Exam 1, EE5350, Fall 2012

1. Convolve h(n) and x(n) to get y(n). Put y(n) in closed form when possible.
(a) $h(n) = 3^{-n} u(-n)$ and $x(n) = 2^{-n} u(-n)$.
(b) $h(n) = 5^n u(n)$, $x(n) = u(n)$.
(c) $h(n) = u(n+3)-u(n-4)$, $x(n) = u(n+1)-u(n-7)$. Express the result in terms of r(n), where $u(n)*u(n) = r(n+1)$.
(d) $h(n) = \delta(\sin((2\pi/N)n)$ and $x(n) = n^2$ where N is even
(e) $h(n) = \delta(n^2-4.5n+2)$ and $x(n) = n^2$

2. A system is described by the recursive difference equation

$$y(n) = \frac{8}{15} y(n-1) - \frac{1}{15} y(n-2) + x(n) + \frac{1}{15} x(n-1)$$

(a) Find $H(e^{jw})$ in closed form. Give $H(e^{j0})$ and $H(e^{j\pi})$.
(b) Give the homogeneous solution to the difference equation above.
(c) Re-write the difference equation so that it generates the impulse response h(n). Give numerical values for h(0) and h(1).
(d) Using your answers to parts (b) and (c), give the impulse response h(n).
(e) Is the system stable ? (Yes or No)

3. Let x(n), h(n) and y(n) denote complex sequences with DTFTs $X(e^{jw})$, $H(e^{jw})$ and $Y(e^{jw})$. Find frequency domain expressions for the following;

(a) $C = \sum_{n=-\infty}^{\infty} x(n) \cdot y(-n)$. Give the substitution you made for y(-n).

(b) $y(n) = \sum_{k=-\infty}^{\infty} h(k) x(-k-n)$

(c) Find the numerical value of

$$\sum_{n=-\infty}^{\infty} \frac{\sin(\pi n/2)}{3\pi n} \bullet \frac{\sin(\pi n/3)}{\pi n}$$

4. The analog signal $x_c(t)$ has a cut-off frequency of 5 radians/sec. A C/D converter samples $x_c(t)$ with a sampling period T to produce x(n). An ideal lowpass digital filter h(n), with cut-off frequency w_6 is applied to x(n) to produce y(n). A D/C converter generates $y_c(t)$ from y(n), again using the sampling period T.

(a) Find the cut-off frequency of x(n) in radians, as a function of T.

(b) Find the largest sampling period T such that y(n) has no aliasing. (Hint: x(n) may still be aliased)

(c) Find the smallest sampling period T so that the filter h(n) modifies the spectrum of x(n).

(d) Give an expression for h(n) that is causal, FIR, and such that the time delay is n_6.

5. For $|w| \le \pi$, and a real value for d, assume that

$$X(e^{jw}) = \cos(d \cdot w)$$

(a) Find x(0).

(b) Find lim x(n) as n approaches infinity.

(c) Is x(n) even, odd, or neither ?

(d) Find an expression for x(n).