Exam 1, EE5350, Spring 2013

1. Convolve h(n) and x(n) to get y(n). Put y(n) in closed form when possible.
(a) $h(n) = 3^n u(n)$ and $x(n) = 5^n u(n)$.
(b) $h(n) = 2^{-n} u(n)$, $x(n) = u(n)$.
(c) $h(n) = u(n+4)-u(n-6)$, $x(n) = u(n+2)-u(n-8)$. Express the result in terms of r(n), where $u(n)*u(n) = r(n+1)$.
(d) $h(n) = \delta(\sin(1 + |n|))$ and $x(n) = \sin(n^2)$.
(e) $h(n) = \delta(n^2 + n - 30)$ and $x(n) = \cos(n)$.

2. A system is described by the recursive difference equation

$$y(n) = \frac{8}{15}y(n-1) - \frac{1}{15}y(n-2) + 3x(n) - \frac{11}{15}x(n-1)$$

(a) Find $H(e^{jw})$ in closed form. Give $H(e^{j0})$ and $H(e^{j\pi})$.
(b) Give the homogeneous solution to the difference equation above.
(c) Re-write the difference equation so that it generates the impulse response h(n). Give numerical values for h(0) and h(1).
(d) Using your answers to parts (b) and (c), give the impulse response h(n).
(e) Is the system stable ? (Yes or No)

3. Let x(n), h(n) and y(n) denote complex sequences with DTFTs $X(e^{jw})$, $H(e^{jw})$ and $Y(e^{jw})$. Find frequency domain expressions for the following;

(a) $C = \sum_{n=-\infty}^{\infty} x(n) \cdot h(-n).$ Give the substitution you made for h(-n).

(b) $y(n) = \sum_{k=-\infty}^{\infty} h(k)x(k-n)$

(c) Using part (a), find the numerical value of

$$C = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{1-\frac{1}{2}e^{-jw}} \bullet \frac{1}{1-\frac{1}{6}e^{-jw}} dw$$

4. The discrete time signal x(n) has a cut-off frequency that may be as large as π radians.

(a) Assuming that x(n) comes from ideal sampling of $x_a(t)$ at a rate of $2\pi/T$ radians/sec., with no aliasing, give the highest possible Nyquist frequency Ω_N for $X_a(j\Omega)$.

(b) If we decimate x(n) with an integer decimation rate N_1 , so that the resulting signal y(n) has a sampling period of $N_1 \cdot T$, give the new sampling rate and the new Nyquist frequency in radians per second.

(c) Given this new Nyquist frequency, what cut-off frequency must our lowpass anti-aliasing filter h(n) have in radians ? (Remember, we have to lowpass filter x(n) before subsampling it to get y(n))

(d) Give the impulse response for h(n), assuming it has a time delay of zero.

5. For $|w| \le \pi$, and a real value for d, assume that

$$X(e^{jw}) = e^{-d \cdot |w|}$$

(a) Find x(0).
(b) Find lim x(n) as n approaches infinity.
(c) Is x(n) even, odd, or neither ?
(d) Is x(n) causal, anti-causal, or non-causal ?
(e) Find an expression for x(n).