MAU11601:Introduction To Programming - Tutorial 6
Gaussian Elimination

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Question 1

NOTE: Part (c) of this question can be done simultaneously with parts (a) and/or (b) to help debug your code.

- (a) Write a MATLAB function

  `function [A,b] = gaussian_elimination(A,b)`

  which takes in an $n \times n$ matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ and a vector $\mathbf{b} \in \mathbb{R}^n$ and performs Gaussian elimination on the augmented matrix $\begin{bmatrix}\mathbf{A} & \mathbf{b}\end{bmatrix}$. The relevant row operations can be done on both the matrix $\mathbf{A}$ and $\mathbf{b}$ separately without explicitly constructing the augmented matrix. The function should overwrite the matrix $\mathbf{A}$ with an upper triangular matrix and should also overwrite the vector $\mathbf{b}$. The function should return $\mathbf{A}$ and $\mathbf{b}$.

  Perform an error check to ensure the matrix input is square.

- (b) Write another MATLAB function

  `function [x] = back_sub(U,v)`

  which takes in an $n \times n$ upper triangular matrix $\mathbf{U}$ and a vector $\mathbf{v} \in \mathbb{R}^n$ and solves the linear system $\mathbf{Ux} = \mathbf{v}$ using backward substitution. The function should return the solution in the vector $\mathbf{x} \in \mathbb{R}^n$.

- (c) Create a run script *run.m* which solves the linear system

$$\begin{aligned} 2x_1 - 3x_2 - x_3 &= 7 \\ 3x_1 + 5x_2 - 3x_3 &= -2 \\ 4x_1 - x_2 + 2x_3 &= 17 \end{aligned}$$

  using a function call to both functions created in (a) and (b).

  You should get as a solution $x_1 = 3, x_2 = -1$ and $x_3 = 2$.