

Geographic Data and Mapping

GEOG380 FA 2018

Statistical foundation

- ▶ Numerical approaches for map analysis
 - ▶ Methods for analyzing spatial data
 - ▶ Numeric summaries for analyzing data
 - ▶ Measures of central tendency, outliers, ranges
 - ▶ Stem-and-leaf plot & histogram
 - ▶ Variance & standard deviation
 - ▶ Rates, proportions, and percentages
 - ▶ Standardizing data



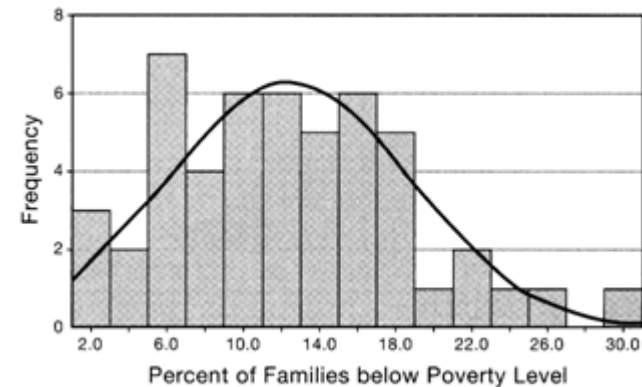
Numerical approaches for map analysis

: Methods for analyzing spatial data

► Graphs

► Histogram

- Class: a group between multiple values
- Height: amount of the values in each class
- Normal distribution
 - Most of the observations locate near the mean (middle of the distribution)
 - Fewer observations locate in both tails
- Positively / negatively skewed distributions (where are the tails?)
 - Examples: figure 3.2 (p.39, next slide)



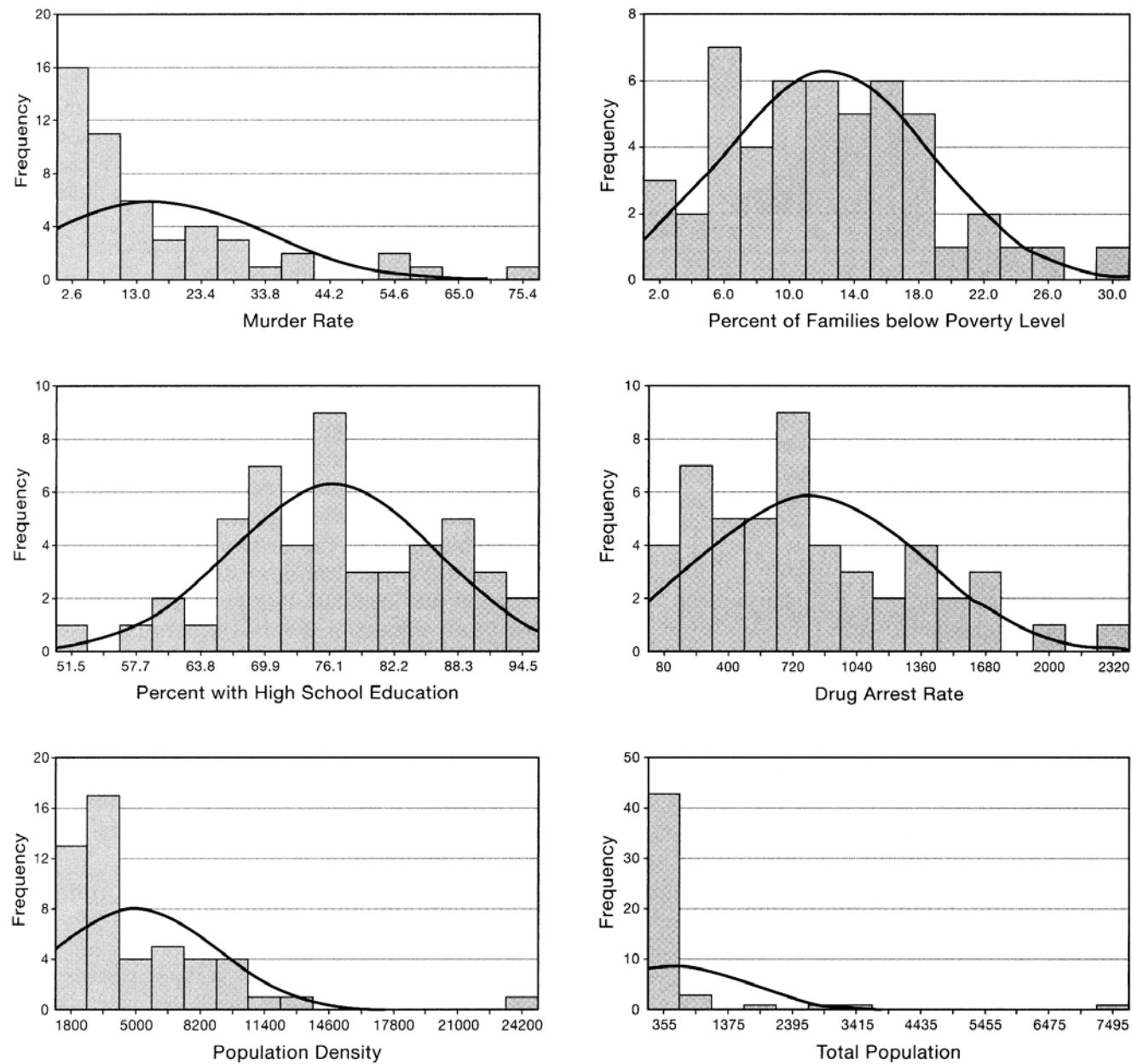


FIGURE 3.2 Histograms for the data presented in Table 3.1.

Methods for analyzing spatial data

► Tables

► Raw table

► Table 3.1 (p.36)

TABLE 3.1 Sample data for 50 U.S. cities (sorted on murder rate)

City	Murder Rate*	Families below Poverty Level (%)	High School Graduates (%)	Drug Arrest Rate [†]	Population Density [‡]	Total Population (in Thousands)
Irvine, CA	0.0	2.6	95.1	780	2607	110
Cedar Rapids, IA	0.9	6.6	84.5	110	2034	109
Overland Park, KS	0.9	1.9	94.1	255	2007	112
Livonia, MI	1.0	1.7	84.7	665	2823	101
Lincoln, NE	1.6	6.5	88.3	294	3033	192
Madison, WI	1.6	6.6	90.6	57	3311	191
Glendale, CA	1.7	12.3	77.2	452	5882	180
Allentown, PA	1.9	9.3	69.4	1078	5934	105
Tempe, AZ	2.1	7.0	89.9	295	3590	142
Boise City, ID	2.4	6.3	88.6	512	2726	126
Lakewood, CO	2.4	5.2	88.2	216	3100	126
Mesa, AZ	3.1	6.9	84.8	223	2653	288
Pasadena, TX	3.4	11.1	69.8	370	2727	119
San Jose, CA	4.5	6.5	77.2	1289	4568	782
Waterbury, CT	4.6	9.9	66.8	1326	3815	109
Springfield, MO	5.0	11.6	77.0	446	2068	140
Chula Vista, CA	5.2	8.6	75.7	808	4661	135

Numeric summaries

2.6, 6.6, 1.9, 1.7, 6.5, 6.6, 12.3, 9.3

- ▶ If you use some kind of numeric information as data, you should also provide an explanation for that numeric such as...

- ▶ **Central tendency measurement**

- ▶ **Mode**: the most frequently occurring value (=6.6)

- ▶ **Median**: the middle value in an ordered set of data
(=6.55) (1.7, 1.9, 2.6, 6.5, 6.6, 6.6, 9.3, 12.3)

- ▶ **Mean**: the average of the data (=5.938)

- ▶ **Standard deviation (SD)**: average of distance between each value and the mean of the data
(=3.749)

- ± 1 SD from mean, ± 2 SD from mean
(later in details)

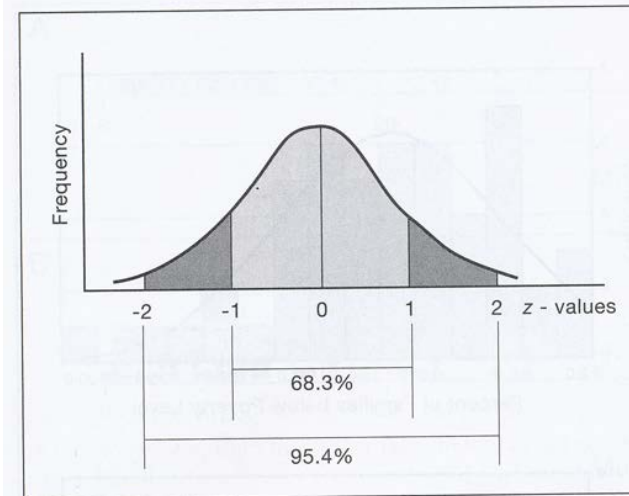
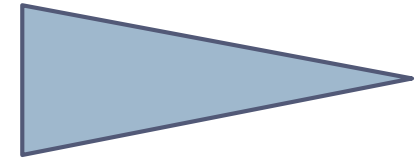


FIGURE 3.3 An example of a normal curve. Histograms will approximate this shape if the data are normal. For a perfectly normal data set, approximately 68 percent and 95 percent of the observations will fall within 1 and 2 standard deviations, respectively, of the mean.

Two types of statistics:

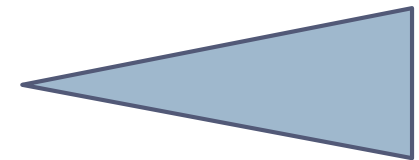
► Descriptive

- Reducing lots of data to manageable and digestible pieces of information — this is what most mapping is all about
 - e.g., generalization, shorelines



► Inferential

- Understanding what conclusions can be drawn from limited information. Often in the form of samples of a population
 - e.g. sampling, geostatistics



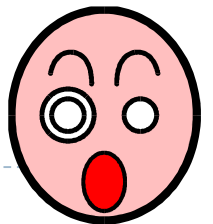
The problem: LOTS of data:

Number of deer-vehicle accidents between counties in Ohio

COUNTY_NAME	DEERVEH02	DEERVEH03	DEERVEH04			
MONROE	32	36	26	MONTGOMERY	415	395
CHAMPAIGN	79	56	98	UNION	332	364
LAWRENCE	202	135	101	MIAMI	327	390
HOCKING	146	127	112	BROWN	382	359
MADISON	149	147	121	WOOD	294	337
BELMONT	160	185	149	LICKING	338	324
OTTAWA	167	151	154	COLUMBIANA	381	447
VINTON	229	233	155	GEAUGA	425	443
PREBLE	214	214	158	ALLEN	404	405
VAN WERT	118	194	158	HIGHLAND	399	394
PIKE	146	119	162	DEFIANCE	335	354
PAULDING	159	181	177	GREENE	493	445
MERCER	179	177	182	SHELBY	339	373
PUTNAM	121	130	183	SENECA	308	381
NOBLE	228	213	186	MEDINA	389	426
CLARK	265	218	191	GUERNSEY	475	435
HARRISON	220	219	191	HOLMES	392	383
FAYETTE	195	223	195	WAYNE	434	506
HENRY	170	186	201	COSHOCTON	612	577
MORGAN	186	191	204	STARK	519	591
GALLIA	361	261	206	HANCOCK	373	500
CARROLL	245	279	214	PORTAGE	494	500
PERRY	279	243	218	ASHTABULA	554	583
MEIGS	204	207	220	WILLIAMS	378	453
FULTON	181	190	234	ASHLAND	432	488
JEFFERSON	270	253	238	MAHONING	443	516
SCIOTO	252	206	238	WARREN	425	482
AUGLAIZE	238	266	259	BUTLER	484	498
HARDIN	199	275	262	CUYAHOGA	476	525
DARKE	224	272	267	FAIRFIELD	493	505
CRAWFORD	238	286	268	TRUMBULL	477	482
ERIE	219	318	268	FRANKLIN	489	511
WASHINGTON	312	336	275	ROSS	518	555
WYANDOT	244	322	277	TUSCARAWAS	502	591
ATHENS	413	388	280	CLERMONT	561	541
PICKAWAY	302	336	283	LORAIN	419	517
ADAMS	323	332	287	DELAWARE	547	577
CLINTON	295	290	290	KNOX	632	612
MARION	296	308	297	LOGAN	561	449
SANDUSKY	256	320	299	SUMMIT	674	642
HURON	320	359	302			618
JACKSON	376	318	305			

Ordering the numbers

26,98,101,112,121,149,154,155,158,158,162,177,
182,183,186,191,191,195,201,204,206,214,218,220,
234,238,238,259,262,267,268,268,275,277,280,
283,287,290,297,299,302,305,308,308,310,325,333,
342,351,357,364,372,374,376,376,378,391,393,
399,400,410,416,440,455,455,460,461,464,472,473,
476,478,483,485,485,506,515,518,529,537,540,
560,576,612,618,670,714,718



Measures of central tendency

▶ The *mean*

- ▶ Add all the values
- ▶ Divide by the number of values

$$\frac{\sum x}{n}$$

339

▶ The *median*

- ▶ Rank all the values

26,98,101,112,121,149,154,155,158,158,162,177,182,183,186,191,191,195,
201,204,206,214,218,220,234,238,238,259,262,267,268,268,275,277,280,
283,287,290,297,299,302,305,308,308,310,325,333,342,351,357,364,372,
374,376,376,378,391,393,399,400,410,416,440,455,455,460,461,464,472,
473,476,478,483,485,485,506,515,518,529,537,540,560,576,612,618,670,
714,718

309

- ▶ Find the *middle value(s)* (between the 44th ~ the 45th values from 88 observations)

▶ The *mode* (more useful for class/category data)

- ▶ The *most common* value/class

158, 191, 238, 268,
376, 455, 485



Outliers

- ▶ An *outlier* is *an extreme value* a long way from the mean or median



- ▶ To identify outliers and other types of characteristics that relate to the *distribution* of data, we need measures of *data spread, variation, or dispersion* such as...
 - ▶ Range, Inter-quartile range (IQR)
 - ▶ Variance, Standard Deviation
(next slides)



Ranges

▶ Range

- ▶ Maximum value – minimum value
 $718 - 26 = 692$

▶ Quantiles

- ▶ Divides ranked observations into **equally-large sets**
 - ▶ Percentiles...into 100 sets (88 observations in total)
 - ▶ Deciles...into 10 sets
 - ▶ Quartiles...into 4 sets

▶ Inter Quartile Range (IQR)

- ▶ A range that includes the **middle-half** of the ranked data or...

$$IQR = P_{75} - P_{25} = 460 - 214 = 246$$

Stem-and-leaf plot: a graphical summary

- ▶ This gives a good **visual summary** of the data, without too much graphical efforts
- ▶ Useful for just getting a rough idea of the whole data

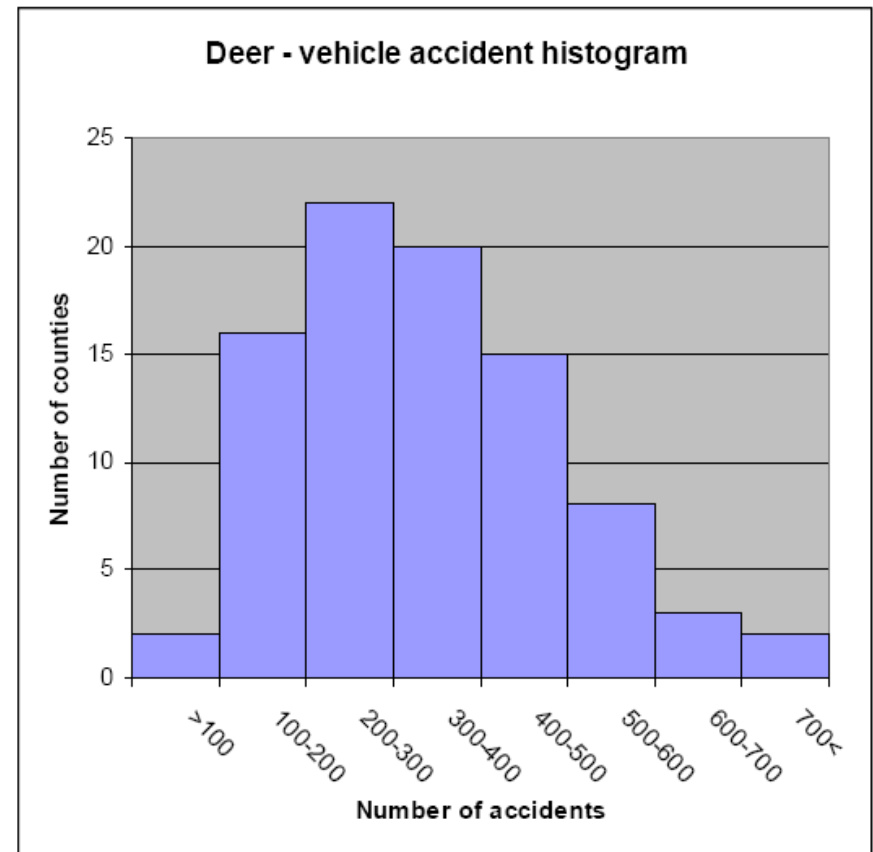
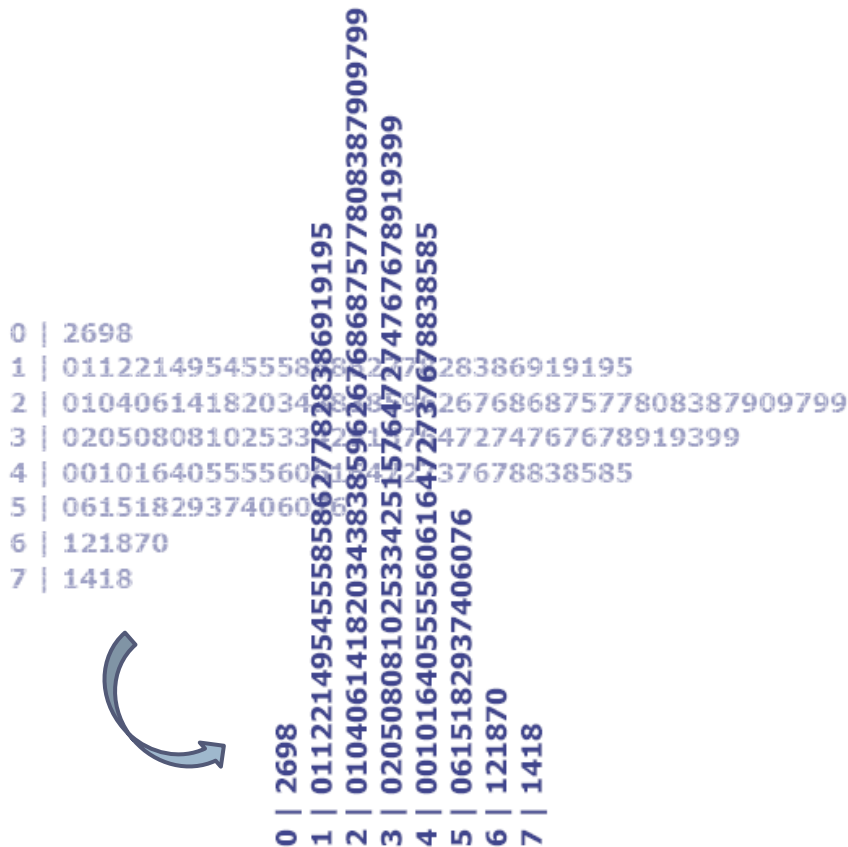
```
0 | 2698
1 | 01122149545558586277828386919195
2 | 01040614182034383859626768687577808387909799
3 | 02050808102533425157647274767678919399
4 | 00101640555560616472737678838585
5 | 0615182937406076
6 | 121870
7 | 1418
```



71, 74, 71, 78

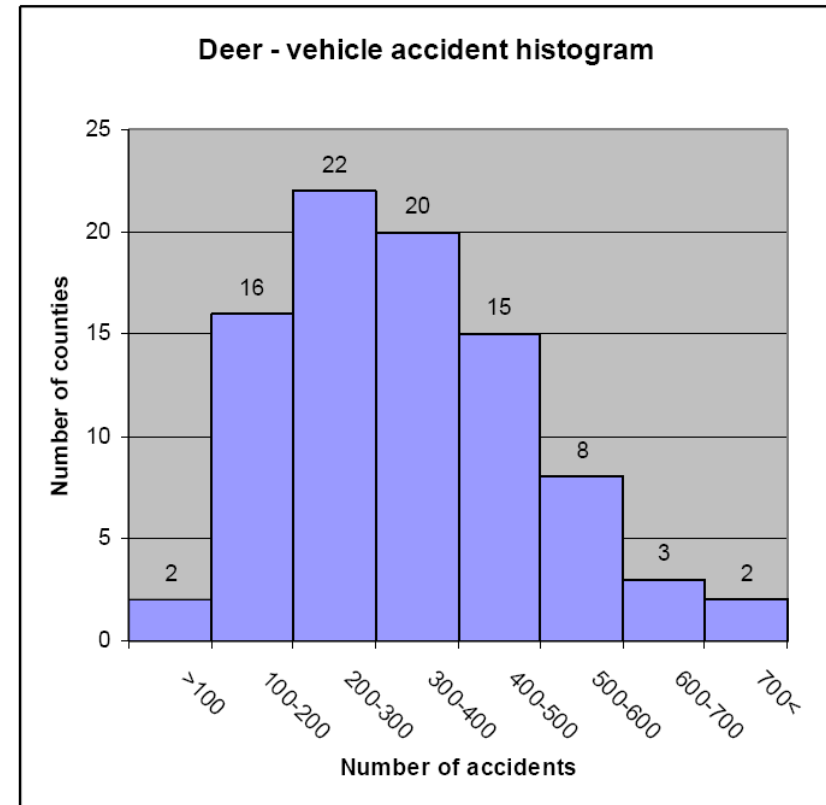
From Stem-and-leaf plot to Histogram

- Turn the stem-and-leaf plot 90 degrees counterclockwise



recap: Histograms

- ▶ A more refined form of stem-and-leaf plots
- ▶ Divide the range of the data into a series of **equal intervals** (ex. 0 to 100, 100 to 200, and so on)
- ▶ Count **how many cases** lie in each interval
- ▶ Plot the counts (or frequencies) as vertical bars



More about Histograms

- ▶ Some important points about the intervals:
 - ▶ Use **simple bounds** (i.e., 0.5-1.0, NOT 0.46-0.98)
 - ▶ Respect **natural breakpoints** (i.e., 0 °C, pH 7, 50%)
 - ▶ **No overlaps** (mutually-exclusive categories) between classes (i.e., NOT 0~10, 9~20)
 - ▶ **Cover all values** (i.e., NOT 0~10, 12~20)
 - ▶ **Same interval-widths** between classes (i.e., NOT 0~10, 11~15)
 - ▶ Appropriate **number of classes** (3 classes, 100 classes)

ALWAYS LABEL EVERYTHING in a histogram!



Variance and Standard Deviation

► Variance

- Calculates an average of **how much each value differs from the mean in squared**

$$\frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n-1}$$

- 1) Sum all differences
- 2) Use **a square** to avoid negative values
- 3) Divide by the number of values
(n-1 for a sample, n≥2) ← **Q. why n≥2?**

► Standard Deviation (SD or Std. Dev., σ , sigma)

- A measure of **how dispersed** numbers are
- Square root of the variance

$$\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n-1}}$$

► Ex) the deer-vehicle accident data

- Variance = 22405, Standard deviation = about 150
- Variance shows much bigger and positive numbers than std. dev.



Standard Deviation (SD or Std.Dev.)

- ▶ So what does the SD mean to us?
- ▶ Measure the SD and add/subtract from the mean, and you get **a range of deviation from the mean**
- ▶ Then often you can apply the statistical empirical rule
 - ▶ Given a set of n measurements of **a normally distributed** variable,
 - ▶ **The mean \pm 1SD** includes roughly **68%** of the observations
 - ▶ **The mean \pm 2SD** includes roughly **95%** of the observations
 - ▶ The mean \pm 3SD includes roughly **99.7%** of the observations

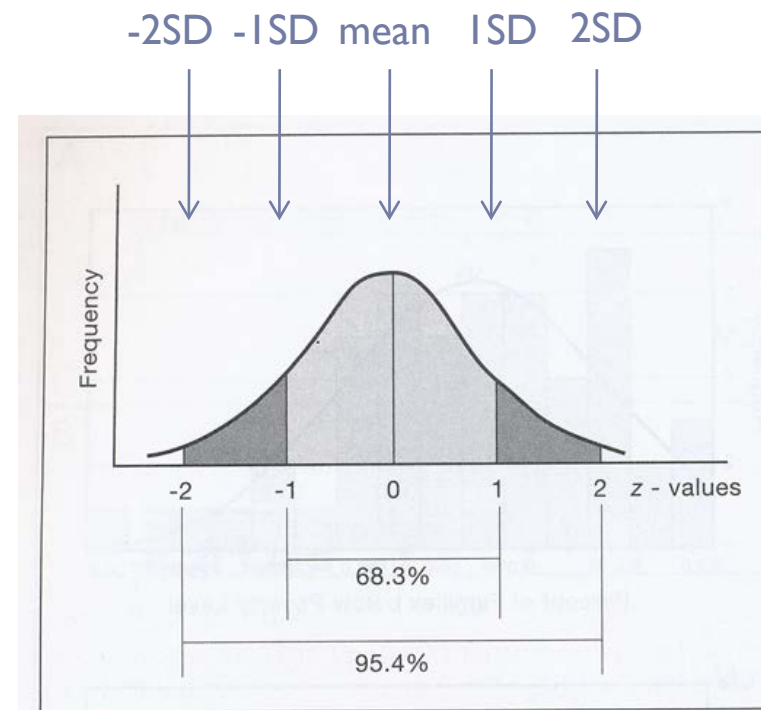
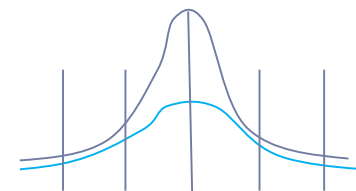


FIGURE 3.3 An example of a normal curve. Histograms will approximate this shape if the data are normal. For a perfectly normal data set, approximately 68 percent and 95 percent of the observations will fall within 1 and 2 standard deviations, respectively, of the mean.

Std.Dev. continued...

- ▶ For the deer-vehicle accidents data we have $mean=158$, $SD=150$, then...
 - ▶ $158 \text{ (mean)} \pm 1 \times 150 \text{ (1SD)} = 8 \sim 308$ (should be $\approx 68\%$ of all counties if the data shows normal distribution)
 - ▶ $158 \text{ (mean)} \pm 2 \times 150 \text{ (2SD)} = 0 \sim 458$ (should be $\approx 95\%$)
 - ▶ $158 \text{ (mean)} \pm 3 \times 150 \text{ (3SD)} = 0 \sim 608$ (should be $\approx 99.7\%$)
 - ▶ Was this close in the real data?
 - ▶ The *range* was **692** ($|26-718|$), so...
 - ▶ $8 \sim 308$ is \approx **50%** of the observations
 - ▶ $0 \sim 458 \approx$ **74%** of the observations
 - ▶ $0 \sim 608 \approx$ **94%** of the observations
- Data does **not** show **normal distribution** but dispersed



Mean Center and Dispersion Measures (p.51)

► Central tendency

► Mean center

$$\bar{s} = (\mu_x, \mu_y) = \left(\frac{\sum_{i=1}^n x_i}{n}, \frac{\sum_{i=1}^n y_i}{n} \right)$$

► Dispersion (measure of spread)

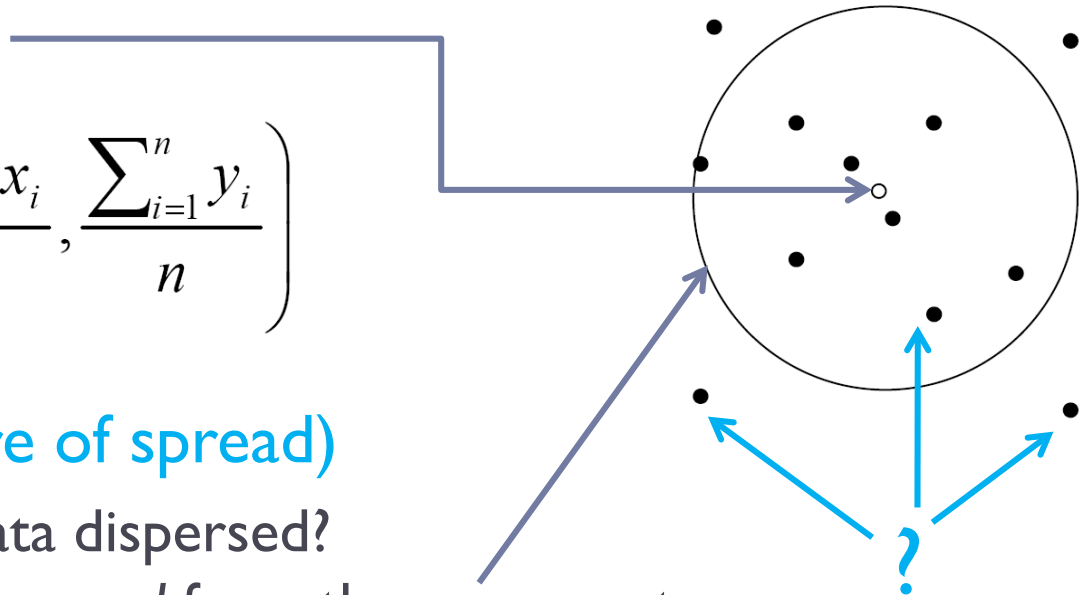
► How much is the data dispersed?

→ use standard distance d from the mean center
larger d : the data are more dispersed

$$d = \sqrt{\frac{\sum_{i=1}^n ((x_i - \mu_x)^2 + (y_i - \mu_y)^2)}{n}}$$

Similar to Std.Dev. =

$$\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n-1}}$$

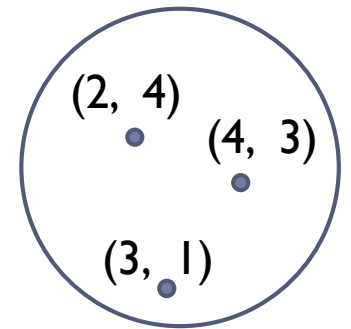


Exercise

- ▶ What is the mean center of the given points in the figure below?



- ▶ What is the dispersion measure?



(map not to scale)



Derived indices

- ▶ **Rates** make data **more comparable** than raw values
 - ▶ E.g. Rate of vehicle accidents *per* population
- ▶ You often hear rates reported as **an index**
 - ▶ Often an index expresses each value as **a percentage** of some base value, or as **standardized z-scores** (in details later)
 - ▶ *Aspatial* examples
 - ▶ Dow Jones, Consumer Price, Poverty, Sustainability, GNP, ...
 - ▶ *Spatial* examples
 - ▶ Location quotient (local economy VS. reference economy), Heat, Wetness, UV, Normalized Difference Vegetation Index (NDVI), The Average Watershed Nitrogen Leaching Index (AWNLI), ...



Rates, Proportions, and Percentages

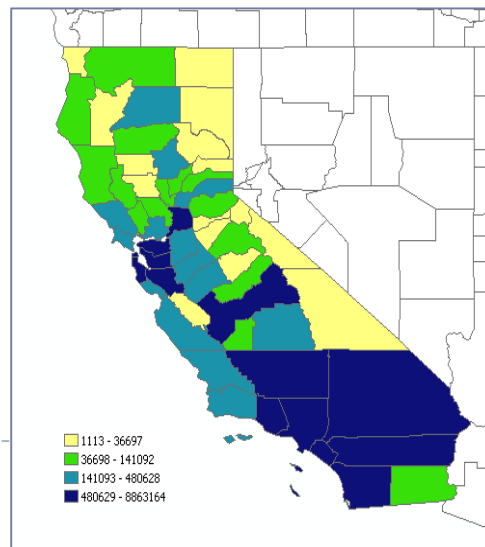
- ▶ Rates are a way of **standardization of data** to a common measure for **comparison** purposes

State	Population	Robbery	Total offenses	Robbery / 1000 p.	Robbery / all offenses
Colorado	4,417,714	3,555	186,379	0.80	1.9 %
Delaware	796,165	1,156	32,267	1.45	3.6 %

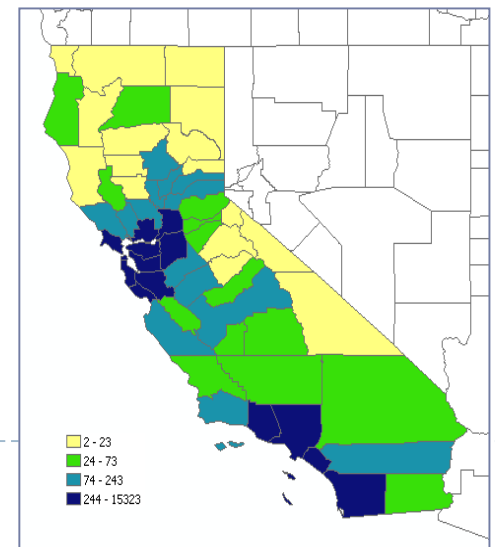
Source: U.S. Department of Justice

- ▶ Geographically... (LA county Vs. San Bernadino county)

1990
Population,
CA
(raw data)



1990
Population
Density,
CA
(rates)



Rate calculations – general notes

- ▶ Choose some **basis** of the unit value
 - ▶ Usually population, area, total income, or number of households... of **an areal unit**
 - ▶ However, some variables are relevant for total count
 - ▶ For example, ...?
- ▶ Decide how to express the **unit** of the results
 - ▶ Per person, per 100,000 people, per unit area, etc.
 - ▶ The calculation is:

Rate = Count / Basis **& Unit**

ex. 10**people/mi²**

ALWAYS LABEL THE UNIT!



September 23rd, 2009

Map of the day, McDonald's edition

Posted by: Felix Salmon
Tags: [consumption](#), [charts](#)

[Post a comment](#)
(3)

Are you happy
with the map...?



This beautiful map comes from [Stephen Von Worley](#), who has mapped the

September 24th, 2009
5:05 am GMT
[\[permalink\]](#)

If you normalised this by dividing by population density I'd imagine it's pretty smooth. Cool that you can see the highways in the West (route 80, 84 and 15).

- Posted by Nic Fulton

For next time

- ▶ Readings

 - ▶ Ch. 3

- ▶ Worksheet I

