GAURI UDAY GODBOLE.

FALL 2001

1] a) $h(n) = a^n u(n) \quad x(n) = u(n)$

By def^n of convolution

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K)$$

$$= \sum_{K=-\infty}^{\infty} a^K u(K)\, u(n-K)$$

$$= \sum_{K=0}^{n} a^K u(K)\, u(n-K)$$

$$= \sum_{K=0}^{n} a^K$$

$$\therefore y(n) = \left[\frac{1-a^{n+1}}{1-a}\right] u(n)$$

b) $h(n) = a^n u(-n) \quad x(n) = b^n u(n)$

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K)$$

$$= \sum_{K=-\infty}^{\infty} a^K u(-K)\, b^{n-K} u(n-K)$$

$$= b^n \sum_{K=-\infty}^{\infty} a^K u(-K)\, b^{-K} u(n-K)$$

$-K > 0$

$n-K > 0$

$K < 0$

$n > K$

$K < n$

Case I:-

$$= b^n \sum_{K=0}^{\infty} a^K b^{-K}$$

$$= b^n \sum_{K=0}^{\infty} (a b^{-1})^K$$

$$y_1(n) = b^n \left[\frac{1}{1-\left(\frac{a}{b}\right)}\right] u(n)$$

Also case II:-

$$y(n) = b^n \sum_{-\infty}^{n} (a/b)^K$$

$$= b^n \sum_{n}^{\infty} (ab)^{-K} \quad \text{ask?}$$

c] $h(n) = u(n-2) - u(n-4) \quad x(n) = u(n+1) - u(n-7)$

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K)$$

$$= \boxed{\sum_{K=-\infty}^{\infty} [u(K-2) - u(K-4)]\,[u(n-K+1) - u(n-K-7)]}$$

$$y(n) = [u(n-2) - u(n-4)] * [u(n+1) - u(n-7)]$$

$$y(n) = [\delta(n) - \delta(n-8) - \delta(n-2) + \delta(n-10)]$$

d) $h(n) = (\cos an)\, u(n) \qquad x(n) = u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \cos(ak)\, u(k)\, u(n-k)$$

$$= \sum_{k=0}^{n} \cos(ak)$$

$$= \sum_{k=0}^{n} \left(e^{jak} + e^{-jak}\right)/2$$

$$= \frac{1}{2}\left[\frac{1-e^{ja(n+1)}}{1-e^{ja}} + \frac{1-e^{-ja(n+1)}}{1-e^{-ja}}\right] \quad //$$

Spring 2001

1] a) $h(n) = a^n u(n) \qquad x(n) = u(n-4)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k-4)$$

$$= \sum_{k=0}^{n-4} (a^k)$$

$$\therefore \quad y(n) = \left[\frac{1-a^{n-3}}{1-a}\right] \quad //$$

b) $h(n) = a^n u(n) \qquad x(n) = b^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} a^k u(k)\, b^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} a^k\, b^{n-k}$$

$$= b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k$$

$$\therefore \quad y(n) = b^n \left[\frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\frac{a}{b}}\right] \quad //$$

c) $h(n) = u(n-1) - u(n-5) \qquad x(n) = u(n+1) - u(n-8)$

$$y(n) = x(n) * h(n)$$

$$y(n) = [u(n-1) - u(n-5)] * [u(n+1) - u(n-8)]$$

$$= [\gamma(n+1) - \gamma(n-8) - \gamma(n-3) + \gamma(n-12)] \quad //$$

d) $h(n) = \sin(\omega_2 n)u(n)$ and $x(n) = u(n)$

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K)$$

$$= \sum_{K=-\infty}^{\infty} \sin(\omega_2 K) u(K)\, u(n-K)$$

$$= \sum_{K=0}^{n} \sin(\omega_2 K)$$

$$= \frac{1}{2j}\left[\sum_{K=0}^{n}\left(e^{j\omega_2 K} - e^{-j\omega_2 K}\right)\right]$$

$$= \frac{1}{2j}\left[\left\{\frac{1-e^{j\omega_2(n+1)}}{1-e^{j\omega_2}}\right\} - \left\{\frac{1-e^{-j\omega_2(n+1)}}{1-e^{-j\omega_2}}\right\}\right] //$$

FALL 1997

1] a) $h(n) = a^n u(n)$ $x(n) = b^n u(n)$

Same as spring 2001.

b) $h(n) = a^n u(n)$ $x(n) = u(n)$

Same as Fall 2001.

c) $h(n) = u(n) - u(n-5)$ $x(n) = u(n-2) - u(n-4)$

$$y(n) = h(n) * x(n)$$

$$= [u(n) - u(n-5)] * [u(n-2) - u(n-4)]$$

$$= [r(n-1) - r(n-3) - r(n-6) + r(n-8)] //$$

d) $h(n) = \cos(\omega_c n)\, u(n)$ $x(n) = a^n u(n)$

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K)$$

$$= \sum_{K=-\infty}^{\infty} \cos(\omega_c K) u(K)\, a^{n-K} u(n-K).$$

$$a^n \sum_{K=0}^{n} \left(\frac{e^{j\omega cK} + e^{-j\omega cK}}{2}\right) a^{-K}$$

$$\frac{a^n}{2} \sum_{K=0}^{n} \left(\frac{e^{j\omega cK}}{a}\right) + \left(e^{j\omega cK} \cdot a\right)^{-K}$$

$$\frac{a^n}{2}\left[\frac{1-\left(e^{j\omega c}/a\right)^{n+1}}{1-\frac{e^{j\omega c}}{a}} + \frac{1-\left(e^{j\omega c}\cdot a\right)^{-(n+1)}}{1-e^{j\omega c}\cdot a}\right] //$$

FALL 2002

1] a) $h(n) = a^n u(n) \quad x(n) = b^n u(n)$

Same as spring 2001.

b) $h(n) = a^n u(n) \quad x(n) = u(n)$

Same as Fall 2001

c) $h(n) = \cos(\omega_c n) u(n) \quad x(n) = u(n-2)$

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K)$$

$$= \sum_{K=-\infty}^{\infty} \cos(\omega_c K)\, u(K)\, u(n-K-2)$$

$$= \sum_{K=0}^{n-2} \cos(\omega_c K)$$

$$= \sum_{K=0}^{n-2} \left(\frac{e^{j\omega_c K} + e^{-j\omega_c K}}{2}\right)$$

$$= \frac{1}{2}\left[\frac{1-e^{j\omega_c (n-1)}}{1-e^{j\omega_c}} + \frac{1-e^{-j\omega_c (n-1)}}{1-e^{j\omega_c}}\right] //$$

d) $h(n) = u(n) - u(n-6) \quad x(n) = u(n-3) - u(n-7)$

$$y(n) = x(n) * h(n)$$

$$= [u(n) - u(n-6)]\,[u(n-3) - u(n-7)]$$

$$= [u(n-2) - u(n-6) - u(n-8) + u(n-12)] //$$

spring 2002

1] a) $h(n) = a^n u(n)$ $\quad x(n) = c^n u(n)$

Same as spring 2001.

b) $h(n) = b^n u(n)$ $\quad x(n) = u(n)$

Same as Fall 2001.

c) $h(n) = u(n)$ $\quad x(n) = r(n)$

$r(n) = n\, u(n)$

$\therefore\ x(n) = n\, u(n)$ $\quad h(n) = u(n)$

$$\therefore\ y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} u(k)\,(n-k)\,u(n-k)$$

$$= \sum_{k=0}^{n} (n-k)$$

$=$

d) $h(n) = u(n) - u(n-5)$ $\quad x(n) = u(n-2) - u(n-6)$

$y(n) = h(n) * x(n)$

$= [u(n) - u(n-5)]\ [u(n-2) - u(n-6)]$

$= [r(n-1) - r(n-5) - r(n-6) + r(n-10)]$ //

FALL 1998

Q.1) a) $h(n) = a^n u(n)$ $\quad x(n) = u(n)$

Same as Fall 2001

b) $h(n) = a^n u(n)$ $\quad x(n) = b^n u(n)$

Same as Spring 2001.

c) $h(n) = u(n) - u(n-3)$

$x(n) = u(n-1) - u(n-5)$.

$y(n) = h(n) * x(n)$

$= [u(n) - u(n-3)] * [u(n-1) - u(n-5)]$

$= [r(n) - r(n-4) - r(n-3) + r(n-7)]$

$y(n) = r(n) - r(n-4) - r(n-3) + r(n-7)$

d) $h(n) = \cos(\omega_c n)\, u(n)$ & $x(n) = u(n)$

Same as Fall 1998.

FALL 1999

Q 1) a) $h(n) = a^n u(n)$ & $x(n) = u(n+1)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k+1)$$

but $n+1-k \geq 0$

$\therefore$ ~~$n+1$~~ $n+1 \geq k$. but $\Rightarrow a \leq k$

$$= \sum_{k=0}^{n+1} (a^k)$$

$$y(n) = \frac{1 - a^{n+2}}{1-a}\, u(n) \; //$$

b) $h(n) = a^n u(n)$ $x(n) = b^n u(n)$

Same as spring 2001

c) $h(n) = u(n+1) - u(n-4)$

$x(n) = u(n+1) - u(n-6)$

$y(n) = [u(n+1) - u(n-4)] * [u(n+1) - u(n-6)]$

$= [r(n+3) - r(n-4) - r(n-2) + r(n-9)] \; //$

d) $h(n) = \cos(\omega_c n) u(n)$ $x(n) = u(n)$

Same as Fall 1998.

FALL 1999

Q.2)

$$y(n) = 2x(n) - \frac{7}{10}x(n-1) + \frac{7}{10}y(n-1) - \frac{1}{10}y(n-2)$$

a) find $H(e^{j\omega})$

$$Y(e^{j\omega}) = 2X(e^{j\omega}) - \frac{7}{10}e^{-j\omega}X(e^{j\omega}) + \frac{7}{10}Y(e^{j\omega})e^{-j\omega} - \frac{1}{10}e^{-2j\omega}Y(j\omega)$$

$$Y(e^{j\omega})\left[1 - \frac{7}{10}e^{-j\omega} + \frac{1}{10}e^{-2j\omega}\right] = \left[2 - \frac{7}{10}e^{-j\omega}\right]X(e^{j\omega})$$

$$\therefore H(e^{j\omega}) = \frac{2 - \frac{7}{10}e^{-j\omega}}{1 - \frac{7}{10}e^{-j\omega} + \frac{1}{10}e^{-2j\omega}}$$

$$H(e^{j\omega}) = \frac{20 - 7e^{-j\omega}}{10 - 7e^{-j\omega} + e^{-2j\omega}} \quad //$$

b) $y(n)\left(10 - 7z^{-1} + z^{-2}\right)$

$y(n)\left(10 - 5z^{-1} - 2z^{-1} + z^{-2}\right)$

$y(n)\left(5(2 - z^{-1}) - z^{-1}(2 - z^{-1})\right)$

$y(n)\ (2 - z^{-1})(5 - z^{-1})$

$y(n)\ \left(1 - (\frac{1}{2})z^{-1}\right)\left(1 - (\frac{1}{5})z^{-1}\right)$

$y(n) = (\frac{1}{2})^n c_1 + (\frac{1}{5})^n c_2$

$c_1 + c_2 = 2$

$\frac{1}{2}c_1 + \frac{1}{5}c_2 = \frac{7}{10}$

$\frac{1}{2} \quad \frac{1}{5}$

$\frac{2}{10}$

$\frac{1}{2}c_1 + \frac{1}{2}c_2 = 1$

$-\frac{1}{2}c_1 + \frac{1}{5}c_2 = -7/10$

$\frac{3}{10}c_2 = 3/10$

$\therefore c_2 = 1$

$\therefore c_1 = 1$

c) given sys is causal sys.

$y(0) = 2$

$y(1) = 2x(1) - \frac{7}{10}x(0) + \frac{7}{10}y(0) - \frac{1}{10}y(-1)$

$= 0 - \frac{7}{10} + \frac{14}{10}$

$y(1) = 7/10$.

d) $\therefore h(n) = \left\{(\frac{1}{2})^n + (\frac{1}{5})^n\right\}u(n)$

Q.2) $y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + 2x(n-1)$

$Y(e^{j\omega}) = \frac{3}{4} e^{-j\omega} Y(e^{j\omega}) - \frac{1}{8} e^{-2j\omega} Y(e^{j\omega}) + 2 e^{-j\omega} X(e^{j\omega})$

$Y(e^{j\omega}) \left[1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-2j\omega} \right] = 2 e^{-j\omega} X(e^{j\omega})$

$\therefore \quad H(e^{j\omega}) = \dfrac{2 e^{-j\omega}}{1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-2j\omega}}$

$y(n) \left(1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2} \right)$

$y(n) \left(1 - \frac{1}{4} z^{-1} - \frac{1}{2} z^{-1} + \frac{1}{8} z^{-2} \right)$

$y(n) \left[\left(1 - \frac{1}{4} z^{-1} \right) \right] - \frac{1}{2} z^{-1} \left(1 - \frac{1}{4} z^{-1} \right) \Big]$

$y(n) \left(1 - \frac{1}{2} z^{-1} \right) \left(1 - \frac{1}{4} z^{-1} \right)$

b) $y(n) = \left(\frac{1}{2} \right)^n c1 + \left(\frac{1}{4} \right)^n c2$

c) given sys is causal sys

Now

$y(0) = \frac{3}{4} y(-1) - \frac{1}{8} y(-2) + 2x(-1)$

$\therefore y(0) = 0$

$y(1) = \frac{3}{4} y(0) - \frac{1}{8} y(-1) + 2x(0)$

$\therefore y(1) = 2$

$\therefore \quad c1 + c2 = 0 \quad$ for $n = 0$

$\frac{1}{2} c1 + \frac{1}{4} c2 = 2 \quad$ for $n = 1$

Now solving this two eq^ns simultaneously we get

$\frac{1}{2} c1 + \frac{1}{2} c2 = 0$

$\underline{-\frac{1}{2} c1 \pm \frac{1}{4} c2 = 2}$

$\frac{1}{4} c2 = -2$

$\therefore \quad c2 = -8$

$\& \quad c1 = 8$

d) $h(n) = \left\{ 8 \left(\frac{1}{2} \right)^n - 8 \left(\frac{1}{4} \right)^n \right\} u(n-1)$

FALL 1998

Q.2) Same as spring 2001

FALL 2002

Q.1) Same as spring 2001

9

5

FALL 1997

Q.2) $y(n) = \frac{8}{15} y(n-1) - \frac{1}{15} y(n-2) + x(n)$

$$Y(e^{j\omega}) = \frac{8}{15} e^{-j\omega} Y(e^{j\omega}) - \frac{1}{15} e^{-2j\omega} Y(e^{j\omega}) + X(e^{j\omega})$$

$$Y(e^{j\omega}) \left[1 - \frac{8}{15} e^{-j\omega} + \frac{1}{15} e^{-2j\omega} \right] = X(e^{j\omega})$$

a) $$H(e^{j\omega}) = \frac{1}{1 - \frac{8}{15} e^{-j\omega} + \frac{1}{15} e^{-2j\omega}}$$

b) $y(n) \left(1 - \frac{8}{15} z^{-1} + \frac{1}{15} z^{-2}\right)$

$y(n) \left(1 - \frac{3}{15} z^{-1} - \frac{5}{15} z^{-1} + \frac{1}{15} z^{-2}\right)$

$y(n) \left(1 - \frac{1}{5} z^{-1}\right) \left(1 - \frac{1}{3} z^{-1}\right)$

$y(n) = \left(1 - \frac{1}{5} z^{-1}\right)\left(1 - \frac{1}{3} z^{-1}\right)$ $\quad y(n) = \left(\frac{1}{5}\right)^n c_1 + \left(\frac{1}{3}\right)^n c_2.$

$y(0) = 1$

$y(1) = \frac{8}{15} y(0)$

$y(1) = 8/15$

c) given sys is causal sys.

$\frac{1}{5} c_1 + \frac{1}{3} c_2 = 1 \Rightarrow \frac{1}{5} c_1 + \frac{1}{5} c_2 = 1/5$

$\frac{1}{5} c_1 + \frac{1}{3} c_2 = 8/15 \Rightarrow \frac{1}{5} c_1 + \frac{1}{3} c_2 = 8/15$

$\frac{-2}{15} c_2 = \frac{-5}{15}$

$\therefore c_2 = \frac{5}{2}$

$\therefore c_1 = \frac{-3}{2}$

$\frac{1}{5} \quad \frac{1}{3}$

15

$\frac{1}{5} \quad \frac{8}{15}$

15

$y(0) = 1, \quad y(1) = \frac{8}{15} y(0)$

$\therefore y(1) = 8/15$

$$\therefore h(n) = \left[\left(\frac{1}{5}\right)^n \left(\frac{-3}{2}\right) + \left(\frac{1}{3}\right)^n \left(\frac{5}{2}\right) \right] u(n)$$

FALL 2001

$-b^{-1}$ ...

Q.2) $y(n) = a x(n) - b\, y(n-2)$

$$Y(e^{j\omega}) = a X(e^{j\omega}) - b\, e^{-2j\omega} Y(e^{j\omega})$$

$$Y(e^{j\omega})\left[1 - b\, e^{-2j\omega}\right] = a X(e^{j\omega})$$

a) $$H(e^{j\omega}) = \frac{a}{(1 - b\, e^{-2j\omega})}$$

c) given sys is causal sys.

b) $y(n)\ (1 - b z^{-2})$ $\Rightarrow$ $(1 - \sqrt{b}\, z^{-1})(1 + \sqrt{b}\, z^{-1})$

$y(n) = (b)^n\ \cancel{z}\ c_1$

for $n = 0$

$y(0) = a\, x(0) - b\, y(-2)$

$\therefore y(0) = a$

$\therefore\ a = (b)^0 c_1$

$\therefore\ c_1 = a.$

$\therefore$ $\boxed{h(n) = a\, (b)^n\, u(n)}$ //

d) sys will be stable if $|b| << 1$

Spring 2002

Q.2) Same as Fall 1997.

Spring 2001

Q.3) find $Y(e^{j\omega})$ in terms of DTFT's.

$$y(n) = \sum_{k=-\infty}^{\infty} h(-k)\, x(k-n)$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} h(-k)\, x(k-n)\right] e^{-j\omega n}$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left[\sum_{K=-\infty}^{\infty} h(-K)\, x(K-n) \right] e^{-j\omega n} \underbrace{e^{j\omega K}\, e^{-j\omega K}}_{1}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{K=-\infty}^{\infty} h(-K)\, e^{-j\omega K} \right] x(K-n)\, e^{-j\omega n + j\omega K}$$

$$= \sum_{K=-\infty}^{\infty} h(-K)\, e^{-j\omega K} \sum_{n=-\infty}^{\infty} x(K-n)\, e^{j\omega (K-n)}$$

$$= H(e^{-j\omega K})\; X(e^{-j\omega K})$$

$$\therefore Y(e^{j\omega}) = X(e^{-j\omega})\; H(e^{-j\omega})$$ //

Q3) b) $y(n) = x(n/5)$ if n is multiple of 5 & $y(n) = 0$ otherwise.

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{\substack{n \text{ is} \\ \text{multiple} \\ \text{of } 5}} x\left(\frac{n}{5}\right) e^{-j\omega n}$$

Let $\frac{n}{5} = K$

$\therefore n = 5K$

$$= \sum X(K) \left(e^{-j\omega 5K} \right)$$

$$\boxed{Y(e^{j\omega}) = X(e^{5j\omega})}$$ //

FALL 1997

Q 3) find $Y(e^{j\omega})$ in terms of DTFT's.

$$y(n) = \sum_{-\infty}^{\infty} h(k)\, x(-k-n)$$

from the Def^n we have

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} h(k)\, x(-k-n) \right] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} h(k)\, x(-k-n)\, e^{-j\omega n} \underbrace{e^{j\omega k}\, e^{-j\omega k}}_{1}$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, e^{+j\omega k} \sum_{n=-\infty}^{\infty} x(-k-n)\, e^{-j\omega n}\, e^{-j\omega k}.$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, e^{+j\omega k} \sum_{n=-\infty}^{\infty} x(-k-n)\, e^{j\omega(-n-k)}$$

$$\boxed{Y(e^{j\omega}) = H(e^{-j\omega})\; X(e^{-j\omega})} \;//$$

Fall 2001

Q.4)

a) $$y(n) = \sum_{k=-\infty}^{\infty} h(-k)\, x(k+n)$$

from the Def^n of fourier transform we have.

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \cancel{y(n)\, e} \left[\sum_{k=-\infty}^{\infty} h(-k)\, x(k+n) \right] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} h(-k)\, x(k+n)\, e^{-j\omega n}\, e^{j\omega k}\, e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} h(-k)\, e^{+j\omega k} \sum_{n=-\infty}^{\infty} x(k+n)\, e^{-j\omega n}\, e^{-j\omega k}$$

$$= \sum_{K=-\infty}^{\infty} h(-K) e^{j\omega K} \sum_{n=-\infty}^{\infty} x(n+K) e^{-j\omega(n+K)}$$

$$Y(e^{j\omega}) = \underset{\Downarrow}{H(e^{j\omega})} X(e^{j\omega})$$

Doubt

b) $y_1(n) = x(\frac{n}{3})$ if n is multiple of 3 & 0 else.

$$Y_1(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y_1(n) e^{-j\omega n}$$

$$= \sum_{\substack{n=-\infty \\ \text{is multiple of } 3}}^{\infty} x(\tfrac{n}{3}) e^{-j\omega n}$$

let $\frac{n}{3} = K \quad n = 3K$

$$= \sum_{K} x(K) e^{-j\omega 3K}$$

$$\therefore Y_1(e^{j\omega}) = X(e^{3j\omega})$$

$y_2(n) = h(n/4)$ if n is multiple of 4 & 0 otherwise.

$$Y_2(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y_2(n) e^{-j\omega n}$$

$$= \sum_{\substack{n \text{ is} \\ \text{multiple of } 4}} h(n/4) e^{-j\omega n}$$

let $n/4 = K \quad j = 4K$

$$= \sum_{K} h(K) e^{-j\omega 4K}$$

$$\therefore Y_2(e^{j\omega}) = H(e^{4j\omega})$$

$$\therefore y(n) = y_1(n) + y_2(n) \quad \therefore Y(e^{j\omega}) = Y_1(e^{j\omega}) + Y_2(e^{j\omega})$$

$$\therefore \boxed{Y(e^{j\omega}) = X(e^{3j\omega}) + H(e^{4j\omega})} \quad //$$

Spring 2002

Q.3) $y(n) = \sum_{K=-\infty}^{\infty} h(-K)\, x(-K-n)$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{K=-\infty}^{\infty} h(-K)\, x(-K-n) \right] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{K=-\infty}^{\infty} h(-K)\, x(-K-n)\, e^{-j\omega n} \underbrace{e^{-j\omega K}\, e^{j\omega K}}_{1}$$

$$= \sum_{K=-\infty}^{\infty} h(-K)\, e^{j\omega K} \sum_{n=-\infty}^{\infty} x(-K-n)\, e^{-j\omega n}\, e^{-j\omega K}$$

$$= \sum_{K=-\infty}^{\infty} h(-K)\, e^{j\omega K} \sum_{n=-\infty}^{\infty} x(-K-n)\, e^{j\omega(-n-K)}$$

$$Y(e^{j\omega}) = H(e^{j\omega})\, X(e^{-j\omega}) \quad //$$

Fall 2002

Q 3) $y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(K-n)$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{K=-\infty}^{\infty} h(K)\, x(K-n) \right] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{K=-\infty}^{\infty} h(K)\, x(K-n)\, e^{-j\omega n}\, e^{j\omega K}\, e^{-j\omega K}$$

$$= \sum_{K=-\infty}^{\infty} h(K)\, e^{-j\omega K} \sum_{n=-\infty}^{\infty} x(K-n)\, e^{-j\omega n}\, e^{j\omega K}$$

$$= \sum_{K=-\infty}^{\infty} h(K)\, e^{-j\omega K} \sum_{n=-\infty}^{\infty} x(K-n)\, e^{j\omega(K-n)}$$

$$\boxed{Y(e^{j\omega}) = H(e^{j\omega})\, X(e^{-j\omega})} \quad //$$

Fall 1999

Q3)

a) $$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(K+n)$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{K=-\infty}^{\infty} h(K)\, x(K+n) \right] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{K=-\infty}^{\infty} h(K)\, x(n+K)\, e^{-j\omega n} \underbrace{e^{j\omega K}\, e^{-j\omega K}}_{1}$$

$$= \sum_{K=-\infty}^{\infty} h(K)\, e^{j\omega K} \sum_{n=-\infty}^{\infty} x(n+K)\, e^{-j\omega(n+K)}$$

$$Y(e^{j\omega}) = H(e^{-j\omega})\, X(e^{j\omega}) \;//$$

b) $y(n) = x(n/3)$ if n is multiple of 3 & $y(n) = 0$ otherwise.

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{\substack{n \text{ is} \\ \text{multiple of} \\ 3}} x(n/3)\, e^{-j\omega n}$$

let $n/3 = K \quad \therefore n = 3K$

$$= \sum_{K} x(K)\, e^{-j\omega 3K}$$

$$\therefore Y(e^{j\omega}) = X(e^{3j\omega}) \;//$$

FALL 1997

Q. 3) $y(n) = \sum_{-\infty}^{\infty} h(K)\, x(-K-n)$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{K=-\infty}^{\infty} h(K)\, x(-K-n)\; e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{K=-\infty}^{\infty} h(K)\, x(-K-n)\, e^{-j\omega n}\, e^{j\omega K}\, e^{-j\omega K}$$

$$= \sum_{K=-\infty}^{\infty} h(K)\, e^{j\omega K} \sum_{n=-\infty}^{\infty} x(-K-n)\, e^{j\omega(-n-K)}$$

$$Y(e^{j\omega}) = H(e^{-j\omega})\, X(e^{-j\omega}) \quad //$$

FALL 2002

Q. 5 $X(e^{j\omega}) = e^{a|\omega|}$

a) find $x(0)$

$$x(n) = 1/2\pi \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= 1/2\pi \int_{-\pi}^{\pi} e^{a|\omega|}\, e^{j\omega n}\, d\omega$$

$$= 1/2\pi \left[\int_{0}^{\pi} e^{a\omega}\, e^{j\omega(0)}\, d\omega + \int_{-\pi}^{0} e^{-a\omega}\, e^{j\omega(0)}\, d\omega \right]$$

$$= 1/2\pi \left[\int_{0}^{\pi} e^{a\omega}\, d\omega + \int_{-\pi}^{0} e^{-a\omega}\, d\omega \right]$$

$$= \frac{1}{2\pi}\left\{ \left[\frac{e^{a\omega}}{a} \right]_{0}^{\pi} + \left[\frac{e^{-a\omega}}{-a} \right]_{-\pi}^{0} \right\}$$

$$= \frac{1}{2\pi}\left\{ \left[\frac{e^{a\pi}-1}{a} + \frac{1-e^{a\pi}}{-a} \right]\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{a\pi}-1}{a} - \left(\frac{-e^{a\pi}+1}{a}\right)\right]\right\}$$ $\Rightarrow$ ~~ask~~!?

$$= \frac{(e^{a\pi}-1)}{2\pi}\left[\frac{1}{a} - \frac{(-1)}{a}\right] = \frac{(e^{a\pi}-1)}{2\pi}\left[\frac{2}{a}\right] \quad \therefore X(0) = \frac{(e^{a\pi}-1)}{a\pi} \; //$$

c) $$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{a|\omega|}\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\left[\int_{0}^{\pi} e^{a\omega}\, e^{j\omega n}\, d\omega + \int_{-\pi}^{0} e^{-a\omega}\, e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} e^{(jn+a)\omega}\, d\omega + \int_{-\pi}^{0} e^{(jn-a)\omega}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{(jn+a)\omega}}{(jn+a)}\right]_0^{\pi} + \left[\frac{e^{(jn-a)\omega}}{(jn-a)}\right]_{-\pi}^{0}\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{(jn+a)\pi}-1}{(jn+a)} + \frac{1-e^{-(jn-a)\pi}}{(jn-a)}\right]\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{jn\pi}e^{a\pi}-1}{(jn+a)} + \frac{1-e^{-jn\pi}e^{a\pi}}{(jn-a)}\right]\right\}$$

$$\because e^{\pm jn\pi} = (-1)^n$$

$$= \frac{1}{2\pi}\left\{\left[\frac{(-1)^n e^{a\pi}-1}{(jn+a)} - \frac{(-1)^n e^{a\pi}-1}{(jn-a)}\right]\right\}$$

$$= \frac{[(-1)^n e^{a\pi}-1]}{2\pi}\left[\frac{1}{jn+a} - \frac{1}{jn-a}\right]$$

$$= \frac{(-1)^n e^{a\pi}-1}{2\pi}\left[\frac{jn-a-jn-a}{-n^2-a^2}\right]$$

$$= \left(\frac{(-1)^n e^{a\pi}-1}{2\pi}\right)\left(\frac{-2a}{-n^2-a^2}\right) \quad \therefore x(n) = \left[\frac{(-1)^n e^{a\pi}-1}{2\pi}\right]\frac{2a}{(n^2+a^2)}$$

$$\therefore x(n) = \frac{[(-1)^n e^{a\pi} - 1]\, a}{\pi (n^2 + a^2)}$$

Spring 2002

Q 5) $X(e^{jw}) = \cosh(3w)$

$\cos h(x) = 0.5 [e^{x} + e^{-x}]$

$\cosh(3w) = 0.5 [e^{3w} + e^{-3w}]$

c) $x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) e^{jwn} dw$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} 0.5 [e^{3w} + e^{-3w}] e^{jwn} dw$$

$$= \frac{1}{4\pi} \int_{-\pi}^{\pi} \left[e^{(3+jn)w} + e^{(jn-3)w} \right] dw$$

$$= \frac{1}{4\pi} \left\{ \left[\frac{e^{(jn+3)w}}{(jn+3)} \right]_{-\pi}^{\pi} + \left[\frac{e^{(jn-3)w}}{(jn-3)} \right]_{-\pi}^{\pi} \right\}$$

$$= \frac{1}{4\pi} \left\{ \left[\frac{e^{(3+jn)\pi} - e^{(jn+3)-\pi}}{(3+jn)} \right] + \left[\frac{e^{(jn-3)\pi} - e^{(jn-3)-\pi}}{(jn-3)} \right] \right\}$$

$$= \frac{1}{4\pi} \left\{ \left[\frac{e^{jn\pi} e^{3\pi} - e^{-jn\pi} e^{-3\pi}}{(3+jn)} \right] + \left[\frac{e^{jn\pi} e^{-3\pi} - e^{-jn\pi} e^{3\pi}}{(jn-3)} \right] \right\}$$

$$= \frac{1}{4\pi} \left\{ \left[(-1)^n \left(\frac{e^{3\pi} - e^{-3\pi}}{(3+jn)} \right) \right] + \left[(-1)^n \left(\frac{e^{-3\pi} - e^{3\pi}}{jn-3} \right) \right] \right\}$$

$$= \frac{1}{2\pi} \left\{ \left[(-1)^n \right. \right.$$

$\Rightarrow$ ask?

sdr1794

10

a) $$x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} 0.5\left[e^{3\omega} + e^{-3\omega}\right] e^{j\omega(0)}\, d\omega$$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi} \left(e^{3\omega} + e^{-3\omega}\right) d\omega$$

$$= \frac{1}{4\pi}\left\{\left[\frac{e^{3\omega}}{3}\right]_{-\pi}^{\pi} + \left[\frac{e^{-3\omega}}{-3}\right]_{-\pi}^{\pi}\right\}$$

$$= \frac{1}{4\pi}\left\{\left[\frac{e^{3\pi} - e^{-3\pi}}{3}\right] + \left[\frac{e^{-3\pi} - e^{3\pi}}{-3}\right]\right\}$$

Q 5) $X(e^{j\omega}) = \cos(2\omega)$

$$\cos 2\omega = \left(e^{2j\omega} + e^{-2j\omega}\right)/2$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}\left(\frac{e^{2j\omega}+e^{-2j\omega}}{2}\right)e^{j\omega n}\,d\omega.$$

$$= \frac{1}{4\pi}\int_{-\pi/2}^{\pi/2}\left[e^{j\omega(n+2)} + e^{j\omega(n-2)}\right]d\omega$$

$$= \frac{1}{4\pi}\left\{\left[\frac{e^{j\omega(n+2)}}{j(n+2)}\right]_{-\pi/2}^{\pi/2} + \left[\frac{e^{j\omega(n-2)}}{j(n-2)}\right]_{-\pi/2}^{\pi/2}\right\}$$

$$= \frac{1}{4\pi}\left[\frac{e^{j\pi/2(n+2)} - e^{-j\pi/2(n+2)}}{j(n+2)} + \frac{e^{j\pi/2(n-2)} - e^{-j\pi/2(n-2)}}{j(n-2)}\right]$$

$$= \frac{1}{4\pi}\left[\frac{e^{jn\pi/2}\,e^{j\pi} - e^{-jn\pi/2}\,e^{-j\pi}}{j(n+2)} + \frac{e^{jn\pi/2}\,e^{-j\pi} - e^{-jn\pi/2}\,e^{j\pi}}{j(n-2)}\right]$$

$$x(0) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \left(\frac{e^{2j\omega} + e^{-2j\omega}}{2} \right) e^{j\omega n} d\omega.$$

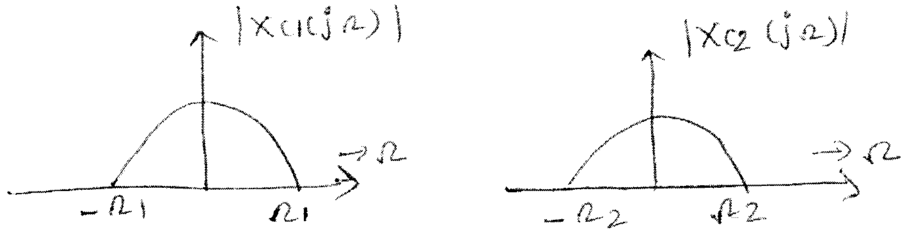
$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \left(\frac{e^{2j\omega} + e^{-2j\omega}}{2} \right) d\omega$$

$$= \frac{1}{4\pi} \left[\frac{e^{2j\omega}}{2j} + \frac{e^{-2j\omega}}{-2j} \right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{4\pi} \left[\frac{e^{j\pi} - e^{-j\pi}}{2j} + \frac{e^{-j\pi} - e^{j\pi}}{-2j} \right]$$

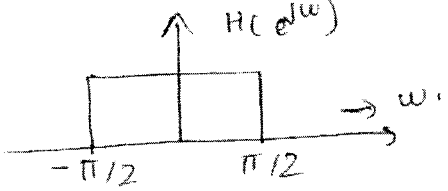
FALL 2002

Q.4) $X_{c1}(j\Omega)$ & $X_{c2}(j\Omega)$ are represented as follows.

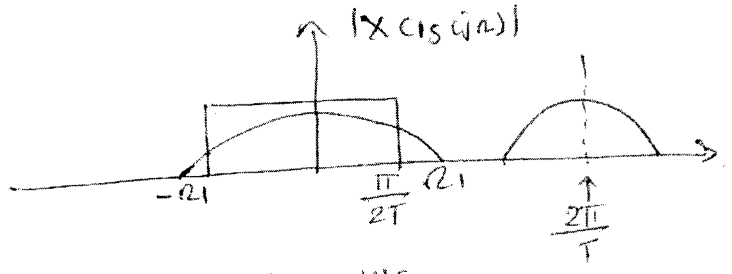


$X_{c1}(j\Omega) = F.T[X_{c1}(t)]$

a) Digital filter transfer fn $H(e^{j\omega})$ & having spectrum as.



after passing $X_{c1}(j\Omega)$ thro' LPF.



$\Omega_c \cdot T = \omega_c$

$\omega_c =$ cutoff freq.

$\Omega_c \cdot T = \frac{\pi}{2}$

$\therefore \Omega_c = \frac{\pi}{2T}$

a) $y_c(t)$ is the o/p of LPF

$\frac{\pi}{2T} < \Omega_1$

$\therefore$ for $y_c(t)$ to be unaliased condition is that

1) $\frac{\pi}{2T} < \Omega_1$

ie $T > \frac{\pi}{2\Omega_1}$

2) $\frac{\pi}{2T} < \frac{2\pi}{T} - \Omega_1$

$\Omega_1 < \frac{2\pi}{T} - \frac{\pi}{2T}$ $\quad \Omega_1 < \frac{3\pi}{2T}$

$T < \frac{3\pi}{2\Omega_1}$

so the limit for T is.

$$\boxed{\frac{\pi}{2\Omega_1} < T < \frac{3\pi}{2\Omega_1}}$$

b) $y(n) = x_1(n) \cdot x_2(n)$

$\therefore y_c(t) = x_{c1}(t) \cdot x_{c2}(t)$

Now by taking Fourier transform

$Y_c(j\Omega) = \frac{1}{2\pi} X_{c1}(j\Omega) * X_{c2}(j\Omega)$

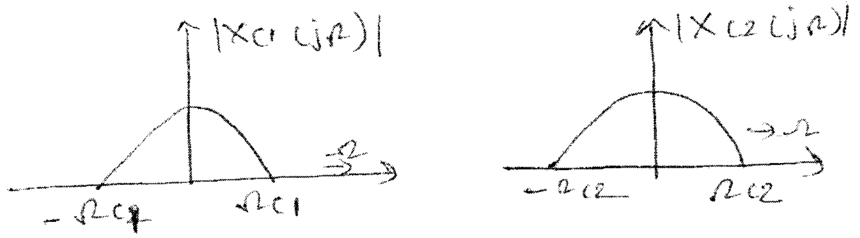
cutoff $= \Omega_{c1} + \Omega_2$

$\frac{2\pi}{T} \geq 2(\Omega_1 + \Omega_2)$

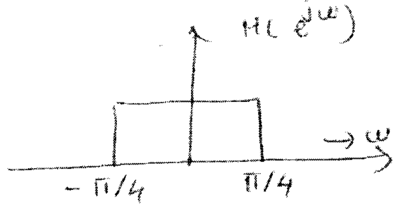
$$\boxed{T \leq \frac{\pi}{\Omega_1 + \Omega_2}}$$

spring 2001

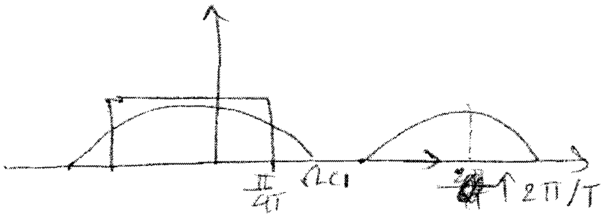
Q.5) The Fourier Transform of $x_{c1}(t)$ & $x_{c2}(t)$ are $X_{c1}(j\Omega)$ & $X_{c2}(j\Omega)$ will be as follows.



a) freq. response of ideal LPF. $\omega_c = \pi/4$.



$y_c(t)$ is LPF o/p.



$\Omega_c \cdot T = w_c \Rightarrow$ cutoff freq

$\Omega_{c1}\, T = \frac{\pi}{4}$

$\therefore \Omega_c = \frac{\pi}{4T}$

$\therefore \frac{\pi}{4T} < \Omega_{c1}$

1) $\therefore \frac{\pi}{4\Omega_{c1}} < T$

~~2) Also $\frac{\pi}{4T} < \frac{2\pi}{4T} - \Omega_{c1}$~~

~~$\therefore \Omega_{c1} < \frac{2\pi}{4T} - \frac{\pi}{4T}$~~ $\frac{2\pi}{T}$

2) $\frac{\pi}{4T} < \frac{2\pi}{T} - \Omega_{c1}$

$\Omega_{c1} < \frac{2\pi}{T} - \frac{\pi}{4T}$

$\Omega_{c1} < \frac{7\pi}{4T}$

$T < \frac{7\pi}{4\Omega_{c1}}$

$\therefore$ Range of sampling period T over which y_{cT} a lowpass filtered, unaliased version of $x_{c1}(t)$ is.

$\therefore$

$$\boxed{\frac{\pi}{4\Omega_{c1}} < T < \frac{7\pi}{4\Omega_{c1}}}$$

b) Now $y(n) = x_1(n) \cdot x_2(n)$

$y_c(t) = x_{c1}(t) \cdot x_{c2}(t)$

Now by F.T

$Y_c(j\Omega) = \frac{1}{2\pi}\, X_{c1}(j\Omega) * X_{c2}(j\Omega)$

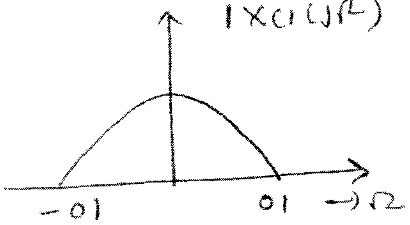
cut off = $\Omega_{c1} + \Omega_{c2}$

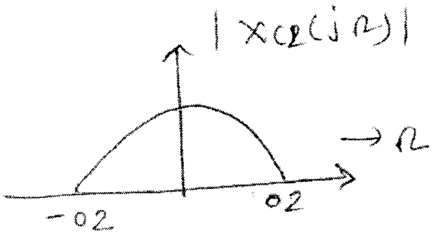
$\frac{2\pi}{T} \geq 2(\Omega_{c1} + \Omega_{c2})$

$$\boxed{T \leq \frac{\pi}{(\Omega_{c1}+\Omega_{c2})}}$$

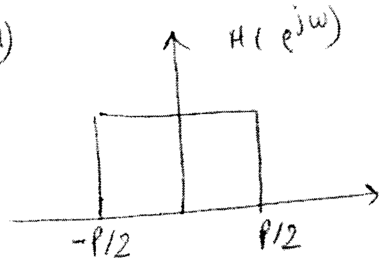
FALL 1997

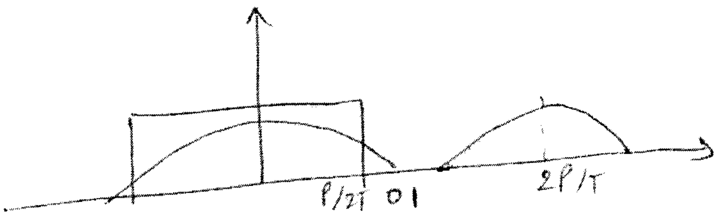
Q.4) F.T. of $x_{c1}(t)$ & $x_{c2}(t)$ are $X_{c1}(j\Omega)$ & $X_{c2}(j\Omega)$ respectively





a)





$\Omega_c \cdot T = w_c \Rightarrow$ cut off

$\Omega_c T = \rho/2$

$\Omega_c = \rho/2T$

1) $\frac{\rho}{2T} < \Omega_1$

$\therefore \frac{\rho}{2\Omega_1} < T$

2) $\frac{\rho}{2T} < \frac{2\rho}{T} - \Omega_1$

$\Omega_1 < \frac{2\rho}{T} - \frac{\rho}{2T}$

$\Omega_1 < \frac{3\rho}{2T}$ $T < \frac{3\rho}{2\Omega_1}$

$\therefore$ Range of T for part a is

$$\boxed{\frac{\rho}{2\sigma_1} < T < \frac{3\rho}{2\sigma_1}}$$

13 back.

b) $y(n) = x_1(n) \cdot x_2(n)$

$y_c(t) = x_{c1}(t) \cdot x_{c2}(t)$

Taking F.T.

$$Y_c(j\Omega) = \frac{1}{2\pi}\left[X_{c1}(j\Omega) * X_{c2}(j\Omega)\right]$$

cutt off $\Rightarrow (\sigma_1 + \sigma_2)$

$$\frac{2\rho}{T} \geq 2(\sigma_1 + \sigma_2)$$

$$T \leq \frac{\rho}{(\sigma_1 + \sigma_2)}$$ //

<u>Spring 2001</u>

Q.4) $F\{x(2n)\}$

a) $$F\{x(2n)\} = \sum_{n=-\infty}^{\infty} x(2n)\, e^{-j\omega n}$$

b) we want sum is over even values of the variable m.

let $m = 2n$ ie $n = \frac{m}{2}$

$$\therefore F\{x(2n)\} = F\{x(m)\} = \sum_{n=-\infty}^{\infty} x(m)\, e^{-j\omega m/2}$$

c) let $x(m) = x(m)\, g(m)$

but $g(m) \begin{cases} = 1 & \text{for } m = \text{even} \\ = 0 & \text{for } m = \text{odd} \end{cases}$

$$\therefore g(m) = \frac{1 + e^{jm\pi}}{2}$$

$$\therefore F\{x(m)\} = \sum_{m=-\infty}^{\infty} \left(\frac{1+e^{jm\pi}}{2}\right) x(m)\, e^{\frac{-j\omega m}{2}}$$

$$= \frac{1}{2} \sum_{m=-\infty}^{\infty} x(m) \, e^{-j\omega m/2} \cdot X(m) \, e^{jm(\pi - \frac{\omega}{2})}$$

$$= \frac{1}{2} \left[X(e^{j\omega/2}) + X(e^{-j(\pi - \frac{\omega}{2})}) \right] //$$

FALL 1998

Q.5) $X(e^{j\omega}) = e^{|\omega|}$ for $|\omega| \leq \pi/2$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \, e^{j\omega n} \, d\omega .$$

$$x(0) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} X(e^{j\omega}) \, d\omega = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{|\omega|} \, d\omega$$

$$= \frac{1}{2\pi} \left\{ \int_{-\pi/2}^{0} e^{-\omega} \, d\omega + \int_{0}^{\pi/2} e^{\omega} \, d\omega \right\}$$

$$= \frac{1}{2\pi} \left\{ \left[\frac{e^{-\omega}}{-1} \right]_{-\pi/2}^{0} + \left[e^{\omega} \right]_{0}^{\pi/2} \right\}$$

$$= \frac{1}{2\pi} \left\{ \frac{1 - e^{+\pi/2}}{-1} + e^{\pi/2} - 1 \right\}$$

$$= \frac{1}{2\pi} \left\{ e^{+\pi/2} - 1 + e^{\pi/2} - 1 \right\}$$

$$= 2 \, (e^{\pi/2} - 1) / 2\pi$$

a) $\therefore x(0) = \dfrac{e^{\pi/2} - 1}{\pi}$ //

b) $$x(n) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{|\omega|} \, e^{j\omega n} \, d\omega$$

$$= \cancel{\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2}}$$

$$= \frac{1}{2\pi}\left\{ \int_{-\pi/2}^{0} e^{-\omega} e^{j\omega n}\, d\omega + \int_{0}^{\pi/2} e^{\omega} e^{j\omega n}\, d\omega \right\}$$

$$= \frac{1}{2\pi}\left\{ \left[\frac{\cancel{e^{-\omega}}\, e^{-(1-jn)\omega}}{-1(1-jn)} \right]_{-\pi/2}^{0} + \left[\frac{e^{(1+jn)\omega}}{1+jn} \right]_{0}^{\pi/2} \right\}$$

$$= \frac{1}{2\pi}\left\{ \frac{1 - e^{(1-jn)\pi/2}}{-(1-jn)} + \frac{e^{(1+jn)\pi/2} - 1}{1+jn} \right\}$$

$$= \frac{1}{2\pi}\left\{ \frac{e^{(1-jn)\pi/2} - 1}{(1-jn)} + \frac{e^{(1+jn)\pi/2} - 1}{1+jn} \right\}$$

$$= \frac{1}{2\pi}\left\{ \frac{e^{\pi/2} e^{-jn\pi/2} - 1}{1-jn} + \frac{e^{\pi/2} e^{jn\pi/2} - 1}{1+jn} \right\} = \text{ask}$$

$$= \frac{1}{2\pi}\left\{ (j) \right.$$

spring 2001

Q6) a) $X(e^{j\omega}) = e^{-7|\omega|} \quad |\omega| \leq \pi$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

a) $$x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-7|\omega|}\, d\omega = \frac{1}{2\pi}\left\{\int_{-\pi}^{0} e^{7\omega}\, d\omega + \int_{0}^{\pi} e^{-7\omega}\, d\omega\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{7\omega}}{7}\right]_{-\pi}^{0} + \left[\frac{e^{-7\omega}}{-7}\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{1-e^{-7\pi}}{7} + \frac{e^{-7\pi}-1}{-7}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{1-e^{-7\pi}}{7} + \frac{1-e^{-7\pi}}{7}\right\}$$

$$= \frac{2\left[1-e^{-7\pi}\right]}{2\pi(7)} \qquad \therefore x(0) = \frac{(1-e^{-7\pi})}{7\pi} \;//$$

b) $\lim_{n\to\infty} x(n) \doteq 0 \;//$

c)