Smolarkiewicz
A Simple Positive Definite Advection Scheme with small Implicit Diffusion

Gerhard Burger   Jelmer Wolterink

Scientific Computing
Utrecht University

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Outline

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The scheme We start with the following upstream advection equation on staggered grid:

$$\psi_i^{N+1} = \psi_i^N - \left(F\left(\psi_i^N, \psi_{i+1}^N, u_{i+1/2}^N\right) - F\left(\psi_{i-1}^N, \psi_i^N, u_{i-1/2}^N\right) \right),$$

where

$$F\left(\psi_i^N, \psi_{i+1}^N, u_{i+1/2}^N\right) = \\ \left(\left(u_{i+1/2}^N + u_{i+1/2}^N\right)\psi_i^N + \left(u_{i+1/2}^N - u_{i+1/2}^N\right)\psi_{i+1}^N \right)\frac{\Delta t}{2\Delta x}.$$

Writing it out Inserting this and collecting terms gives us

$$
\begin{aligned}
\psi_i^{N+1} = {} & \frac{\Delta t}{2\Delta x} \left(u_{i-1/2}^N + u_{i-1/2}^N \right) \psi_{i-1}^N \\
& + \left(1 - \frac{\Delta t}{2\Delta x} \left(u_{i+1/2}^N + u_{i+1/2}^N - u_{i-1/2}^N + u_{i-1/2}^N \right) \right) \psi_i^N \\
& - \frac{\Delta t}{2\Delta x} \left(u_{i+1/2}^N - u_{i+1/2}^N \right) \psi_{i+1}^N
\end{aligned}
$$

Writing it out We can write this as

$$\psi_i^{N+1} = \alpha_i \psi_{i-1}^N + \beta_i \psi_i^N + \gamma_i \psi_{i+1}^N, \quad \text{for } i = 1, \ldots, M-1,$$

where we have that

$$\alpha_i = \frac{\Delta t}{2\Delta x} \left(u_{i-1/2}^N + u_{i-1/2}^N \right),$$

$$\beta_i = \left(1 - \frac{\Delta t}{2\Delta x} \left(u_{i+1/2}^N + u_{i+1/2}^N - u_{i-1/2}^N + u_{i-1/2}^N \right) \right),$$

$$\gamma_i = -\frac{\Delta t}{2\Delta x} \left(u_{i+1/2}^N - u_{i+1/2}^N \right).$$

Matrix form We can also write this in matrix form

$$
\begin{bmatrix}
\psi_1^{N+1} \\
\psi_2^{N+1} \\
\vdots \\
\psi_{M-2}^{N+1} \\
\psi_{M-1}^{N+1}
\end{bmatrix}
=
\begin{bmatrix}
\beta_1 & \gamma_1 & & & \\
\alpha_2 & \beta_2 & \gamma_2 & & \\
& \ddots & \ddots & \ddots & \\
& & \alpha_{M-2} & \beta_{M-2} & \gamma_{M-2} \\
& & & \alpha_{M-1} & \beta_{M-1}
\end{bmatrix}
\begin{bmatrix}
\psi_1^{N} \\
\psi_2^{N} \\
\vdots \\
\psi_{M-2}^{N} \\
\psi_{M-1}^{N}
\end{bmatrix}
$$

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  - Second item.
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Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.

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📕 A. Author.
Handbook of Everything.
Some Press, 1990.

📄 S. Someone.
On this and that.
Journal of This and That, 2(1):50–100, 2000.