

08	9b	<p>Peter retires with a lump sum of \$100 000. The money is invested in a fund which pays interest each month at a rate of 6% per annum, and Peter receives a fixed monthly payment of \$$M$ from the fund. Thus, the amount left in the fund after the first monthly payment is \$(100 000 - M).</p> <p>(i) Find a formula for the amount, \$$A_n$, left in the fund after n monthly payments</p> <p>(ii) Peter chooses the value of M so that there will be nothing left in the fund at the end of the 12th year (after 144 payments). Find the value of M.</p>	2 3
<p>i. Let $A_1 = 100\,000 \times 1.005 - M$ as 6% pa, then 0.5% = 0.005 per month $= 100\,000 - M$</p> <p>$A_2 = (100\,000 \times 1.005 - M) \times 1.005 - M$ $= 100\,000 \times 1.005^2 - M(1 + 1.005)$</p> <p>$A_n = 100\,000 \times 1.005^n - M(1 + 1.005 + \dots + 1.005^{n-1})$</p> <p>ii. As $n = 144$,</p> <p>$0 = 100\,000 \times 1.005^{144} - M(1 + 1.005 + \dots + 1.005^{143})$</p> <p>$M = 100\,000 \times 1.005^{144} \div (1 + 1.005 + \dots + 1.005^{143})$</p> <p>Now, for $1 + 1.005 + \dots + 1.005^{143}$, $a = 1$, $r = 1.005$ and $n = 144$ and use</p> $S_n = \frac{a(r^n - 1)}{r - 1}$ $= \frac{1(1.005^{144} - 1)}{1.005 - 1}$ <p>$\therefore M = 100\,000 \times 1.005^{144} \div \frac{1(1.005^{144} - 1)}{1.005 - 1}$</p> $= \frac{100\,000 \times 1.005^{144} \times 0.005}{1.005^{144} - 1}$ $= 975.8502136$ $= 975.85 \quad (2 \text{ dec pl})$ <p>\therefore value of M is 975.85</p>			

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

- (i) The most successful candidates were those that developed the correct expression, starting from the given expression for A_1 , then finding A_2 , A_3 and on to A_n . Candidates are reminded that working needs to be shown, as incorrect expressions for A_n without supporting working could not be awarded marks that were available for intermediate steps. Apart from those responses where candidates could not successfully develop the correct expression for A_n , another common error was in calculating the interest rate to be used, despite the fact that the expression for A_1 was given.
- (ii) Candidates who developed the correct expression for A_n were generally successful in this part. In better responses, candidates recognised and were able to sum a geometric series and few calculator errors were evident. Common mistakes were in using $n = 143$ in the sum of the geometric series, substituting $n = 12$ into the expression for A_n and incorrect manipulation of the equation in M .

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/