Exam 1, EE5350, Spring 2005

1. Convolve h(n) and x(n) to get y(n) below. Put y(n) in closed form when possible.

(a) $h(n) = a^n u(n)$ and $x(n) = b^n u(n)$.

(b) $h(n) = a^{-n} u(-n)$ and $x(n) = u(n)$.

(c) $h(n) = u(n)$ and $x(n) = r(n)$ where $r(n) = n \cdot u(n)$.

(d) $h(n) = u(n)-u(n-4)$, $x(n) = u(n-3)-u(n-4)$. Express the result in terms of r(n), where $u(n)*u(n) = r(n+1)$.

2. A system is described by the recursive difference equation

$$y(n) = \frac{8}{15} y(n-1) - \frac{1}{15} y(n-2) + x(n)$$

$\delta(n)$

(a) Find $H(e^{jw})$ in closed form.

(b) Find the homogeneous solution for y(n).

(c) Find the impulse response h(n).

(d) State whether or not the given difference equation is causal.

3. Find $Y(e^{jw})$ in terms of the DTFT's of h(n) and x(n) if

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(-k-n)$$

4. A C/D converter samples $x_c(t)$ with a sampling period T to produce x(n). An ideal lowpass digital filter h(n), with cut-off frequency w_c is applied to x(n) to produce y(n). A D/C converter generates $y_c(t)$ from y(n), again using the sampling period T. The cut-off frequency of $x_c(t)$ is 9 radians/sec.

(a) Find the cut-off frequency of x(n) in radians, as a function of T.

(b) Find the largest sampling period T such that y(n) has no aliasing. (Hint: x(n) may still be aliased)

(c) Find the smallest sampling period T so that the filter h(n) modifies the spectrum of x(n).

(d) Give an expression for h(n) that is causal, FIR, and such that the time delay is n_1.

To do

5. Let y(n) = x(K·n), where K is a positive integer. We want to find $Y(e^{jw})$ in terms of $X(e^{jw})$.

(a) Express x(K·n) in terms of the inverse transform of $X(e^{jw})$.

(b) Starting with the DTFT expression for $Y(e^{jw})$ and using part (a), write $Y(e^{jw})$ as an integral that has a summation of exponentials in its integrand.

(c) Re-write part (b) after making a change of variable so that the sum of exponentials becomes

$$\sum_{k=-\infty}^{\infty} e^{jnu}$$

(d) What are the limits on the integral now ?

(e) Using the sifting property of the Dirac delta function and fact that

$$\sum_{n=-\infty}^{\infty} e^{jnu} = 2\pi \sum_{n=-\infty}^{\infty} \delta(u - 2\pi n)$$

find a final expression for $Y(e^{jw})$.

Exam 1, EE5350, Fall 2005

1. Convolve h(n) and x(n) to get y(n) below. Put y(n) in closed form when possible.

(a) $h(n) = a^n u(n)$ and $x(n) = e^{jwn}u(n)$. Is x(n) an eigenfunction of the system ?

(b) $h(n) = a^n u(n)$ and $x(n) = e^{jwn}$. Is x(n) an eigenfunction of the system ?

(c) $h(n) = u(n-5)$ and $x(n) = r(n)$ where $r(n) = n \cdot u(n)$.

(d) $h(n) = u(n+2)-u(n-4)$, $x(n) = u(n-3)-u(n-5)$. Express the result in terms of r(n), where $u(n)*u(n) = r(n+1)$.

2. A system is described by the recursive difference equation

$$y(n) = \frac{9}{20}y(n-1) - \frac{1}{20}y(n-2) + 5x(n) - \frac{23}{20}x(n-1)$$

(a) Find $H(e^{jw})$ in closed form.

(b) Find the homogeneous solution for y(n).

(c) Find the impulse response h(n).

(d) State whether or not the given difference equation is causal.

3. Find $Y(e^{jw})$ in terms of the DTFT's of h(n) and x(n) if

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(-k+n)$$

4. A C/D converter samples $x_c(t)$ with a sampling period T to produce x(n). An ideal lowpass digital filter h(n), with cut-off frequency w_c is applied to x(n) to produce y(n). A D/C converter generates $y_c(t)$ from y(n), again using the sampling period T. The cut-off frequency of $x_c(t)$ is 4 radians/sec.

(a) Find the cut-off frequency of x(n) in radians, as a function of T.

(b) Find the largest sampling period T such that y(n) has no aliasing. (Hint: x(n) may still be aliased)

(c) Find the smallest sampling period T so that the filter h(n) modifies the spectrum of x(n).

(d) Give an expression for h(n) that is causal, FIR, and such that the time delay is n_1.

5. Let y(n) = x(L·n), where L is a positive integer. We want to find $Y(e^{jw})$ in terms of X().

(a) Express x(L·n) in terms of the inverse transform of $X(e^{jw})$.

(b) Starting with the DTFT expression for $Y(e^{jw})$ and using part (a), write $Y(e^{jw})$ as an integral that has a summation of exponentials in its integrand.

(c) Re-write part (b) after making a change of variable so that the sum of exponentials becomes

$$\sum_{n=-\infty}^{\infty} e^{jnu}$$

(d) What are the limits on the integral now ?

(e) Using the sifting property of the Dirac delta function and fact that

$$\sum_{n=-\infty}^{\infty} e^{jnu} = 2\pi \sum_{n=-\infty}^{\infty} \delta(u - 2\pi n)$$

find a final expression for $Y(e^{jw})$.

6. The DTFT, $X_m(e^{jw})$, of an N-sample window starting at time m, can be written as

$$X_m(e^{jw}) = \sum_{n=m}^{N+m-1} x(n) e^{-jw(n-m)}$$

(a) Given a value for w, and given that $e^{-jw(n-m)}$ is already pre-calculated, how many real multiplies are required to calculate $X_m(e^{jw})$ for x(n) real ?

(b) Give an efficient method for calculating $X_m(e^{jw})$ from $X_{m-1}(e^{jw})$

(c) How many real multiplies are required in part (b) ?

$\sum_{k=0}^{n} k = ? \ \frac{k(k+1)}{2} \ ?$ → h(n) when to use u(n)

Exam 1, EE5350, Fall 2004

1. Convolve h(n) and x(n) to get y(n) below. Put y(n) in closed form when possible.

(a) $h(n) = b^n u(n)$ and $x(n) = u(-n)$.

(b) $h(n) = a^n u(-n)$ and $x(n) = c^n u(-n)$.

(c) $h(n) = u(n+2)-u(n-5)$, $x(n) = u(n+1)-u(n-4)$. Express the result in terms of r(n), where $u(n)*u(n) = r(n+1)$.

(d) $h(n) = r(n)$ and $x(n) = u(n)$. Express the result in terms of u(n), r(n), and $r^2(n)$.

2. An LSI system is described by the recursive difference equation

$$y(n) = a \bullet x(n) - b \bullet y(n-2)$$

(a) Find $H(e^{jw})$ in closed form.

(b) Find the impulse response h(n). → ?

(c) State whether or not the given difference equation is causal.

(d) Under which conditions is the system stable ?

3. Let x(n) and y(n) denote complex sequences and let $X(e^{jw})$ and $Y(e^{jw})$ denote their DTFT's.

(a) Find a frequency domain expression for the constant;

$$C = \sum_{n=-\infty}^{\infty} x(n) \bullet y^*(n)$$

(b) Find the numerical value of

$$\sum_{n=-\infty}^{\infty} \frac{\sin(\pi n/2)}{3\pi n} \bullet \frac{\sin(\pi n/8)}{7\pi n}$$

$w_c = \pi/2$ $\quad w_c = \pi/8$

$\frac{\sin w_c n}{\pi n} \xrightarrow{FT}$ 1 w_c, 0 elsewhere

4. A C/D converter samples $x_c(t)$ with a sampling period T to produce x(n). An ideal lowpass digital filter h(n), with cut-off frequency w_c is applied to x(n) to produce y(n). A D/C converter generates $y_c(t)$ from y(n), again using the sampling period T. The cut-off frequency of $x_c(t)$ is 9 radians/sec.

(a) Find the cut-off frequency of x(n) in radians, as a function of T.

(b) Find the largest sampling period T such that y(n) has no aliasing. (Hint: x(n) may still be aliased)

(c) Find the smallest sampling period T so that the filter h(n) modifies the spectrum of x(n).

(d) Give an expression for h(n).

$t = nT$

4. C/D converters sample analog signals $x_{c1}(t)$ and $x_{c2}(t)$ at a sampling rate $1/T$ to produce $x_1(n)$ and $x_2(n)$ respectively. The Fourier transforms of $x_{c1}(t)$ and $x_{c2}(t)$ have cut-off frequencies Ω_1 and Ω_2 respectively.

(a) Assume that $x_1(n)$ is filtered by an ideal lowpass digital filter having a cut-off frequency of $\pi/6$ radians, producing an output $y(n)$. $y(n)$ is passed through a D/C converter with a sampling rate $1/T$ to produce the output $y_c(t)$. For what range of sampling periods T is $y_c(t)$ a lowpass filtered, unaliased version of $x_{c1}(t)$?

(b) Assume that there is no digital filter, but that $y(n)$ is formed as $y(n) = [x_1(n)\cdot x_2(n)]^4$ before going through the D/C converter to produce the output $y_c(t)$. For what sampling periods T is $y_c(t) = [x_{c1}(t)\cdot x_{c2}(t)]^4$?

5. Let $y(n) = x(L\cdot n)$, where L is a positive integer. We want to find $Y(e^{jw})$ in terms of $X()$.

(a) Express $x(L\cdot n)$ in terms of the inverse transform of $X(e^{jw})$.

(b) Starting with the DTFT expression for $Y(e^{jw})$ and using part (a), write $Y(e^{jw})$ as an integral that has a summation of exponentials in its integrand.

(c) Re-write part (b) after making a change of variable so that the sum of exponentials becomes

$$\sum_{n=-\infty}^{\infty} e^{jnu}$$

?,

(d) What are the limits on the integral now ?

(e) Using the sifting property of the Dirac delta function and fact that

$$\sum_{n=-\infty}^{\infty} e^{jnu} = 2\pi \sum_{n=-\infty}^{\infty} \delta(u - 2\pi n)$$

↳ how many values of n.

find a final expression for $Y(e^{jw})$.

6. Assume that

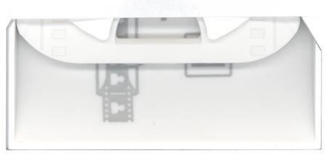
$$X(e^{jw}) = \cosh(3w) \quad for\ |w| \le \pi$$

where $\cosh(x) = .5[e^x + e^{-x}]$.

(a) Find $x(0)$.

(b) Find lim $x(n)$ as n approaches infinity.

(c) Find an expression for $x(n)$.



Kranthi Kotha.
XXX-XX-5524.

Exam 1, EE5350, Spring 2003

1. Convolve h(n) and x(n) to get y(n). Put y(n) in closed form when possible.
(a) $h(n) = c^n u(n-3)$ and $x(n) = b^n u(n-2)$.
(b) $h(n) = d^n u(n)$ and $x(n) = u(n-5)$.
(c) $h(n) = \cos(w_c n)u(n)$ and $x(n) = u(n-8)$.
(d) $h(n) = u(n-1)-u(n-9)$, $x(n) = u(n-4)-u(n-7)$. Express the result in terms of r(n), where $u(n)*u(n) = r(n+1)$.

2. An LTI (linear time invariant) system is described by the recursive difference equation

$$y(n) = 2x(n) - \frac{7}{12}x(n-1) + \frac{7}{12}y(n-1) - \frac{1}{12}y(n-2)$$

(a) Find $H(e^{jw})$ in closed form.
(b) Find the homogeneous solution.
(c) Find h(0) and h(1)
(d) Find the impulse response h(n).
(e) State whether or not the given difference equation is causal.

3. Here we derive F{x(2n)}.
(a) First, set up this Fourier transform as a sum over n, with no simplifications.
(b) Next, what do we substitute for n so that the sum is over even values of the variable m ? (n = f(m). what is f(m) ?) Rewrite the sum.
(c) Next, we replace x(m) by g(m)x(m) where g(m) = 1 for m even and 0 for m odd. Give g(m). Rewrite the sum so that it is over all values of m.
(d) Now, give F{x(2n)} in terms of X().

4. C/D converters sample analog signals $x_{c1}(t)$ and $x_{c2}(t)$ at a sampling rate 1/T to produce $x_1(n)$ and $x_2(n)$ respectively. The Fourier transforms of $x_{c1}(t)$ and $x_{c2}(t)$ have cut-off frequencies Ω_1 and Ω_2 respectively.
(a) Assume that $x_1(n)$ is filtered by an ideal lowpass digital filter having a cut-off frequency of $\pi/5$ radians, producing an output y(n). y(n) is passed through a D/C converter with a sampling rate 1/T to produce the output $y_c(t)$. For what range of sampling periods T is $y_c(t)$ a lowpass filtered, unaliased version of $x_{c1}(t)$?
(b) Assume that there is no digital filter, but that y(n) is formed as $y(n) = [x_1(n) \cdot x_2(n)]^4$ before going through the D/C converter to produce the output $y_c(t)$. For what sampling rates T is $y_c(t) = [x_{c1}(t) \cdot x_{c2}(t)]^4$?

NAME:- CHANDRASEKARAN, SIDDARTH
SSN :- XXX-XX-5696

Exam 1, EE5350, Summer 2006

1. Convolve h(n) and x(n) to get y(n) below. Put y(n) in closed form when possible.

(a) $h(n) = a^{-n}u(-n)$ and $x(n) = b^{-n}u(-n)$.

(b) $h(n) = a^{n}u(n)$ and $x(n) = u(n)$.

(c) $h(n) = u(n-5)$ and $x(n) = r(n)$ where $r(n) = n \cdot u(n)$.

(d) $h(n) = u(n+1)-u(n-4)$, $x(n) = u(n-2)-u(n-5)$. **Express the result in terms of r(n).**

2. A linear time invariant) system is described by the recursive difference equation

$$y(n) = 2x(n) - x(n-1) + \frac{7}{12}y(n-1) - \frac{1}{12}y(n-2)$$

(a) Find $H(e^{jw})$ in closed form.

(b) Find the homogeneous solution.

(c) Find h(0) and h(1)

(d) Find the impulse response h(n).

(e) State whether or not the given difference equation is causal.

3. Let x(n), y(n), and h(n) denote complex sequences.

(a) Find $Y(e^{jw})$ in terms of $X(e^{jw})$ and $H(e^{jw})$ if

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(k-n)$$

(b) Express the quantity E using $H(e^{jw})$ and $X(e^{jw})$ if

$$E = \sum_{k=-\infty}^{\infty} h^{*}(k)x(k)$$

EXAM 1: EE5350 FALL 2004

1. (a) $h(n) = b^n u(n) \qquad x(n) = u(-n)$

$$y(n) = \sum_{k=-\infty}^{\infty} u(-k)\, b^{n-k}\, u(\underbrace{n-k}_{n-k \geq 0})$$

$-k \geq 0$, $k \leq 0$ $\qquad$ $n-k \geq 0$, $k \leq n$

Case 1: $n > 0$

$$y(n) = \sum_{k=-\infty}^{0} b^{n-k} = b^n \sum_{k'=0}^{\infty} b^{k'} = \frac{b^n}{1-b} \;;\; |b|<1 \;,\; n>0$$

$(k' \leftarrow -k)$

Case 2: $n < 0$

$$y(n) = \sum_{k=-\infty}^{n} b^{n-k} = b^n \sum_{k=-\infty}^{n} b^{-k} \quad \text{let } k' \leftarrow -k$$

$$b^n \sum_{k'=\infty}^{-n} b^{k'} = b^n \sum_{k'=-n}^{\infty} b^{k'} = \frac{(b)^{-n}}{1-b} \qquad |b|<1 \;,\; n<0$$

(b) $h(n) = a^n u(-n) \qquad x(n) = c^n u(-n) \longrightarrow \bar{h}(n) = a^{-n} u(n) \;;\; \bar{x}(n) = c^{-n} u(n)$;

$$\bar{y}(n) = \bar{h}(n) * \bar{x}(n) = \sum_{k=-\infty}^{\infty} a^{-k} \underbrace{u(k)}_{k \geq 0} c^{-(n-k)} \underbrace{u(n-k)}_{n-k \geq 0}$$

$k \leq n$

$$\bar{y}(n) = \sum_{k=0}^{n} a^{-k} c^{-n} c^{k} = c^{-n} \sum_{k=0}^{n} \left(\frac{c}{a}\right)^k = c^{-n} \frac{1-(c/a)^{n+1}}{1-(c/a)} \cdot u(n) \quad c/a \neq 1$$

$$\boxed{\therefore\; y(n) = c^{n} \cdot \left\{ \frac{1-(c/a)^{-n+1}}{1-(c/a)} \right\} u(-n)}$$

(c) $h(n) = u(n+2) - u(n-5) \qquad x(n) = u(n+1) - u(n-4)$

$y(n) = h(n) * x(n)$

$= \{u(n+2) - u(n-5)\} * \{u(n+1) - u(n-4)\}$

$$\boxed{y(n) = r(n+4) - r(n-3) - r(n-1) + r(n-8)}$$

(d) $h(n) = r(n) \qquad x(n) = u(n)$

$= n\, u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} k\, \underbrace{u(k)}_{k \geq 0}\, \underbrace{u(n-k)}_{n-k \geq 0} = \sum_{k=0}^{n} k = \frac{n(n+1)}{2} \cdot u(n)$$

$k \leq n$

(2)

2. $y(n) = a \cdot x(n) - b \cdot y(n-2)$

(a) $y(n) + b y(n-2) = a x(n)$

$Y(e^{j\omega}) + b e^{-j2\omega} Y(e^{j\omega}) = a X(e^{j\omega})$

$H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{a}{1 + b e^{-j2\omega}}$

(b) Let $x(n) = 0$

$y(n) = -b y(n-2)$

$y(n) + b y(n-2) = 0$

$Y(z) + b z^{-2} Y(z) = 0$

$Y(z)\{1 + b z^{-2}\} = 0$

$z^2 + b = 0$

$(z - j\sqrt{b})(z + j\sqrt{b})$ ie $z = j\sqrt{b}, -j\sqrt{b}$

$$\boxed{y_h(n) = A_1 (j\sqrt{b})^n + A_2 (-j\sqrt{b})^n}$$

Let $x(n) = \delta(n)$

$y(n) + b y(n-2) = a\delta(n)$

$y(0) = a\delta(0) - b y(-2) = 1$

$A_1 + A_2 = 1$

$y(1) = a\delta(1) - b y(-1) = 0$

$j(A_1\sqrt{b} - A_2\sqrt{b}) = 0$

$A_1 = A_2$

$\therefore A_1 = \frac{1}{2}, A_2 = \frac{1}{2}$

$$\therefore \boxed{h(n) = \left\{\frac{1}{2}(j\sqrt{b})^n + \frac{1}{2}(-j\sqrt{b})^n\right\} \cdot u(n)}$$

(c) It is causal

(d) system is stable if $h(n) = 0, n < 0$

3. (a) $C = \sum_{n=-\infty}^{\infty} x(n) \cdot y^*(n)$

$y(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(e^{j\omega}) e^{j\omega n} d\omega \longrightarrow y^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y^*(e^{j\omega}) e^{-j\omega n} d\omega$

$C = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi}\int_{-\pi}^{\pi} Y^*(e^{j\omega}) e^{-j\omega n} d\omega \cdot x(n)$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} Y^*(e^{j\omega}) \underbrace{\sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}}_{X(e^{j\omega})} \cdot d\omega$

$$\boxed{C = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y^*(e^{j\omega}) X(e^{j\omega}) d\omega}$$

3. $y = x(2n)$

$y(n) = x(2n)$

(a)

$$F(x(2n)) = \sum_{n=-\infty}^{\infty} x(2n)\, e^{-j\omega n}$$

(b) let $m = 2n$ i.e. $f(m) = 2n$

for all n i.e. $1, 2, 3 \ldots$ $m = 2, 4, 6 \ldots$

$$F(x(2n)) = \sum_{m=-\infty}^{\infty} x(m)\, e^{-j\omega(m/2)}$$

(c) $x(m) \longleftarrow g(m)\, x(m)$

(d)

$$x(2n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\, e^{j(2n)\theta}\, d\theta$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\, e^{j(2n)\theta}\, e^{-j\omega n}\, d\theta$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta}) \underbrace{\sum_{n=-\infty}^{\infty} e^{-jn(\omega-2\theta)}}\, d\theta$$

$$\sum_{n=-\infty}^{\infty} e^{jn(2\theta-\omega)} \longrightarrow u \to \omega - 2\theta \qquad du = -2d\theta \qquad d\theta = -\frac{du}{2}$$

θ	$-\pi$	π
u	$\omega+2\pi$	$\omega-2\pi$

$$= \frac{-1}{4\pi}\int_{\omega+2\pi}^{\omega-2\pi} X(e^{j(\omega-u)/2})\, 2\pi\delta(u-2\pi n)\, du$$

$$\sum_{n=-\infty}^{\infty} e^{jnu} = 2\pi\delta(u-2\pi n) \qquad \left[\sum_{n=-\infty}^{\infty} e^{j\omega n T} = \frac{2\pi}{T}\delta(\omega - 2\pi n/T)\right]$$

$$= -\frac{1}{2}\int_{\omega+2\pi}^{\omega-2\pi} X(e^{+j(\omega-u)/2})\, \delta(u-2\pi n)\, du \longrightarrow \frac{1}{2}\int_{\omega-2\pi}^{\omega+2\pi} X(e^{j(\omega-u)/2})\, \delta(u-2\pi n)\, du$$

$\longrightarrow n = 0, 1$

$$= \underbrace{\frac{-1}{2} X(e^{j\omega/2})}_{n=0} + \left(\frac{-1}{2}\right) X(e^{j\frac{\omega}{2} - j\pi})$$

$$= \frac{-1}{2} X(e^{j\omega/2}) \; \frac{-1}{2} X(e^{j(\omega/2-\pi)})$$

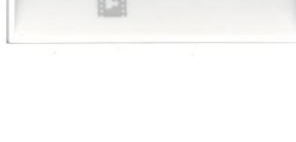
$$= \frac{+1}{2} \cdot \left[X(e^{j\omega/2}) + X(-e^{j\omega/2})\right]$$

$\longrightarrow \cos\left(\frac{\omega}{2}-\pi\right) + j\sin\left(\frac{\omega}{2}-\pi\right)$

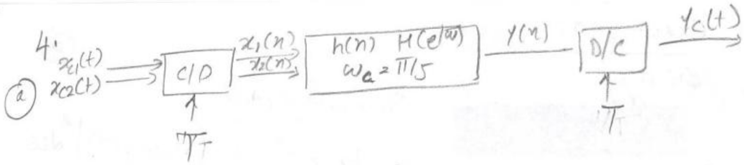
$-\cos\left(\pi - \frac{\omega}{2}\right) - j\sin\left(\pi - \frac{\omega}{2}\right)$

$-\cos\left(\frac{\omega}{2}\right) - j\sin\frac{\omega}{2}$

$= -e^{j\omega/2}$



4. (a)



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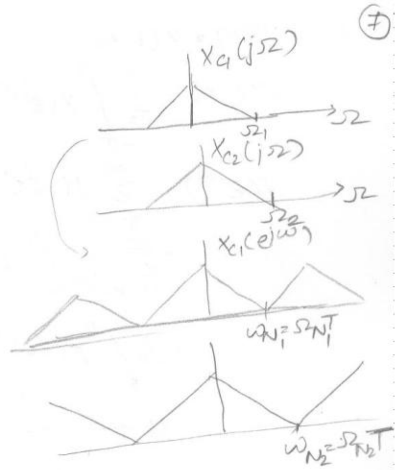
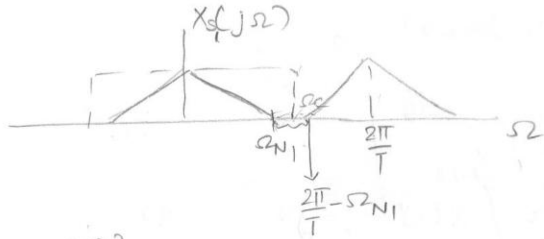


$\Omega_{N_1} = \Omega_1 \quad \Omega_{N_2} = \Omega_2$

$\boxed{\omega_{N_1} = \Omega_1 T \quad \omega_{N_2} = \Omega_2 T}$

$\Omega = \dfrac{\omega}{T}$



(Ω_c)

$\dfrac{\omega_c}{T} \le \dfrac{2\pi}{T} - \Omega_1 \qquad \omega_c \le 2\pi - \Omega_1 T$

$T \le \dfrac{2\pi - \omega_c}{\Omega_1} \rightarrow T \le -\left(\dfrac{\omega_c - 2\pi}{\Omega_1}\right) \qquad T \ge \dfrac{\omega_c - 2\pi}{\Omega_1}$

$\dfrac{\omega_c}{T} \ge \Omega_{N_1} \rightarrow \dfrac{\omega_c}{T} \ge \Omega_1 \qquad T \le \dfrac{\omega_c}{\Omega_1}$

$$\boxed{\frac{\omega_c - 2\pi}{\Omega_1} \le T \le \frac{\omega_c}{\Omega_1}}$$

(b)

$y(n) = [x_1(n) \cdot x_2(n)]^4$

$y(n) = x_1(n) \cdot x_2(n)$

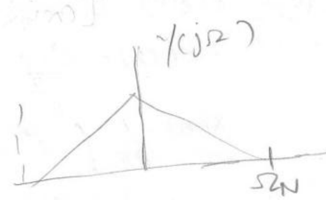
$\Omega_{N} \rightarrow \Omega_1 + \Omega_2$

$y(n) = [x_1(n) \cdot x_2(n)]^2$

$\Omega_N \rightarrow 2(\Omega_1 + \Omega_2)$

$y(n) = [x_1(n) \cdot x_2(n)]^4$

$\Omega_N \rightarrow 4(\Omega_1 + \Omega_2)$



$\boxed{\Omega_s \ge 2\Omega_N}$

$\dfrac{2\pi}{T} \ge 2 \cdot 4(\Omega_1 + \Omega_2)$

$$\boxed{T \le \frac{\pi}{4(\Omega_1 + \Omega_2)}}$$

⑤. $y(n) = x(L\cdot n)$ Exam 2006 Summer ⑧

$$x(L\cdot n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\, e^{j(Ln)\theta}\, d\theta$$

Parseval's theorem

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\, e^{+j(Ln)\theta}\, d\theta\, e^{-j\omega n}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta}) \sum_{n=-\infty}^{\infty} e^{-jn(\omega - L\theta)}\, d\theta$$

$u \leftarrow \omega - L\theta \qquad \frac{-1}{L} du = d\theta$

θ	$-\pi$	π
u	$\omega+L\pi$	$\omega-L\pi$

$$= \frac{-1}{2L\pi}\int_{\omega+L\pi}^{\omega-L\pi} X(e^{j\frac{\omega-u}{L}}) \sum_{n=-\infty}^{\infty} e^{-jnu}\, du$$

$$e^{-j\omega n T} = \frac{2\pi}{T}\,\delta\!\left(\omega - \frac{2\pi n}{T}\right)$$

$$= \frac{+1}{2L\pi}\int_{\omega-L\pi}^{\omega+L\pi} X(e^{j(\omega-u)/L})\, 2\pi\, \delta(u - 2\pi n)\, du$$

$n = 0, 1$

$$= \frac{1}{L}\Big\{ \underbrace{X(e^{j\omega/L})}_{n=0} + \underbrace{X(e^{j(\omega/L - 2\pi/L)})}_{n=1} \Big]$$

⑥. $X(e^{j\omega}) = \cosh(3\omega)$ for $|\omega| \le \pi$

$\sin(jx) = \frac{e^{j(jx)} - e^{-j(jx)}}{2j} = \frac{e^{-x} - e^{x}}{2j}$

ⓐ $\cosh(x) = \frac{e^{x} + e^{-x}}{2}$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{3\omega} + e^{-3\omega}}{2}\, e^{j\omega n}\, d\omega = \frac{1}{4\pi}\int_{-\pi}^{\pi} e^{(3+jn)\omega} d\omega + \frac{1}{4\pi}\int_{-\pi}^{\pi} e^{-(3+jn)\omega} d\omega$$

$$= \frac{1}{4\pi}\left\{ \frac{e^{(3+jn)\pi} - e^{-(3+jn)\pi}}{3+jn} - \frac{e^{(-3+jn)\pi} - e^{(3+jn)\pi}}{3-jn} \right\}$$

$$x(0) = \frac{1}{4\pi}\left[\left(\frac{e^{3\pi} - e^{-3\pi}}{3}\right) - \frac{e^{-3\pi} + e^{3\pi}}{3}\right] = \frac{2}{2\cdot 4\pi}\,\frac{e^{3\pi} - e^{-3\pi}}{3} = \frac{j}{\pi 3}\sinh 3\pi$$

ⓑ $\lim_{n\to\infty} x(n) = 0$ ~~for finite~~

ⓒ $x(n) =$