2022 Exam - Techniques in Therefol Physics Liam Burke (burkel 8 Ptcd.ie)  $Q200 \times (X-1)y^{+} + 3xy^{+} + y = 0$ Charse Same paint to to expond Salution around. X0 =0 4'' + 3x + 9 = 0X (X-1)  $p(x) = 3 \qquad q(x) = 1$   $\sqrt{X} \qquad \qquad X(x)$ y Non onalytic @ X= 9 X (X-1) X = 0 is Singular point Analytic @ X.=0  $X p(X) = \frac{3X}{X-1}$  -p Analytic  $e^{-X_0} = 0$  $\chi^2 q(x) = \frac{x}{x-1}$   $\rightarrow$  Analytic Q(x) = 0Xa = 0 is a regular Singular point so we con assume a somtion of the form  $g(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$  Such that  $y'(x) = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1}$   $y'(x) = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-2}$   $y''(x) = \sum_{n=0}^{\infty} a_n (n+r) (n+r-1) x^{n+r-2}$   $xy'' = \sum_{n=0}^{\infty} a_n (n+r) (n+r-1) x^{n+r-1}$   $xy'' = \sum_{n=0}^{\infty} a_n (n+r) (n+r-1) x^{n+r-1}$ 

$$\sum_{n=0}^{\infty} G_{n}(n+r)(n+r-1) \times^{n+r} - \sum_{n=0}^{\infty} G_{n}(n+r)(n+r-1) \times^{n+r-1} \times^{n$$

r=0 r=1

$$Q_{n+1} = Q_n \left[ (n+r)(n+r-1) + 3(n+r) + 1 \right]$$

$$(n+r+1)(n+r)$$

$$C = 0$$
  $C = 0$   $C = 0$   $C = 0$   $C = 0$ 

$$Q_{r=1}$$

$$Q_{n+1} = a_n [(n+1)n + 3(n+1) + 1]$$

$$(n+2)(n+1)$$

$$a_2 = a_1 \left[ \frac{2 + 3(2) + 1}{3(2)} \right] = \frac{9}{6} a_1 = 3a_9$$

$$49 \quad y(x) = x = x \quad (1 + 2x + 3x^{7} + --) = x \\ y(x) = x = 0 \quad (1-x)^{2}$$

Construct a Second Solution

Rook for a second Solution ya of the form

$$y_{\alpha}(x) = y(x)v(x) = \frac{x}{(1-x)^2}v(x)$$

o Sub ya into ODE

· Fist compute ya = y'v + y v'

y2" = yav+y'v + y'v' + y v''

x(x-1)[y''v+2y'v'+yv'']+3x(y'v+yv')+yv=0

X(X-1)y''v + 3xy'v + yv + X(X-1)[2y'v' + yv'']+ 3xyv' = 0

 $(x-1)^{2}$   $(x-1)^{2}$ 

$$x(x-1)2y'v' + x(x-1)yv'' + 3xyv' = 0$$

$$v'' + 29v' - 3v' = 0$$

$$ln(y) = ln\left(\frac{x}{(1-x)^2}\right) = ln(x) - 2ln(1-x)$$

$$d ln(y) = ln\left(\frac{x}{(1-x)^2}\right) = ln(x) - 2ln(1-x)$$

$$Qn(y) = \frac{1}{x} + \frac{2}{(1-x)} = \frac{1}{x} + \frac{2}{(1-x)}$$

$$d_{N}(1-x) = \frac{1}{1-x}$$

$$y'' + \left(\frac{2}{x} + \frac{4}{(1-x)} - \frac{3}{(1-x)}\right)y' = 0$$

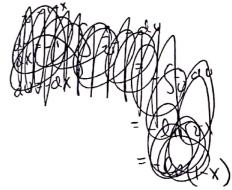
$$V'' + \left(\frac{2}{x} + \frac{1}{(1-x)}\right)^{-1} = 0$$

variables
$$\frac{dv'}{v} = -\left(\frac{z}{x} + \frac{1}{(1-x)}\right) dx$$

$$-200(x)+00(x-1)+0$$

$$ln(v') = -2ln(x) + ln(x-1) + Ci$$







$$\begin{array}{l}
\text{En} \text{ on } (x-1) + C_1 \\
\text{In } (x^{-2} + \ln(x-1) + C_1 \\
\text{In } (C) \\
\text{Some some cost}
\end{array}$$

So 
$$\overrightarrow{v} = C(x-1) = C - C \times 7^2$$
  
Integration  $\overrightarrow{v} = C \cdot (x-1) = C \times 7^2$   
 $\overrightarrow{v} = C \cdot (x-1) = C \times 7^2$   
Then  $y_a = y_v = x \times 7^2$   $C \cdot (2n(x) + \frac{1}{x}) + C_2$ 

Integration 
$$\sqrt{x^2} \times x \times x^2$$

Integration  $\sqrt{x} = C \ln(x) + \frac{C}{x} + \frac{C}{x}$ 

Then  $\sqrt{y} = \sqrt{x} = \frac{x}{(1-x)^2} \left[ C \left( \ln(x) + \frac{1}{x} \right) + C_2 \right]$