Introduction To Programming: Tutorial 5 Solutions

Liam Burke
School Of Mathematics, Trinity College Dublin

November 5, 2022

(a)

$$T_m = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & \ddots & \ddots & \ddots \\ & & & -1 & 2 \end{bmatrix}$$

$$A = T_m \otimes I_m + I_m \otimes T_m$$

$$= \begin{bmatrix} 2I_m & -I_m & & & \\ -I_m & 2I_m & -I_m & & \\ & -I_m & \ddots & \ddots & \\ & & \ddots & \ddots & -I_m \\ & & & -I_m & 2I_m \end{bmatrix} + \begin{bmatrix} T_m & & & \\ & T_m & & \\ & & \ddots & \\ & & & T_m \end{bmatrix}$$

$$= \begin{bmatrix} T_m + 2I_m & -I_m & & \\ -I_m & T_m + 2I_m & -I_m & \\ & -I_m & \ddots & -I_m \\ & & -I_m & T_m + 2I_m \end{bmatrix} = P_m$$

(b)

$$v = Ab = \begin{bmatrix} T_m + 2I_m & -I_m & & & & \\ -I_m & T_m + 2I_m & -I_m & & & \\ & -I_m & T_m + 2I_m & -I_m & & \\ & & \ddots & \ddots & \ddots & \\ & & & & & -I_m \\ & & & & -I_m & T_m + 2I_m \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$= \begin{bmatrix} (T_m + 2I_m)b_1 - b_2 \\ -b_1 + (T_m + 2I_m)b_2 - b_3 \\ -b_2 + (T_m + 2I_m)b_3 - b_4 \\ \vdots \\ -b_{i-1} + (T_m + 2I_m)b_i - b_{i+4} \\ \vdots \\ -b_m + (T_m + 2I_m)b_m \end{bmatrix} \text{row } i$$