

## Laplace Transformation

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt \quad (1)$$

There will be a formula sheet on the final for these formulae:

$$\mathcal{L}\{1\} = \frac{1}{s} \quad (2)$$

$$\mathcal{L}\{t\} = \frac{1}{s^2} \quad (3)$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \quad (4)$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \quad (5)$$

$$\mathcal{L}\{\sin(kt)\} = \frac{k}{s^2 + k^2} \quad (6)$$

$$\mathcal{L}\{\cos(kt)\} = \frac{s}{s^2 + k^2} \quad (7)$$

Not every function has a Laplace Transform.

**Definition:** A function is said to be of exponential order  $c$  if there exist constants  $c$ ,  $M > 0$ , and  $T > 0$ , such that  $|f(t)| \leq Me^{ct}$ .

**Theorem:** If  $f(t)$  is piecewise continuous on  $[0, \infty)$  and of exponential order  $c$  for  $t \geq T$ , then  $\mathcal{L}\{f(t)\}$  exists for  $s > c$ .

$$\mathcal{L}\{a + b + c\} = \mathcal{L}\{a\} + \mathcal{L}\{b\} + \mathcal{L}\{c\} \quad (8)$$