

1/27/24

Super position lab

Dirac Notation

a.1) $|0\rangle$ (Normalized) \checkmark

a.2) $|1\rangle$ (Normalized) \checkmark

a.3) $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ (Normalized) \checkmark

a.4) $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ (Normalized) \checkmark

a.5) $3|0\rangle + 7|1\rangle$ $3^2 + 7^2 = 9 + 49 \Rightarrow \sqrt{58} = \sqrt{3^2 + 7^2}$ $\frac{3}{\sqrt{58}}|0\rangle + \frac{7}{\sqrt{58}}|1\rangle$

a.6) $2|0\rangle - 3|1\rangle$ $2^2 + 3^2 = 4 + 9 = \sqrt{13}$ $\frac{2}{\sqrt{13}}|0\rangle - \frac{3}{\sqrt{13}}|1\rangle$

a.7) $|0\rangle + e^{i\pi/2}|1\rangle$ $\frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/2}|1\rangle)$ \checkmark

b.1) $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b.2) $|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

b.3) $\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) = \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) - \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle))$
 $= \frac{1}{2}(\cancel{|0\rangle} + |1\rangle - \cancel{|0\rangle} + |1\rangle) = \frac{1}{2}(2|1\rangle) = |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

b.4) $\frac{1}{\sqrt{2}}(|1\rangle + |-\rangle) = \frac{1}{\sqrt{2}}(|1\rangle + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle))$
 $= \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{2}|0\rangle - \frac{1}{2}|1\rangle = \frac{1}{2}|0\rangle + (\frac{1}{\sqrt{2}} - \frac{1}{2})|1\rangle = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} - \frac{1}{2} \end{bmatrix}$

$$\frac{1}{\sqrt{2}}(|1\rangle + |-\rangle) = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle))$$

 $= \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{2}|0\rangle - \frac{1}{2}|1\rangle$

Inner products

$$\langle 0 | + \rangle = \langle 0 | \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \rangle = \frac{1}{\sqrt{2}}(\langle 0 | 0 \rangle + \langle 0 | 1 \rangle) = \frac{1}{\sqrt{2}}$$

$$\langle - | + \rangle = 0 \checkmark$$

$$\langle 0 | \psi \rangle = \langle 0 | 2|0\rangle + \beta|1\rangle \rangle = 2\langle 0 | 0 \rangle + \beta\langle 0 | 1 \rangle = 2 \checkmark$$

$$\langle \phi | \psi \rangle = \langle \gamma^* \langle 0 | + \delta^* \langle 1 | 2|0\rangle + \beta|1\rangle \rangle$$

 $= \gamma^* 2\langle 0 | 0 \rangle + \delta^* \beta \langle 1 | 1 \rangle \checkmark$

Outer product

$$|0\rangle\langle 0| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \checkmark$$

$$|1\rangle\langle +| = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \checkmark$$

$$|0\rangle\langle \psi| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix} = \begin{bmatrix} \alpha^* & \beta^* \\ 0 & 0 \end{bmatrix} \checkmark$$

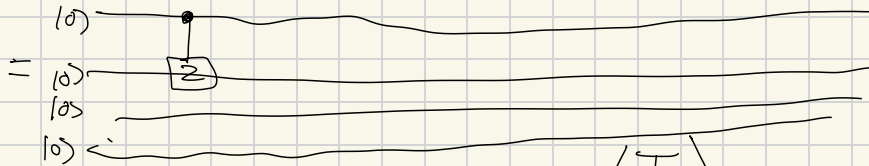
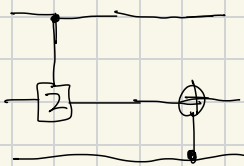
$$|0\rangle\langle \psi| = \begin{bmatrix} \gamma \\ \delta \end{bmatrix} \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix} = \begin{bmatrix} \gamma\alpha^* & \gamma\beta^* \\ \delta\alpha^* & \delta\beta^* \end{bmatrix} \checkmark$$

Hackathon 2025 Feb 1, 25

goal: Reduce movement, reduce CZ gate

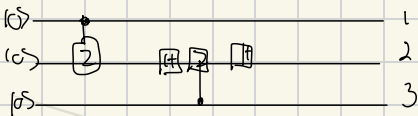
Qiskit (transpile)
MC (compile)

$$CNOT_{c \rightarrow t} = H_t CZ_{c,t} H_t$$



$$\text{cost} = 2(G_2 + G_{xy}) + B(L_2 + L_{xy}) + \gamma CZ + \delta \left(\frac{T}{T_0} \right)^{\epsilon} \eta M$$

0 0 0 ...



$$H = e^{i\pi/2} R_z(\pi/2) R_x(\pi/2)$$

0 0 0 0

Steps

1 Move 12 Hot (2)

2 CZ 00 (12)

3 Move (back St (3)

4 apply H to 2

5 move 3 in to Hot next to 2 (4)

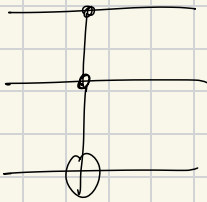
6 apply CZ gate to 23

Move 12 Hot

CZ

Move 7 back

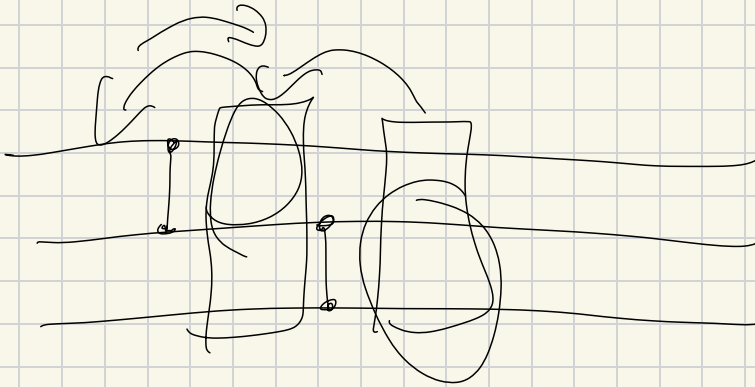
7 apply 1 to 2



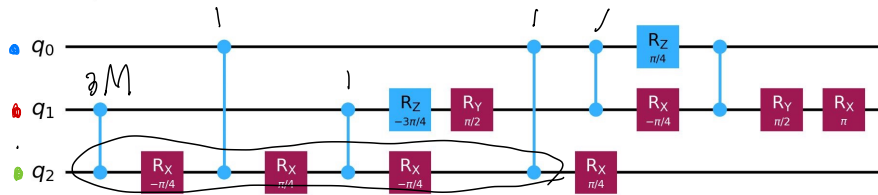
0 0 0 0 0 0

00 00

Steps: Translate gates to native gate set



Global Phase: $\pi/8$



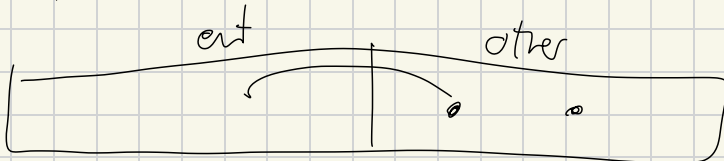
6C2	6C2
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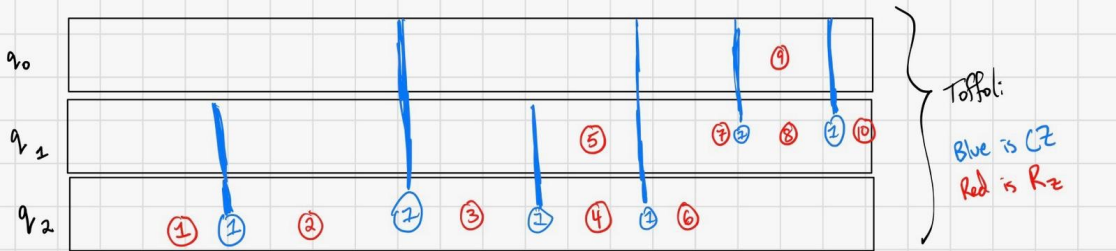
IGM	7M
-----	----

10 L 10 L

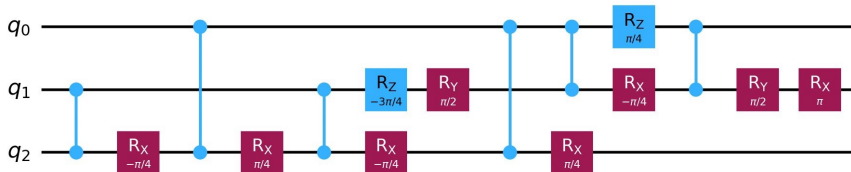
Steps

1	Move (q_1, q_2) to hot	(2 moves)
2	Move q_1 to storage	(1 move)
3	Move q_0 to hot (next to q_2)	(1 move)
4	Apply LRx to q_2	(1L)
5	Apply C2	(1C2)
6	Apply LRx to q_2	(1L)
7	Move q_0 to storage	(1M)
8	Move q_1 to hot, next to q_2	(1M)
9	Apply C2	(C2)
10	Apply LR2 LRy q_1 and LRx q_2	(3L)
11	Move q_1 to stor	(1M)
12	Move q_0 to hot	(1M)
13	C2	(C2)
14	Move q_2 to stor	
15	Move q_1 to hot	
	Apply C2	
	App LRx q_2	

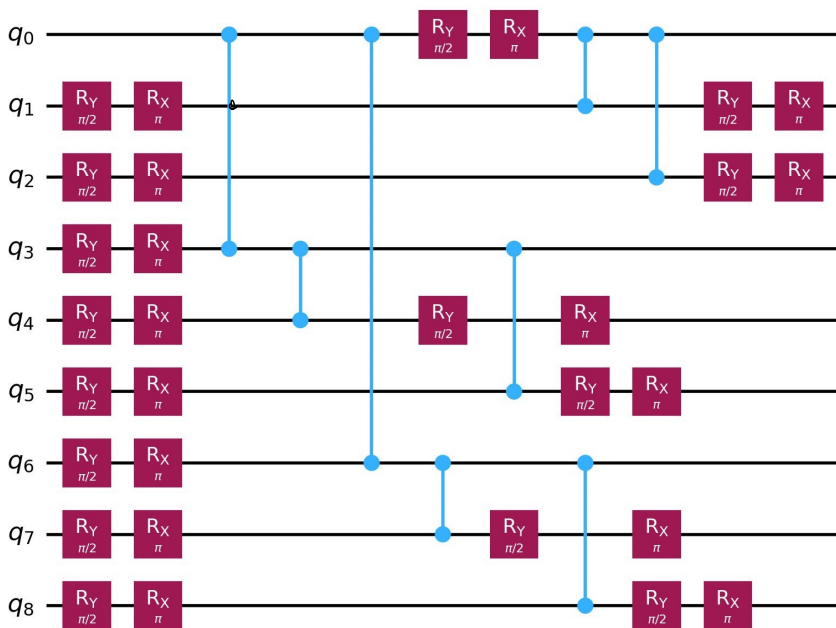




Global Phase: $\pi/8$



Global Phase: $3\pi/2$



Steps: $GRY[q_1 - q_8]$
 $LR_i q_0$

$GRX[q_1 - q_8]$
 $LR'_i q_0$

Step $q_0, q_3, q_4, q_5, q_6, q_7$ into mem

2 q_0, q_3 q_4 q_5 q_6 q_7 (GL, 1M)
1 2 3 4 5

3 C2 [q_0, q_3] (C2)

4 $q_3 \rightarrow$ slot 2 (2M)

q_0 q_4, q_3 q_5 q_6 q_7 (C2)
1 2 3 4 5

5 C2 (C2)

6 $q_3 \rightarrow$ slot 3 and $q_0 \rightarrow$ slot 4 (1M, 2T)

q_4 q_3, q_5 q_0, q_6 q_7
1 2 3 4 5

7 LRU [q_4] (LR)

8 C2 (C2)

9 LRU q_0 (LR)

10 Move q_6 to slot 5 and $q_3 \rightarrow$ slot 4 and $q_1 \rightarrow$ slot 1 (2M, 3T)

q_0, q_1 q_4 q_5 q_7, q_6
1 2 3 4 5

11 C2 (C2)

12 $R_y [q_7], R_x [q_4], R_y [q_5], R_x [q_3]$

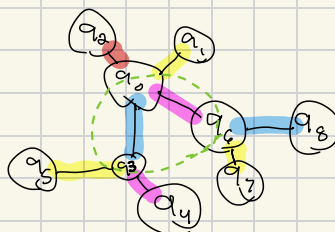
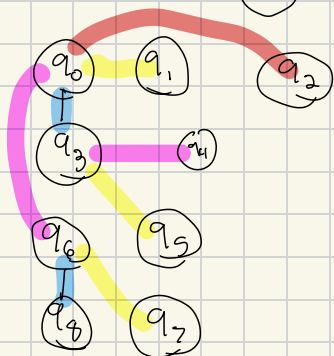
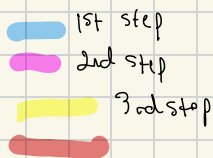
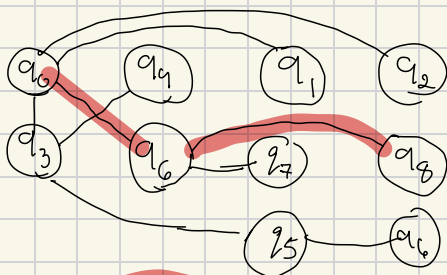
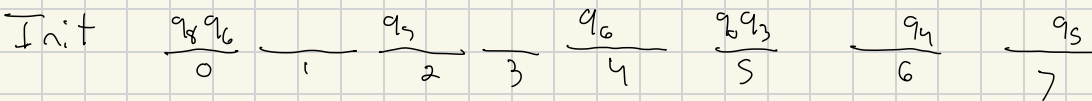
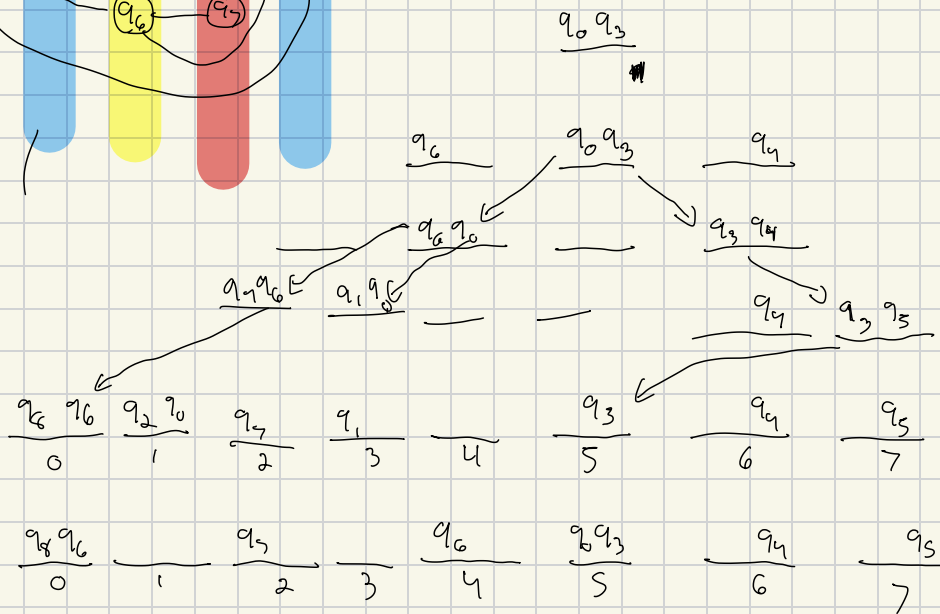
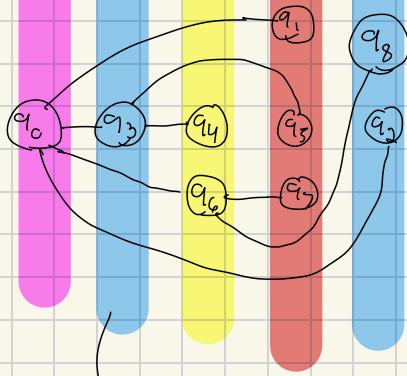
13 Move q_7, q_4 to slot 5 (1M, 2T)

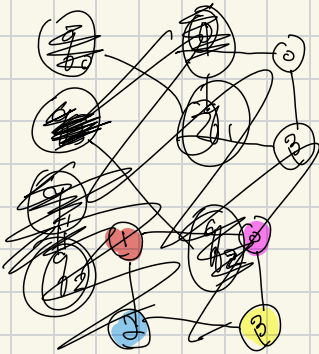
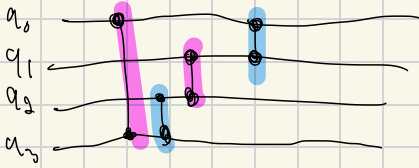
13 and $q_2 \rightarrow$ slot 1, and $q_6 \rightarrow$ slot 5 (1M, 2T)

14 C2 (C2)

15 $R_y R_x [q_1], R_y R_x [q_2], R_x [q_7], R_y R_x [q_8]$

q_0, q_2 q_4 q_5 0 q_6, q_8 (7LR)



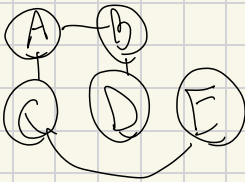


$n=4$

$color = [-1, -1, -1, -1]$

$sortVert = [0, 1, 2, 3]$

$avail = [True, True, True, True]$



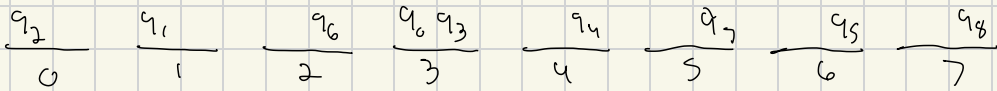
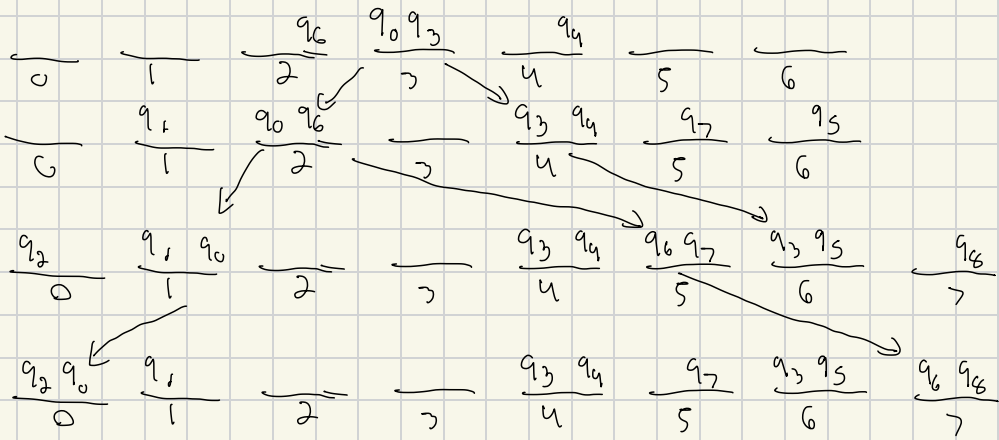
$A \rightarrow 0$

$B \rightarrow 1$

$C \rightarrow 1$

$D \rightarrow 0$

$E \rightarrow 0$



Step

1

Init

2 L PhX₂(a=.5, x=.5, z=0)[q₃, q₄, q₆]

3 CZ

4 L PhX₂(a=0, x=0, z=1)[q₃]

5 q₀ → slot 2, q₃ → slot 4

6 CZ

7 PhX₂(a=0.5, x=.5, z=1)[0]

8 PhX₂(a=0, x=0, z=1)[6]

9 PhX₂(a=.5, x=.5, z=0)[1, 5, 7]

10 q₀ → slot 2, q₃ → slot 5, q₆ → slot 6

11 CZ

12 Global PhX₂(a=.5, x=.5, z=0)

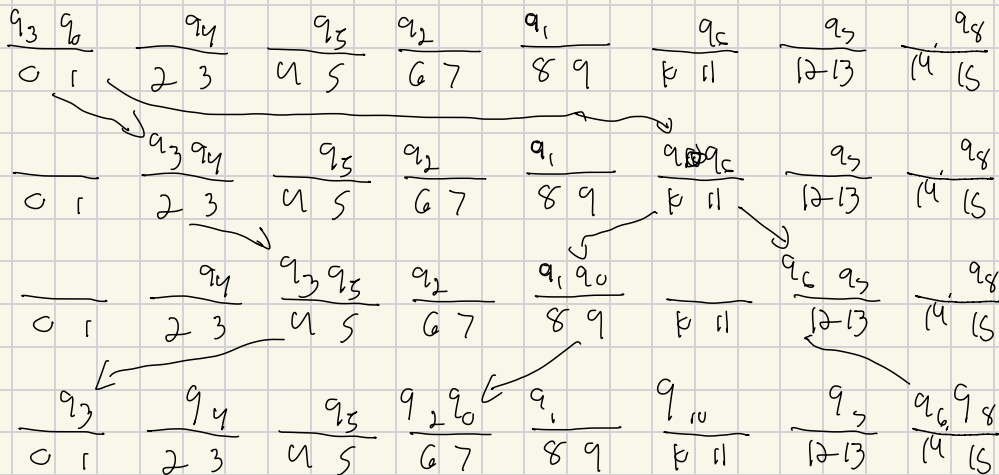
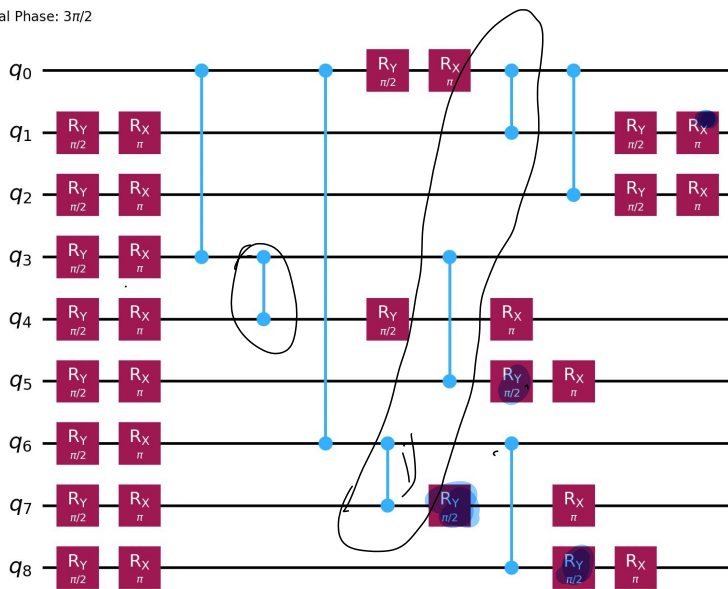
13 PhX₂(a=.5, x=.5, z=0)[0, 3, 6]

14 q₀ → slot 0, q₆ → slot 7

15 CZ

16 PhX₂(a=.5, x=.5, z=0)[2, 6]

Global Phase: $3\pi/2$



	$q_3 q_4$	q_5	q_2	q_1	$q_0 q_6$	q_7	q_8	
0 1	2 3	4 5	6 7	8 9	10	11 12	13 14	15 16
	q_4	$q_3 q_5$	q_2	$q_1 q_6$		q_6	$q_7 q_8$	q_8
0 1	2 3	4 5	6 7	8 9	10	11 12	13 14	15 16
	q_4	$q_3 q_5$	$q_2 q_6$	q_1		q_6	q_7	$q_8 q_6$
0 1	2 3	4 5	6 7	8 9	10	11 12	13 14	15 16

GRY($\pi/2$)

GRX(π)

LRV($-\pi/2$) [q_6]

LRX($-\pi$) [q_0]

CZ

$q_3 \rightarrow s_2$, $q_0 \rightarrow s_{12}$

CZ

$q_3 \rightarrow s_4$	$q_4 q_6 \rightarrow s_5 q_4$	q_6						
$q_0 \rightarrow s_7$	3	4	5	6	7	8	9	
	$q_4 q_3$		$q_6 q_6$					
1	2	3	4	5	6	7	8	9
	$q_3 q_3$	q_4		q_6	$q_6 q_7$			
1	2	3	4	5	6	7	8	9

1 GRV GRX

2 LRV [q_0] LRV [q_0]

3

$q_0 q_3$

4 CZ

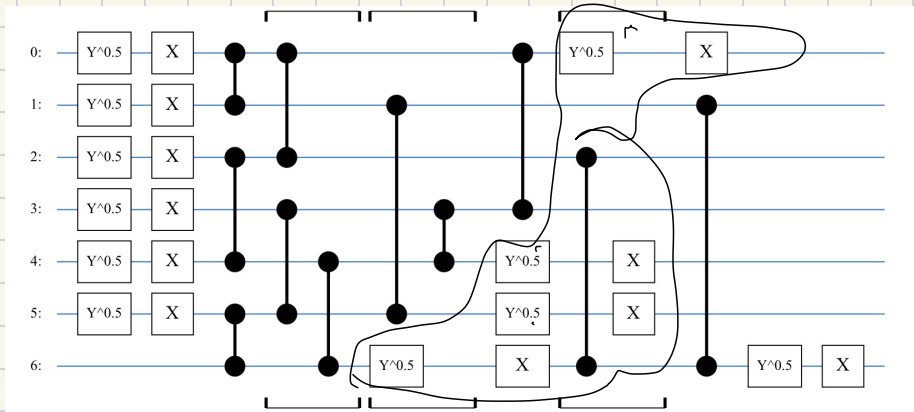
5 $q_3 \rightarrow s_3$, $q_0 \rightarrow s_5$

6 CZ

7 $q_3 \rightarrow s_2$, $q_6 \rightarrow s_6$

Index	0	1	2	3	4	5
1	a_2		a_3	$a_0 a_1$	✓	
2	a_2	$a_0 a_3$		a_1	✓	
3	$a_2 a_0$	a_3		a_1	✓	
4	a_0	a_3	$a_2 a_1$		✓	
5	a_0	$a_1 a_3$	a_2		✓	
6	$a_1 a_0$		$a_2 a_3$		✓	
7	a_1	$a_0 a_3$	a_2		✓	
8	a_1	a_3	$a_2 a_0$		✓	
9	$a_1 a_2$	a_3	a_0		✓	
10	a_2	$a_1 a_3$	a_0		✓	
11	$a_3 a_2$	a_1	a_0		✓	

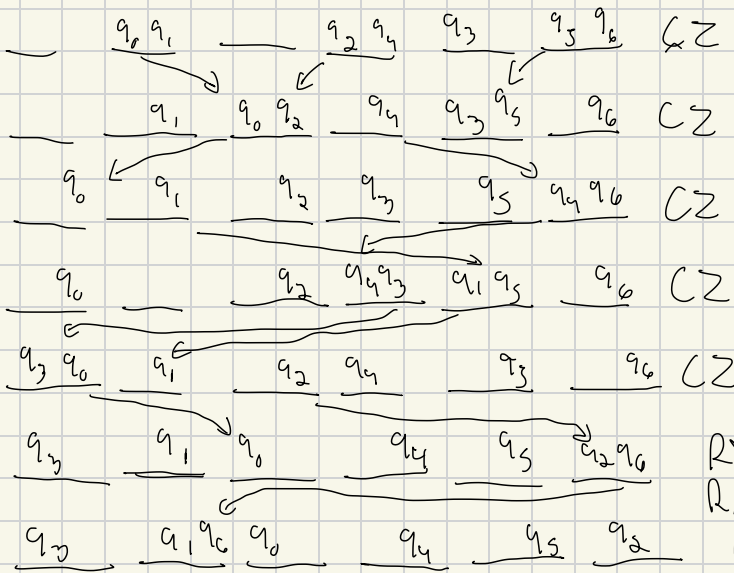
Index	0	1	2	3	4	5
1	a_2		a_3	$a_0 a_1$	✓	
2	a_2	$a_0 a_3$		a_1	✓	
3	$a_2 a_0$	a_3		a_1	✓	
4	a_0	a_3	$a_2 a_1$		✓	
5	a_0	$a_1 a_3$	a_2		✓	
6	$a_1 a_0$		$a_2 a_3$		✓	
7	a_1	$a_0 a_3$	a_2		✓	
8	a_1	a_3	$a_2 a_0$		✓	
9	$a_1 a_2$	a_3	a_0		✓	
10	a_2	$a_1 a_3$	a_0		✓	
11	$a_3 a_2$	a_1	a_0		✓	



GRV(.5)

GRX

GRX⁻¹ GRV⁻¹



GRV(.5)[q6, q5, q4, q3, q2, q1]

GRX[q6, q5, q4, q3, q2, q1] CZ

CZ

GRV(.5)[q6]

GRX[q6]