Exam 1, EE5350, Summer 2014

1. Convolve h(n) and x(n) to get y(n). Put y(n) in closed form when possible.

(a) $h(n) = a^n u(n)$ and $x(n) = b^n u(n)$.

(b) $h(n) = c^n u(n)$, $x(n) = c^n u(n)$.

(c) h(n) = u(n+5)-u(n-7), x(n) = u(n+1)-u(n-8). Express the result in terms of r(n), where u(n)*u(n) = r(n+1).

(d) $h(n) = \cos(w_c n)u(n)$, x(n) = u(n).

2. A system is described by the recursive difference equation

$$y(n) = \frac{5}{12} y(n-1) - \frac{1}{24} y(n-2) - x(n) + \frac{5}{12} x(n-1)$$

(a) Find $H(e^{jw})$ in closed form. Give $H(e^{j0})$ and $H(e^{j\pi})$.

(b) Give the homogeneous solution to the difference equation above.

(c) Re-write the difference equation so that it generates the impulse response h(n). Give numerical values for h(0) and h(1).

(d) Using your answers to parts (b) and (c), give the impulse response h(n).

(e) Is the system stable ? (Yes or No)

(f) In the given difference equation, assume that legal arguments of x() and y() range from 0 to N-1, and that x(n) is already available. For the pseudocode below that implements the difference equation, give values or expressions for A, B, C, D, and E. (Hint: n starts at 0 and counts up to N-1.)

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y(0) = A
y(1) = B
For n = C to D
y(n) = E
End
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3. Let x(n), h(n) and y(n) denote complex sequences with DTFTs $X(e^{jw})$, $H(e^{jw})$ and $Y(e^{jw})$. Find frequency domain expressions for the following;

$$(a) \quad C = \sum_{n=-\infty}^{\infty} x(n)\cdot y(-n) \qquad (b) \quad y(n) = \sum_{k=-\infty}^{\infty} h(-k)x(k-n)$$

(c) Find the numerical value of

$$\sum_{n=-\infty}^{\infty} \frac{-\sin(\pi n/7)}{6\pi n} \bullet \frac{-\sin(\pi n/11)}{3\pi n}$$

4. C/D converters sample analog signals $x_{c1}(t)$ and $x_{c2}(t)$ at a sampling rate $1/T$ to produce $x_1(n)$ and $x_2(n)$ respectively. The Fourier transforms of $x_{c1}(t)$ and $x_{c2}(t)$ have cut-off frequencies Ω_1 and Ω_2 respectively.

(a) Assume that $x_1(n)$ is filtered by an ideal lowpass digital filter having a cut-off frequency of $\pi/3$ radians, producing an output y(n). y(n) is passed through a D/C converter with a sampling rate $1/T$ to produce the output $y_c(t)$. For what range of sampling periods T is $y_c(t)$ a lowpass filtered, unaliased version of $x_{c1}(t)$?

(b) Assume that there is no digital filter, but that y(n) is formed as $y(n) = [x_1(n) \cdot x_2(n)]^4$ before going through the D/C converter to produce the output $y_c(t)$. For what sampling periods T is $y_c(t) = [x_{c1}(t) \cdot x_{c2}(t)]^4$?

5. Assume that

$$X(e^{jw}) = \cos(a \cdot w) \qquad for\,|w| \le \pi/2$$

(a) Find x(0)

(b) Using Parseval's theorem, find the numerical value of

$$\sum_{n=-\infty}^{\infty} x^2(n)$$

(c) Given your answer in part (b), give lim x(n) as n approaches infinity.

(d) Find a real expression for x(n) and state whether or not it is absolutely summable.

6. A moving average filter has the impulse response

$$h(n) = \begin{Bmatrix} \frac{1}{N}, 0 \le n \le N-1 \\ 0, \;\; \text{else} \end{Bmatrix}$$

(a) Is this an FIR or an IIR filter ?

(b) Give a recursive difference equation for y(n) for this filter.

(c) Give a closed form for $H(e^{jw})$

(d) Suppose that the cut-off frequency w_c for this filter is the smallest positive w for which $H(e^{jw}) = 0$. Give w_c in terms of N.

7. A signal x(n) has N samples numbered 0 to N-1. The pseudoCode below should calculate the magnitude and phase responses from x(n) for M frequencies w(k), where k varies from 0 to M-1. These frequencies are evenly spaced and include 0 and π. The only complex variables are z and H.

$\Delta w = A$
$w = -\Delta w$
For $0 \le k \le M-1$
$w = w+\Delta w$
$H = B$
$z = e^{-jw}$

For $0 \le n \le N-1$
$H = H+C$
End
$Amp(k) = |H|$
$E = Real\{H\}$
$F = Im\{H\}$
$Phi(k) = G$
$ww(k) = w$
End

(a) Give the value of A, so that w varies from 0 to π as k varies from 0 to M-1.
(b) Give values for B and C, so that samples of the frequency response are temporarily stored in the variable H.
(c) Give expression G in terms of E and F, so that the phase is stored as Phi(k).