2016 16 b Some yabbies are introduced into a small dam. The size of the population, y, of yabbies can be modelled by the function $y = \dfrac{200}{1+19e^{-0.5t}}$, where t is the time in months after the yabbies are introduced into the dam.

(i) Show that the rate of growth of the size of the population is $\dfrac{1900e^{-0.5t}}{(1+19e^{-0.5t})^2}$. **2**

(ii) Find the range of the function y, justifying your answer. **2**

(iii) Show that the rate of growth of the size of the population can be rewritten as $\dfrac{y}{400}(200 - y)$. **1**

(iv) Hence, find the size of the population when it is growing at its fastest rate. **2**

(i) $y = 200(1 + 19e^{-0.5t})^{-1}$

$\dfrac{dy}{dt} = -200[1 + 19e^{-0.5t})^{-2} \times -9.5e^{-0.5t}$

$= \dfrac{1900e^{-0.5t}}{(1+19e^{-0.5t})^2}$

State Mean: **1.24**

(ii) As $t \geq 0$, consider $t = 0$:

$y(0) = \dfrac{200}{1+19e^{-0.5(0)}}$

$= \dfrac{200}{1+19}$

$= 10$

As $t \to \infty$, $y(\infty) = \dfrac{200}{1+19e^{-0.5(\infty)}}$

$= \dfrac{200}{1+0}$

$= 200$

State Mean: **0.42**

As y is monotonic increasing, then range is $10 \leq y < 200$.

(iii) From $y = \dfrac{200}{1+19e^{-0.5t}}$

$\therefore 1 + 19e^{-0.5t} = \dfrac{200}{y}$

$19e^{-0.5t} = \dfrac{200}{y} - 1$

$19e^{-0.5t} = \dfrac{200-y}{y}$

$\therefore 1900e^{-0.5t} = 100\left[\dfrac{200-y}{y}\right]$

As $\dfrac{dy}{dt} = \dfrac{1900e^{-0.5t}}{(1+19e^{-0.5t})^2}$

$= 100\left[\dfrac{200-y}{y}\right] \div \left[\dfrac{200}{y}\right]^2$

$= 100\left[\dfrac{200-y}{y}\right] \times \dfrac{y^2}{40000}$

$= \dfrac{y(200-y)}{400}$

$= \dfrac{y}{400}(200 - y)$

State Mean: **0.18**

(iv) Let $R = \dfrac{y}{400}(200 - y)$

$\therefore R = \dfrac{y}{2} - \dfrac{y^2}{400}$

$\dfrac{dR}{dy} = \dfrac{1}{2} - \dfrac{y}{200} = 0$

$\therefore \dfrac{y}{200} = \dfrac{1}{2}$

$y = 100$

$\dfrac{d^2R}{dy^2} = -\dfrac{1}{200} < 0 \quad \therefore$ maximum.

$\therefore$ fastest rate when 100 yabbies.

State Mean: **0.28**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

BOSTES: Notes from the Marking Centre

This information is released by BOSTES in late Term 1 2017.