Exam 1, EE5350, Summer 2008

1. Convolve h(n) and x(n) to get y(n) below. Put y(n) in closed form when possible.
(a) $h(n) = a^{-n}u(-n)$ and $x(n) = b^{-n}u(-n)$.
(b) $h(n) = a^{n}u(n)$ and $x(n) = e^{jwn}$. Is x(n) an eigenfunction of the system ?
(c) h(n) = u(n+3)-u(n-4), x(n) = u(n-2)-u(n-5). Express the result in terms of r(n),

2. A system is described by the recursive difference equation

$$y(n) = \frac{8}{15}y(n-1) - \frac{1}{15}y(n-2) + 5x(n) - \frac{7}{5}x(n-1)$$

(a) Find $H(e^{jw})$ in closed form.
(b) Find the homogeneous solution for y(n).
(c) Find h(0) and h(1)
(d) Find the impulse response h(n).
(e) State whether or not the given difference equation is causal.

3. Let x(n), y(n), and h(n) denote complex sequences.
(a) Find $Y(e^{jw})$ in terms of $X(e^{jw})$ and $H(e^{jw})$ if

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(k-n)$$

(b) If

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n+s_2k)$$

find $Y(e^{jw})$ in terms of $X(e^{jw})$ and $H(e^{jw})$ if s_2 can equal either +1 or -1.
(Hint: you already know the answer for s_2 = -1.)

(c) Let

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(s_1n+s_2k)$$

We want to find $Y(e^{jw})$ in terms of $X(e^{jw})$ and $H(e^{jw})$ where if s_1 can equal either +1 or -1 and s_2 can equal either +1 or -1. Multiplying both sides of the equation by $\sum e^{-jwn}$, we can write $\exp(-jwn)=\exp(-jf(w)(s_1n + s_2k))\cdot \exp(-j(-(s_2f(w))\cdot k)$.
Looking only at terms above having "n", give an expression for f(w).
(d) Continuing part (c), give an expression for $Y(e^{jw})$.

4. Let $x_c(t)$ denote an analog signal with upper cut-off frequency Ω_o rad/sec. A system has a C/D converter that samples input $x_c(t)$, producing x(n). x(n) is then convolved with h(n) which is a digital lowpass filter impulse response with cut-off frequency w_c radians. The filtered output y(n) passes through a D/C converter, producing the analog output $y_c(t)$. The C/D and D/C converters both use a sampling rate 1/T.

(a) If we want $y_c(t)$ to be a lowpass filtered version of $x_c(t)$, where the filter's cut -off frequency is Ω_c rad/sec., what should w_c be equal to, for the digital filter ?

(b) For the system of part (a), give an upper limit on the sampling period T so that $y_c(t)$ is not aliased.

(c) Assume that h(n) now has a constant cut-off frequency of $w_c = \pi/8$radians. For what range of sampling periods T is $y_c(t)$ a lowpass filtered, unaliased version of $x_c(t)$?

5. Assume that

$$X(e^{jw}) = e^{-d|w|} \quad for\,|w| \le \pi$$

(a) Find x(0)

(b) Using Parseval's theorem, find the numerical value of

$$\sum_{n=-\infty}^{\infty} x^2(n)$$

(c) Given your answer in part (b), give lim x(n) as n approaches infinity.

(d) Find a real expression for x(n) and state whether or not it is absolutely summable.

6. A signal x(n), which has N_3 samples numbered 0 to N_3-1, is to be convolved with a causal filter h(n), which has N_4 samples numbered 0 to N_4-1. The output is y(n).

(a) In the time domain convolution pseudocode below, give expressions for X and Y.

$y(n) = 0$ for $0 \le n \le N_3 + N_4 - 2$
For $0 \le k \le N_3 - 1$
For $0 \le i \le N_4 - 1$
$n = X$
$y(n) = y(n) + Y$
End
End

(b) In the time domain convolution pseudocode below, give expressions for X and Y.

For n = 0 to $N_3 + N_4 - 2$
$y(n) = 0$
For $0 \le k \le N_3 - 1$
If(X) $y(n) = y(n) + Y$
End
End

(c) In the time domain convolution pseudocode below, give expressions for YY and ZZ.

For $0 \le n \le N_3 + N_4 - 2$
XX = x(n)
Call Update(XX,X,h, N_4,Y)
y(n) = Y
End

Function Update does the following operations

X(0) = XX
Y=0.
For $0 \le k \le N_4 - 1$
Y=Y+YY
End
For $N_4 - 1 \ge n \ge 1$ (counting backwards from (N_4–1) to 1)
X(n) = ZZ
End