Exam 1, EE5350/4318, Spring 2012

1. Convolve h(n) and x(n) to get y(n). Put y(n) in closed form when possible.
(a) $h(n) = \delta(n-n_1) - \delta(n-n_3)$ and $x(n) = \cosh(n^2)$.
(b) $h(n) = 3^{-n} u(-n)$ and $x(n) = 2^{-n} u(-n)$.
(c) $h(n) = 2^n u(n)$, $x(n) = u(-n)$.
(d) $h(n) = \delta(\sin((2\pi/N)n)$ and $x(n) = n^2$ where N is odd.

2. A linear time invariant system is described by the recursive difference equation

$$y(n) = 2x(n) - x(n-1) + \frac{7}{12} y(n-1) - \frac{1}{12} y(n-2)$$

(a) Find $H(e^{jw})$ in closed form.
(b) Find the homogeneous solution.
(c) Find h(0) and h(1)
(d) Find the impulse response h(n).
(e) State whether or not the given difference equation is causal.

3. Let x(n), h(n) and y(n) denote complex sequences with DTFTs $X(e^{jw})$, $H(e^{jw})$ and $Y(e^{jw})$. Find frequency domain expressions for the following;

$$(a) \quad C = \sum_{n=-\infty}^{\infty} x(n) \cdot y^*(-n) \qquad (b) \quad y(n) = \sum_{k=-\infty}^{\infty} h(k) x(-k-n)$$

(c) Find the numerical value of

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{1-.5e^{-jw}} \bullet \frac{1}{1-.8e^{-jw}} \, dw$$

4. A C/D converter samples $x_c(t)$ with a sampling period T to produce x(n). An ideal lowpass digital filter h(n), with cut-off frequency w_5 is applied to x(n) to produce y(n). A D/C converter generates $y_c(t)$ from y(n), again using the sampling period T. The cut-off frequency of $x_c(t)$ is 3 radians/sec.
(a) Find the cut-off frequency of x(n) in radians, as a function of T.
(b) Find the largest sampling period T such that y(n) has no aliasing. (Hint: x(n) may still be aliased)
(c) Find the smallest sampling period T so that the filter h(n) modifies the spectrum of x(n).
(d) Give an expression for h(n).

5. A signal x(n) has N samples numbered 0 to N-1. The pseudocode below should calculate y(n) by using the causal difference equation, y(n) = x(n) - .5 y(n-1). Assume that legal arguments n in x(n) and y(n) are numbered 0 to N-1.

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y(A) = B
For n = C to D
y(n) = x(n) - .5 y(n-1)
End
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(a) For the first statement, give B, and a legal value for A, so that y(C) in the third statement can be calculated.
(b) In the second statement, give values for C and D so that the samples y(n) are calculated in the proper order, using only legal values for n.
(c) Is the filter stable (yes or no) ?

6. Assume that

$$X(e^{jw}) = e^{a|w|}$$

(a) Find x(0).
(b) Find lim x(n) as n approaches infinity.
(c) Find an expression for x(n).