

# Multi-Weighted Tree Based Query Optimization Method for Parallel Relational Database Systems<sup>1</sup>

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## Abstract

A multi-weighted tree based query optimization method for parallel relational databases is proposed in this paper. The method consists of a multi-weighted tree based parallel query plan model, a cost model for parallel query plans and a query optimizer. The parallel query plan model models three types of parallelism of query execution, processor and memory allocation to operations, memory allocation to buffers in pipelines and data redistribution among processors. The cost model takes the waiting time of operations in pipelining execution into consideration and is computable in a bottom-up fashion. The query optimizer addresses the query optimization problem in the context of Select-Project-Join queries. Heuristics for determining the processor allocation to operations and the memory allocation to operations and buffers in pipelines are derived and used in the query optimizer. In addition, the query optimizer considers multiple join algorithms, and can make an optimal choice of join algorithm for each join operation in a query.

## 1. Introduction

In recent years, multiprocessor-based parallel database systems have drawn considerable attention due to high performance, high availability and scalability for data intensive applications. A key to the success of parallel database systems is parallel query optimization. During the last years much attention has been paid to the development of parallel query optimization techniques. Various methods have been proposed<sup>[1-26]</sup>. While

considerable progress has been made in parallel query optimization, many problems still remain open. Each of the existing parallel query optimization methods only investigated one or a few aspects of the parallel query optimization. All of them have the following restrictions:

A. The models of parallel plans, such as right-deep, left-deep, bushy, segmented right-deep and zigzag tree, are not enough to model the processor and memory allocation to operations, memory allocation to buffers between operations in pipelines and data redistribution among processors.

B. The cost models of query plans did not take the waiting time of the operations in pipelining execution into consideration so that the quality of query plans chosen as the best plans by optimizers can not be guaranteed.

C. Almost all the methods address optimization problems only in the context of multi-way join queries. Few methods considered the optimization problems of complex queries, such as selection-projection-join (SPJ for short) queries.

D. Almost all the methods are based one parallel join algorithm only and most of the methods dealt with only parallel hash join algorithm so that no choice of the optimal algorithm can be made for operations in a query. The choice of the optimal algorithms for operations in a query has significant effect on the performance of the query processing.

E. Most methods are not aware of memory resources in order to generate good-quality plans, especially the memory allocation to the buffers between operations in pipelining execution.

To counter the five restrictions above, a multi-weighted tree based query optimization method is proposed in this

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paper. The method consists of a multi-weighted tree based parallel query plan model, a cost model for parallel query plans and a query optimizer. The parallel query plan model is the first one to model all basic relational operations, all three types of parallelism of query execution, processor and memory allocation to operations, memory allocation to the buffers between operations in pipelining execution and data redistribution among processors. The query plan cost model takes the waiting time of the operations in pipelining execution into consideration and is computable in bottom-up fashion. A mathematics model of waiting time of the operations in pipelining execution is proposed. The query optimizer addresses the query optimization problem in the context of SPJ queries that are widely used in commercial DBMSs. Several heuristics for determining the processor allocation to operations are derived and used in the query optimizer. The query optimizer is aware of memory resources in generating good-quality plans. It includes the heuristics for determining the memory allocation to operations and buffers between operations in pipelining execution so that the memory resource is fully exploit. Multiple join algorithms are considered in the query optimizer also. The query optimizer can make an optimal choice of join algorithm for each join operation in a query.

The rest of the paper is organized as follows. Section 2 describes the parallel query plan representation model. Section 3 presents the cost model for parallel query plans. Section 4 illustrates the parallel query optimizer. Section 5 concludes the paper and discusses the future research.

## 2. Parallel query plan model

Let  $R$  be a relation and  $\psi$  be a proposition expression with the attributes of  $R$  as variables. For any  $t \in R$ ,  $\psi(t)$  is a proposition produced by substituting the variables in  $\psi$  with the attribute values of  $t$ . Let  $\pi$  be an subset of attributes of  $R$ ,  $\pi(t)$  be a tuple with attributes set  $\pi$  whose values on  $\pi$  are the same with  $t$ , and  $\theta \in \{=, \neq, <, >, \geq, \leq\}$ . Follows are the definition of the basic operators in the query plan model proposed in this paper.

**Scan**( $R, \psi, \pi$ ): compute  $\{t \mid \exists r (r \in R \text{ and } \psi(r)=\text{true and } \pi(r)=t)\}$ .

**Sort**( $R, \pi$ ): sort  $R$  according to the order of the values on the attributes specified by  $\pi$ .

**Merge**( $R, S, \pi, \theta$ ): compute  $\{r \mid r \in R, s \in S \text{ and } \pi(r) \theta \pi(s)\}$ .

**Nested\_Loop**( $R, S, \pi, \theta$ ): use parallel nested-loops-join algorithm to compute the join of  $R$  and  $S$ <sup>[27]</sup>.

**Build-Hash-Table**( $R, \pi$ ): build the hash table of  $R$  by applying a hash function to the values on the attributes specified by  $\pi$ <sup>[27]</sup>.

**Probe-Hash-Table**( $T, S, \pi$ ): probe the hash table  $T$  using the tuples of  $S$  by the same hash function used in building  $T$  to perform a join of  $T$  and  $S$ <sup>[27]</sup>.

**CMD-Join**( $R, S$ ): join two relations  $R$  and  $S$  that have been declustered by the CMD method<sup>[35]</sup>.

**B<sup>+</sup>-Tree-Join**( $R, S$ ): join two relations  $R$  and  $S$  one of which has a parallel B<sup>+</sup>-tree index<sup>[34]</sup>.

Let  $A$  be the set of join attributes of relations  $R$  and  $S$ . The algorithms for implementing the relational operations can be easily expressed by the basic operation above. The selection, projection operations or their combination can be expressed by the Scan operator. The parallel sort-merge join algorithm, parallel nested-loops join algorithm, parallel hash join algorithm, parallel CMD-join algorithm and parallel B<sup>+</sup>-tree-join algorithm can be expressed by Merge(Sort( $R, \pi$ ), Sort( $S, \pi$ ),  $\pi, \theta$ ), Nested-Loop( $R, S, \pi, \theta$ ), Probe-Hash-Table(Build-Hash-Table( $R, \pi$ ),  $S, \pi$ ), CMD-Join( $R, S$ ) and B<sup>+</sup>-Tree-Join( $R, S$ ) respectively.

**Definition 2.1.** Let  $G = (V, E)$  be a tree,  $E' \subseteq E$ ,  $V' \subseteq V$ , and  $S_1, \dots, S_m, T_1, \dots$ , and  $T_k$  be sets. A *multi-weighted tree* is a system  $ET = (G, NF, EF)$ , where  $NF$  is a function  $V' \rightarrow (S_1 \times \dots \times S_m)$ , and  $EF$  is a function  $E' \rightarrow (T_1 \times \dots \times T_k)$ .

**Definition 2.2.** Let  $Q$  be a query,  $N$  be the number of processing nodes,  $M$  be the memory size in bytes,  $OP$  be the set of basic operators in  $Q$ , and  $\phi$  be the set of operand relations in  $Q$ . A *parallel query plan* of  $Q$  is a multi-weighted tree,  $((V, E), F_1, F_2)$ , where  $V = OP \cup \phi$ ,  $E = \{(X, Y) \mid (X, Y \in OP) \text{ or } (Y \in OP \text{ and } X \in \phi)\}$ ,  $F_1$  is the function  $OP \rightarrow \{0, 1, \dots, N\} \times \{0, 1, \dots, M\}$ ,  $F_2$  is the function  $E \rightarrow \{(X, Y) \mid (Y \in OP \text{ and } X \in \phi)\} \rightarrow \{0, 1, \dots, M\} \times \{P, S\}$ .

It is obvious that the allocation of processing nodes and memory to operators is modeled by function  $F_1$ .  $F_1$  also models the intra-operation parallelism by assigning multi-processing nodes to an operator. The pipelining parallelism is modeled by function  $F_2$ . If  $F_2$  puts weight  $(m, P)$  on edge  $(op_i, op_j)$  then the operators  $op_i$  and  $op_j$  are pipelining-parallel executable and the buffer size assigned to the pipeline is  $m$ . Otherwise, the weight  $(0, S)$  on edge  $(op_i, op_j)$  forces  $op_i$  and  $op_j$  to be executed sequentially. From the definition of the edge set  $E$ , all independent operators must be on different paths of the parallel query plan tree, which models the inter-operator parallelism.

## 3. Cost model of parallel query plans

The cost model of a multi-weighted tree based parallel query plans consists of two parts. One is the response time of individual operations in the plan. The other is to combine the response time of individual operations to obtain the response time of the entire plan. The response time of individual operations is a function  $rtime(op, A, n, m, \rho, \theta)$ , where  $op$  is operation,  $A$  is the algorithm to implement  $op$ ,  $n$  is number of processing nodes assigned to  $op$ ,  $m$  is the memory size assigned to  $op$ ,  $\rho$  is the set of sizes of the operand relations, and  $\theta$  is the size of the result. A lot of research work has been done to model the response time of individual operations. We concentrate on the cost model of entire parallel query plans in this paper.

### 3.1. Waiting time of operators

Let  $op_y$  and  $op_x$  be two operations in pipelining execution,  $op_y$  be the producer, and  $op_x$  be the consumer. By the definition of the query plan,  $op_x$  must be the parent of  $op_y$  and there is a pipelining buffer which can hold  $m$  tuples.  $op_y$  sends tuples to the buffer and  $op_x$  retrieves tuples from the buffer. The buffer may overflow or underflow. In case of underflow,  $op_x$  has to wait. In case of overflow,  $op_y$  has to wait. Let  $\lambda$  be the average rate at which  $op_y$  produces a tuple,  $\mu$  be the average rate at which  $op_x$  consumes a tuple, and  $output(op_y)$  be the set of tuples produced by  $op_y$  and consumed by  $op_x$ . Since the function of each relational operation is to scan its input relations and generate new relation, it is reasonable to assume that

$$\mu = |output(op_y)| / rtime(op_x, A_x, n_x, m_x, \rho_x, \theta_x),$$

$$\lambda = |output(op_y)| / rtime(op_y, A_y, n_y, m_y, \rho_y, \theta_y).$$

**Theorem 3.1.** The average waiting time of  $op_x$  and  $op_y$ ,  $WT_{xy}^c$  and  $WT_{xy}^p$ , are

$$WT_{xy}^c = \begin{cases} 1/\lambda & \lambda \geq \mu \\ 1/\lambda + (|output(op_y)| - 1)(1/\lambda - 1/\mu) & \lambda < \mu \end{cases}$$

$$WT_{xy}^p = \begin{cases} 0 & \lambda \leq \mu \\ (|output(op_y)| - \lambda m / (\lambda - \mu))(1/\mu - 1/\lambda) & \lambda > \mu \end{cases}$$

Due to the length limitation of the paper, the proof of theorem 3.1 is omitted. Let  $WT_x$  be the waiting time of an operator,  $op_x$ , in a parallel query plan tree. We define the waiting time of  $WT_x$  in five cases using theorem 3.1.

**Case 1.**  $op_x$  is the root and has one son  $op_y$  and  $w$  is the weight on edge  $(op_x, op_y)$ . In the case,  $WT_x$  is

$$\begin{cases} WT_{xy}^c & \text{if } w = (m, P) \\ 0 & \text{if } w = (0, S) \end{cases}$$

**Case 2.**  $op_x$  is the root and has two sons  $op_y$  and  $op_z$ , and  $w_1$  and  $w_2$  are the weights on edges  $(op_y, op_x)$  and  $(op_z, op_x)$ ,  $m_1$  and  $m_2$  are the sizes of the buffers on edges  $(op_y, op_x)$  and  $(op_z, op_x)$ . In the case,  $WT_x$  is

$$\begin{cases} 0 & \text{if } w_1 = w_2 = (0, S) \\ WT_{xy}^c & \text{if } w_1 = (m_1, P), w_2 = (0, S) \\ WT_{xz}^c & \text{if } w_1 = (0, S), w_2 = (m_2, P) \\ WT & \text{if } w_1 = (m_1, P), w_2 = (m_2, P) \end{cases}$$

Since  $\max\{WT_{xy}^c, WT_{xz}^c\} \leq WT_x \leq WT_{xy}^c + WT_{xz}^c$ , we use

$WT = (\max\{WT_{xy}^c, WT_{xz}^c\} + (WT_{xy}^c + WT_{xz}^c))/2$  to estimate  $WT_x$ .

**Case 3.**  $op_x$  is an inner node and has a parent  $op_y$  and a son  $op_z$ , and  $w_1$  and  $w_2$  are the weights on edges  $(op_x, op_y)$  and  $(op_z, op_x)$ . In this case,  $WT_x$  is

$$\begin{cases} 0 & \text{if } w_1 = w_2 = (0, S) \\ WT_{yx}^p & \text{if } w_1 = (m_1, P), w_2 = (0, S) \\ WT_{xz}^c & \text{if } w_1 = (0, S), w_2 = (m_2, P) \\ (\max\{WT_{yx}^p, WT_{xz}^c\} + (WT_{yx}^p + WT_{xz}^c))/2 & \text{if } w_1 = (m_1, P), w_2 = (m_2, P) \end{cases}$$

**Case 4.**  $op_x$  is inner node and has parent  $op_y$  and sons

$op_z$  and  $op_w$ , and  $w_1, w_2$  and  $w_3$  are the weights on edges  $(op_x, op_y)$ ,  $(op_z, op_x)$  and  $(op_w, op_x)$ . In this case,  $WT_x$  is

$$\begin{cases} 0, & \text{if } w_1 = w_2 = w_3 = (0, S) \\ WT_{xz}^c, & \text{if } w_1 = (0, S), w_2 = (m_2, P), w_3 = (0, S) \\ WT_{xw}^c, & \text{if } w_1 = (0, S), w_2 = (0, S), w_3 = (m_3, P) \\ WT_1, & \text{if } w_1 = (0, S), w_2 = (m_2, P), w_3 = (m_3, P) \\ WT_{yx}^p, & \text{if } w_1 = (m_1, P), w_2 = (0, S), w_3 = (0, S) \\ WT_2, & \text{if } w_1 = (m_1, P), w_2 = (m_2, P), w_3 = (0, S) \\ WT_3, & \text{if } w_1 = (m_1, P), w_2 = (0, S), w_3 = (m_3, P) \\ WT_4, & \text{if } w_1 = (m_1, P), w_2 = (m_2, P), w_3 = (m_3, P) \end{cases}$$

where,  $WT_1 = (\max\{WT_{xz}^c, WT_{xw}^c\} + (WT_{xz}^c + WT_{xw}^c))/2$ ,

$WT_2 = (\max\{WT_{yx}^p, WT_{xz}^c\} + (WT_{yx}^p + WT_{xz}^c))/2$ ,  $WT_3 =$

$(\max\{WT_{yx}^p, WT_{xw}^c\} + (WT_{yx}^p + WT_{xw}^c))/2$ , and  $WT_4 = (\max$

$\{WT_{yx}^p, WT_{xz}^c, WT_{xw}^c\} + (WT_{yx}^p + WT_{xz}^c + WT_{xw}^c))/2$ .

**Case 5.**  $op_x$  is a leaf node and has a parent  $op_y$ , and  $w$  is the weight on edge  $(op_x, op_y)$ . In this case,  $WT_x$  is

$$\begin{cases} WT_{yx}^p & \text{if } w = (m, P) \\ 0 & \text{if } w = (0, S) \end{cases}$$

It should be noted that  $WT_x = 0$  if  $op_x$  is not executed in pipelining manner with any operation.

### 3.2. Cost model for entire query plans

In the following discussion, let  $P$  be a parallel query plan,  $\alpha = \{a_1, \dots, a_k\}$ ,  $\eta = \{n_1, \dots, n_k\}$ ,  $\sigma = \{m_1, \dots, m_k\}$ ,  $\rho = \{s_1, \dots, s_k\}$ , and  $\theta = \{o_1, \dots, o_k\}$ , where,  $a_i, n_i$  and  $m_i$  are the algorithm, number of processing nodes and memory size assigned to the  $i^{th}$  operator in  $P$ ,  $s_i$  and  $o_i$  are the set of sizes of the operand relations and the size of the result of the  $i^{th}$  operator in  $P$ .

**Definition 3.1.** Let  $op$  be an operator in  $P$ . The elapse time of  $op$  is the elapse time from the beginning of the execution of  $P$  to the termination of the execution of  $op$ , denoted by  $etime(op)$ .

**Definition 3.2.** The response time of a parallel query plan  $P$  is the elapse time from the time of  $P$  being started to the termination time of the execution of the root operator in  $P$ .

Now, we give a recursive algorithm to compute the response time of  $P$ . For each node directly connected to relation nodes (leaf nodes)  $op_x$  in  $P$ ,  $etime(op_x) = rtime(op_x, a_x, n_x, m_x, s_x, o_x) + WT_x$ . For each other operator  $op_x$  in  $P$ , six cases need to be considered.

**Case 1.**  $op_x$  has a son  $op_y$ , and the weight on edge  $(op_y, op_x)$  is  $(0, S)$ , i.e.,  $op_x$  and  $op_y$  are sequentially executed. In this case,  $etime(op_x) = rtime(op_x, a_x, n_x, m_x, s_x, o_x) + etime(op_y) + WT_x$ .

**Case 2.**  $op_x$  has a son  $op_y$ , and the weight on edge  $(op_y, op_x)$  is  $(m, P)$ , i.e.,  $op_x$  and  $op_y$  are executed in pipelining manner. In this case,  $etime(op_x) = (etime(op_y) - rtime(op_y) - WT_y) + WT_x + rtime(op_x, a_x, n_x, m_x, s_x, o_x)$ , where  $(etime(op_y) - rtime(op_y) - WT_y)$  is the start time of  $op_x$ .

**Case 3.**  $op_x$  has two sons  $op_y$  and  $op_z$ , and the weights on edge  $(op_y, op_x)$  and  $(op_z, op_x)$  are  $(0, S)$  and  $(0, S)$ , i.e., the execution of  $op_x$  and  $op_y$  is sequential, the execution of  $op_x$  and  $op_z$  is sequential, and  $op_y$  and  $op_z$  are executed in parallel. In this case,  $etime(op_x) = rtime(op_x, a_x, n_x, m_x, s_x, o_x) + WT_x + \max\{etime(op_y), etime(op_z)\}$ .

**Case 4.**  $op_x$  has two sons  $op_y$  and  $op_z$ , and the weights on edge  $(op_y, op_x)$  and  $(op_z, op_x)$  are  $(m_1, P)$  and  $(m_2, P)$ , i.e.,  $op_x$  and  $op_y$  are executed in pipelining manner,  $op_x$  and  $op_z$  are executed in pipelining manner, and  $op_y$  and  $op_z$  are executed in parallel. In this case,  $etime(op_x) = rtime(op_x, a_x, n_x, m_x, s_x, o_x) + WT_x + \min\{etime(op_y) - rtime(op_y) - WT_y, etime(op_z) - rtime(op_z) - WT_z\}$ .

**Case 5.**  $op_x$  has two sons  $op_y$  and  $op_z$ , and the weights on edge  $(op_y, op_x)$  and  $(op_z, op_x)$  are  $(0, S)$  and  $(m, P)$ , i.e.,  $op_x$  and  $op_y$  are executed sequentially,  $op_x$  and  $op_z$  are executed in pipelining manner, and  $op_y$  and  $op_z$  are executed in parallel. In this case,  $etime(op_x) = rtime(op_x, a_x, n_x, m_x, s_x, o_x) + WT_x + \max\{etime(op_y), etime(op_z) - rtime(op_z) - WT_z\}$ .

**Case 6.**  $op_x$  has two sons  $op_y$  and  $op_z$ , and the weights on edge  $(op_y, op_x)$  and  $(op_z, op_x)$  are  $(m, P)$  and  $(0, S)$ . This case is similar to case 5.

Finally, the response time of  $P$  is  $etime(op_{root})$ , where  $op_{root}$  is at the root of  $P$ .

## 4. Parallel query optimizer

The parallel query optimizer (PQO) consists of six phases. Given a query  $Q$ , phase 1 is to generate join trees of  $Q$ . Phase 2 chooses appropriate parallel join algorithms for join operations in each join tree. Phase 3 transformed the join trees with join algorithms assigned to all join operations into operator trees. Phase 4 determines the pipelines in each operator tree and the execution strategy of the pipelines. Phase 5 assigns memory and processing nodes to the operators and assigns memory to the buffers in each operator tree. Finally, phase 6 chooses optimal query plan from the set of operator trees resulted in phase 5. We present the techniques in the 6 phases in the following subsections.

### 4.1. Generating join trees

Join tree is a model of multi-join query. The internal nodes express join operations. The leaf nodes are relations. Given a query  $Q$  with  $n$  join operations, we first generate the join trees of  $Q$  ignoring the selection and projection

operations. The selection and projection operations will be considered in the generation of operator trees. The algorithm for generating join trees uses the dynamic programming technique. The set of the join trees with one join operation,  $Join\_tree[1]$ , are generated first. Generally, after the set of join trees with  $k$  join operations,  $Join\_tree[k]$ , being generated from  $Join\_tree[k-1]$ , the set of join trees with  $k+1$  join operations,  $Join\_tree[k+1]$ , are generated from  $Join\_tree[k]$ . Finally, the set of join trees with  $n$  join operations,  $Join\_tree[n]$ , are generated from  $Join\_tree[n-1]$ . The size of the join results are estimated while the join trees are constructed. Given the number of join operations,  $n$ , and the set of join relations,  $R\_SET$ , the following **G-Join-Tree** returns all the sets,  $Join\_tree[k]$  with  $k$  join operations for  $0 \leq k \leq n$ , and the size,  $size[t]$ , of each join tree  $t$ .

#### ALGORITHM G-Join-Tree( $n, R\_SET$ )

- (1)  $Join\_tree[0] := \{Join\_tree(0, R) \mid R \in R\_SET\}$ ;  
/\*  $Join\_tree(0, R)$  generates special join trees with only one join relation  $R$  \*/
- (2) FOR  $i = 1$  TO  $n+1$  DO  $size[Join\_tree(0, R_i)] := |R_i|$ ;
- (3) FOR  $i = 1$  TO  $n$  DO  $Join\_tree[i] := \text{empty set}$ ;
- (4) FOR  $k = 1$  TO  $n$ ,  $i = 0$  TO  $k-1$  DO
- (5)   FOR  $\forall (Ltree, Rtree) \in Join\_tree[i] \times Join\_tree[k-i]$ ;
- (6)   IF  $leaves(Ltree) \cap leaves(Rtree) = \text{empty set}$
- (7)   THEN  $Join\_tree[k+1] := Join\_tree[k+1] \cup \{Join\_tree(Ltree, Rtree)\}$ ;
- (8)    $size[Join\_tree(Ltree, Rtree)] := Join\_size(size[Ltree], size[Rtree])$ .

Function  $leaves(T)$  gives the set of leaf nodes in tree  $T$ . Function  $Join\_tree(L, R)$  combines join trees  $L$  and  $R$  to form a new tree of which  $L$  and  $R$  are the left and right subtrees. Function  $Join\_size(x, y)$  computes the size of join result of relations with sizes  $x$  and  $y$ . There are several methods of estimating the sizes of join results<sup>[28, 29]</sup>. We do not discuss them again in the paper.

### 4.2. Choosing parallel join algorithms

PQO adopts a heuristic method to choose appropriate parallel join algorithm for each join operation in the join tree generated by algorithm **G-Join-Tree**. Let  $J$  be a join operation,  $R$  and  $S$  be the join relations of  $J$ ,  $A$  be the join attribute set, the size of  $R$  be smaller than that of  $S$ ,  $C_0$ =(one of  $R$  or  $S$  can be fit into memory),  $C_1$ =( $R$  or  $S$  has ordered index on  $A$ ),  $C_2$ =( $R$  or  $S$  is sorted in the order of the values of  $A$ ),  $C_3$ =(the operation after  $J$  needs sorted input relation),  $C_4$ =(Number of processing nodes  $< \delta$ ),  $C_5$ =(the skew of  $R$  or  $S$  on  $A$  is great than  $\vartheta$ ),  $C_6$ =( $J$  is equal join),  $C_7$ =(The difference of the sizes of  $R$  and  $S$  is greater than  $\Delta$ ), where  $\vartheta$ ,  $\delta$  and  $\Delta$  are variable with different parallel computing systems. They need to be decided by experiments.  $C_4$  expresses the number of processing nodes is very small.  $C_5$  expresses the data of  $R$  or  $S$  is very skew on  $A$ . Let  $PR$  be the result of projection of  $R$  on  $A$ ,  $n$  is the

size of  $PR$ ,  $K_i$  is the number of tuples whose value on  $A$  is the  $i^{th}$  value in  $PR$ ,  $|R|$  is the size of  $R$ . The *skew* of  $R$  on  $A$  is defined as  $\sum_{i=1}^n \left| k_i - \frac{|R|}{n} \right|$ .  $C_7$  expresses the difference

of the sizes of  $R$  and  $S$  is very big. From the experimental result and theoretic analysis<sup>[30-35]</sup>, we derived the following heuristic rules.

- Rule 1**  $C_0 \rightarrow$  (choose parallel nested\_loops join algorithm)
- Rule 2**  $(R \text{ or } S \text{ has parallel } B^+ \text{ tree index on } A) \rightarrow$  (choose parallel  $B^+$  tree based join algorithm)
- Rule 3**  $(R \text{ or } S \text{ is stored in parallel CMD file}) \rightarrow$  (choose parallel join algorithm based on CMD files).
- Rule 4**  $C_6 \wedge (C_1 \vee C_2 \vee C_3) \rightarrow$  (choose parallel sort\_merge join algorithm).
- Rule 5**  $C_6 \wedge \neg(C_1 \vee C_2 \vee C_3) \wedge C_5 \wedge C_4 \rightarrow$  (choose parallel sort\_merge join algorithm).
- Rule 6**  $C_6 \wedge \neg(C_1 \vee C_2 \vee C_3) \wedge C_5 \wedge \neg C_4 \rightarrow$  (choose parallel nested\_loops join algorithm).
- Rule 7**  $C_6 \wedge \neg(C_1 \vee C_2 \vee C_3) \wedge C_7 \rightarrow$  (choose parallel nested\_loops join algorithm).
- Rule 8**  $\neg C_6 \wedge C_4 \rightarrow$  (choose parallel sort\_merge join algorithm).
- Rule 9**  $\neg C_6 \wedge \neg C_4 \rightarrow$  (choose parallel nested\_loops join algorithm).

Whenever PQO chooses an appropriate algorithm for a join operation, it first try to choose one of the algorithms of parallel sort\_meger join, parallel nested\_loops join, parallel join based on  $B^+$  tree index and parallel join based on CMD files using the nine rules. If all the conditions of the rules can not be satisfied, the parallel hash join algorithm is choosen.

### 4.3. Transform join tree into operator tree

Let  $T$  be a join tree of a query  $Q$ , and all the join operations in  $T$  have been assigned parallel algorithms.  $T$  is transformed to an operator tree in two steps.

The first step is to insert the selection and projection operations of  $Q$  into appropriate positions of  $T$  to form a SPJ tree. Since selection and projection are expressed by scan operators, actually the first step is to insert scan operators into  $T$ . Let  $v$  and  $w$  be a pair of parent-son nodes of  $T$ ,  $v$  be the parent and  $w$  be the son. The heuristic rules of inserting scan operators is as follows.

- Rule 1.** If  $w$  is a leaf node (relation), then insert a scan operator between  $v$  and  $w$ .
- Rule 2.** If a selection or projection operation need to be executed on the output of  $w$ , then insert a scan operator between  $v$  and  $w$ .
- Rule 3.** If  $v$  is the root of  $T$  and a selection or projection operation need to be executed on the output of  $v$ , then insert a scan operator as the new root of  $T$  and the parent of  $v$ .
- Rule 4.** Move the Scan operator as close as possible to the leaf nodes according to the algebra rule of the relational operations.

When a Scan operator is inserted into the join tree, the output size of the Scan node,  $size[Scan]$ , can be estimated. During the execution of a query, some intermediate results may need to be distributed over multiple processing nodes. A new operator,  $Dis(R, P)$ , which distributes relation  $R$  among  $P$  processing nodes is needed. The following rule is for inserting the Dis operators.

**Rule 5.** If  $w$  is an internal join operation node and the output of  $w$  need to be distributed over multiple processing nodes, then insert a Dis operation between  $v$  and  $w$ .

The second step replaces each join node in SPJ tree with a subtree of basic operators according to the algorithm assigned to the node to form a operator tree. Let join operation node  $J$  have been assigned algorithm  $Alg$ . The input,  $R$  and  $S$ , of  $J$  may be initial relations or intermediate result relations, and  $R$  is smaller than  $S$ . The rules for constructing an operator tree from a SPJ tree are as follows.

**Rule 1.** If  $Alg$  is parallel hash join, then replace  $J$  according to Fig. 4.1 (A).

**Rule 2.** If  $Alg$  is sort-merge join, then replace  $J$  according to Fig. 4.1 (B).

**Rule 3.** If  $Alg$  is nested-loops join, then replace  $J$  according to Fig. 4.1 (C).

**Rule 4.** If  $Alg$  is parallel  $B^+$  tree based join (B-Tree-Join), then replace  $J$  according to Fig. 4.1 (D).

**Rule 5.** If  $Alg$  is parallel CMD join (CMD-Join), then replace  $J$  according to Fig. 4.1 (E).

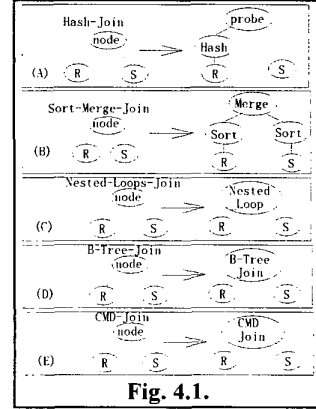


Fig. 4.1.

### 4.4. Determine pipelines in an operator tree

Given a operator tree  $T$ , the first task of determining pipelines in  $T$  is to compute the weight,  $(m, x)$ , on each edge  $(w, v)$  of  $T$ , where  $v$  is the parent of  $w$ . We first discuss how to compute  $m$ . If  $v$  and  $w$  are executed sequentially then  $m=0$ , otherwise  $m$  is computed by the following theorem 4.1 that is derived from theorem 3.1.

**Theorem 4.1.** Let  $\lambda$  be the average rate at which  $w$  produces tuples, and  $\mu$  be the average rate at which  $v$  consumes the tuples produced by  $w$ . Assume that  $w$  and  $v$  are executed in pipelining manner. If  $m$  is computed as

$$m = \begin{cases} \text{Size of one tuple} & \text{if } \lambda \leq \mu \\ (\lambda - \mu) | \text{output}(w) | / \lambda & \text{if } \lambda > \mu \end{cases}$$

then the average waiting times of  $w$  and  $v$  are minimized.

The second component,  $x$ , of the weight  $(m, x)$  on the edge  $(w, v)$  can be computed as follows.

- If  $v = \text{Scan}$ ,  $x=P$ .
- If  $v = \text{Sort}$ ,  $x=S$ .
- If  $v = \text{Hash}$ ,  $x=P$ .
- If  $v = \text{Probe}$  and  $w = \text{Hash}$ ,  $x=S$ .
- If  $v = \text{Probe}$  and  $w \neq \text{Hash}$ ,  $x=P$ .
- If  $v = \text{Merge}$ ,  $x=P$ .
- If  $v = \text{Nested-loop}$ ,  $x=P$ .
- If  $w = \text{Dis}$ ,  $x=S$ .
- If  $v = \text{CMD-Join}$ ,  $x=S$ .
- If  $v = \text{B-Tree-Join}$ ,  $x=S$ .

**Definition 4.1.** Let  $T$  be a operator tree with weight on each edge. A *natural pipeline* of  $T$  is a subtree,  $ST$ , of  $T$  satisfying (1) all edges have weight of type  $(m, P)$ , and (2) there is no subtree  $ST'$  in  $T$  such that  $ST'$  satisfies (1) and  $ST$  is a subtree of  $ST'$ .

It is obvious that each subtree in the forest obtained by removing the edges with weight  $(0, S)$  from  $T$  is a natural pipeline. Thus, the set of natural pipelines of  $T$  can be obtained by removing the edges with weight  $(0, S)$  from  $T$ . If the natural pipelines are regarded as nodes and the removed edges are regarded as edges, we obtain a special tree. This tree is called *natural pipeline tree*.

Due to the limitation of the memory size, all the operation on a pipeline may not be executed in parallel. If the total size of all the pipeline buffers on a pipeline is greater than the size of available memory, the pipeline should be partitioned into multiple subpipelines such that the total size of all the buffers on each subpipeline is smaller than the size of available memory. The resulting pipelines are called *executable pipelines*. All the operations on a executable pipeline can be executed in parallel. The algorithm to generate the *executable pipeline tree* is as follows.

#### Algorithm Partition

**Input:** natural pipeline tree  $T$ , available memory size  $M$ .

**Output:** executable pipelines tree  $T'$ .

- (1) FOR each natural pipeline  $F$  in  $T$  DO
- (2) IF total size of all buffers on  $F$  is greater than  $M$
- (3) THEN Partition  $F$  into multiple subpipelines in top down manner so that the total size of all the buffers on each subpipeline is not greater than  $M$ ;
- (4) Change partition edges into dashed lines with weight  $(0, S)$

### 4.5. Determine strategies of executing Pipelines

Some executable pipelines in an executable pipeline tree can be executed in parallel. Give an executable pipeline tree  $EPT$ , a strategy of executing pipelines in  $EPT$  can be expressed by a graph, called *execution strategy graph*. A node of the graph is a set of pipelines in  $EPT$  that can be executed in parallel, where “can be executed in parallel” means that all the pipelines contained in the node are in different paths of  $EPT$  and the total memory required by them is smaller than the available memory. An edge  $(x, y)$  of the execution strategy graph expresses that the pipelines in  $x$  must be executed before the pipelines in  $y$ . The algorithm to determine the execution strategy and to generate execution strategy graph is as follows.

**Input:** Executable pipeline tree  $EPT=(V, E)$ , and size  $M$  of available memory in bytes.

**Output:** Execution strategy graph  $G=(V', E')$ .

- (1) FOR each  $v \in V$  DO  $W(v) := 0$ ;
- (2) FOR each  $v \in V$  DO
- (3)  $W(v) :=$  time of executing pipeline of  $v$  by one processing node; /\*  $W(v)$  is called the work of  $v$  \*/

- (4) Construct graph  $G=(V', E')$  that is isomorphic with the longest path  $P=(V'', E'')$  of  $EPT$ :
  - A. For  $\forall v \in V''$ , add one and only one node  $N_v$  to  $V'$ ,  $N_v$  initially contains the pipelines in  $v$ ,  $W(N_v) := W(v)$ ;
  - B. For  $\forall (v, w) \in E''$ , add one and only one edge  $(N_v, N_w)$  to  $E'$ ;
- (5) /\* Insert pipelines in nodes of  $(V-V')$  into  $V'$  of  $G$  \*/
  - /\* Assume that  $EPT$  has  $L$  levels and the level numbers from the root of  $EPT$  are  $L, L-1, \dots, 2$  and  $1$  \*/
  - FOR  $i = 1$  TO  $L$  DO
    - FOR  $\forall v \in (V-V')$  that has the largest work  $W(v)$  and at level  $i$  of  $EPT$  DO
      - A. Find a node  $N_w$  in  $G$  that satisfies the following two conditions:
        - (a). (the level of  $N_w$  in  $EPT$ )  $\leq$  (the level of  $v$  in  $EPT$ );
        - (b).  $N_w$  is the node that satisfies condition (a) and has the smallest work  $W(N_w)$ ;
      - B.  $N_w := N_w \cup \{\text{pipelines in } v\}$ ;
      - $W(N_w) := W(N_w) + W(v)$ ;
      - remove  $v$  and related edges from  $EPT$ ;
  - (6) FOR each  $N_v \in G$  DO
    - A. If the required memory size by all pipelines in  $N_v$  is greater than  $M$ , then  $N_v$  is partitioned into  $n$  nodes,  $N_{v1}, N_{v2}, \dots$  and  $N_{vn}$ , which satisfy the following conditions:
      - (a). For  $1 \leq i \leq n$ , the memory size required by all the pipelines in  $N_{vi}$  is not greater than  $M$ ;
      - (b).  $n$  is the minimal integer that satisfies condition (a);
    - B. substitute path  $(N_{vp}, N_v, N_{vs})$  in  $G$  by the path  $(N_{vp}, N_{v1}, \dots, N_{vn}, N_{vs})$ .
    - /\*  $N_{vp}$  and  $N_{vs}$  are the parent and son of  $N_v$  \*/

### 4.6. Memory assignment

The method of assigning memory to pipeline buffers has been discussed in section 4.4. This section describes the algorithm,  $M\_Assign$ , of assigning memory to pipelines and operators. Given a executable pipeline tree  $EPT$ , algorithm  $M\_Assign$  assigns memory to the pipelines and operators in  $EPT$  based on the execution strategy graph of  $EPT$ , the ratio of the data volumes processed by the pipelines, and the ratio of the data volumes processed by the operators. The *data volume processed by a pipeline* is the total size of all the relations and intermediate results on the pipeline. The *data volume processed by a operator* is the total size of all the input relations of the operator. Since the nodes in an execution strategy graph are executed sequentially, we assume that each node of the execution strategy graph has been assigned whole available memory. Algorithm  $M\_Assign$  is as follows.

#### Algorithm M\_Assign( $EPT, G, M, m, size$ )

**Input:** Executable pipeline tree  $EPT$ , execution strategy graph  $G$ , available memory size  $M$ , and array *size*, where *size*[*op*] is the size of the output of *op* in  $EPT$  which has been evaluated by algorithms in sections 4.1 and 4.3.

**Output:** Array  $PMS$ , where  $PMS[P]$  is the size of memory assigned to pipeline  $P$ , array  $PM$ , where  $PM[P]$  is the memory assigned to pipeline  $P$ , array  $OMS$ , where  $OMS[op]$  is the size of memory assigned to operator node *op*. and array  $OM$ , where  $OM[op]$  is the memory assigned to operator node *op*.

- (1) FOR  $\forall v \in G$  DO /\* Assign memory to each pipeline in each  $v \in G$  \*/
- (2)  $Data := 0$ ;

```

(3) FOR each pipeline  $P$  in  $v$  DO
(4)    $PD[P] :=$  sum of the sizes of all relations on  $P$ ;
(5)   FOR each operation  $op$  on  $P$  DO  $PD[P] := PD[P] + size[op]$ ;
(6)    $Data := Data + PD[P]$ ;
(7) FOR each pipeline  $P$  in  $v$  DO /* Assign memory to  $P$  */
(8)    $PR[P] := PD[P]/Data$ ,  $PMS[P] := \lfloor M \times PR[P] \rfloor$ ;
(9)   IF  $PMS[P] < 1$  THEN  $PMS[P] := 1$ ;
(10)   $PM[P] :=$  location of  $PMS[P]$  bytes of memory assigned to  $P$ ;
(11) IF there is residual memory
    THEN Assign the residual memory to some pipelines so that the
         ratios,  $(PMS[P]/PD[P])$ 's, of all the pipelines are
         as close as possible;
(12) FOR each  $P$  in  $v$  DO /* Assign memory to operation nodes */
(13)    $Data := 0$ ;
(14)   FOR each  $op$  on  $P$  DO
(15)     IF  $op$  has two input relations  $R$  and  $S$ 
(16)     THEN  $OD[op] := size(R) + size(S)$ ; /*  $size(X) =$  size of  $X$  */
(17)     ELSE  $OD[op] := size(R)$ ; /*  $op$  has one input relation  $R$  */
(18)      $Data := Data + OD[op]$ ;
(19)   FOR each  $op$  on  $P$  DO /* Assign memory to  $op$  */
(20)      $OR[op] := OD[op]/Data$ ;
(21)      $OMS[op] := \lfloor PMS[P] \times OR[op] \rfloor$ ;
(22)     IF  $OMS[op] < 1$  THEN  $OMS[op] := 1$ ;
(23)      $OM[op] :=$  the location of the  $OMS[op]$  bytes of memory
         in  $PM[P]$  assigned to  $P$ 
(24)   IF the memory assigned to  $P$  has not used up
(25)   THEN Assign the residual memory to some operator nodes so
         that the ratios,  $(OMS[op]/OD[op])$ 's, of all the operator
         nodes on  $P$  are as close as possible.

```

#### 4.7. Processing node assignment

Given a executable pipeline tree  $EPT$ , processing node assignment algorithm,  $P\_Assign$ , assigns processing nodes to the pipelines and operators in  $EPT$  based on the execution strategy graph of  $EPT$ , the ratio of the costs of the pipelines, and the ratio of the costs of the operators. The *cost of an operator* is the execution time of the operator using one processing node. The *cost of a pipeline* is the total cost of all the operators on the pipeline. Since the nodes in an execution strategy graph are executed sequentially, each node of the execution strategy graph has been assigned all the processing nodes initially. The algorithm  $P\_assign$  is similar to  $M\_Assign$ .

#### 4.8. Selecting and executing optimal query plan

Given a query  $Q$ , PQO uses the algorithms in sections 4.1, 4.2, 4.3, 4.4 and 4.5 to generate a set of executable pipeline trees and execution strategy graphs. For each pair of executable pipeline tree and execution strategy graph, PQO uses the algorithms in sections 4.6 and 4.7 to assign processing nodes and memory to the operators in the executable pipeline trees. The memory assignment for pipeline buffers and the determination of the weights on the edges of the executable pipeline trees have been performed in section 4.4. Let  $OM(v)$  and  $OPROC(v)$  be

the memory and processing nodes assigned to the operator node  $v$ . For each  $v$  in the executable pipeline tree, let  $(OM(v), OPROC(v))$  be the weight of  $v$ . Finally, PQO changes each dashed line in every executable pipeline tree into a normal edge with weight  $(0, S)$  to obtain the set,  $QPS$ , of multi-weighted tree based parallel query plans of  $Q$ . For each plan  $P$  in  $QPS$ , PQO computes the parallel execution time of  $P$  using the recursive algorithm in section 3.2, chooses the plan  $P_0$  with the minimum parallel execution time, and executes  $P_0$  according to its execution strategy graph to perform the processing of  $Q$ .

### 5. Summary and Future Research

A multi-weighted tree based query optimization method is proposed in this paper. The parallel query plan model of the method is the first one to model processor and memory allocation to operations, memory allocation to the buffers between operations in pipelines and data redistribution among processors. The cost model of the method takes the waiting time of the operations in pipelining execution into consideration. The query optimizer of the method addresses the query optimization problem in the context of SPJ queries. Several heuristics determining the processor and memory allocation to operators and buffers are derived and used in the query optimizer. Multiple algorithms for implementing join operations are also considered by the query optimizer. The query optimizer can make an optimal choice of join algorithm for each join operation in a query.

In the future research, we will address the issues of heterogeneous parallel computing architectures, dynamic/pre-emptive optimization, space-time tradeoffs, evaluation of optimization algorithms, and accuracy of the cost model.

### References

- [1] W. Hong and M. Stonebraker, "Optimization of Parallel Query Execution Plans in XPRS", in Proc. of the 1st Int'l Conf. on Parallel and Distributed Information Systems, IEEE CS Press, 1991, pp..
- [2] M.S. Chen, P.S. Yu, and K.L. Wu, "Scheduling and Processor Allocation for Parallel Execution of Multi-Join Queries", in Proc. of IEEE Int'l Conf. on Data Engineering, 1992, pp. 58-67.
- [3] H. Lu, M.C. Shan, and K.L. Tan, "Optimization of Multi-Way Join Queries for Parallel Execution", in Proc. of Int'l Conf. on Very Large Data Bases, 1991.
- [4] M.C. Murphy and M.C. Shan, "Execution Plan Balancing: A Practical Technique for Multiprocessor Query Optimization", in Proc. of Int'l Conf. on Data Engineering, 1991.
- [5] M.S. Chen, et al., "Using Segmented Right-Deep Tree for the Execution of Pipelined Hash Joins", in Proc. of Int'l Conf. on Very Large Data Bases, 1992.
- [6] K.A. Hua, Y.L. Lo, and H.C. Young, "Including the Load Balancing Issue in the Optimization of Multi-Way Join Queries

- for Shared-Nothing Database Computers", in Proc. of the 2nd Int'l Conf. Parallel and Distributed Information Systems, IEEE CS Press, Los Alamitos, Calif., 1993, pp. 74-83.
- [7] R.S.G. Lanzelotte, P. Valduriez, and M. Zait, "On the Effectiveness of Optimization Search Strategies for Parallel Execution Spaces", in Proc. of Int'l Conf. on Very Large Data Bases, 1993.
- [8] D. Schneider and D.J. DeWitt, "Tradeoffs in Processing Complex Join Queries via Hashing in Multiprocessor Database Machines", in Proc. of Int'l Conf. on Very Large Data Bases, 1990.
- [9] E.J. Shekita, H.C. Young, and K.L. Tan, "Multi-Join Query Optimization for Symmetric Multi-Processors", in Proc. Of the 19th Int'l Conf. on Very Large Data Bases, Morgan Kaufman, San Mateo, Calif., 1993.
- [10] M. Ziane, M. Zait and P. Borla-Salamet, "Parallel Query Processing in dbs3," in Proc. of the 2nd Int'l Conf. Parallel and Distributed Information Systems, IEEE CS Press, Los Alamitos, Calif., 1993.
- [11] L. Haas, et al., "Extensible Query Processing in Starburst", in Proc. of ACM SIGMOD Int'l Conf. on Management of Data, 1989.
- [12] J. Srivastava, and G. Elssesser, "Optimizing Multi-Join Queries in Parallel Relational databases", in Proc. of the 2nd Int'l Conf. Parallel and distributed Information Systems, IEEE CS Press, Los Alamitos, Calif., 1993, pp.84-92.
- [13] S. Ganguly, W. Hasan, and R. Krishnamurthy, "Query Optimization for Parallel Execution", in Proc. of ACM SIGMOD Int'l Conf. on Management of Data, 1992.
- [14] W. Hong, "Exploiting Inter-Operator Parallelism in XPRS", in Proc. of ACM SIGMOD Int'l Conf. on Management of Data, 1992.
- [15] M.L. Lo, et al., "On Optimal Processor Allocation to Support Pipelined Hash Joins", in Proc. of ACM SIGMOD Int'l Conf. on Management of Data, 1993.
- [16] N. Duppel, "Modelling and Optimization of Complex Database Queries in a Shared-Nothing System", in Proc. of 3rd IEEE Symp. Parallel and Distributed Processing, IEEE CS Press, Los Alamito, Calif., 1991.
- [17] H.I. Hsiao, M.S. Chen and P.S. Yu, "Processor Allocation on Multicomputer Database Systems", in Proc. of IEEE Int'l Conf. on Data Engineering, 1995.
- [18] H.I. Hsiao, M.S. Chen and P.S. Yu, "Processor Allocation for Parallel Execution of Hash Joins," in Proc. of Int'l Conf. Parallel and Distributed Systems, 1993.
- [19] M. Mehta, V. Soloview, and D.J. Dewitt, "Batch Scheduling in Parallel Database Systems," in Proc. of IEEE Int'l Conf. on Data Engineering, 1993.
- [20] E. Rahm and R. Marek, "Analysis of Dynamic Load Balancing Strategies for Parallel Shared Nothing Database Systems", in Proc. of Int'l Conf. on Very Large Data Bases, 1993.
- [21] K. L. Tan and H. Lu, "Pipeline Processing of Multi-Way Join Queries in Shared Memory Systems," in Proc. of the 22nd Int'l Conf. on Parallel Processing, Vol.1, 1993.
- [22] K.L. Tan and H. Lu, "On Resource Scheduling of Multi-Join Queries in Parallel Database Systems", Information Processing Letters, Vol. 48, No. 4, Nov. 1993.
- [23] K.A. Hua, W. Tavanapong, and H.C. Young, "A Performance Evaluation of Load Balancing for Join Operations on Parallel Databases", Chinese Journal of Software, Vol.9, No.4, 1998.
- [24] H-I. Hsiao, M-S. Chen, and P.S. Yu, "On Parallel Execution of Multiple Pipelined Hash Joins", in Proc. of ACM SIGMOD Int'l Conf. on Management of Data, 1994.
- [25] A.N. Wilschut, J. Flokstra, and M.G. Apers, "Parallel Evaluation of Multi-Join Queries", in Proc. of ACM SIGMOD Int'l Conf. on Management of Data, 1995.
- [26] C. Lee and Z-A. Chang, "Workload Balance and Page Access Scheduling for Parallel Joins in Shared-Nothing Systems", in Proc. of Int'l Conf. on Data Engineering, 1993.
- [27] D.A. Schneider and D. DeWitt, "A Performance Evaluation of Four Parallel Join Algorithms in a Shared-Nothing Multiprocessor Environment", in Proc. ACM SIGMOD, 1989.
- [28] R.J. Lipton and J.F. Naughton, "Query Size Estimation by Adaptive Sampling", J. Computer and System Science, 51(1), 1995.
- [29] J.D. Ullman, Principles of Database and Knowledge-Base Systems, Volume II, Computer Science Press, Inc., 1989.
- [30] D.A. Schneider and D.J. DeWitt, "A Performance Evaluation of Four Parallel Join Algorithms in a Shared-Nothing Multiprocessor Environment", in Proc. of ACM Int'l Conf. on Management Data, 1989, pp.110-121.
- [31] P.V. Valduriez and G. Gardarin, "Join and Semijoin Algorithms for a Multiprocessor Database Machine", ACM Transactions on Database Systems, Vol.9, No.1, 1984.
- [32] D. DeWitt, J. Naughton, and J. Burger, "Nested Loops Revisited", in Proc. of the 2nd Parallel and Distributed Information Systems Conf., IEEE CS Press, Los Alamitos, Calif., 1993, pp.230-242.
- [33] D. BittON, "Parallel Algorithms for the Execution of Relational Database Operations", ACM Trans. database Systems, Vol.8, No.3, 1983, pp.324-353.
- [34] Wunjun Sun, Jianzhong Li, and Hong Chang, "Design, Analysis and Implementation of B<sup>+</sup>-Tree Based Parallel Join Algorithms", Computer Journal, Vol. 21, No. 1, 1998.
- [35] Jianzhong Li and Wei Du, "Parallel CMD Join Algorithms on Parallel Databases", Chinese Journal of Software, Vol.9, No.4, 1998.