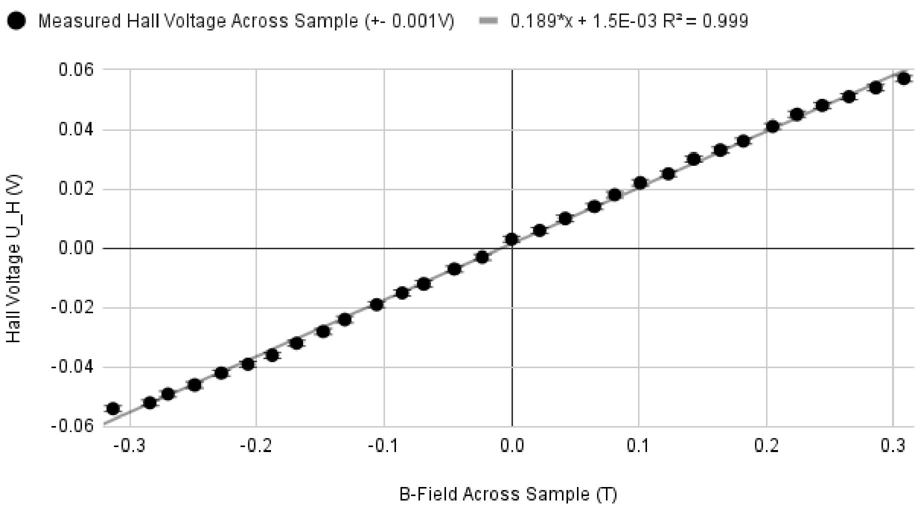

Figure 6: Graph of Hall Voltage against Magnetic Field (B), p-Ge sample, fixed 0.025A

The plot of the Hall Voltage U_H against the magnetic field B across the semiconductor is clearly linear in nature, and the direct proportionality of these constants is evident due to how close the y-intercept of the trend line lies to the origin. The slope of the graph above (Figure 6) provides the value of $\frac{U_H}{B_z}$ in equation 13, and is given by:

$$\frac{U_H}{B_z} = (0.189 \pm 0.001)[VT^{-1}] \tag{21}$$

As the quantities of the thickness d of the Germanium sample and the current I are known to be $10^{-3}[m]$ and $0.025[A]$ respectively, all data necessary to compute the hall coefficient of Germanium is known. Substituting the respective values of the slope $\frac{U_H}{B_z}$, current I and thickness d into equation 13, one achieves:

$$R_H = \frac{U_H}{B_z} \cdot \frac{d}{I} = (0.189)[VT^{-1}] \cdot \frac{10^{-3}[m]}{0.025[A]} = (0.007556 \pm 0.000306)[VmAT^{-1}] \tag{22}$$

As one can immediately observe from the sign of the Hall Coefficient, the sign of the charge carriers in the semiconductor are positive. Applying formulas 14 and 10, one can additionally calculate the hall mobility μ and the carrier charge density n as follows:

$$\mu = |R_H|\sigma = |7.56[VmAT^{-1}]| \cdot 36.14[Sm^{-1}] = (0.2731 \pm 0.0112)[m^2V^{-1}s^{-1}] \tag{23}$$

$$n = \frac{1}{qR_H} = \frac{1}{1.6 \cdot 10^{-19}[C] \cdot 7.56[VmAT^{-1}]} = (8.26 \pm 0.335)10^{20}[m^{-3}] \tag{24}$$

Alternatively, one can compute the Hall Coefficient R_H of p-doped Germanium (p-Ge) by analyzing the relationship between the Hall Voltage U_H and a varied current I across the sample for a fixed B-Field. The same method is repeated for a fixed B-field of density 0.3T, and the current is varied from 0-25mA. For clarity, only the graph of the values of hall voltage against the current is included to avoid clutter. All values obtained in the table can be seen graphically in figure 7.

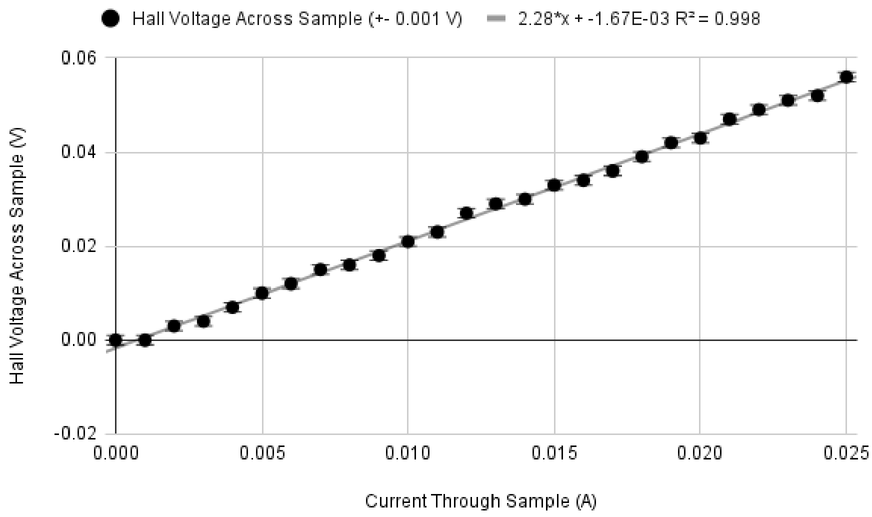

Figure 7: Plot of Hall Voltage Against Current for Fixed B-Field (0.3T) for p-Ge sample

Extracting the slope $\frac{U_H}{I}$ from this graph, the ratio of these two quantities is given explicitly by:

$$\frac{U_H}{I} = (2.28 \pm 0.02)[VA^{-1}] \tag{25}$$

Substituting this value into equation 13, one can calculate the resultant hall coefficient of p-doped Germanium (p-Ge) by substituting values of $0.3T$ and $10^{-3}m$ for the incident b-field and thickness of the sample respectively. Below is the aforementioned calculation in detail.

$$R_H = \frac{U_H}{I} \cdot \frac{d}{B_z} = (2.28)[VA^{-1}] \cdot \frac{10^{-3}[m]}{0.3[T]} = (0.007593 \pm 0.000073)[VmAT^{-1}] \tag{26}$$

Similar to before, the consequent calculated values of the hall mobility μ and the charge carrier density n can be determined. Below are the calculations of these values.

$$\mu = |R_H|\sigma = |7.6[VmAT^{-1}]| \cdot 36.14[Sm^{-1}] = (0.2744 \pm 0.0032)[m^2V^{-1}s^{-1}] \tag{27}$$

$$n = \frac{1}{qR_H} = \frac{1}{1.6 \cdot 10^{-19}[C] \cdot 7.6[VmAT^{-1}]} = (8.22 \pm 0.0796) \cdot 10^{20}[m^{-3}] \tag{28}$$

Comparing the two results for the Hall Coefficients R_H and their derived quantities of the hall mobility μ and carrier density n, the two results agree very nicely with one another, only deviating by a minimal amount. The similarity in these two coefficients effectively demonstrates the consistency of the method employed during this investigation.

Moving on, the effects of temperature on the p-doped Germanium semiconductor on the Hall Voltage U_H and Electrical Conductivity σ will be measured for a fixed incident magnetic field strength B of $0.3T$ and a current I of $0.025A$. The temperature range used (in C°) varied from $25C^{\circ}$ to $140C^{\circ}$ in steps of 5°. For each temperature, the reciprocal of the temperature was calculated, alongside the Hall Voltage across the sample U_H, the Voltage Across the sample, the Current across the sample, and the resultantly calculated conductivity σ through formula 3. Below is the data table which contains these raw and calculated data measurements. The green highlighted section of the table accentuates the range of reciprocal temperatures that satisfy the temperature requirements necessary such that the semiconductor exhibits intrinsic semiconductor properties. These temperatures will be used later on a restricted domain such that the slope of the linear section of the conductivity graph will be used to determine the energy band gap between the conduction and valence bands.

Table 5: Raw Data for Hall Voltage and Conductivity against the Temperature of the p-Ge Sample

Temperature of Sample (+- 1 C)	Temperature of the Sample (+- 1K)	1/Temp. of Sample (+- 1K^-1)	Hall Voltage of Sample (V)	Voltage Across Sample (+-0.001V)	Calculated Conductivity of Sample
25	298.15	0.0034	0.052	1.38	36.23
30	303.15	0.0033	0.052	1.41	35.46
35	308.15	0.0032	0.052	1.46	34.25
40	313.15	0.0032	0.052	1.5	33.33
45	318.15	0.0031	0.052	1.54	32.47
50	323.15	0.0031	0.051	1.58	31.65
55	328.15	0.0030	0.049	1.61	31.06
60	333.15	0.0030	0.048	1.63	30.67
65	338.15	0.0030	0.045	1.65	30.30
70	343.15	0.0029	0.041	1.64	30.49
75	348.15	0.0029	0.034	1.62	30.86
80	353.15	0.0028	0.03	1.54	32.47
85	358.15	0.0028	0.024	1.49	33.56
90	363.15	0.0028	0.016	1.36	36.76
95	368.15	0.0027	0.01	1.25	40.00
100	373.15	0.0027	0.005	1.05	47.62
105	378.15	0.0026	0.002	0.97	51.55
110	383.15	0.0026	0	0.88	56.82
115	388.15	0.0026	0	0.74	67.57
120	393.15	0.0025	0	0.65	76.92
125	398.15	0.0025	0	0.57	87.72
130	403.15	0.0025	-0.002	0.5	100.00
135	408.15	0.0025	-0.002	0.43	116.28
140	413.15	0.0024	-0.002	0.41	121.95

The data in the table above serves to represent many important relationships between specific properties of the p-doped semiconductor. To begin, the relationship between the Hall Voltage U_H and the temperature of the sample T is plotted to qualitatively analyze the effects of temperature T on the Hall Voltage across the sample U_H.

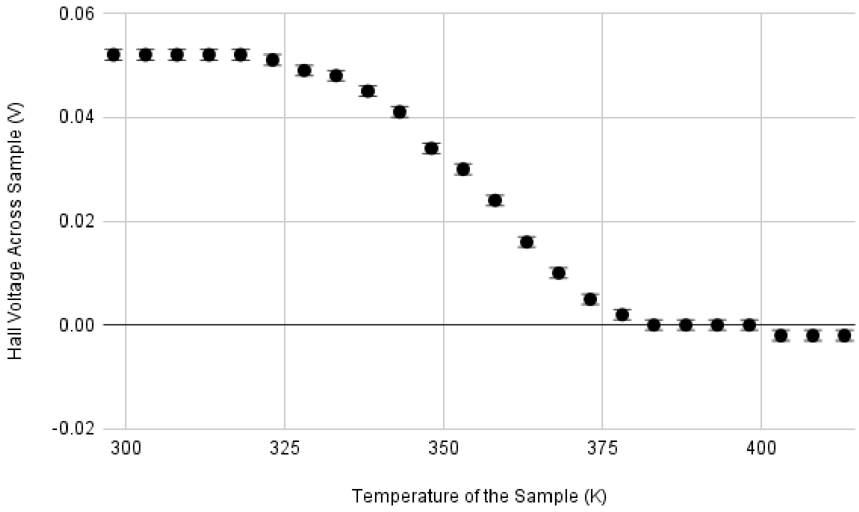

Figure 8: Plot of Measured Hall Voltage against Temperature for Fixed B-Field (0.3T) and Fixed Current (0.025A)

The graph above demonstrates very interesting behavior, which can best be explained by the behavior of the charge carrier density in p-doped type semiconductors. When the temperature of a p-doped semiconductor increases, the charge density also increases due to the ionization of the impurities in the semiconductor. By applying formula 10, it is known that the Hall Coefficient decreases inversely to the temperature. As the current I, b-field B_z and the thickness of the sample are all fixed, one can show that when the Hall Coefficient decreases inversely in the intrinsic range, so will the hall voltage by applying formula 13. As seen in the graph, this is exactly what is occurring. As the temperature moves into the intrinsic domain of temperatures, the Hall Voltage begins to drop, directly caused by the increase in charge carrier density.

Using the data from the table, one can similarly investigate the relationship between the electrical conductivity and the temperature of the sample. From pre-established theory regarding the Debye Temperature, the Electrical Conductivity σ is predicted to remain constant in materials under the Debye temperature, however after reaching it, begin to grow in a 'pseudo-exponential' fashion. Below denoted the plot of raw data extracted for the electrical conductivity against the temperature of the sample in Kelvin.

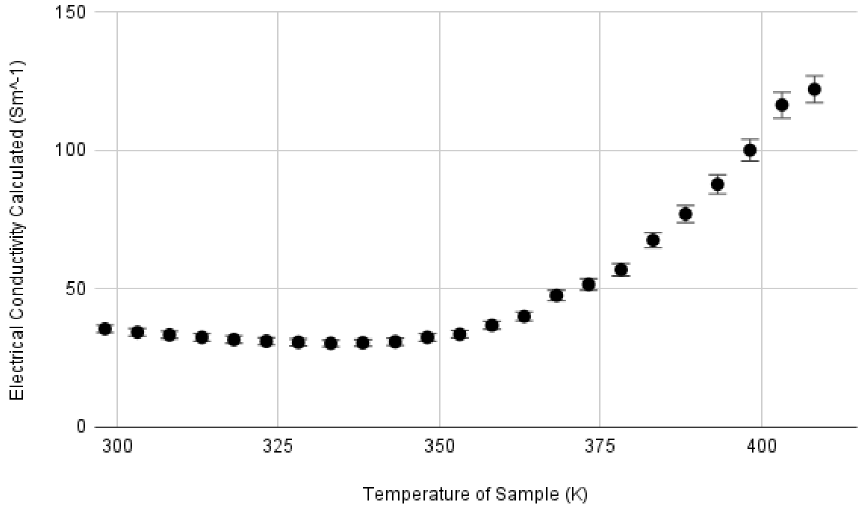

Figure 9: Plot of conductivity against temperature of the p-Ge sample

As one can see, the empirical measurements are consistent with the preestablished theory. A clear exponential trend is present after approximately 350K, indicating the presence of the Debye temperature somewhere in that temperature neighborhood. So far, all gathered results from the data collected has fit with the established theory qualitatively, however no meaningful numerical results have been established. However, one can use the relationship between the electrical conductivity and the temperature of the sample with scaled axes as a means of calculating the energy band gap E_g using formula 4. Below is the plot of the logarithm of the electrical conductivity $\log \sigma$ against the reciprocal of the temperature $\frac{1}{T}$ of the sample. Using the slope of the graph for the intrinsic range where the graph appears linear will be used to calculate the energy band gap.

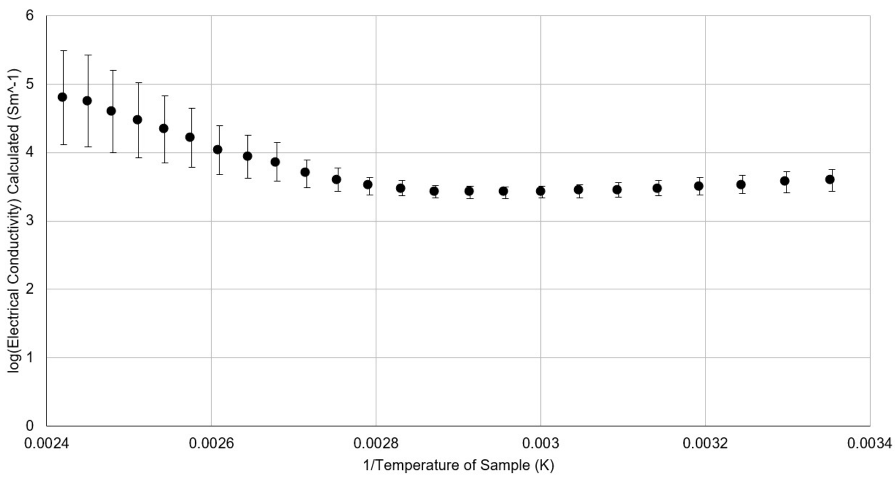

Figure 10: Plot of log(conductivity) against reciprocal of temperature of the p-Ge sample

As one can see, there is a clear range of temperature reciprocals such that linear behavior is exhibited. This range lies from x-axis values of $0.024K^{-1}$ to approximately $0.0275K^{-1}$.

Restricting the graph to the intrinsic range will yield a linear graph with a slope directly related to the energy gap of the semiconductor through formula 4. Below is the linear fit to the data extracted from the intrinsic temperature range of the semiconductor.

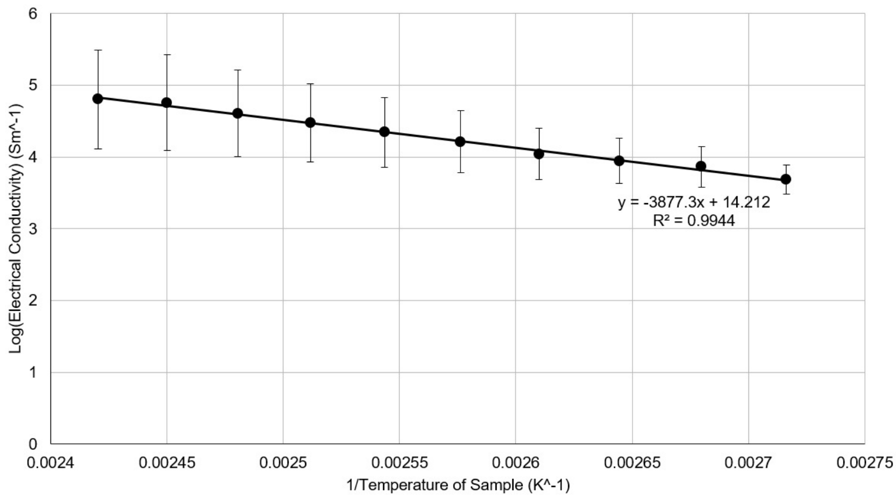

Figure 11: Plot of log(Electrical Conductivity) against reciprocal of Temperature p-Ge

As one can see in the plot, as the reciprocal of the temperature increases, the logarithm of the electrical conductivity in the material (p-Ge) appears to decrease linearly. The slope of this linear graph appears to be approximately:

$$\text{Slope} = \log(\sigma) \cdot T = (-3877 \pm 100)[Sm^{-1}K] \tag{29}$$

Applying this slope to equation 4, the energy band gap of the semiconductor can be calculated. Below is the calculation:

$$E_g = -2k_B T \log(\sigma) = -2(1.380649 \cdot 10^{-23}) \cdot (-3877) = 1.07 \cdot 10^{-19}[J] = (0.67 \pm 0.02)[eV] \tag{30}$$

Therefore, the energy gap between the valence and conduction band of the p-doped Germanium semiconductor is approximately 0.67 eV. As most semiconductors have energy band gaps of approximately 1-2eV, it is clear that the result obtained agrees with the theory to a commensurable degree.

With all relevant properties of the p-doped Germanium sample measured, one can confidently make remarks on all aspects of the material related to the hall coefficient and its derived quantities. With a similar method, the hall coefficient R_H and electrical conductivity of a copper sample and its derived quantities can also be measured. To begin, the sample was fixed between the gap in the iron core electromagnet, and the power source was upgraded to allow for currents up to 15A to flow through the sample. As the conductivity σ of copper is theoretically much greater than that of p-doped Germanium, a much larger current is necessary to yield a measurable voltage. Before any measurements of voltage or current were taken, the geometrical measurements for the length and cross sectional area of the copper sample were examined. Given by the Advanced Physics Lab I manual, the length and cross-sectional area of the sample are:

$$l_{cu} = 150 \cdot 10^{-3}[m] \tag{31}$$

$$A = d \cdot w = 0.025[m] \cdot 18 * \cdot 10^{-6}[m] = 4.5 \cdot 10^{-7}[m^2] \tag{32}$$

Below is the plot of values of voltage across the sample against the current across the sample for currents from 0A ranging to 15A. Note the scale of axes, and how for such a large current, a very small voltage is observed. This phenomenon limited some of the experimental accuracy in the measurements taken, as the voltage across the sample was at times deviating on intervals smaller than the smallest scale division of the instrument, leading to some digital-form data which will be shown. However, as such a large current was used during the measurements of conductivity, the graph below demonstrates a clear and smooth trend.

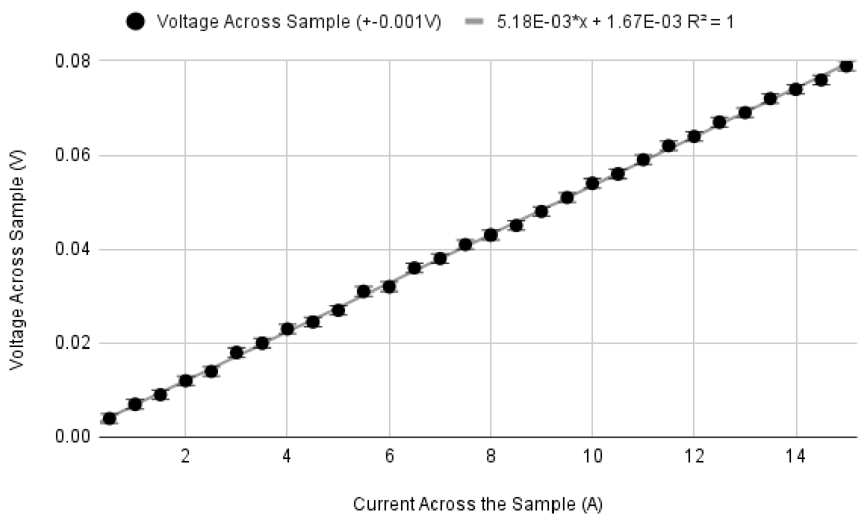

Figure 12: Plot of the Voltage across the sample against the Current across the sample to determine the Electrical Conductivity of Copper

The slope of this graph yields the resistance of the Copper sample, due to Ohm's Law. Below is the measured slope of the relationship between the voltage and current across the sample respectively.

$$\text{Slope} = R = \frac{V}{I} = (5.18 \pm 0.02 \cdot 10^{-3})[\Omega] \tag{33}$$

Applying equation 3 to calculate the conductivity of Copper, the resulting conductivity value was found to be:

$$\sigma = \frac{l}{R \cdot A} = \frac{150 \cdot 10^{-3}[m]}{5.18[\Omega] \cdot 4.5 \cdot 10^{-7}[m^2]} = (6.43 \pm 0.0262)10^7[Sm^{-1}] \tag{34}$$

As one can observe, the conductivity of copper is much larger than that of the conductivity of p-doped Germanium semiconductor. This is likely due to the much larger free mean path of the charge carriers in the Copper. Additionally, as copper is a metal and a conductor, the conduction and valence band overlap, allowing free transport of charge. This means that it is much easier for charges to conduct, thereby explaining the large conductivity.

Next, the current across the Copper sample was fixed at exactly 12A, and the magnetic field was varied from within an approximate range of -0.3T to 0.3T. The graph below demonstrates the linear trend that was observed between the Hall Voltage U_H across the sample and the incident magnetic field. Note that due to how small the Hall Voltage was–on the order of a few micro volts (μV)–the measurements were limited by the precision of the μV-voltmeter. For this reason, the results in the graph appear to be step wise. While this may result in a worse approximation of the real value, as the results are already expected to be linear, the step wise nature should not impact the validity of the results to a large degree.

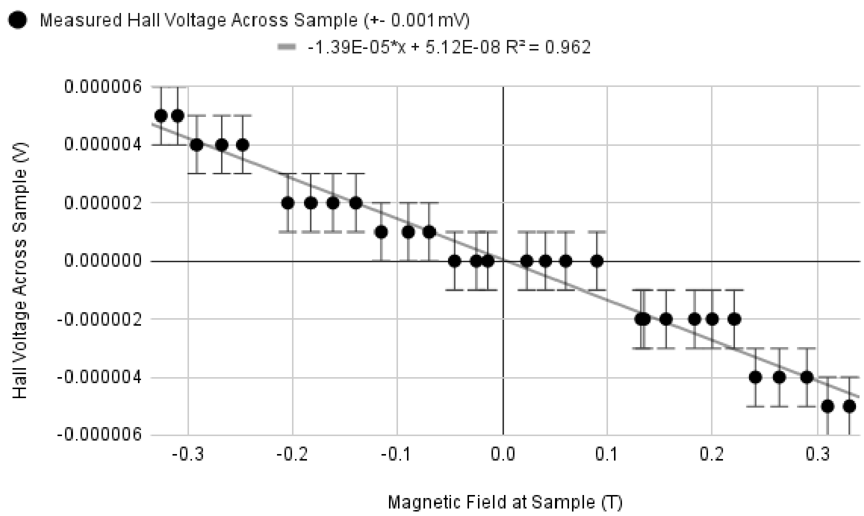

Figure 13: Plot of Hall Voltage U_H against magnetic field B for Copper Sample

The slope of this graph can be used in conjunction with formula 13 to calculate the hall coefficient of copper for a fixed current across the sample. The slope of the graph is given as:

$$\text{Slope} = \frac{U_H}{B_z} = (-1.39 \pm 0.05) \cdot 10^{-5}[VT^{-1}] \tag{35}$$

Substituting the slope into equation 13, the Hall Coefficient R_H of copper calculated with a fixed current of 12A is:

$$R_H = \frac{U_H \cdot d}{I \cdot B_z} = (-1.39 \cdot 10^{-5}[VT^{-1}]) \cdot \frac{18 \cdot 10^{-6}[m]}{15[A]} = (-2.08 \pm 0.08)10^{-11}[VmAT^{-1}] \tag{36}$$

It is immediately clear from the sign of the hall coefficient that the charge carriers of copper are electrons. Furthermore, the hall coefficient R_H allows for the calculation of the mobility and the charge carrier density by applying formulas 10 and 14. Below are the calculated values of the mobility and charge carrier density which were calculated in the exact same way as detailed during the analysis of the p-Germanium data.

$$\mu = (1.34 \pm 0.05)10^{-3}[m^2V^{-1}s^{-1}] \tag{37}$$

$$n = (-2.99 \pm 0.11)10^{29}[m^{-3}] \tag{38}$$

As one can immediately observe, the charge carrier density of copper is much greater than that of germanium, which makes sense due to its very large conductivity. Moving on, a similar measurement was conducted to calculate the hall coefficient R_H only with a fixed B-Field of 0.412T. The current was then varied between 0A and 15A and the trend of the hall voltage U_H was observed. Below is a plot of the results. Again notice that the results appear stepwise solely due to the limited precision of the instruments used.

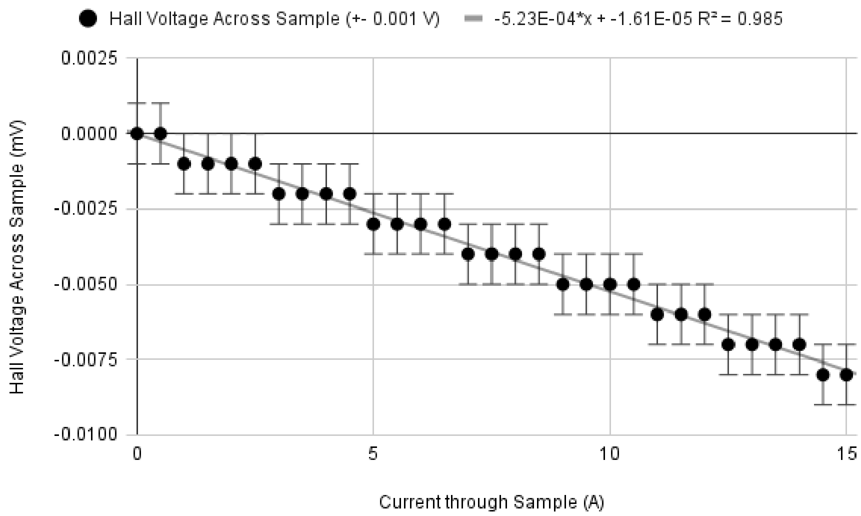

Figure 14: Plot of Hall Voltage against Variable Current through Cu Sample for B-Field of 0.412T

As on can see, a similar negative trend between the hall voltage and the current through the sample is present just as in the first graph. More specifically, the slope calculated as the relationship between the hall voltage U_H and the current I is given as:

$$\text{Slope} = \frac{U_H}{I} = (-5.23 \pm 0.16) \cdot 10^{-7}[VA^{-1}] \tag{39}$$

Applying the slope to formula 13, one achieves a Hall Coefficient R_H value of:

$$R_H = \frac{U_H}{I} \cdot \frac{d}{B_z} = (-2.28 \pm 0.05)10^{-11}[VmAT^{-1}] \tag{40}$$

From the calculated Hall Coefficient from a fixed B-Field of 0.412T, the mobility μ

and charge carrier density n can be found through the same application of formulas 10 and 14. Below are the calculated values of the hall mobility and charge carrier density respectively:

$$\mu = (1.47 \pm 0.03)10^{-3}[m^2V^{-1}s^{-1}] \tag{41}$$

$$n = (-2.73 \pm 0.06)10^{29}[m^{-3}] \tag{42}$$

From the sign of the hall coefficient one can again reaffirm that the sign of the charge carriers is negative. Furthermore, both calculated values for the carrier density and mobility agree with one another to a commensurable degree.

For the last aspect of the investigation, the effects of temperature T on the electrical conductivity σ and hall voltage U_H were measured across the copper sample. A range of temperatures from 305K to 415K was used, and the values of the electrical conductivity were calculated from empirical measurements of the current and voltage across the sample respectively. Below is the table which showcases the collected data.

Table 6: Raw Data for Hall Voltage and Conductivity against the Temperature of the p-Ge Sample

Temperature of Sample (+- 1 C)	Temperature of the Sample (+- 1K)	Hall Voltage of Sample (+- 0.001 mV)	Voltage Across Sample (+-0.001V)	Calculated Conductivity of Sample	Error of conductivity
31.85	305	-0.014	0.097	5.15E+07	5.31E+05
36.85	310	0.013	0.099	5.05E+07	5.10E+05
41.85	315	0.041	0.101	4.95E+07	4.90E+05
46.85	320	0.075	0.101	4.95E+07	4.90E+05
51.85	325	0.098	0.102	4.90E+07	4.81E+05
56.85	330	0.119	0.102	4.90E+07	4.81E+05
61.85	335	0.132	0.103	4.85E+07	4.71E+05
66.85	340	0.146	0.104	4.81E+07	4.62E+05
71.85	345	0.154	0.104	4.81E+07	4.62E+05
76.85	350	0.166	0.105	4.76E+07	4.54E+05
81.85	355	0.174	0.105	4.76E+07	4.54E+05
86.85	360	0.181	0.106	4.72E+07	4.45E+05
91.85	365	0.188	0.106	4.72E+07	4.45E+05
96.85	370	0.192	0.106	4.72E+07	4.45E+05
101.85	375	0.194	0.106	4.72E+07	4.45E+05
106.85	380	0.199	0.108	4.63E+07	4.29E+05
111.85	385	0.201	0.108	4.63E+07	4.29E+05
116.85	390	0.205	0.108	4.63E+07	4.29E+05
121.85	395	0.206	0.108	4.63E+07	4.29E+05
126.85	400	0.207	0.109	4.59E+07	4.21E+05
131.85	405	0.199	0.109	4.59E+07	4.21E+05
136.85	410	0.206	0.109	4.59E+07	4.21E+05
141.85	415	0.188	0.111	4.50E+07	4.06E+05

From the table above, the relationships between Conductivity and Temperature, and Hall Voltage and Temperature were extracted and plotted into graphs with their corresponding errors. Note that the plots contain error bars, but due to the order of magnitude of difference between uncertainty and measurement, are not fully visible in the plot. First, the conductivity against temperature is plotted. To hypothesize about the expected behavior of the electrical conductivity with respect to the temperature, one can associate the relevant relationship between the resistivity and the temperature. As the temperature of a metal increases, the resistivity increases as well. As the electrical conductivity is defined as the reciprocal of the resistivity, it is clear that the electrical conductivity should decrease with respect to the temperature in an inversely proportional manner. Below is the plot of electrical conductivity against temperature.

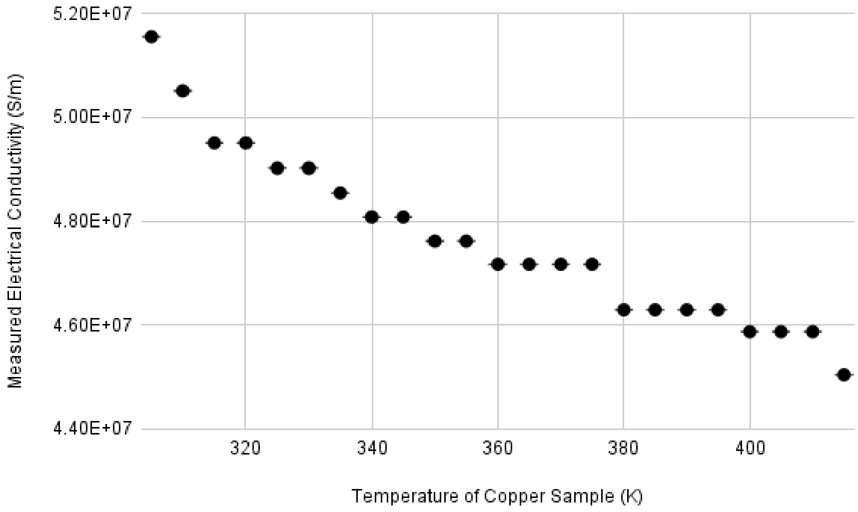

Figure 15: Plot of Electrical Conductivity against Temperature of Sample (K)

As one can observe in the plot, the electrical conductivity behaves as expected from pre-established theory. As the temperature of the sample increases, the measured electrical conductivity of the sample appears to decrease in an inversely proportional manner. Moving on, one can extract the relationship between the measured hall voltage and the temperature of the sample. As the hall voltage is inversely proportional to the charge carrier density n for constant current and B-field, one can analyze the behavior of the charge carrier density and then apply its behavior inversely to the hall voltage to determine its behavior. As the conductivity decreases with temperature, so does the charge carrier density, by formula 2. As the hall voltage U_H is inversely proportional to the charge carrier density, by equations 10 and 13, it is expected that as the temperature increases, so does the hall voltage. The plot of the hall voltage against temperature is detailed below:

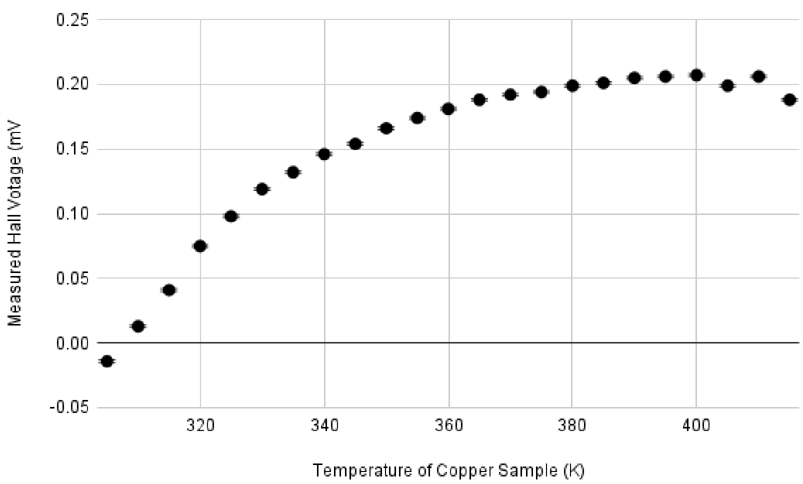

Figure 16: Plot of Electrical Conductivity against Temperature of Sample (K)

As one can see, the trend of the hall voltage agrees with pre-established theory. As the temperature of the copper sample increases, the charge carrier density decreases, and due to the inversely proportional relationship between the hall voltage U_H and the charge carrier density in the sample n, the hall voltage increases.

All measurements taken throughout the investigation adhere to the expected behavior to a strong degree. There exists a clear correlation between related measurements of the hall coefficients for both solid materials, and meaningful conclusions were derived from the associated calculated quantities.

5 Error Analysis

For all errors involving the slope of the graph, the uncertainty of the slope was approximated using the built-in Excel function (LINEST), which works by approximating the linear slope of a set of data points through the least squares method. As a byproduct, the error of the slope is given in absolute form. Due to the efficiency of this method, and the large quantity of graphs, it was the clear choice for this investigation.

For all errors involving numerical calculations with propagated error, such as for the uncertainties of the final measurements, the root sum of squares method (RSS) was employed. This method involves calculating the propagated error of a formulaic value by computing all partial derivatives with respect to all error prone parameters, and taking it under a square root. The formula for this is given below.

$$\Delta y = \sqrt{\sum_{i=0}^{n} \left(\frac{\partial y}{\partial x_i} \cdot \Delta x_i \right)^2} = \sqrt{\left(\frac{\partial y}{\partial x_1} \cdot \Delta x_1 \right)^2 + \cdots + \left(\frac{\partial y}{\partial x_n} \cdot \Delta x_n \right)^2} \tag{43}$$

The calculated propagation formula for the error of the conductivity is given by:

$$\Delta \sigma = \frac{l}{w \cdot d \cdot U} \sqrt{(\Delta I)^2 + \left(\frac{I}{U} \Delta U \right)^2} \tag{44}$$

Here, l, w and d are the physical dimensions of the sample and are considered to have no error since they are taken from the literature values of the manual. The voltage U and current I were measured using the CASSY system, hence their errors are instrumental errors of the CASSY system. The uncertainties ΔU and I reflect the precision limits of the CASSY measurement system.

The error in all Hall coefficients R_H, is given by the error propagation formula:

$$\Delta R_H = \sqrt{\left(\frac{d}{I} \Delta (\frac{U_H}{B_z}) \right)^2 + \left(\frac{dU_H}{B_z I^2} \Delta I \right)^2} \tag{45}$$

For R_H, the term U_H/B_z was treated as a single variable because it was obtained from the graph and its error was determined using the LINEST function. Again, d had no associated error and ΔI was the instrumental error from the CASSY system.

The error in mobility, $\Delta\mu$, is derived from the errors in conductivity σ and the Hall coefficient R_H:

$$\Delta\mu = \sqrt{(\sigma\Delta R_H)^2 + (R_H\Delta\sigma)^2} \tag{46}$$

Since μ is a product of σ and R_H, the error propagation formula involves the relative errors of both σ and R_H, which were previously determined.

The error in charge carrier density, Δn is given by:

$$\Delta n = \sqrt{\left(\frac{\Delta R_H}{qR_H^2}\right)^2} \tag{47}$$

The error of the charge carrier density n is only dependent on R_H, as the elementary charge q does not have any error as it is a fundamental constant.

The CASSY system, like all measurement devices, has a finite precision. The smallest step units observed on some graphs indicate the limits of this precision. During the experiment, the current may not have been perfectly stable, causing fluctuations that would affect the accuracy of measurements. This instability could result in perturbations in the measured values of voltage and current. Factors such as temperature variations, magnetic field interference or vibrations in the setup could have introduced additional errors.

6 Discussion

Reflecting upon the investigation, it is clear than an extensive analysis of the intrinsic properties of both solid materials was successfully performed. The Conductivities of each material reflect their real world behavior to a commensurable degree. In addition, the Hall Coefficients yield derived values of the hall mobility μ and charge carrier density n that match with the expected properties of each medium. Lastly, the temperature dependent relationships examined for the p-doped Germanium sample clearly demonstrated the behavior of the conductivity in the intrinsic temperature range, and allowed for the calculation of a literarily consistent value for the energy gap between the Valence and Conduction bands. Furthermore, the temperature dependent relationships examined for the Copper sample similarly demonstrated the effects of temperatures greater than the Debye temperature, and the inverse relationship between the Hall Voltage and the Charge carrier density in the conductor.

While most of the experimental method was fruitful in allowing the appropriate examination of these intrinsic phenomena, there are various places where the experiment could be improved. To begin, a clear fault in the experimental design was the lack of consideration of the order of magnitude of certain results during instrumental preparation. As seen in the graphs which measure the hall voltage U_H of copper against magnetic field and current, the measurements were limited in precision to the smallest scale division of the micro-volt μV sensor. This lack of precision led to digital-like data which appeared step-wise in nature rather than the expected linear behavior. This of course is an issue as the finer detail of the relationship between the hall voltage U_H and magnetic field B cannot be examined to a precise degree. To account for the small values of the hall voltage in Copper, it may be beneficial in future experimentation to add a voltage amplifier between the CASSY and the sample such that the measurement is not limited to the smallest scale division of the CASSY μV sensor. Alternatively, an even more precise voltage box could be used to avoid the potential electric noise produced by a voltage amplifier.

As mentioned during the data analysis section, the values of the Hall Coefficients R_H for both solid material samples accurately reflected the expected behavior of each material. To begin, the hall coefficient R_H of the p-doped Germanium sample can be deemed accurate as all derived quantities from it match the expected description of the material. To begin, the sign of the hall coefficient for p-Germanium is positive, which matches with the p-doped nature of the semiconductor. In addition, the charge carrier density of p-Ge is small in relation to copper, which makes sense due to it's lower charge concentration and semiconductor-like nature. It is also important to note that the hall coefficient was calculated in two different ways, each method fixing one of the associated quantities of the hall coefficient: the current through the sample, and the incident b-field. By contrast, the hall coefficient R_H of the Copper sample and its derived values also accurately describe its intrinsic properties. To begin, the sign of the hall coefficient are negative, indicating the presence of negative charges. As copper is a very good conductor of electrons–and is in fact used in electrical technologies all the time–its principal charge carrier is the electron. In addition, due to Copper's much larger mean free path of the electrons due to its crossed valence and conduction bands, electrons move much more freely through the material and are the main mediators of conduction. Note also that the charge carrier density n calculated for the copper sample in both examples is much larger than that of the p-doped Germanium sample. This can be correlated to the much larger conductivity of copper to that of Germanium, applying formula 2.

On the grounds of conductivity, the conductivity of copper was also observed to be much larger than that of p-doped Germanium. This is clear again due to the semiconductor nature of the Germanium sample, and the conductor-like nature of the Copper sample. In both cases, the numerical value of the conductivity and its uncertainty agree to the theoretical values to a commensurable degree. During all measurements, the temperature of the sample was kept at the same room temperature to ensure that no fluctuations in the conductivities were observed and the two values could be compared efficiently.

Overall, all values collected during the experiment match with the theoretical predictions from pre-established theory. While some improvements could be made to the experimental method to ensure that the measurements taken are not limited by the instrumental precisions of the setup, the overall design of the experiment is sufficient in analyzing the intrinsic properties of the two selected media.

7 Conclusion

The results of the experiment are as follows. First the constant of proportionality calculated between the current in the two electromagnetic coils and the magnetic field strength is given by:

$$\text{Slope} = 0.2328 \pm 0.0008[TA^{-1}] \tag{48}$$

The literature value of the proportionality constant as calculated from equation 7 is given by:

$$\frac{\mu_0 N_{tot}}{\delta} = \frac{4\pi \cdot 10^{-7}[TmA^{-1}](1200)}{0.6 \cdot 10^{-2}[m]} \approx 0.2513[TA^{-1}] \tag{49}$$

While the calculated proportionality constant does not fall within the range of absolute error of the measured value, it is clear that the two values agree with one another due to the minimal difference between them $\approx 0.0185[TA^{-1}]$. While they cannot be considered statistically coincident, it is clear that the trend predicted is followed to some degree. After the measurement of this proportionality constant was taken, the conductivity σ of both the p-doped Germanium (p-Ge) and Copper samples were examined. The values of the conductivity of the p-doped Germanium and Copper are given as follows:

$$\sigma_{pGe} = \frac{l}{R \cdot A} = \frac{20 \cdot 10^{-3}[m]}{55.34[\Omega] \cdot 10^{-5}[m^2]} = (36.14 \pm 0.25)[\Omega^{-1} m^{-1}] \tag{50}$$

$$\sigma_{copper} = \frac{l}{R \cdot A} = \frac{150 \cdot 10^{-3}[m]}{5.18[\Omega] \cdot 4.5 \cdot 10^{-7}[m^2]} = (6.43 \pm 0.0262) \cdot 10^7 [Sm^{-1}] \tag{51}$$

As briefly discussed in the discussion, it is evident why the conductivity σ of Copper is much larger than that of p-doped Germanium: copper is a conductor, and Germanium is doped as a semiconductor. After measuring the conductivities of both materials the Hall Coefficients were measured as a means of determining the charge sign, hall mobility and charge carrier density of the samples. The measured Hall Coefficient R_H of the Germanium sample for a fixed current, and the associated hall mobility μ and charge carrier density n are given as:

$$R_H = (0.007556 \pm 0.000306)[VmAT^{-1}] \tag{52}$$

$$\mu = (0.2731 \pm 0.0112)[m^2 V^{-1} s^{-1}] \tag{53}$$

$$n = (8.26 \pm 0.335)10^{20}[m^{-3}] \tag{54}$$

Similarly, the measured Hall Coefficient R_H of the Germanium sample for a fixed incident B-Field of 0.3T and the associated hall mobility μ and charge carrier density n are given as:

$$R_H = (0.007593 \pm 0.000073)[VmAT^{-1}] \tag{55}$$

$$\mu = (0.2744 \pm 0.0032)[m^2V^{-1}s^{-1}] \tag{56}$$

$$n = (8.22 \pm 0.0796) \cdot 10^{20}[m^{-3}] \tag{57}$$

The two measured values for the hall coefficient R_H of Germanium do not agree statistically–they do not fall within each others absolute error–but this is primarily due to how small the error of each coefficient is. In practice, one could still associate a very strong correlation between the two values, as they are equivalent up to the fifth decimal place and only differ by ≈ 0.000037. For this reason, one can still claim that the two calculated values of the hall coefficient for germanium are accurate to one another. Similarly, due to the agreement between both measured values of the hall coefficient, the values for the hall mobility and charge carrier density are similarly close.

Moving on, the measured hall coefficient R_H for the copper sample for a fixed current, and the associated hall mobility μ and charge carrier density n are given as:

$$R_H = (-2.08 \pm 0.08) \cdot 10^{-11}[VmAT^{-1}] \tag{58}$$

$$\mu = (1.34 \pm 0.05) \cdot 10^{-3}[m^2V^{-1}s^{-1}] \tag{59}$$

$$n = (-2.99 \pm 0.11) \cdot 10^{29}[m^{-3}] \tag{60}$$

Again, the measured Hall Coefficient R_H of the Copper sample for a fixed incident B-Field of 0.412T and the associated hall mobility μ and charge carrier density n are given as:

$$R_H = (-2.28 \pm 0.05) \cdot 10^{-11}[VmAT^{-1}] \tag{61}$$

$$\mu = (1.47 \pm 0.03) \cdot 10^{-3}[m^2V^{-1}s^{-1}] \tag{62}$$

$$n = (-2.73 \pm 0.06) \cdot 10^{29}[m^{-3}] \tag{63}$$

As one can see, both hall coefficients are negative and are $\approx 0.2[VmAT^{-1}]$ different from one another. Both values fall outside of each others error, however for a similar reason to before this is likely due to the small absolute error of each measurement which prohibits statistical agreement except for when the values are very close to each other. Due to the similarity of the two measured hall coefficients R_H, the resultant calculated values of the hall mobility and the charge carrier density appear consistent with one another for both cases.

As discussed before, the temperature dependence of the conductivity for the p-doped Germanium semiconductor was measured as a means of calculating the energy band gap of the semiconductor. The final value for the energy gap of the semiconductor (in eV) is given as:

$$E_g = 1.07 \cdot 10^{-19}[J] = (0.67 \pm 0.02)[eV] \tag{64}$$

As stated in the manual, typical semiconductor energy band gaps range from 1-2eV, so a value of 0.67eV is in the same order of magnitude and area as expected for a typical semiconductor. In addition, the band gap may vary between different semiconductors, and so 0.67eV is a very reasonable value to have obtained. Other than the quantitative value of the slope extracted from the temperature conductivity plots, the plots of the hall voltage against temperature for both the semiconductor and conductor agree with the expected behavior. The rationale behind the curves exhibited in the plots is detailed extensively during the data analysis section, however lies in the phenomena exhibited by semiconductors in the intrinsic range of temperatures, and conductors past the Debye temperature. As an extension to the current experimental design, it may be interesting to also examine the behavior of p-Ge for low temperatures near the freeze-out range. This could be performed through the use of an electric cooler, which would allow for the full range of temperature behavior to be examined, and a more broad understanding of the material to be developed. While this may come with experimental limitations, it is a clear choice of improvement to the method to further the pursuit of knowledge in regard to these solid materials.

References

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