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Upstream advection equation on staggered grid:

$$\psi_i^{N+1} = \psi_i^N - \Big(F\left(\psi_i^N, \psi_{i+1}^N, u_{i+1/2}^N\right) - F\left(\psi_{i-1}^N, \psi_i^N, u_{i-1/2}^N\right) \Big), \tag{1}$$

where

$$\begin{multline} F\left(\psi_i^N, \psi_{i+1}^N, u_{i+1/2}^N\right) = \\ \Big(\left(u_{i+1/2}^N + \left|u_{i+1/2}^N\right|\right) \psi_i^N + \left(u_{i+1/2}^N - \left|u_{i+1/2}^N\right|\right) \psi_{i+1}^N \Big) \frac{\Delta t}{2\Delta x}. \end{multline} \tag{2}$$

This gives

$$\begin{multline} \psi_i^{N+1} = \psi_i^N - \frac{\Delta t}{2\Delta x} \Bigg(\Big(\left(u_{i+1/2}^N + \left|u_{i+1/2}^N\right|\right) \psi_i^N + \left(u_{i+1/2}^N - \left|u_{i+1/2}^N\right|\right) \psi_{i+1}^N \Big) \\ - \Big(\left(u_{i-1/2}^N + \left|u_{i-1/2}^N\right|\right) \psi_{i-1}^N + \left(u_{i-1/2}^N - \left|u_{i-1/2}^N\right|\right) \psi_i^N \Big) \Bigg). \end{multline} \tag{3}$$

Collecting terms gives the method of lines representation

$$\begin{aligned} \psi_i^{N+1} &= \frac{\Delta t}{2\Delta x} \left(u_{i-1/2}^N + \left|u_{i-1/2}^N\right|\right) \psi_{i-1}^N \\ &+ \left(1 - \frac{\Delta t}{2\Delta x} \left(u_{i+1/2}^N + \left|u_{i+1/2}^N\right| - u_{i-1/2}^N + \left|u_{i-1/2}^N\right|\right)\right) \psi_i^N \\ &- \frac{\Delta t}{2\Delta x} \left(u_{i+1/2}^N - \left|u_{i+1/2}^N\right|\right) \psi_{i+1}^N \end{aligned} \tag{4}$$