Taylor Determinacy and Labor Supply

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Abstract

How important is labor supply for the ability of monetary policy to influence inflation and employment? Hiring costs alter the response of inflation to monetary policy. As shown in Kurozumi and Van Zandweghe (2010), adjustments in employment can make it difficult for monetary policy to reach its price stability and full employment targets. As the policy response is more vigorous in maintaining inflation around a target, that target becomes impossible to maintain. Recent fluctuations in the participation rate has lead to a growing concern about the role of labor supply in monetary policy. This paper shows that as labor supply becomes more elastic, the monetary authority is more likely to be able to stabilize the economy around its steady state targets. The Central bank response to cyclical unemployment is important for price level stability regardless of business cycle goals.

1 Introduction

Understanding unemployment and inflation are central topics in economics. The Federal Reserve has a dual mandate of ensuring price stability and near full employment. The Philips curve, the idea unemployment and inflation are inversely related over short time horizons, has historically been a key concern for central banks. Monetary authorities around the world are given a mission to prevent large changes in the overall price level of the economy. Concerns about a Philips curve cause central banks to face a dilemma balancing price stability goals with concerns about short-term labor market impacts. Most central banks accomplish this goal by picking a target for average price growth. The Phillips curve suggests that if a central bank takes action to contain inflation from rising above its target, then it should it expect those actions to cause a temporary increase in unemployment. To accomplish a reduction in inflation the central bank reduces the supply of money, it does this by selling government debt and calibrates this sale by trying to hit a target interest rate for that debt on the market. The Taylor rule seeks to quantify empirically how central banks have historically balanced the apparent trade-off embodied in the Phillips curve in choosing these interest rate targets.

In many New Keynesian macroeconomic models[1], a Taylor rule is used to model how a monetary authority will respond to changes in underlying macroeconomic variables. A common approach, that is also used in this paper, is to model the central bank as directly setting the interest rate based on how large deviations of expected inflation and output are from a target inflation level and steady state output. A key component in these New Keynesian models is that prices don't adjust freely to clear markets. Instead, most use Calvo pricing that has firms with market power that face a constant probability of being unable to adjust prices every quarter. A Phillips curve relation in these models arise from how changes in the decisions of agents in the model impact the setting of prices in those firms. The Taylor rule then closes the model by setting interest rates to influence the price equilibrium in the economy.[2] In many of these models the main channels from interest rate changes to price changes are a demand channel and an investment channel.

[1] The cashless New Keynesian model used in this paper is fairly typical. A good introduction of models of this type is Galí (2015)

[2] These new Keynesian models generally have multiple equilibria and the Taylor rule is used as an equilibrium selection criterion. See Cochrane (2011)

In the demand channel, an increase in interest rates causes increased savings demand and the resulting reduced aggregate demand causes downward pressure on prices. In the investment channel rising interest rates increase borrowing costs and therefore lower firm investment.

An important question for central banks is how responsive they need to be to deviations in inflation from their target rate in order to ensure they are able to get the economy to stay on the path of stable price growth. Stability around the steady state of the model based on choices of the Taylor rule parameters is called Taylor determinacy. In the baseline New Keynesian model, the model is generally determined if the central bank responds to deviations in inflation by increasing the interest rate more than one to one.

Kurozumi and Van Zandweghe (2010) adds a frictional labor market to the standard new Keynesian model. The labor market in Kurozumi and Van Zandweghe (2010) uses a Diamond, Mortenson, Pissarides matching function.[3] This matching function rations the number of new employment relationships that can be formed each period based on labor supply and labor demand. The matching function itself is an ad-hoc Cobb Douglas function that takes the number of job postings and the number of job seekers as inputs to determine how many new employment relationships are formed each period. The idea behind these models is that in the labor market it takes time to find a match and there are transaction costs involved that make labor market adjustments sluggish. The matching function conveniently generates a Beveridge curve, the ratio of unemployment to job vacancies, and captures intuitive notions search costs in a labor market.[4] Because of the search cost caused by matching in the labor market, firms face adjustment costs. These adjustment costs cause instability around the steady state for various choices of parameters in a Taylor rule. If monetary policy causes a decrease in employment today, next period there is more costly hiring for firms since they have to hire more to recover to their long run employment level. Kurozumi and Van Zandweghe (2010) find calls this novel channel, the 'Vacancy channel'. When interest rates increase, aggregate demand falls and unemployment increases. When the economy recovers, firms anticipate large hiring costs. This increase in future firm costs causes cost push inflation as firms raise prices today in order to cover those anticipated

[3]See Pissarides (2000) for an introduction.

[4]There is a large literature about the labor market with matching frictions. A good introduction is Pissarides (2000), Petrongolo and Pissarides (2001) is a good overview of the general literature and Yashiv (2007) is a good overview of the empirical literature.

hiring costs in the future. This can cause Taylor indeterminacy if labor adjustment is slow relative to changes in consumption or if the central bank is very aggressive to inflation rate deviations relative to unemployment.

Is labor supply important for business cycles and monetary policy? Since the 2009 recession, a new focus on labor supply shows that it can be important. Erceg and Levin (2014) finds that in the 2009 recession, fifty percent of the decline in the employment to population ratio was due to the fall in the participation rate. Their model has an adjustment cost for labor supply and households that don't doesn't have matching frictions in the labor market. This allows their model to match.Elsby et al. (2009) show that accounting for changes in participation is important to understanding unemployment dynamics. The literature on participation decisions in macro-models with matching has been growing in recent years including Furlanetto and Groshenny (2016) and Campolmi and Gnocchi (2016).

The key innovation in this paper is to consider how labor supply responses effect Taylor determinacy. In Kurozumi and Van Zandweghe (2010), labor is supplied inelastically. If participation changes as a result of changing labor market conditions, then the hiring costs of firms could be different than predicted in Kurozumi and Van Zandweghe (2010). Additionally Kurozumi and Van Zandweghe (2010) find that the as consumption is more variable relative to employment changes the effect is strengthened. To address this a model similar to Galí (2010) is used. The model has Calvo price setting, labor market matching with Nash bargaining, households which allocate members between employment and out of the labor force, and a central bank that operates according to a Taylor rule. Because of the matching in the labor market there is unemployment in the model for unmatched workers looking for work.

The findings in this paper show that relative to a setting with inelastic labor supply, the inflationary response from an increase in interest rates from the vacancy channel is reduced. As the disutility function becomes less convex and participation more elastic, wages are more volatile. With the option to leave the labor force, temporarily low wages create a discouraged worker effect like in Lucas and Rapping (1969).

Since wages don't move as much with participation rate changes, there is relatively less hiring cost pressure to firms adjusting back to the full employment level. Despite the reduction in labor supply, job finding rates still fall lowering the bargaining position of households, real wages fall. Taken together, this causes the choices of parameters that ensure determinacy to increase. In particular, high responsiveness to inflation deviations with lower responsiveness unemployment is less likely to cause instability around the steady state when accounting for participation changes. The results are similar when changing the disutility of unemployment. As unemployment becomes more costly, with household wage bargaining power increased to hit similar steady values, then wages and participation fall by more from an increase in interest rates and the the effect of the vacancy channel in Kurozumi and Van Zandweghe (2010) is mitigated. Similar to the effects in the model from increased labor supply elasticity. In both cases only responding to expected deviations in inflation almost guarantee Taylor indeterminacy.

2 Model

The model is inspired by the representative household design from Merz (1995) and Andolfatto (1996). The model is inhabited by three kinds of decision makers, each a unit mass of infinitely lived agents. They consist of households, wholesale firms, and Calvo firms. There is also a government that conducts monetary policy specified by a Taylor rule. The model has discrete timing with each period indexed by $t = 0, 1, 2, ..., \infty$.

Households make decisions about how their unit mass of members are allocated between looking for work, working, or in leisure. After the labor market clears, households purchase consumption goods from each of the unique Calvo firms and trade one period bonds with other households.

Wholesale firms produce wholesale goods depending on the number of workers they employ. New workers are hired based on responses to job postings.

There are two kinds of firms in the model, Calvo firms and wholesale firms. Calvo firms have a competitive monopoly buying wholesale goods and then reselling them as a differentiated good indexed by j. Calvo firms, as their name implies, face Calvo (1983) style price frictions. Calvo firms do not hire labor, their only input in production is the generic wholesale good produced by wholesale firms.

Wholesale firms produce their generic good using labor as the sole input. The labor market has Diamond, Mortensen, and Pissarides style matching frictions. The number matches is determined using a Cobb Douglas function that takes the number of vacancies posted by wholesale firms and the number of searching members from households to give the number of new hires each period. The wage for all workers is determined by Nash bargaining between firms and households on the new hires. There are no capital goods in the model.

The timing of the model is as follows: At the beginning of the period a δ share of all employment relationships end. Then agents make their decisions–households decide how to allocate their time between search and leisure and how many bonds to purchase. Wholesale firms determine how many vacancies are posted. The labor market resolves and wages are settled by Nash bargaining between households and wholesale firms. Calvo firms that are able, change their price, the other Calvo firms keep the price they had last period. The Calvo then repackage wholesale goods to meet demand for their individuals from households.

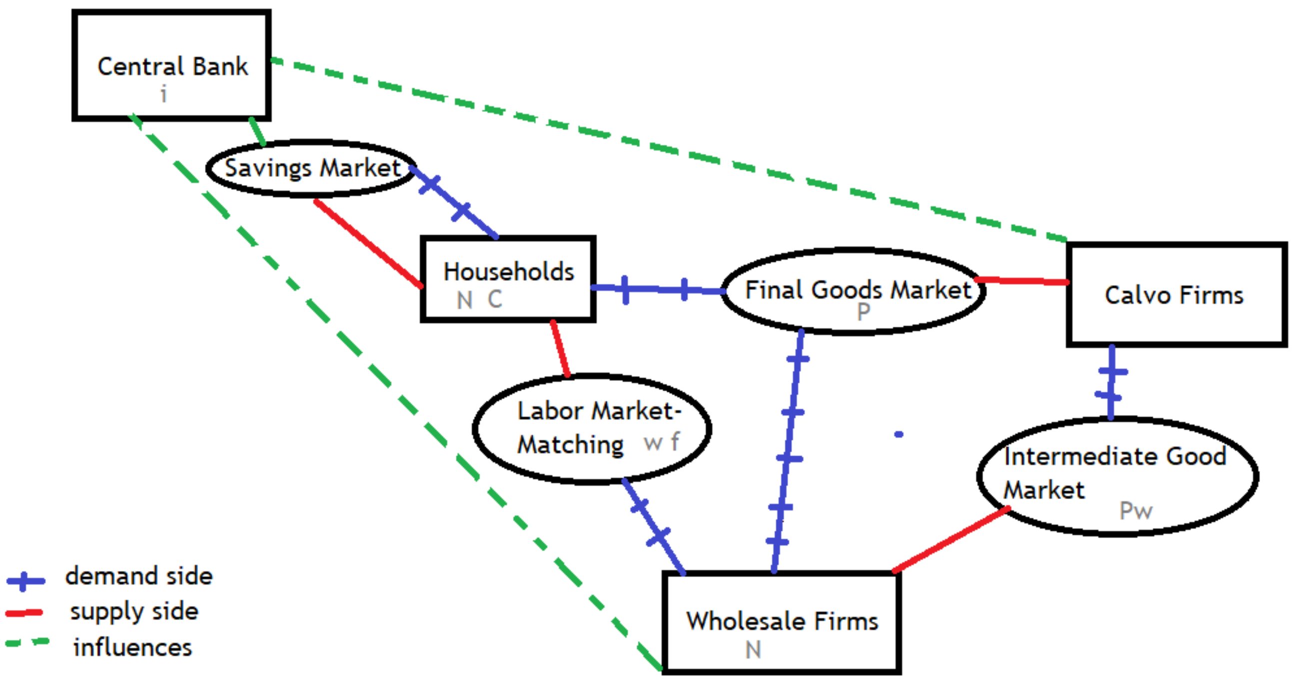

Figure 1: The Agents in the model and their associated markets

2.1 Households

Each household contains a unit mass of members and consumption is shared between members. Each household is atomistic and take as given the current nominal interest rate i_t, real wage w_t, the dividend payment D_t, and the job finding rate f_t

Each household buys goods from each Calvo firm, with a constant elasticity of substitution of ϵ. The number goods demanded from firm j $c_t(j)$ at price $P_t(j)$ is given by $c_t(j) = (\frac{P_t(j)}{P_t})^{\epsilon} C_t$ with $C_t = (c_t(j)^{\frac{1+\epsilon}{\epsilon}})^{\frac{\epsilon}{1+\epsilon}}$ and $P_t = [\int_0^1 P_t(j)^{1+\epsilon}]^{\frac{1}{1+\epsilon}}$.

The household uses labor income from N_t workers with the negotiated wage w_t to buy 1 period bonds from other households which pay a return of $(1 + i_t)$ in the next period and to pay for their consumption of final goods C_t. The inflation rate π_t is defined as $1 + \pi_t \equiv P_t/P_{t-1}$ and define the real rate of interest using the Fischer rule as $R_{t+1} = \frac{1+i_t}{1+\pi_{t+1}}$. The budget constraint in each period written in real terms is therefore given by $C_t = R_{t-1}B_{t-1} - B_t + w_t N_t + D_t$.

Given the job finding rate, the household sends S_t members to look for work in order to reach the target employment level for this period N_t. For a given target N_t, the household sends $S_t = (1/f_t)(N_t - (1-\delta)N_{t-1})$ members to look for work. Since households are large and contain an infinite number of members, a law of large applies and there is no uncertainty about the resulting level of employment for households. The members who do search and fail become unemployed: $U_t = (1 - f_t)S_t = (1 - f_t)/f_t(N_t - (1 - \delta)N_{t-1})$.

The household has a forward looking, additively separable utility function that is a sum of the individual period consumption utility $u(C_t)$ less disutility from time spent in unemployment or employment given by the disutility function $\Phi(L)$. The composite effort variable is defined as $L_t \equiv N_t + U_t = (1 - f_t)/f_t N_t - (1 - \delta)(1 - f_t)/f_t N_{t-1}$. The effort variable weights the relative disutilities of effort in employment and unemployment into a single measure. Households have a discount rate of β. Each individual period utility is monotonic, strictly concave, and each satisfy an Inada condition. Specifically I assume $u'(x) > 0$, $\Phi'(x) > 0$ $u''(x) < 0$, $\Phi''(x) < 0$, and $\lim_{x\to 0} u'(x) = \infty$.

Writing the problem recursively and letting Z_t be a vector of the prices $\pi_t, i_t, w_t, D_t, P_t^w, f_t$, households solve equation 5.

$$W(N_{t-1}, B_{t-1}, Z_t) = \max_{N_t \in [0,1], B_t, C_t \geq 0} \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\phi}}{1+\phi} + \beta E_t[W(N_t, B_t, Z_{t+1})] \tag{1}$$

Subject to

$$C_t = R_{t-1}B_{t-1} - B_t + w_t N_t + D_t \tag{2}$$

2.1.1 Household first order conditions

$$C_t^{-\sigma} = \beta E_t[\frac{1+i_t}{1+\pi_{t+1}} C_{t+1}^{-\sigma}]]$$

$$w_t = (\chi + \chi\frac{1-f_t}{f_t})\frac{L_t^\phi}{C_t^{-\sigma}} - \chi(1-\delta)E_t[\beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{1-f_{t+1}}{f_{t+1}}\frac{L_{t+1}^\phi}{C_{t+1}^{-\sigma}}]$$

Since it follows a geometric distribution, the fraction $\frac{1-f}{f}$ is the average number of unemployed people per new hire. Since it appears frequently, μ_t will be defined as $\mu_t = \frac{1-f_t}{f_t}$. $\chi\frac{L_t^\phi}{C_t^{-\sigma}}$ is the marginal disutility of effort for newly employed people $\chi\frac{1-f}{f}\frac{L_t^\phi}{C_t^{-\sigma}}$ is the marginal disutility for unemployed times the number of unemployed required per new hire. $\chi(1-\delta)E_t[\beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{1-f_{t+1}}{f_{t+1}}\frac{L_{t+1}^\phi}{C_{t+1}^{-\sigma}}$ accounts for the effort savings of having to on the margin hire $(1-\delta)$ less people next period with the associated marginal disutility for unemployed per new hire required next period. As another shorthand, the modified stochastic discount factor $\hat{\beta}_{t+1}$ is defined as the normal stochastic discount factor using the intertemporal rate of substitution times the probability of remaining employed in the next period, namely: $\hat{\beta}_{t+1} = (1-\delta)\beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$.

2.2 Wholesale Firms

There is a unit mass of wholesale firms in the model. They produce wholesale goods which are sold to the Calvo firms at the relative price P_t^w. Each firm is large and employs a unit mass of workers. Of the N_{t-1} workers employed at the firm each period, the firm only retains $(1-\delta)N_{t-1}$ next period. To employ more workers, wholesale firms must post vacancies to match with prospective employees. For each vacancy that firms decide to post they must pay a real cost γ per vacancy posted. As discussed in the matching function section, this can be rewritten as the cost $\Gamma f_t^{\frac{1-\omega}{\omega}} H_t$ where $\Gamma = \gamma M^{(-1-\omega)/\omega}$ is the adjusted real cost , $f_t^{\frac{1-\omega}{\omega}}$ is proportional to the number of vacancies posted, and $H_t = N_t - (1-\delta)N_{t-1}$ is the number of new hires. Firms are each large enough that a law of large numbers in hires applies and each must post vacancies of $V_t = H_t/q_t$ to reach a target employment level N_t. The number of hires firms need each period is $H_t = N_t - (1-\delta)N_{t-1}$. The prevailing wage rate is set through Nash bargaining between households and wholesale firms. Firms take wages and prices as given when making vacancy posting decisions. Firms have decreasing returns to scale.

Wholesale firms pay dividends to households and have a discount rate $\tilde{\beta}_{t+1} = \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$.

$$\Pi_t(N_{t-1}, Z_t) = \max_{N_t} P_t^w A_t N_t^{1-\alpha} - w_t N_t - \Gamma f_t^{\frac{1-\omega}{\omega}} H_t + E_t[\tilde{\beta}_{t+1} \Pi_{t+1}(N_t, Z_{t+!})] \tag{3}$$

Labor demand is given by the following first order condition for Wholesale Firms in equation 8 .

$$w_t = (1-\alpha) P_t^w A_t N_t^{-\alpha} - \Gamma f_t^{\frac{1-\omega}{\omega}} + E_t[\tilde{\beta}_{t+1}(1-\delta)\Gamma f_{t+1}^{\frac{1-\omega}{\omega}}] \tag{4}$$

Relative to the standard labor demand functions in simple macroeconomic models, the new terms here are the hiring cost smoothing terms related to the job finding rate. The firm is forward looking and may push hiring into the future if hiring is realatively costly today and vice versa.

2.3 Matching

The number of hires in the labor market is rationed by a Cobb-Douglas function that takes the number of vacancies V_t posted by wholesale firms and the number of people S_t looking for work as inputs. The matching function has an exponent of ω on vacancies and $1-\omega$ exponent on the number of people searching for work.

$$H_t = M V_t^{\omega} S_t^{1-\omega}$$

The job finding rate, f_t, is defined as the number of hires per searching worker and tightness Θ_t is the number of vacancies per searcher.

$$f_t \equiv \frac{H_t}{S_t} = M(\frac{V_t}{S_t})^{\omega} = M\Theta_t^{\omega}$$

Likewise the vacancy filling rate is defined as hires per vacancy posted.

$$q_t = \frac{H_t}{V_t} = \frac{f_t}{\Theta_t} = M\Theta_t^{\omega-1}$$

Wholesale firms pay a real cost γ for each vacancy posted, for the rest of the paper this cost function will be given in terms of the job finding rate as follows.

$$\gamma V_t = \frac{\gamma H_t}{q_t} = \gamma M^{-1} \Theta_t^{1-\omega} H_t$$

Then using $\Theta = M^{-1/\omega} f_t^{1/\omega}$ and defining $\Gamma = \gamma M^{(-1-\omega)/\omega}$.

$$\gamma V_t = \Gamma f_t^{\frac{1-\omega}{\omega}} H_t$$

2.4 Nash Bargaining

Wages are set every period through Nash bargaining over the net surplus of a potential employment relation between households and firms labeled S_t^H and S_t^W respectively. If a match is found effort cost is sunk – matched individuals incur the effort cost of employed individuals regardless of the result of negotiation. The benefit of keeping a match is the additional labor income less the employed disutility and labor search savings next period. Altogether that implies that the net surplus generated from a match to households is given by equation:

$S_t^H = w_t C_t^{-\sigma} + \chi\beta(1-\delta)E_t[\frac{1-f_{t+1}}{f_{t+1}} L_{t+1}^{\phi}]$.

So at equilibrium, $S_t^H = \chi(1 + \frac{1-f_t}{f_t})L_t^{\phi}$

The benefit of employing an additional matched worker for wholesale firms is the marginal productivity increase minus the wage payment and search cost saving for next period. The first order condition of the firm implies this is equal to the sunk search cost paid in the current period.

$$S_t^W \equiv (1-\alpha)P_t^w N_t^{-\alpha} - w_t + E_t[\frac{1+\pi_{t+1}}{1+i_t}(1-\delta)\Gamma f_{t+1}^{\frac{1-\omega}{\omega}}] = \Gamma f_t^{\frac{1-\omega}{\omega}}$$

The Nash bargaining problem with bargaining power η is :

$$\max_w [S_t^H]^{\eta}[S_t^w]^{1-\eta}$$

2.4.1 Nash bargaining solution

$$\eta[S_t^H]^{-1}C_t^{-\sigma} = (1-\eta)[S_t^w]^{-1}$$

$$\Gamma f_t^{\frac{1-\omega}{\omega}} C_t^{-\sigma} = \frac{(1-\eta)}{\eta}(w_t C_t^{-\sigma} - \beta(1-\delta)\chi E_t[\frac{1-f_{t+1}}{f_{t+1}} L_{t+1}^{\phi}])$$

$$\frac{\eta}{1-\eta}\Gamma f_t^{\frac{1-\omega}{\omega}} = \chi(1 + \frac{1-f_t}{f_t})\frac{L_t^{\phi}}{C_t^{\sigma}} \tag{5}$$

2.5 Calvo firms

Calvo firms are monopolistically competitive, and buy wholesale goods at P_t^w and resell it as a differentiated product. The Calvo firms pay their profits as dividends to the household. Calvo firms discount the future at $\tilde{\beta}_{t+1} = \beta C_{t+1}^{-\sigma}/C_t^{-\sigma}$

These firms face Calvo style price frictions. Each period Calvo firms are allowed to reoptimize on price with probability $1 - \theta_P$, otherwise each firm remains at their price from last period.

To solve this the decision problem is written recursively with two value functions. J in the following equation is the expected profit to the firm when the firm is able to pick a price this period. $F(P_{t-1})$ is the expeted profit when the firm remains at the price from the previous period.

$$J = \max_{P_t(j)} (\frac{P_t(j)}{P_t} - P_t^w)(\frac{P_t(j)}{P_t})^{\epsilon} C_t + E_t[(1-\theta_p)J + \theta_p F(P_t(j))]$$
$$F(P_{t-1}(j)) = (\frac{P_{t-1}(j)}{P_t} - P_t^w)(\frac{P_{t-1}(j)}{P_t})^{\epsilon} C_t + E_t[\tilde{\beta}_{t+1}(1-\theta_p)J + \theta_p F(P_{t-1}(j))]$$

$$P_t(j) = \frac{\epsilon}{1+\epsilon} \frac{\sum_{s=0}^{\infty} (\tilde{\beta}_{t+s}\theta_p)^s P_{t+s}^w P_{t+s}^{-(1+\epsilon)} C_{t+s}}{\sum_{s=0}^{\infty} (\tilde{\beta}_{t+s}\theta_p)^s P_{t+s}^{-(1+\epsilon)} C_{t+s}}$$

Focusing on the symmetric equilibrium where all firms that choose new prices, pick the same re-optimization price P_t^* yields the aggregate inflation rate given by equation 11.

$$P_t = [(1-\theta_p)(P_t^*)^{1+\epsilon} + \theta_p P_{t-1}^{1+\epsilon}]^{\frac{1}{1+\epsilon}}$$
$$1 + \pi_t = [(1-\theta_p)(\frac{P_t^*}{P_{t-1}})^{1+\epsilon} + \theta_p]^{\frac{1}{1+\epsilon}} \tag{6}$$

The reset price inflation in equation 11 P_t^*/P_{t-1} can be written as the following set of recursive equations in equilibrium.

$$\frac{P_t^*}{P_{t-1}} = \frac{\epsilon}{1+\epsilon} \frac{g_t}{h_t}$$

where

$$g_t = (1+\pi_t)[\frac{P_t^w}{P_t} C_t + \theta_p E_t[\frac{P_{t+1}}{P_t(1+i_t)}(1+\pi_{t+1})^{-(1+\epsilon)} g_{t+1}]$$
$$h_t = C_t + \theta_p E_t[\frac{P_{t+1}}{P_t(1+i_t)}(1+\pi_{t+1})^{-(1+\epsilon)} h_{t+1}]$$

This inherently assumes $\theta_p \frac{1}{1+r} < 1$. In economic terms this implies that firms set price level according to a weighted time average of real marginal cost where the weights correspond to time discounting, the probability of maintaining that price for that period, quantity sold, and aggregate price level.

2.6 Taylor Rule

The government sets a nominal interest rate target to achieve its policy objectives given by a Taylor rule. Following Kurozumi and Van Zandweghe (2010) and evidence from Orphanides and Wieland (2008), the baseline Taylor rule in the model depends on expected inflation instead of the more common rules the use actual inflation. The Taylor rule also depends on past interest rates to smooth policy responses. Altogether, the Taylor rule weights are given by ϕ_R on the previous interest rate, a weight on expected inflation deviations of Φ_Π, and a weight on the employment level deviations of ϕ_U. There is also a stochastic monetary shock z_t that shifts the interest rate. The shock follows an AR(1) process, $z_t = \rho z_{t-1} + \zeta_t$ where ζ_t is an iid mean zero random variable.

$$1 + i_t = (1 + i_{t-1})^{\phi_R}[(1 + i)(\frac{E_t[\pi_{t+1}]}{\pi})^{\phi_\pi}(\frac{N_t}{N})^{\phi_U}]^{1-\phi_R}]e^{z_t} \tag{7}$$

2.7 Steady State

There exists a zero inflation, no growth steady state where every variable is constant. This implies that the steady state interest rate is $\beta(1 + i) = 1$ from households first order condition in equation 6. Solving for the steady state markup of Calvo firms implies that the relative wholesale price, is the inverse of that markup and depends on the elasticity of substitution in the households demand function. $P^w = \frac{1+\epsilon}{\epsilon}$.

In order to hit reasonable steady state values for the job finding rate f and the employment level N, the steady state is solved by fixing those variables to reasonably steady state values and then adjusting the choices for the parameters on the bargaining power, η, the steady state consumption level C, and the relative weight given to labor disutility, χ to hit those targets. This has added benefit of making the solution to the steady state a linear system .

1. $C = N^{1-\alpha} - \delta N \Gamma f^{\frac{1-\omega}{\omega}}$
2. $w = (1-\alpha)\frac{1+\epsilon}{\epsilon}N^{-\alpha} - (1-(1-\delta)\beta)\Gamma f^{\frac{1-\omega}{\omega}}$
3. $\chi = \frac{\frac{\eta}{1-\eta}\Gamma f^{\frac{1-\omega}{\omega}}}{(1+\frac{1-f}{f})\frac{L^\phi}{C^{-\sigma}}}$
4. $\eta = \frac{\frac{\chi}{f}L^\phi C^\sigma}{\frac{\chi}{f}L^\phi C^\sigma + \Gamma f^{\frac{1-\omega}{\omega}}}$

The first equation is from the good market clearing condition. The second combines the first order condition for wholesale firms with the Nash bargaining solution and the last equation combines the Nash equilibrium with the participation first order condition of households. In steady state $L = (f + \delta(1-f))/fN$, the steady state wage is found using the participation first order condition of households given in equation 7.

3 Linear approximation around Steady State

This paper is focused on the stability of the model around the steady state given different choices of the policy parameters in the Taylor rule. The model is log linearized around the steady state and then the policy rules are solved using an implementation of algorithm from Sims (2002) [5]. The full linear system is given in the appendix.

4 Calibration

The parameters of the model are calibrated to match empirical evidence from other papers or to hit certain steady state values. The Calvo price probability is set to the standard $\theta_p = 0.7$ which corresponds to a price change every three quarters. The elasticity of substitution between the differentiated Calvo goods is set to $\epsilon = -10$, which yields a steady state price markup of of about 1.11 for the Calvo firms. For the wholesale firms, the exponent in the productivity function $Y = N^{1-\alpha}$ is set to the common assumption of $\alpha = 1/3$ which in a normal constant return to scale Cobb Douglas production function would give a wage share of income of about a third. However, In this model there is no capital.

Household's discount future periods with a rate of $\beta = .98$ yielding a steady state real interest rate of about 1%. The household's coefficient of relative risk aversion is set at $\sigma = 1$ for convenience but it also matches the choice in the closely related Galí (2010) and Kurozumi and Van Zandweghe (2010) papers and serves to make comparisons easier. The choice of the curvature parameter for the disutility of effort can be controversial since the parameter chosen in most macro-models diverges significantly from results given in

[5] The implementation is a slightly modified version of the code from the Julia version of the Quantitative Economics project.

quasi-experimental microeconemetric studies. The baseline in this paper of $\phi = 4/3$ is motivated in part from the discussions in Keane and Rogerson (2012) and Chetty et al. (2011). This parameter determines the response of participation from changes in the path of wages, so it is both central to the analysis and the results are sensitive to the parameter choice. Alternative calibration choices for the disutility function are discussed in the results section.

Following evidence in Orphanides and Wieland (2008), the Taylor rule estimates are set to 2.4 for the expected inflation response and 1.5 for the labor market (employment) response. The total hiring cost $\delta\Gamma f^{\frac{1-\omega}{\omega}}$ is set to be 2.3% of steady state output to match evidence from Yashiv (2000). This implies that $\Gamma = .0274$. The matching function exponent ω is set to 0.72 to match with the results from the literature described in Yashiv (2000).

Most of the remaining parameters are set to hit a steady state values for the employment to population ratio, the job finding rate, and the unemployment rate. The employment to population ratio is set to 0.62, the job finding rate is set to 0.70, and the unemployment rate is set to 4.5%. This yields a separation rate of 0.17, a bargaining power of 0.998 for firms, and the weight given to disutility in the combined household utility function $\chi = 20.75$. Hagedorn and Manovskii (2008) supports a calibration that puts high weight on firm bargaining power, but a strong outside option for workers.

5 Results

5.1 Baseline Impulse response

To understand the dynamics in the model we look at the impulse response to a one time positive interest rate shock of $z_0 = e_0 = .25$.

A positive interest rate shock sends the economy into contraction. On impact employment, wages, participation, and inflation all fall. Interestingly unemployment also falls on impact. Since wages contract sharply, leisure becomes relatively cheap and the number of people that leave the labor force is larger than the number of people that leave employment. As interest rates, wages, and prices begin to recover labor supply increases faster than employment. Unemployment rises drastically. Unemployment stays above its

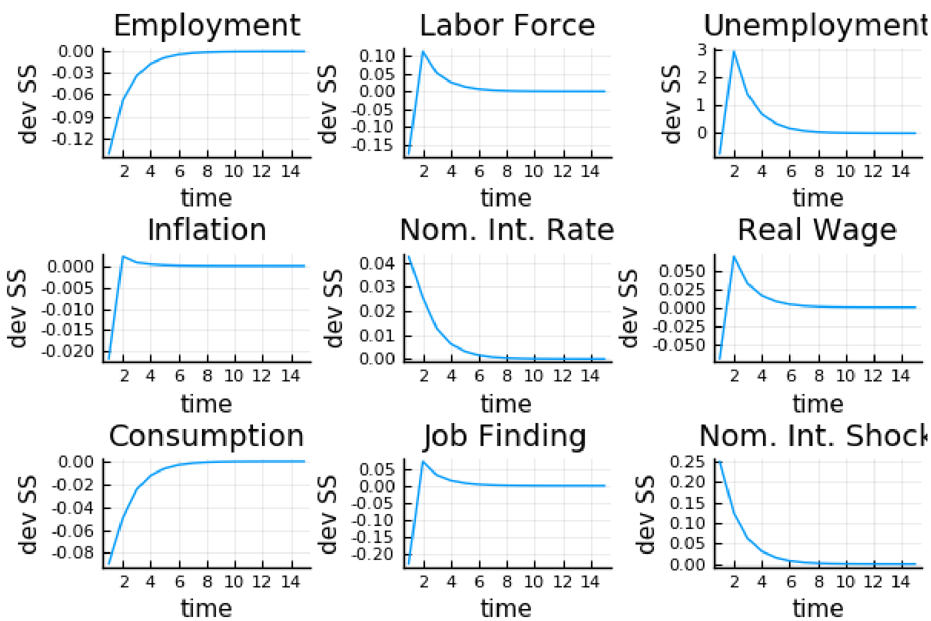

Figure 2: The impulse function of the baseline model with a positive unexpected shock to the nominal interest rate

steady state value as the recovery continues, slowly decaying towards the steady state as wages, employment, and prices return to trend. In some calibrations unemployment rises on impact.

In this model wages, which are bargained every period, are likely to be too flexible and are the likely cause of the poor performance of models that use this matching function structure as emphasized in Shimer (2005) and Galí (2010). However, employment is downwardly rigid in this model. Firms and households can't elect to end employment relationships they can only choose not to enter new employment relationships. Together this should have the effect of forcing more variability in wages then should be expected in the short run. Previous literature such as Galí (2010) and Erceg et al. (2000) use the ambiguity in wage setting created by the ex-post economic surplus in firm-worker matches to motivate a staggered wage setting model similar to the Calvo pricing model used to generate price rigidity. Other papers such as Hall (2005) look at bounds in wage setting including fixed real wages. A more recent paper Christiano et al. (2016) is able to generate more stable wages in bargaining environment using a slightly different parameterization and model matching the results from Hagedorn and Manovskii (2008).

5.2 Taylor Determinacy

For a given choice of the Taylor policy parameters whether or not the resulting linear approximation around the steady state is stable and therefore returns to the steady after a deviation.

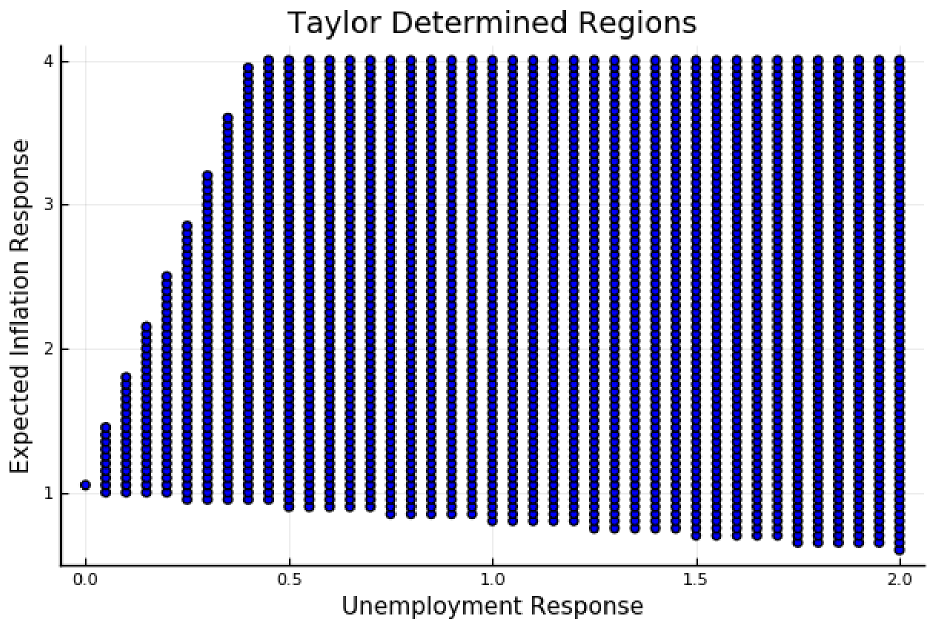

Figure 3: Shaded regions of indeterminacy in the baseline model

In order for the model to have Taylor determinacy the Taylor rule must respond to inflation changes by more than 1:1. For example if the inflation rate is 10% percent above target then the nominal interest rate needs to be set by more than 10% above target to insure the inflation rate returns to the target level. However if monetary policy is insufficiently responsive to unemployment, then even strong responses to inflation don't guarantee determinacy. In fact, as the response to inflation deviations get larger the minimum response to unemployment deviations get larger as well. The results in this paper echo some of the findings from other papers on models in this class. Blanchard and Galí (2010) show that strict unemployment targeting from the central bank preforms better welfare terms than strict inflation targeting. Kurozumi and Van Zandweghe (2010) show that Taylor indeterminacy is caused when the response to unemployment is insufficiently strong. This is caused by hiring costs and the relative sluggish adjustment of labor market variables.

6 Labor supply elasticity and the absence of the participation decision

When $\phi = 5.0$ then the results are more similar to the results in Kurozumi and Van Zandweghe (2010).

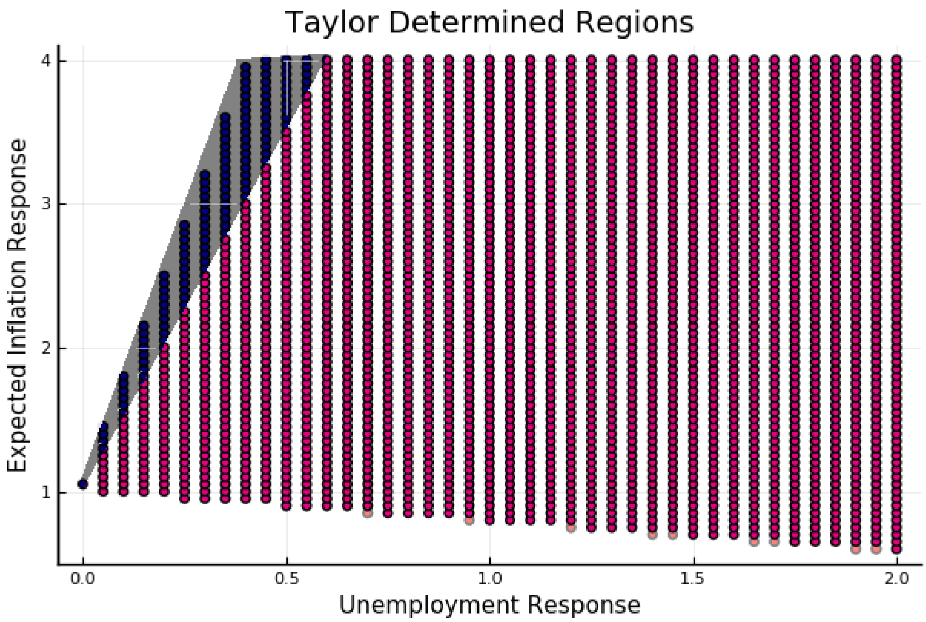

Figure 4: Taylor determinacy regions with an inelastic labor supply in red. The shaded area in blue is the additional Taylor determinacy region for the baseline model.

How important is labor participation dynamics for the response of unexpected monetary policy interest rate adjustments and Taylor determinacy of a central bank's An increase in the curvature of distuility of effort implies less changes in the participation rate [6]. As the results show below, increasing the curvature of the disutility function increases the set of parameters for which the model doesn't have Taylor determinacy.

Additionally, the choice of the curvature parameter for the disutility of effort is controversial since the parameter chosen in most macro-models diverges significantly from results given in quasi-experimental microeconemetric studies. The baseline in this paper of $\phi = 1$ is motivated in part from the discussions in

[6] This version of the model with less elastic labor supply also serves as good comparison to Kurozumi and Van Zandweghe (2010) which has inelastic labor supply. As their results predict, since this paper shows that the addition of labor supply makes the adjustment of employment less sluggish relative to consumption the range of parameters that induce stability is larger as labor elasticity increases.

Keane and Rogerson (2012) and Chetty et al. (2011). This parameter determines the response of participation from changes in the path of wages, so it is both central to the analysis and the results are sensitive to the parameter choice. The results given below show that as the disutility function becomes less convex and therefore Frisch elasticity, the marginal utility constant wage elasticity of labor supply, increases then on impact wages and participation fall by more and there is less disinflation. Additionally, the indeterminate Taylor rule region when the monetary authority has low interest rate responsiveness to changes in employment and high interest responses to inflation changes is weakened. This implies the effect found in Kurozumi and Van Zandweghe (2010) is strengthened when using more convex labor supply disutility function that is more in line with the quasi-experimental microeconomic studies. As the ϕ decreases and the discouraged worker effect increases and the region for which the linearization around the steady state is stable increases. When $\phi = 0.5$ you get the following results keeping the response to employment changes to $\phi_y = 0.045$.

Comparing the impulse response functions from both indicate that the differing path of wages and their direct impact on supply side costs are strongly impacted by the change in participation. In part this mitigation of the effect in Kurozumi and Van Zandweghe (2010) is from the increased fall in wages decreasing the hiring costs in the future. This is likely in part due to increased sensitivity to changes in wages that one sees in the typical real business cycle model such as in Lucas and Rapping (1969). The central bank response also plays a role, when the convexity is large any positive increase in interest rates seems to generate indeterminacy.

7 The role of voluntary unemployment

As the probability of finding a job approaches one, the number of unemployed as traditionally defined those who expended effort to obtain but were unable to be matched with work, declines. When the job finding rate is one the first order condition for labor supply of households become similar to what is seen in standard RBC models. The wage rate is such that it equates the marginal utility of additional consumption of working more with the disutility that the work would entail. In this model a job finding rate of 1 means that matching function and Nash bargaining components don't make a lot of sense. When the steady is very high then the impulse response function responds differently.

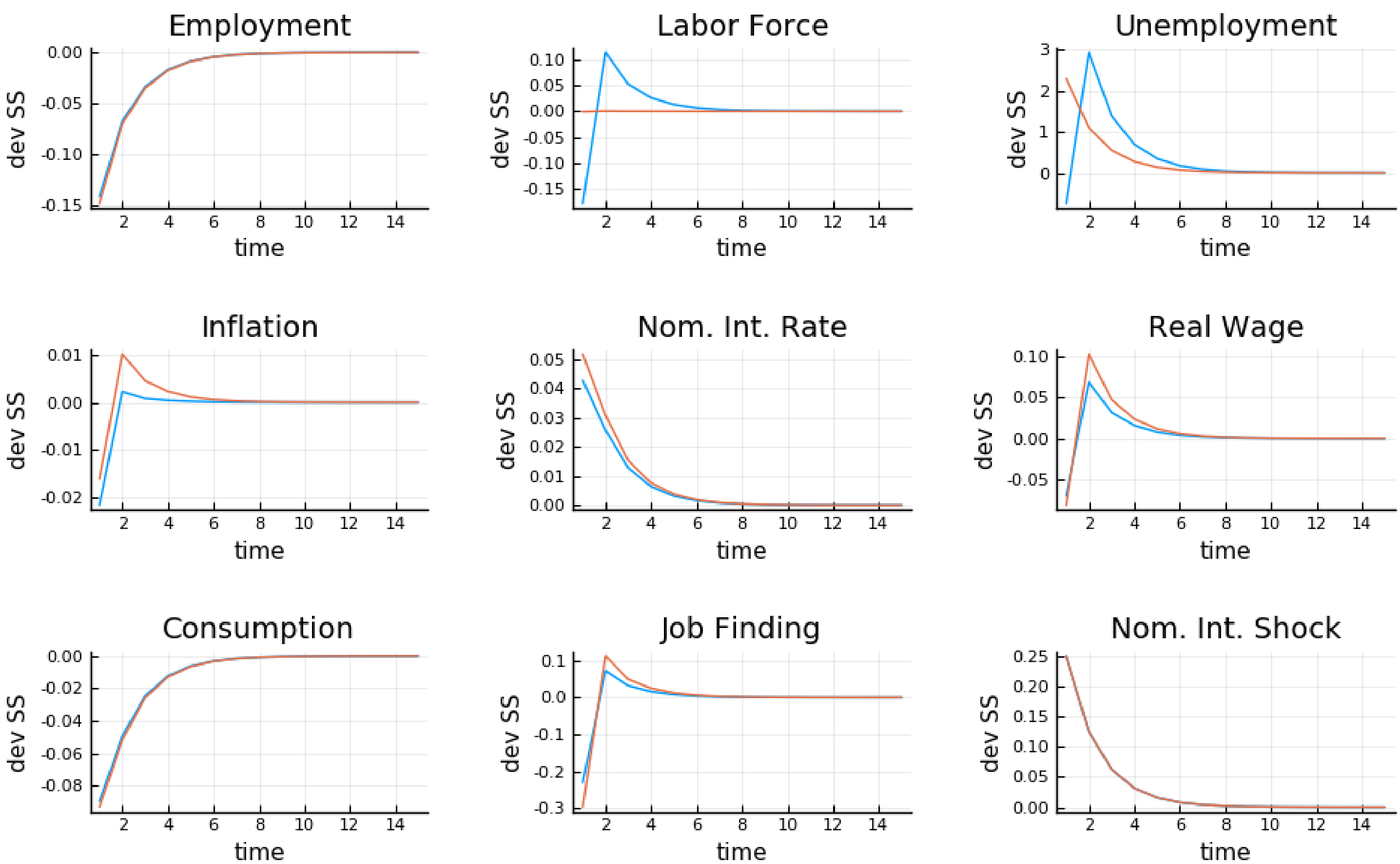

Figure 5: Impulse response function with inelastic labor supply in red, compared to baseline model in blue. When labor supply is less elastic the larger wage changes induce more inflation.

Comparing a modified model with wages that are set to clear labor demand with supply. Then the following equations apply: Households labor supply

$$w_t = \chi \frac{L_t^{\phi}}{C_t^{-\sigma}} \tag{8}$$

Firms labor demand, which is a bit different from the standard since it includes the hiring cost

$$\Gamma = (1-\alpha) P_t^w A_t N_t^{-\alpha} - w_t + E_t[\tilde{\beta}_{t+1}(1-\delta)\Gamma] \tag{9}$$

Then the new equations for the models are

$$\tilde{w}_t = \phi \tilde{L}_t - \sigma \tilde{C}_t \tag{10}$$

$$\tilde{P}_t^w = \frac{w}{\tau} \tilde{w}_t + \alpha \tilde{N}_t - \frac{\beta}{\tau} E[r_{t+1}] \tag{11}$$

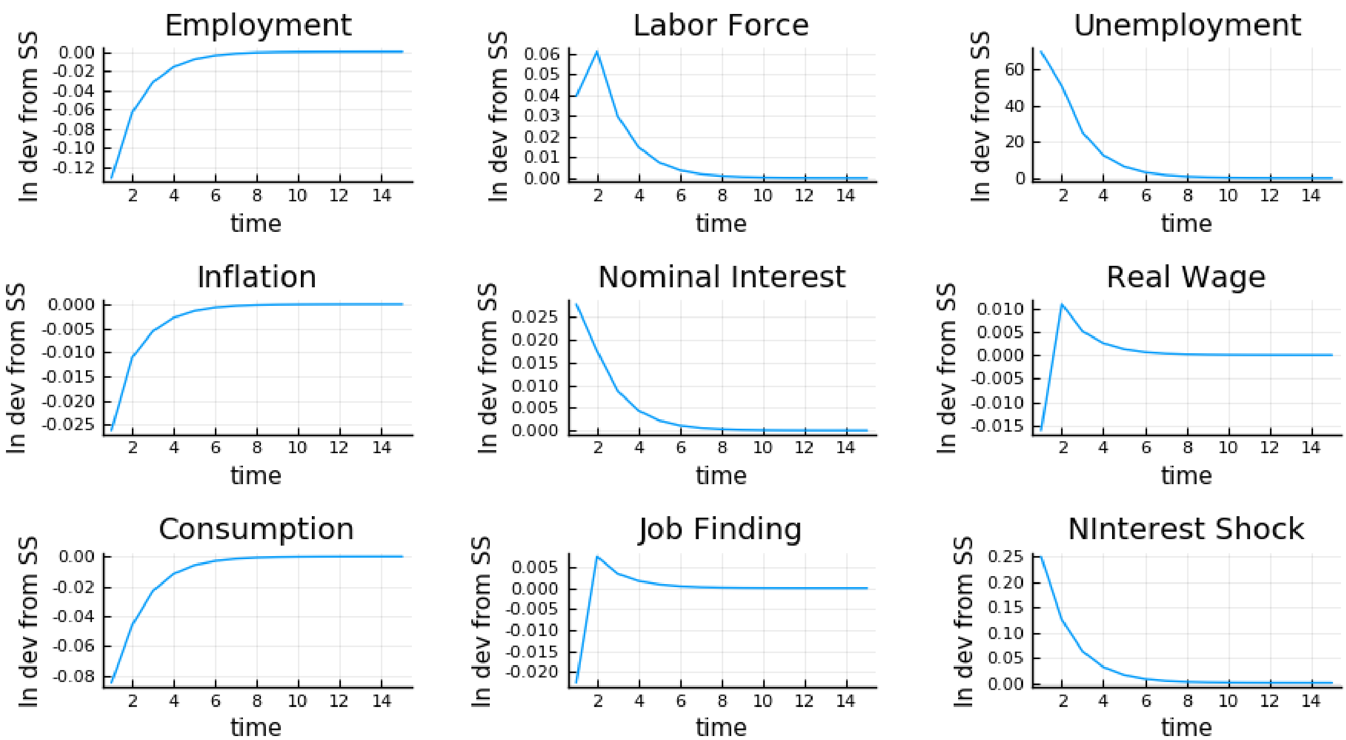

Figure 6: Impulse response function when the steady state job finding rate is near one.

where again $\tau = w + (1 - \beta(1 - \delta))\Gamma$

While in general the response remains the same, the magnitude of the recession is smaller than in the model with matching frictions. Additionally, the pattern of wages is very different when the wages are being set to clear the labor market instead of by Nash bargaining. The Taylor determinancy is the same as the textbook model case, as long as the response to inflation is strong enough, more than 1:1, then the model is determined. It's clear that the determinancy results depend on the movement of labor supply. The results here conform to what was anticipated in the Kurozumi and Van Zandweghe (2010) , the additional flexibility from a labor supply decision help the adjustment in employment be less sluggish and closer to the adjustment in consumption.

Figure 8 shows the comparison between the three models. The lower region is similar between all three and corresponds to the greater than 1:1 rule of thumb for monetary policy. Even though hiring cost and slow depreciation of employment are present in all models, the upper region of indeterminacy is only present in models with matching frictions. In the models with matching frictions more flexible labor supply leads to

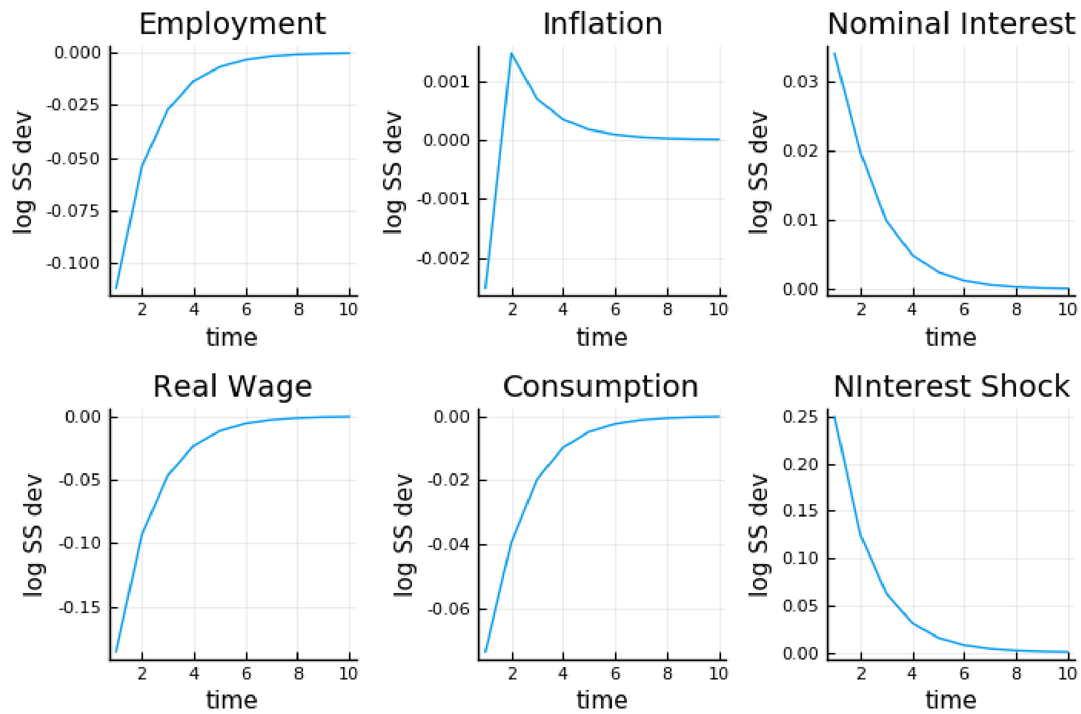

Figure 7: Impulse response to a nominal interest rate shock in the baseline model without matching frictions

an decrease in the upper region of indeterminacy. Countercyclical movements in labor supply baked into the model, since it follows household employment targets which follow demand, soak up some of the volatility in wages not unlike what would happen in a simple supply and demand graph as a more elastic curve would induce more quantity changes relative to price changes.

8 Conclusion

This paper showed that hiring costs can cause monetary policy to become unpredictable and lead to instability in the economy. This isn't caused only by monetary policy not responding strongly enough to inflation deviating from target, but also by not responding enough to disruptions in the labor market. When the central bank is aggressive in pinning the inflation to its long run target it is important to account for the disruptions this policy causes in the labor market. Holding the response to cyclical unemployment fixed, the response to inflation that guarantees stability is bounded, the policy response can also be too strong as well as being too weak. The likelihood that a particular response to price instability is effective increases

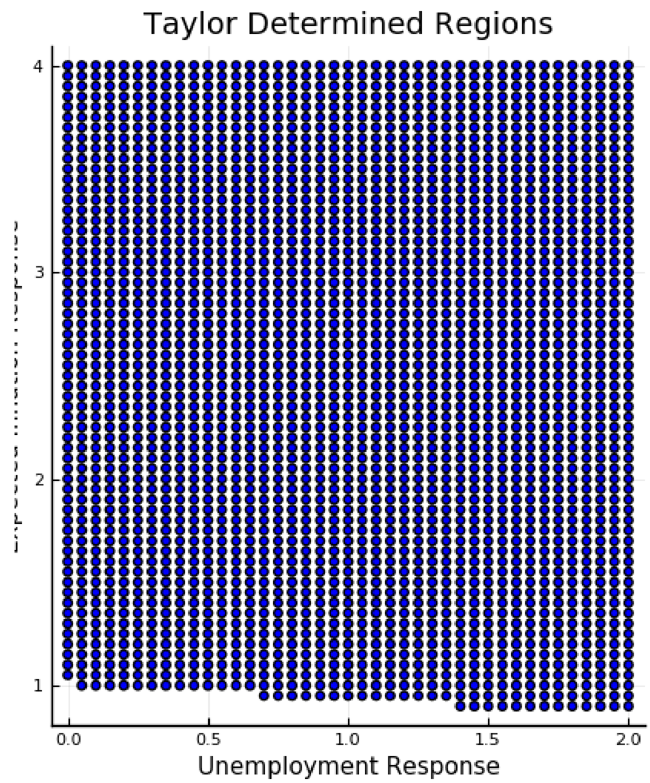

Figure 8: Taylor determinant regions in the baseline model without matching frictions

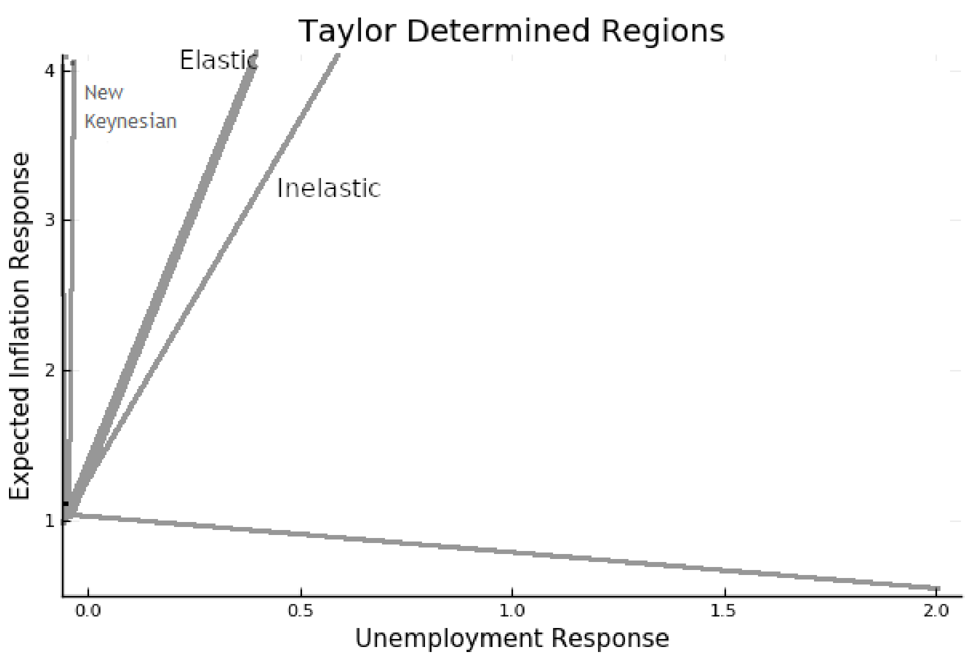

Figure 9: Taylor determinacy boundary for the three models

as the policy is more sensitive to labor market disruptions. Estimates of Taylor rule parameters that match historical Federal Reserve behavior imply these bounds are unlikely to be violated for the United States. Still, the results in this paper show that responding aggressively to employment deviations is crucial in meeting the price stability goals of a central bank regardless of its other goals.

Responding to measures of unemployment is more effective for monetary policy than measures of employment or output. Because of the addition of labor supply; employment and output don't correlate strongly with unemployment or labor market costs. This is partly because of the greater volatility in unemployment compared against output or employment itself. But unemployment is the measure directly connected to adjustment costs in the labor market and therefore the important measure to target to counteract inflation caused by those adjustment costs.

In this model labor supply has a mitigating effect on adjustment costs and inflation. The greater elasticity of labor supply reduces the variability in wages which reduces the resulting response in inflation. The dynamics in this model match the inflow from non-participation into the labor force, which is countercyclical. However the outflow from the labor force into participation is strongly procyclical and leads to the participation rate being mildly procyclical. As noted in Elsby et al. (2015) this outflow is mainly compositional. During recession the population of unemployed mostly contains individuals with high labor attachement. In a sense the people with high relative reservation wages leave the market early on when wage offers start stagnating.

This concurs with results in Kurozumi and Van Zandweghe (2010) which showed that labor hiring costs can put upward pressure on inflation as the result of unexpected interest rate increases. With a Taylor rule based on expectations or as adjustment of labor variables are slow relative to consumption then the economy is no longer stable around the steady state. The addition of labor supply mitigates this effect, but the stability still depends crucially on the relative strength of the responses to inflation to labor market variables. When the discouraged worker effect dominates and labor supply falls in response to an increase in interest rates, the model is more likely to be stable around the steady state. This is due to a variety of mechanisms, the strongest of which is the greater reactivity of wages as labor participation falls, similar to labor supply dynamics in Lucas and Rapping (1969). For policy this implies that it is important to consider hiring cost and it is possible to create a situation where interest rate increases raise the path of future inflation growth.

This paper notably ignores several effects that could be fruitful for further research. One of which is a precautionary motive arising out of income risk for individuals in the households. Precautionary savings implies more consumption volatility relative to a permanent income benchmark as unemployment induces greater savings demand for individuals to self insure themselves against long unemployment spells. The same motive exists for labor supply as noted in Acemoglu and Shimer (1999). This would create an additional added worker effect and the results here would imply less wage variability and a stronger vacancy channel than implied in this paper. Another is accounting for labor market skill growth and decay based on employment history. Finally, this paper does not impose a zero lower bound on nominal interest rates. In fact in the example impulse response function with a high curvature, using the Taylor rule prescribed the central bank sets negative nominal interest rates on impact. Presumably, accounting for a zero lower pound would increase the regions of indeterminacy since it would mitigate the amount the central bank could respond to changes, in effect reducing the policy weights on inflation and employment when near the zero lower bound. Finally a richer set of heterogeneity of workers within families could help participation match outflow characteristics from the labor force. Erceg and Levin (2014) has a model that accomplishes part of this.

These results also imply that like in many papers before such as Galí (2010) and Shimer (2005) that in macro-labor models with matching frictions, labor dynamics in themselves don't seem to have large effects on inflation. In part this seems to rest on the flexibility on wages given by period by period Nash bargaining. These results also imply however that ignoring labor supply often leads to misleading results. A message that finds support with the findings of Elsby et al. (2009) and Krusell et al. (2012).

References

Acemoglu, Daron and Robert Shimer (1999), "Efficient Unemployment Insurance." *Journal of Political Economy*, 107, 893–928.

Andolfatto, David (1996), "Business Cycles and Labor-Market Search." *American Economic Review*, 112–132.

Blanchard, Olivier and Jordi Galí (2010), "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment." *American Economic Journal: Macroeconomics*, 2, 1–30.

Calvo, Guillermo A (1983), "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12, 383–398.

Campolmi, Alessia and Stefano Gnocchi (2016), "Labor Market Participation, Unemployment and Monetary Policy." *Journal of Monetary Economics*, 79, 17–29.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber (2011), "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins." *American Economic Review*, 101, 471–75.

Christiano, Lawrence, Martin Eichenbaum, and Mathias Trabandt (2016), "Unemployment and Business Cycles." *Econometrica*, 84, 1523–1569.

Cochrane, John H (2011), "Determinacy and Identification with Taylor Rules." *Journal of Political economy*, 119, 565–615.

Elsby, Michael WL, Bart Hobijn, and Ayşegül Şahin (2015), "On the Importance of the Participation Margin for Labor Market Fluctuations." *Journal of Monetary Economics*, 72, 64–82.

Elsby, Michael WL, Ryan Michaels, and Gary Solon (2009), "The ins and outs of Cyclical Unemployment." *American Economic Journal: Macroeconomics*, 1, 84–110.

Erceg, Christopher, Dale Henderson, and Andrew Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts." *Journal of Monetary Economics*, 46, 281–313.

Erceg, Christopher J and Andrew T Levin (2014), "Labor Force Participation and Monetary Policy in the Wake of the Great Recession." *Journal of Money, Credit and Banking*, 46, 3–49.

Furlanetto, Francesco and Nicolas Groshenny (2016), "Mismatch Shocks and Unemployment during the Great Recession." *Journal of Applied Econometrics*, 31, 1197–1214.

Galí, Jordi (2010), "Monetary Policy and Unemployment." In *Handbook of Monetary Economics*, volume 3, 487–546, Elsevier.

Galí, Jordi (2015), *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and its Applications*. Princeton University Press.

Hagedorn, Marcus and Iourii Manovskii (2008), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review*, 98, 1692–1706.

Hall, Robert (2005), "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, 95, 50–65.

Keane, Michael and Richard Rogerson (2012), "Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom." *Journal of Economic Literature*, 50, 464–76.

Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, and Ayşegül Şahin (2012), "Is Labor Supply Important for Business Cycles?" Technical report, National Bureau of Economic Research.

Kurozumi, Takushi and Willem Van Zandweghe (2010), "Labor Market Search, the Taylor Principle, and Indeterminacy." *Journal of Monetary Economics*, 57, 851–858.

Lucas, Robert and Leonard Rapping (1969), "Real Wages, Employment, and Inflation." *Journal of Political Economy*, 77, 721–754.

Merz, Monika (1995), "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics*, 36, 269–300.

Orphanides, Athanasios and Volker Wieland (2008), "Economic Projections and Rules of Thumb for Monetary Policy." *Federal Reserve Bank of St. Louis Review*, 90, 307–24.

Petrongolo, Barbara and Christopher A Pissarides (2001), "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*, 39, 390–431.

Pissarides, Christopher A (2000), *Equilibrium Unemployment Theory*. MIT press.

Shimer, Robert (2005), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review*, 95, 25–49.

Sims, Christopher A (2002), "Solving Linear Rational Expectations Models." *Computational Economics*, 20, 1.

Yashiv, Eran (2000), "The Determinants of Equilibrium Unemployment." *American Economic Review*, 90, 1297–1322.

Yashiv, Eran (2007), "Labor Search and Matching in Macroeconomics." *European Economic Review*, 51, 1859–1895.

A Linearized Model

The full system is listed below, lower case variables are log deviations from the steady state. For example $\tilde{y}_t \equiv (Y_t - Y)/Y$.

1. $(1-\lambda_f)\tilde{w}_t - \sigma(1-\lambda_f)\tilde{c}_t = \phi\tilde{L}_t - \tilde{f}_t - (\phi\lambda_f E_t[\tilde{L}_{t+1}] - \beta(1-\delta)E_t[\tilde{f}_{t+1}])$

   $\lambda_f = \beta(1-\delta)(1-f)$

2. $\tilde{P}^w_t = \alpha\tilde{n}_t + \frac{w}{\tau}\tilde{w}_t + \frac{\Gamma_f}{\tau}\tilde{f}_t - \beta(1-\delta)\frac{\Gamma_f}{\tau}E_t[\tilde{f}_{t+1}] - \frac{\beta}{\tau}E_t[r_{t+1}]$

   $\tau = w + (1-\beta(1-\delta)\Gamma f^{\frac{1-\omega}{\omega}}$

   $\Gamma_f = \frac{1-\omega}{\omega}\Gamma f^{\frac{1-\omega}{\omega}}$

3. $\frac{1}{\omega}\tilde{f}_t = \phi\tilde{l}_t + \sigma\tilde{c}_t$

4. $\tilde{L}_t = \frac{N}{L}\tilde{N}_t + \frac{U}{L}\tilde{U}_t$

5. $\tilde{U}_t = \frac{1}{\delta}\tilde{N}_t - \frac{1-\delta}{\delta}\tilde{N}_{t-1} - \frac{1}{1-f}\tilde{f}_t$

6. $E_t[\tilde{C}_{t+1}] = \tilde{C}_t + \frac{1}{\sigma} E_t[r_{t+1}]$

7. $E_t[r_{t+1}] = i_t - E_t[\pi_{t+1}]$

8. $\pi_t = \beta E_t[\pi_{t+1}] + \lambda_p P_t^w$

9. Define the share of hiring cost as a percent of gdp as $\Theta = \frac{\Gamma f^{\frac{1-\omega}{\omega}}}{N^{1-\alpha}}$. Then the linearized market clearing condition for final goods is: $(1-\alpha-\Theta)\tilde{N}_t = (1-\Theta)\tilde{C}_t + \delta\Theta\tilde{f}_t - (1-\delta)\Theta\tilde{N}_{t-1}$

10. $\tilde{w}_t = \tilde{w}_{t-1} + \frac{\delta_w}{1-\delta_w}\tilde{b}_t$

11. Baseline Taylor Rule $i_t = \phi_\pi E_t[\pi_{t+1}] + \phi_U \tilde{N}_t + z_t$