The index laws

Law 1: Multiplying terms with the same base

Consider $2^4 \times 2^3 = (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^7$

But $2^{4+3} = 2^7$

$\therefore 2^4 \times 2^3 = 2^{4+3} = 2^7$

When multiplying terms with the same base, add the powers:

$$a^m \times a^n = a^{m+n}$$

Proof:

$$a^m \times a^n = \underbrace{a \times a \times \ldots \times a}_{m \text{ factors}} \times \underbrace{a \times a \times \ldots \times a}_{n \text{ factors}}$$

$$= \underbrace{a \times a \times \ldots \times a}_{m+n \text{ factors}}$$

$$= a^{m+n}$$

Example 4

Simplify the following, expressing your answers in index form.

a $6^3 \times 6^7$ b 5×5^3 c $y^4 \times y^8$

Solution

a $6^3 \times 6^7 = 6^{3+7}$

$= 6^{10}$

b $5 \times 5^3 = 5^1 \times 5^3$

$= 5^{1+3}$

$= 5^4$

c $y^4 \times y^8 = y^{4+8}$

$= y^{12}$

Example 5

Simplify the following.

a $3p^4 \times 2p^6$ b $5e^2f \times 3e^4f^5$

Solution

a $3p^4 \times 2p^6 = (3 \times 2) \times (p^4 \times p^6)$

$= 6p^{4+6}$

$= 6p^{10}$

b $5e^2f \times 3e^4f^5 = (5 \times 3) \times (e^2 \times e^4) \times (f \times f^5)$

$= 15e^{2+4}f^{1+5}$

$= 15e^6f^6$

Exercise 7-02

1 Simplify (giving answers in index notation):

a $10^3 \times 10^2$ **b** 10×10^4 **c** $3^2 \times 3^5$ **d** $7^4 \times 7$

e $8 \times 8^3 \times 8^4$ **f** $5^4 \times 5 \times 5^4$ **g** $6 \times 6^2 \times 6^3 \times 6^4$ **h** $4^4 \times 4^4 \times 4^4$

i $11^7 \times 11^{13}$ **j** 2×2^3 **k** $3^4 \times 3 \times 3^7$ **l** $7^2 \times 7^5 \times 7$

---

2 Simplify:

a $x \times x^4$ **b** $g^4 \times g^4$ **c** $w^7 \times w$ **d** $b^3 \times b^{10}$

e $p^{10} \times p^{10}$ **f** $r \times r$ **g** $y \times y^3 \times y^2$ **h** $m^3 \times m \times m^4$

---

3 Simplify:

a $3p^2 \times 2p^5$ **b** $4y^{10} \times 3y^2$ **c** $6m \times 3m^8$

d $h^3 \times 5h^8$ **e** $3q^3 \times 8q^8$ **f** $2a^2 \times 5a^5$

g $5n^8 \times 6n^8$ **h** $2b^3 \times 15b^6$ **i** $3e^4 \times e^6$

j $10p^3 \times 5p^2q$ **k** $8a^3b^2 \times 2b^3a$ **l** $4w^5y^2 \times 5w^4y^3$

m $5a^3c^2 \times 2b^4c$ **n** $10p^3q^8 \times qp^2$ **o** $4g^3h^2 \times 5gh^4$

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4 Write true (T) or false (F) for each of the following.

a $5^3 \times 3^7 = 15^{10}$ **b** $7^2 \times 8^2 = 56^4$ **c** $3 \times 7^2 = 21^2$ **d** $4^3 \times 4^7 = 4^{10}$

e $3^2 \times 2^4 \times 3^2 \times 2^5 = 3^4 \times 2^9$ **f** $5^2 \times 5^3 = 25^{15}$ **g** $2^7 \times 2^8 = 2^{15}$

h $7^3 \times 7^5 = 49^8$ **i** $4^2 \times 3^3 = 12^6$ **j** $5^4 \times 3^2 \times 3^7 \times 5 = 3^9 \times 5^5$

---

5 Simplify and evaluate:

a $2^3 \times 2^5$ **b** $2^3 \times 5^2$ **c** $10^2 \times 2^{10}$ **d** $5^3 \times 3^5$

e $3^3 \times 3^3$ **f** $5^3 \times 2^3$ **g** $10^2 \times 10^3$ **h** $2^{10} \times 10^3$

6 Simplify:

a $x^4 \times x^3 \times x^2$ **b** $y^6 \times x^3 \times y$ **c** $5 \times 3n \times 4n^2$ **d** $5 \times m \times 4n^2$

e $5qp \times 4q^2 \times 5p^3$ **f** $(a^4 \times b^3) \times (a^4 \times b^2)$ **g** $4^a \times 4^b$ **h** $2^{x+1} \times 2^x$

i $3^{2y} \times 3^y$ **j** $(p+q)^2 \times (p+q)^3$ **k** $(x-y) \times (x-y)^2$ **l** $(a+3)^n \times (a+3)$

Answers

Exercise 7-02

1 **a** 10^5 **b** 10^5 **c** 3^7 **d** 7^5 **e** 8^8 **f** 5^9
g 6^{10} **h** 4^{12} **i** 11^{20} **j** 2^4 **k** 3^{12} **l** 7^8

2 **a** x^5 **b** g^8 **c** w^8 **d** b^{13} **e** p^{20} **f** r^2
g y^6 **h** m^8

3 **a** $6p^7$ **b** $12y^{12}$ **c** $18m^9$ **d** $5h^{11}$
e $24q^{11}$ **f** $10a^7$ **g** $30n^{16}$ **h** $30b^9$
i $3e^{10}$ **j** $50p^5q$ **k** $16a^4b^5$ **l** $20w^9y^5$
m $10a^3b^4c^3$ **n** $10p^5q^9$ **o** $20g^4h^6$

4 **a** F **b** F **c** F **d** T **e** T **f** F
g T **h** F **i** F **j** T

5 **a** 256 **b** 200 **c** 102 400
d 30 375 **e** 729 **f** 1000
g 100 000 **h** 1 024 000

6 **a** x^9 **b** x^3y^7 **c** $60n^3$ **d** $20mn^2$
e $100p^4q^3$ **f** a^8b^5 **g** 4^{a+b} **h** 2^{2x+1}
i 3^{3y} **j** $(p+q)^5$ **k** $(x-y)^3$ **l** $(a+3)^{n+1}$

Law 2: Dividing terms with the same base

Law 2: Dividing terms with the same base

Consider $5^7 \div 5^4 = \dfrac{5^7}{5^4}$

$$= \frac{\cancel{5}^{1} \times \cancel{5}^{1} \times \cancel{5}^{1} \times \cancel{5}^{1} \times 5 \times 5 \times 5}{{}_{1}\cancel{5} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{5}}}$$

$$= 5 \times 5 \times 5$$

$$= 5^3$$

But $5^{7-4} = 5^3$

$$\therefore 5^7 \div 5^4 = \frac{5^7}{5^4} = 5^{7-4} = 5^3$$

When dividing terms with the same base, subtract the powers:

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

Proof:

$$a^m \div a^n = \frac{a^m}{a^n}$$

$$= \frac{{}^{1}\cancel{a} \times {}^{1}\cancel{a} \times a \times a \times a \times \ldots \times a}{{}_{1}\cancel{a} \times {}_{1}\cancel{a} \times a \times a \times \ldots \times a} \qquad \begin{matrix} (m \text{ factors}) \\ (n \text{ factors}) \end{matrix}$$

$$= a \times a \times \ldots \times a (m - n \text{ factors})$$

$$= a^{m-n}$$

Example 6

Simplify the following, expressing your answers in index form.

a $4^5 \div 4^3$ **b** $\dfrac{10^7}{10^4}$ **c** $y^{12} \div y^3$

Solution

a $4^5 \div 4^3 = 4^{5-3}$

$= 4^2$

b $\dfrac{10^7}{10^4} = 10^{7-4}$

$= 10^3$

c $y^{12} \div y^3 = y^{12-3}$

$= y^9$

Example 7

Simplify the following.

a $k^7 \div k$ **b** $15m^8 \div 3m^2$ **c** $30a^5b^7 \div 10a^2b^5$

Solution

a $k^7 \div k = k^7 \div k^1$
$= k^{7-1}$
$= k^6$

b $15m^8 \div 3m^2 = \frac{{}^{5}\cancel{15}m^8}{{}_{1}\cancel{3}m^2}$
$= 5m^{8-2}$
$= 5m^6$

c $30a^5b^7 \div 10a^2b^5 = \frac{{}^{3}\cancel{30}a^5b^7}{{}_{1}\cancel{10}a^2b^5}$
$= 3a^{5-2}b^{7-5}$
$= 3a^3b^2$

Just for the record

Remember that taxi

Indian mathematician Srinivasa Ramanujan (1888–1920) loved working with numbers. One day he was visited by a friend in a taxi numbered 1729. When Ramanujan heard the number, he immediately said '1729 is a very interesting number as it is the smallest number that can be expressed as the sum of two cubes in two different ways.'

This means that we can write

$1729 = x^3 + y^3$

Here is one of the possible ways:

$1729 = 10^3 + 9^3$

Find the other.

Exercise 7-03

1 Simplify, giving your answers in index form:

a $\frac{5^8}{5^2}$ **b** $\frac{9^{12}}{9^3}$ **c** $\frac{2^{27}}{2^3}$ **d** $7^4 \div 7^3$

e $10^5 \div 10^5$ **f** $8^5 \div 8$ **g** $20^{15} \div 20^5$ **h** $\frac{2^{20}}{2}$

i $6^{80} \div 6^{20}$ **j** $8^{15} \div 8^{11}$ **k** $3^{12} \div 3^{6}$ **l** $2^{18} \div 2^{11}$

---

2 Simplify:

a $\frac{h^{20}}{h^4}$ **b** $\frac{y^8}{y^2}$ **c** $\frac{a^{12}}{a^4}$ **d** $b^{16} \div b^{15}$

e $m^{16} \div m^{16}$ **f** $n^7 \div n$ **g** $t^{18} \div t^9$ **h** $\frac{w^{25}}{w}$

i $e^{30} \div e^{10}$ **j** $\frac{d^9}{d^5}$ **k** $p^{15} \div p^{10}$ **l** $w^{24} \div w^6$

---

3 Simplify the following.

a $10y^{15} \div 5y^3$ **b** $20w^9 \div 4w^3$ **c** $24r^8 \div 3r^2$ **d** $\frac{30x^4}{x^3}$

e $\dfrac{10m^{10}}{2m^2}$ **f** $\dfrac{12g^{12}}{6g^6}$ **g** $16h^{10} \div 8h$ **h** $15y^8 \div 15y^4$

i $18g^{60} \div 6g^4$ **j** $\dfrac{a^6b^3}{a^2b^2}$ **k** $\dfrac{36p^8q^3}{4p^4q}$ **l** $\dfrac{100f^2g^4}{5fg^2}$

m $20m^{15}n \div 2m^{14}$ **n** $36y^8x^7 \div 12x^3y$ **o** $44e^4f^{10} \div 4ef^2$ **p** $30k^7m^4 \div 6k^6m^2$

---

4 Write true (T) or false (F) for the following.

a $10^3 \div 2^2 = 5^1$ **b** $8^4 \div 4^4 = 2^2$ **c** $12^{10} \div 12^{10} = 1$ **d** $15^8 \div 15^4 = 15^2$

e $10^9 \div 10^3 = 10^6$ **f** $7^4 \div 7^2 = 1^2$ **g** $\dfrac{20^4}{5^2} \div 4^2$ **h** $12^3 \div 3^3 = 4^1$

5 Evaluate:

a $2^{10} \div 2^5$ **b** $4^5 \div 2^3$ **c** $3^3 \div 2^3$ **d** $\frac{10^3}{2^3}$ **e** $\frac{5^4}{5^4}$ **f** $\frac{2^{10}}{5^2}$

g $4^5 \div 2^{10}$ **h** $20^3 \div 5^3$ **i** $10^6 \div 5^4$ **j** $4^9 \div 8^3$ **k** $\frac{12^5}{6^8}$ **l** $3^{10} \div 27^2$

---

6 Simplify:

a $\frac{x^4 \times x^3}{x^2}$ **b** $\frac{y^{10}}{y^3 \times y}$ **c** $\frac{a^5 \times a^3}{a \times a^4}$

d $4^a \div 4^b$ **e** $2^{x+1} \div 2^x$ **f** $3^{2y} \div 3^y$

g $\frac{4m^3 \times 5m^7}{10m^6}$ **h** $\frac{6n^{16} \times 8n^4}{3n^3 \times 4n^5}$ **i** $\frac{p^6}{6p^2} \times \frac{30p^4}{5p}$

Answers

Exercise 7-03

1 **a** 5^6 **b** 9^9 **c** 2^{24} **d** 7^1
e 10^0 **f** 8^4 **g** 20^{10} **h** 2^{19}
i 6^{60} **j** 8^4 **k** 3^6 **l** 2^7

2 **a** h^{16} **b** y^6 **c** a^8 **d** b^1
e m^0 **f** n^6 **g** t^9 **h** w^{24}
i e^{20} **j** d^4 **k** p^5 **l** w^{18}

3 **a** $2y^{12}$ **b** $5w^6$ **c** $8r^6$ **d** $30x$
e $5m^8$ **f** $2g^6$ **g** $2h^9$ **h** y^4
i $3g^{56}$ **j** a^4b **k** $9p^4q^2$ **l** $20fg^2$
m $10mn$ **n** $3x^4y^7$ **o** $11e^3f^8$ **p** $5km^2$

4 **a** F **b** F **c** T **d** F **e** T **f** F
g F **h** F

5 **a** 32 **b** 128 **c** 3.375 **d** 125
e 1 **f** 40.96 **g** 1 **h** 64
i 1600 **j** 512 **k** $0.1\dot{4}\dot{8}$ **l** 81

6 **a** x^5 **b** y^6 **c** a^3 **d** 4^{a-b} **e** 2
f $3y$ **g** $2m^4$ **h** $4n^{12}$ **i** p^7

Law 3: Raising a power to a power

Consider $(4^2)^5 = 4^2 \times 4^2 \times 4^2 \times 4^2 \times 4^2$

$= (4 \times 4) \times (4 \times 4) \times (4 \times 4) \times (4 \times 4) \times (4 \times 4)$

$= 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

$= 4^{10}$

But $4^{2 \times 5} = 4^{10}$

$\therefore (4^2)^5 = 4^{2 \times 5} = 4^{10}$

When raising a term with a power to another power, multiply the powers:

$$(a^m)^n = a^{m \times n}$$

Proof:

$$(a^m)^n = \underbrace{a^m \times a^m \times \ldots \times a^m}_{n \text{ factors}}$$

$$= \underbrace{\underbrace{a \times a \times \ldots \times a}_{m \text{ factors}} \times \underbrace{a \times a \times \ldots \times a}_{m \text{ factors}} \times \ldots \times \underbrace{a \times a \times \ldots \times a}_{m \text{ factors}}}_{n \text{ lots of } m \text{ factors}}$$

$$= a^{m \times n}$$

Example 8

Simplify the following, expressing your answers in index form.

a $(2^3)^5$ b $(y^2)^{14}$

Solution

a $(2^3)^5 = 2^{3 \times 5}$
$= 2^{15}$

b $(y^2)^{14} = y^{2 \times 14}$
$= y^{28}$

Law 4: Powers of products and quotients

Consider $(2 \times 3)^5 = (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 2^5 \times 3^5$

Also $$\left(\frac{5}{3}\right)^4 = \frac{5}{3}\times\frac{5}{3}\times\frac{5}{3}\times\frac{5}{3}$$

$$= \frac{5\times5\times5\times5}{3\times3\times3\times3}$$

$$= \frac{5^4}{3^4}$$

Powers of products and quotients:

$$(ab)^n = a^n b^n \quad \text{and} \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Proof:

$$(ab)^n = \underbrace{ab\times ab\times \ldots \times ab}_{n \text{ factors}}$$

$$= a\times b\times a\times b\times \ldots \times a\times b$$

$$= \underbrace{a\times a\times \ldots \times a}_{n \text{ factors}} \times \underbrace{b\times b\times \ldots \times b}_{n \text{ factors}}$$

$$= a^n \times b^n$$

Also $$\left(\frac{a}{b}\right)^n = \underbrace{\frac{a}{b}\times\frac{a}{b}\times \ldots \times\frac{a}{b}}_{n \text{ factors}}$$

$$= \frac{a\times a\times \ldots \times a}{b\times b\times b\times \ldots \times b} \quad \begin{matrix}(n \text{ factors})\\(n \text{ factors})\end{matrix}$$

$$= \frac{a^n}{b^n}$$

Example 9

Simplify each of the following.

a $(2k)^5$ **b** $(5m^4)^3$ **c** $\left(\frac{m}{4}\right)^3$ **d** $\left(\frac{2w^3}{3}\right)^4$

Solution

a $(2k)^5 = 2^5 \times k^5$
$= 32k^5$

b $(5m^4)^3 = 5^3 \times (m^4)^3$
$= 125 \times m^{4\times 3}$
$= 125m^{12}$

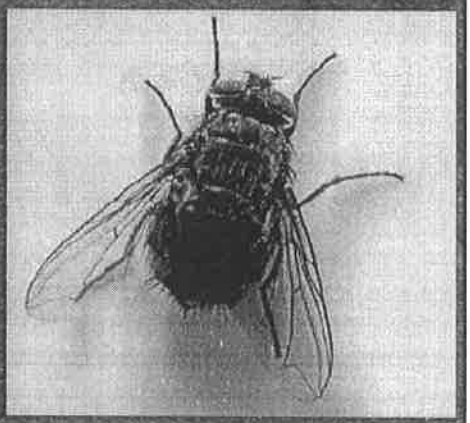
c $\left(\frac{m}{4}\right)^3 = \frac{m^3}{4^3}$
$= \frac{m^3}{64}$

d $\left(\frac{2w^3}{3}\right)^4 = \frac{(2w^3)^4}{3^4}$
$= \frac{2^4 \times (w^3)^4}{3^4}$
$= \frac{16w^{12}}{81}$

Just for the record

The house fly

The female common house fly, *Musca domestica*, can lay up to 1000 eggs at a time. In three weeks these reach maturity and are ready to breed. Huge populations would result if all the descendants of a single pair of house flies survived and reproduced. Fortunately, this is not the case as the mortality rate is very high. The few house flies we see are the true survivors.



Over the 13 weeks of summer, how many descendants could a single pair of house flies produce, assuming that each pair (original and descendants) mates only once?

(Give your answer in index form.)

Exercise 7-04

1 Simplify, giving your answers in index form:

a $(4^3)^2$ **b** $(5^2)^8$ **c** $(3^3)^4$ **d** $(2^7)^4$ **e** $(2^1)^2$

f $(9^4)^3$ **g** $(10^0)^2$ **h** $(6^4)^5$ **i** $(5^3)^5$ **j** $(2^5)^{10}$

k $(3^1)^5$ **l** $(7^3)^0$ **m** $(2^2)^{10}$ **n** $(13^2)^2$ **o** $(4^4)^4$

---

2 Simplify each of the following. Give your answers in index form.

a $(e^2)^4$ **b** $(t^5)^5$ **c** $(y^3)^7$ **d** $(c)^5$ **e** $(m^7)^5$

f $(y^4)^4$ **g** $(h^0)^6$ **h** $(p^6)^3$ **i** $(w^4)^1$ **j** $(x^1)^{10}$

k $(n^3)^8$ **l** $(d^3)^3$ **m** $(k^5)^{10}$ **n** $(d^3)^4$ **o** $(a^8)^8$

---

3 Simplify the following.

a $(2d)^4$ **b** $(5m)^2$ **c** $(4y^5)^2$ **d** $(3x^2)^4$ **e** $(5m^6)^5$

f $(2w^5)^3$ **g** $(10d^5)^4$ **h** $(3e^7)^3$ **i** $(2b^4)^1$ **j** $(6d^6)^2$

k $(3f^4)^5$ **l** $(2c^3)^{10}$ **m** $(3h^5)^4$ **n** $(6k)^2$ **o** $(8w^3)^2$

---

4 Simplify each of the following.

a $\left(\frac{e}{2}\right)^5$ **b** $\left(\frac{x}{7}\right)^2$ **c** $\left(\frac{3m}{2}\right)^3$ **d** $\left(\frac{5h}{6}\right)^2$

e $\left(\frac{f^2}{3}\right)^4$ **f** $\left(\frac{n^5}{p^2}\right)^8$ **g** $\left(\frac{w^2}{t^3}\right)^5$ **h** $\left(\frac{am}{c}\right)^4$

i $\left(\frac{2k^3}{5}\right)^2$ **j** $\left(\frac{3r^4}{c^2}\right)^2$ **k** $\left(\frac{a^2b}{d^5}\right)^4$ **l** $\left(\frac{5c^2}{3x^3}\right)^3$

---

5 Simplify the following, giving your answers in index form.

a $(m^3)^{10}$ **b** $(5f)^3$ **c** $(-2)^8$ **d** $(-x)^3$

e $(y^3)^{12}$ **f** $(4w^5)^4$ **g** $(-2d)^5$ **h** $(2^{10})^{10}$

i $(-3p^2)^3$ **j** $(-5m^3)^2$ **k** $(3f^5)^5$ **l** $(-m^2)^4$

---

6 Evaluate:

a $(2^3)^2$ **b** $(-3^2)^2$ **c** $(10^2)^3$ **d** $(-5)^2$

e $(-2)^3$ **f** $(-4^2)^3$ **g** $(-3^4)^2$ **h** $(-5^2)^3$

i $\left(\frac{3}{2}\right)^2$ **j** $\left(\frac{2}{5}\right)^2$ **k** $\left(\frac{5}{2}\right)^3$ **l** $\left(-\frac{3}{4}\right)^2$

---

7 Simplify each of the following.

a $(l^3m^5)^6$ **b** $(x^2y^4)^5$ **c** $\left(\frac{a^4}{d^3}\right)^7$ **d** $\left(-\frac{m^2}{n}\right)^4$

e $(-we^2k^3)^3$ **f** $\left(\frac{2y^3}{x^2}\right)^3$ **g** $\left(\frac{p^2q^3}{t^4}\right)^5$ **h** $(w^2ak^4)^8$

i $(-3m^2n)^5$ **j** $(-2p^2w^3)^4$ **k** $(a^2d^3y^5)^0$ **l** $\left(\frac{a^2c^4}{d^7}\right)^0$

m $\left(\frac{d^2e^5f}{4}\right)^3$ **n** $\left(-\frac{m^2n^3}{2y^5}\right)^5$ **o** $(4k^3m)^3$ **p** $(8k^4y^5)^2$

q $(3a^3df^4)^5$ **r** $(6d^5p^2)^4$ **s** $\left(-\frac{3ay^4}{b^2}\right)^5$ **t** $\left(\frac{kp^3}{3q^4}\right)^2$

---

8 Simplify:

a $(5^2)^x$ **b** $(5^x)^2$ **c** $((x+1)^2)^3$

d $(b^3)^4 \div (b^4)^2$ **e** $(b^3)^4 \times (b^4)^2$ **f** $(6n^4)^2 \times (3n^2)^2$

g $(6n^4)^2 \div (3n^2)^2$ **h** $\frac{(3x^5)^3}{(x^2)^4}$ **i** $\frac{5y^5 \times (2y)^5}{8y^2 \times (y^3)^2}$

Answers

Exercise 7-04

1 **a** 4^6 **b** 5^{16} **c** 3^{12} **d** 2^{28} **e** 2^2 **f** 9^{12}
g 10^0 **h** 6^{20} **i** 5^{15} **j** 2^{50} **k** 3^5 **l** 7^0
m 2^{20} **n** 13^4 **o** 4^{16}

2 **a** e^8 **b** t^{25} **c** y^{21} **d** c^5
e m^{35} **f** y^{16} **g** h^0 **h** p^{18}
i w^4 **j** x^{10} **k** n^{24} **l** d^9
m k^{50} **n** d^{12} **o** a^{64}

3 **a** $16d^4$ **b** $25m^2$ **c** $16y^{10}$ **d** $81x^8$
e $3125m^{30}$ **f** $8w^{15}$ **g** $10\,000d^{20}$ **h** $27e^{21}$
i $2b^4$ **j** $36d^{12}$ **k** $243f^{20}$ **l** $1024c^{30}$
m $81h^{20}$ **n** $36k^2$ **o** $64w^6$

4 **a** $\frac{e^5}{32}$ **b** $\frac{x^2}{49}$ **c** $\frac{27m^3}{8}$ **d** $\frac{25h^2}{36}$
e $\frac{f^8}{81}$ **f** $\frac{n^{40}}{p^{16}}$ **g** $\frac{w^{10}}{t^{15}}$ **h** $\frac{a^4m^4}{c^4}$
i $\frac{4k^6}{25}$ **j** $\frac{9r^8}{c^4}$ **k** $\frac{a^8b^4}{d^{20}}$ **l** $\frac{125c^6}{27x^9}$

5 **a** m^{30} **b** 5^3t^3 **c** 2^8 **d** $-x^3$
e y^{36} **f** 4^4w^{20} **g** -2^5d^5 **h** 2^{100}
i -3^3p^6 **j** 5^2m^6 **k** 3^5f^{25} **l** m^8

6 **a** 64 **b** 81 **c** 1 000 000 **d** 25
e -8 **f** -4096 **g** 6561 **h** -15 625
i $\frac{9}{4}$ **j** $\frac{4}{25}$ **k** $\frac{125}{8}$ **l** $\frac{9}{16}$

7 **a** $l^{18}m^{30}$ **b** $x^{10}y^{20}$ **c** $\frac{a^{28}}{d^{21}}$
d $\frac{m^8}{n^4}$ **e** $-w^3e^6k^9$ **f** $\frac{8y^9}{x^6}$
g $\frac{p^{10}q^{15}}{t^{20}}$ **h** $w^{16}a^8k^{32}$ **i** $-243m^{10}n^5$
j $16p^8w^{12}$ **k** 1 **l** 1
m $\frac{d^6e^{15}f^3}{64}$ **n** $-\frac{m^{10}n^{15}}{32y^{25}}$ **o** $64k^9m^3$
p $64k^8y^{10}$ **q** $243a^{15}d^5f^{20}$ **r** $1296d^{20}p^8$
s $-\frac{243a^5y^{20}}{b^{10}}$ **t** $\frac{k^2p^6}{9q^8}$

8 **a** 5^{2x} **b** 5^{2x} **c** $(x+1)^6$
d b^4 **e** b^{20} **f** $324n^{12}$
g $4n^4$ **h** $27x^7$ **i** $20y^2$

The zero index

Consider $5^3 \div 5^3 = \dfrac{5^3}{5^3}$

$$= \frac{\cancel{5}^{1} \times \cancel{5}^{1} \times \cancel{5}^{1}}{{}_{1}\cancel{5} \times \cancel{5}_{1} \times \cancel{5}_{1}}$$

$$= 1$$

But $5^3 \div 5^3 = 5^{3-3} = 5^0$

$\therefore 5^0 = 1$

Any number raised to the power of zero is equal to 1:

$$a^0 = 1$$

Proof:

$$a^m \div a^m = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \ldots \times \overset{1}{\cancel{a}}}{{}_{1}\cancel{a} \times \cancel{a}_{1} \times \cancel{a}_{1} \times \ldots \times \cancel{a}_{1}} \quad \begin{matrix} (m \text{ factors}) \\ (m \text{ factors}) \end{matrix}$$

$$= 1$$

but $a^m \div a^m = a^{m-m} = a^0$

$\therefore a^0 = 1$

Example 10

Simplify the following.

a 7^0 b $(-3)^0$ c m^0

Solution

a $7^0 = 1$ b $(-3)^0 = 1$ c $m^0 = 1$

Example 11

Simplify:

a $(ab)^0$ **b** $(5k)^0$

Solution

a $(ab)^0 = 1$ **b** $(5k)^0 = 1$

Example 12

Simplify:

a $5d^0$ **b** $(3y)^0 + 3y^0$

Solution

a $5d^0 = 5 \times d^0$
$= 5 \times 1$
$= 5$

b $(3y)^0 + 3y^0 = 1 + 3 \times y^0$
$= 1 + 3 \times 1$
$= 4$

Exercise 7-05

1 Simplify the following.

a 8^0 **b** $(-2)^0$ **c** d^0 **d** m^0

e $\left(\frac{2}{3}\right)^0$ **f** $(-6)^0$ **g** $(-700)^0$ **h** $(1\ 000\ 000)^0$

i $(-14)^0$ **j** $\left(\frac{5}{4}\right)^0$ **k** a^0 **l** $\left(-\frac{1}{2}\right)^0$

2 Simplify:

a $(km)^0$ **b** $(x^2y)^0$ **c** $(xyw)^0$ **d** $(-ab)^0$

e $\left(\frac{p}{q}\right)^0$ **f** $\left(\frac{3}{4}\right)^0$ **g** $(7y)^0$ **h** $(9cd)^0$

---

3 Simplify the following.

a $7^0 + 2^0$ **b** $3y^0$ **c** $-(4m)^0$ **d** $3 \times (5d)^0$

e $(5t^2)^0$ **f** $(6x)^0 + 2^0$ **g** $2m^0 + (2m)^0$ **h** $2w^0 \times 3p^0$

i $12u^0 \div 3$ **j** $3^2 \times 5^0$ **k** $(5a)^0 + 4$ **l** $8b^0 - (3b)^0$

m $6h^0 - (6h)^0$ **n** $-7c + 4c^0$ **o** $(3e^2)^0 - (10e)^0$ **p** $\left(\frac{1}{2}\right)^0 + \frac{1}{2}y^0$

q $100^0 - 1000^0$ **r** $3f^0 + 4 - (5f)^0$ **s** $36q^5 \div 12q^5$ **t** $(3x^3)^3 \div x^9$

u $60m^5n^3 \div 12mn^3$ **v** $12p^0 \div (2p)^0$ **w** $(a^2b^3)^0$ **x** $7 \times 4k^0$

Answers

Exercise 7-05

1 **a** 1 **b** 1 **c** 1 **d** 1 **e** 1 **f** 1
g 1 **h** 1 **i** 1 **j** 1 **k** 1 **l** 1
2 **a** 1 **b** 1 **c** 1 **d** 1 **e** 1 **f** 1
g 1 **h** 1
3 **a** 2 **b** 3 **c** -1 **d** 3 **e** 1 **f** 2
g 3 **h** 6 **i** 4 **j** 9 **k** 5 **l** 7
m 5 **n** -3 **o** 0 **p** $1\frac{1}{2}$ **q** 0 **r** 6
s 3 **t** 27 **u** $5m^4$ **v** 12 **w** 1 **x** 28

The negative index

Consider $2^4 \div 2^7 = \frac{2^4}{2^7}$

$$= \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times 2 \times 2 \times 2}$$

$$= \frac{1}{2 \times 2 \times 2}$$

$$= \frac{1}{2^3}$$

But $2^4 \div 2^7 = 2^{4-7}$

$$= 2^{-3}$$

$$\therefore 2^{-3} = \frac{1}{2^3}$$

A negative power or index gives a fraction (with numerator 1):

$$a^{-m} = \frac{1}{a^m}$$

Proof:

$$a^0 \div a^n = \frac{a^0}{a^n}$$

$$= \frac{1}{a^n} \quad (\text{since } a^0 = 1)$$

But $a^0 \div a^n = a^{0-n}$

$$= a^{-n}$$

$$\therefore a^{-n} = \frac{1}{a^n}$$

Example 13

Express using positive indices:

a 3^{-1} **b** 4^{-3} **c** k^{-5}

Solution

a $3^{-1} = \frac{1}{3^1}$

$$= \frac{1}{3}$$

b $4^{-3} = \frac{1}{4^3}$

c $k^{-5} = \frac{1}{k^5}$

Example 14

Express using positive indices:

a $3k^{-5}$ **b** a^2b^{-3} **c** $(5m)^{-2}$

Solution

a $3k^{-5} = 3 \times k^{-5}$

$= \frac{3}{1} \times \frac{1}{k^5} \quad (3 = \frac{3}{1})$

$= \frac{3}{k^5}$

b $a^2b^{-3} = a^2 \times b^{-3}$

$= \frac{a^2}{1} \times \frac{1}{b^3}$

$= \frac{a^2}{b^3}$

c $(5m)^{-2} = \frac{1}{(5m)^2}$

$= \frac{1}{25m^2}$

Example 15

Evaluate 2^{-3}, leaving your answer as a fraction.

Solution

$2^{-3} = \frac{1}{2^3}$

$= \frac{1}{2 \times 2 \times 2}$

$= \frac{1}{8}$

Negative powers of quotients

Consider $\left(\frac{2}{3}\right)^{-1} = \frac{1}{\frac{2}{3}}$

$= 1 \div \frac{2}{3}$

$= 1 \times \frac{3}{2}$

$= \frac{3}{2}$

and $\left(\frac{4}{5}\right)^{-2} = \frac{1}{\left(\frac{4}{5}\right)^2}$

$= \frac{1}{\frac{16}{25}}$

$= 1 \div \frac{16}{25}$

$= 1 \times \frac{25}{16}$

$= \frac{5^2}{4^2}$

$= \left(\frac{5}{4}\right)^2$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a} \quad \text{and} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$$

Proof:

$$\left(\frac{a}{b}\right)^{-1} = \frac{1}{\frac{a}{b}} \qquad \text{and} \qquad \left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^{n}}$$

$$= 1 \div \frac{a}{b} \qquad = 1 \div \frac{a^n}{b^n}$$

$$= 1 \times \frac{b}{a} \qquad = 1 \times \frac{b^n}{a^n}$$

$$= \frac{b}{a} \qquad = \frac{b^n}{a^n}$$

$$= \left(\frac{b}{a}\right)^{n}$$

Example 16

Simplify the following and evaluate if possible.

a $\left(\frac{4}{5}\right)^{-1}$ **b** $\left(\frac{3}{5}\right)^{-2}$ **c** $\left(\frac{2a}{b^2}\right)^{-3}$

Solution

a $\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$

$= 1\frac{1}{4}$

b $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{2}$

$= \frac{25}{9}$

$= 2\frac{7}{9}$

c $\left(\frac{2a}{b^2}\right)^{-3} = \left(\frac{b^2}{2a}\right)^{3}$

$= \frac{(b^2)^3}{(2a)^3}$

$= \frac{b^6}{8a^3}$

Exercise 7-06

1 Express using positive indices:

a 5^{-2} **b** 3^{-7} **c** 4^{-1} **d** 8^{-2}

e 10^{-4} **f** m^{-1} **g** h^{-3} **h** w^{-2}

i 20^{-4} **j** $(-11)^{-1}$ **k** k^{-8} **l** c^{-6}

---

2 Express using positive indices:

a $4d^{-1}$ **b** $3x^{-5}$ **c** $2d^{-3}$ **d** $4m^{-2}$ **e** ab^{-2}

f m^2n^{-4} **g** wy^{-2} **h** $4ac^{-1}$ **i** $3p^{-2}$ **j** $15kw^{-4}$

k $12y^2m^{-3}$ **l** $a^{-4}m^2$ **m** $d^{-3}y^3$ **n** $4xy^{-3}$ **o** $v^{-1}m^{-2}$

3 Write each of the following using positive indices.

a $(2m)^{-1}$ **b** $(xy)^{-1}$ **c** $(4h)^{-2}$ **d** $(5k)^{-3}$

e $(3h)^{-2}$ **f** $(4k)^{-3}$ **g** $(2c)^{-4}$ **h** $(8y)^{-1}$

---

4 Evaluate the following, leaving your answers as fractions.

a 3^{-2} **b** 4^{-3} **c** 6^{-1} **d** 7^{-2}

e 11^{-1} **f** 2^{-5} **g** 4^{-2} **h** 10^{-2}

5 Express using negative indices:

a $\frac{1}{m}$ **b** $\frac{1}{w}$ **c** $\frac{1}{8}$ **d** $\frac{1}{9}$ **e** $\frac{1}{2^2}$

f $\frac{1}{n^4}$ **g** $\frac{1}{3^4}$ **h** $\frac{1}{10^{-3}}$ **i** $\frac{1}{e^3}$ **j** $\frac{1}{t^2}$

k $\frac{2}{a}$ **l** $\frac{4}{t^2}$ **m** $\frac{2}{w^5}$ **n** $\frac{5}{d}$ **o** $\frac{1}{2y}$

p $\frac{1}{7e}$ **q** $\frac{1}{3a^2}$ **r** $\frac{5}{3m^4}$ **s** $\frac{1}{8p^3}$ **t** $\frac{2}{3k^6}$

6 Evaluate:

a $\left(\frac{1}{3}\right)^{-1}$ **b** $\left(\frac{1}{4}\right)^{-2}$ **c** $\left(\frac{2}{3}\right)^{-2}$ **d** $\left(\frac{2}{5}\right)^{-3}$

e $\left(\frac{2}{3}\right)^{-1}$ **f** $\left(\frac{3}{4}\right)^{-1}$ **g** $\left(\frac{1}{10}\right)^{-5}$ **h** $\left(\frac{5}{4}\right)^{-1}$

---

7 Simplify the following and evaluate if possible.

a $\left(\frac{4}{w}\right)^{-1}$ **b** $\left(\frac{m}{n}\right)^{-1}$ **c** $\left(\frac{1}{4}\right)^{-1}$ **d** $\left(\frac{4}{5}\right)^{-1}$

e $\left(\frac{k}{3}\right)^{-1}$ **f** $\left(\frac{x}{3}\right)^{-2}$ **g** $\left(\frac{a^2}{4}\right)^{-3}$ **h** $\left(-\frac{4}{3}\right)^{-2}$

i $\left(\frac{2d}{5}\right)^{-2}$ **j** $\left(-\frac{h^2}{m^3}\right)^{5}$ **k** $\left(\frac{a^2c^3}{4}\right)^{-3}$ **l** $\left(\frac{5d^2}{p^3}\right)^{-3}$

---

8 Simplify each of the following, using positive indices.

a $y^5 \times y^{-2}$ **b** $e^{-3} \times e^{7}$ **c** $m \times m^{-1}$ **d** $n^6 \times n^{-5}$ **e** $4g^3 \times 3g^{-1}$

f $5a^{-2} \times 6a^3$ **g** $5x^{-2} \times 2x$ **h** $30e^{-3} \times 2e^{-1}$ **i** $8p^{-1} \div 2p^2$ **j** $8q \div 2q^{-2}$

k $2r^{-2} \div 8r^{-1}$ **l** $2t^{-2} \div 8t^{-1}$ **m** $(h^{-1})^4$ **n** $(b)^{-3}$ **o** $(5x^{-1})^2$

9 Simplify the following and express your answers in positive index form.

a $x^3y^4 \times x^{-3}y^{-5}$ **b** $p^{-4}q^{-1} \times 5p^2q^{-3}$ **c** $(m^2n^3)^{-2}$

d $w^3p^5 \div w^5p^3$ **e** $m^2n^3 \div m^{-5}n^{-1}$ **f** $4a^3bc^2 \times -2a^{-5}b^{-3}c^{-2}$

g $8xy^3 \div 4x^2y^7$ **h** $(6m^4)^{-2} \times 9m^{-3}$ **i** $p^2q \times p^{-3}q^{-1} \div p^4q^3$

j $8a^3h^{-1} \div -4ah \div a^2h^3$ **k** $(a^2k^2)^{-3} \times (a^{-1}k^2)^{-2}$ **l** $4x^{-3}y^{-1} \div 8xy^3 \times 5x^{-1}$

m $4r^4t^{-3} \times 5r^{-5}t^4$ **n** $-15ab^{-2} \div 5a^{-1}b^{-3} \div -6ab^7$ **o** $(d^{-3}h^{-1})^{-1} \div -4d^3h^2$

p $(2v^3w^{-2})^5 \div 8v^2w^{-7}$ **q** $81a^{-3}e^{-4} \div (3a^2e^{-1})^4$ **r** $(c^{-1}d^{-3})^3 \div (c^{-1}d^2)^{-4}$

---

10 Evaluate the following, leaving your answers as fractions.

a $2^3 \times 2^{-4}$ **b** $(3^2)^{-3} \div (3^{-3})^3$ **c** $5^{-1} \div 2^{-1}$

d $3^{-2} \div 2^{-1} \times 6$ **e** $(4^{-2})^2 \div (2^{-2})^3$ **f** $3^2 \times (3^{-2})^2$

Answers

Exercise 7-06

1 **a** $\frac{1}{5^2}$ **b** $\frac{1}{3^7}$ **c** $\frac{1}{4}$ **d** $\frac{1}{8^2}$

e $\frac{1}{10^4}$ **f** $\frac{1}{m}$ **g** $\frac{1}{h^3}$ **h** $\frac{1}{w^2}$

i $\frac{1}{20^4}$ **j** $\frac{1}{-11}$ **k** $\frac{1}{k^8}$ **l** $\frac{1}{c^6}$

2 **a** $\frac{4}{d}$ **b** $\frac{3}{x^5}$ **c** $\frac{2}{d^3}$ **d** $\frac{4}{m^2}$

e $\frac{a}{b^2}$ **f** $\frac{m^2}{n^4}$ **g** $\frac{w}{y^2}$ **h** $\frac{4a}{c}$

i $\frac{3}{p^2}$ **j** $\frac{15k}{w^4}$ **k** $\frac{12y^2}{m^3}$ **l** $\frac{m^2}{a^4}$

m $\frac{y^3}{d^3}$ **n** $\frac{4x}{y^3}$ **o** $\frac{1}{vm^2}$

3 **a** $\frac{1}{2m}$ **b** $\frac{1}{xy}$ **c** $\frac{1}{(4h)^2}$ **d** $\frac{1}{(5k)^3}$

e $\frac{1}{(3h)^2}$ **f** $\frac{1}{(4k)^3}$ **g** $\frac{1}{(2c)^4}$ **h** $\frac{1}{8y}$

4 **a** $\frac{1}{9}$ **b** $\frac{1}{64}$ **c** $\frac{1}{6}$ **d** $\frac{1}{49}$

e $\frac{1}{11}$ **f** $\frac{1}{32}$ **g** $\frac{1}{16}$ **h** $\frac{1}{100}$

5 **a** m^{-1} **b** w^{-1} **c** 8^{-1} **d** 9^{-1} **e** 2^{-2}

f n^{-4} **g** 3^{-4} **h** 10^{-3} **i** e^{-3} **j** t^{-2}

k $2a^{-1}$ **l** $4t^{-2}$ **m** $2w^{-5}$ **n** $5d^{-1}$ **o** $(2y)^{-1}$

p $(7e)^{-1}$ **q** $(3a^2)^{-1}$ **r** $5(3m^4)^{-1}$ **s** $(8p^3)^{-1}$

t $2(3k)^{-6}$

6 **a** 3 **b** 16 **c** $\frac{9}{4} = 2\frac{1}{4}$

d $\frac{125}{8} = 15\frac{5}{8}$ **e** $\frac{3}{2} = 1\frac{1}{2}$ **f** $\frac{4}{3} = 1x\frac{1}{3}$

g 100 000 **h** $\frac{4}{5}$

7 **a** $\frac{w}{4}$ **b** $\frac{n}{m}$ **c** 4 **d** $\frac{5}{4}$ **e** $\frac{3}{k}$

f $\frac{9}{x^2}$ **g** $\frac{64}{a^6}$ **h** $\frac{9}{16}$ **i** $\frac{25}{4d^2}$ **j** $-\frac{m^{15}}{h^{10}}$

k $\frac{64}{a^6c^9}$ **l** $\frac{p^9}{125d^6}$

8 **a** y^3 **b** e^4 **c** 1 **d** n **e** $12g^2$

f $30a$ **g** $\frac{10}{x}$ **h** $\frac{60}{e^4}$ **i** $\frac{4}{p^3}$ **j** $4q^3$

k $\frac{r^3}{4}$ **l** $\frac{1}{4t}$ **m** $\frac{1}{h^4}$ **n** $\frac{1}{b^3}$ **o** $\frac{25}{x^2}$

9 **a** $\frac{1}{y}$ **b** $\frac{5}{p^2q^4}$ **c** $\frac{1}{m^4n^6}$ **d** $\frac{p^2}{w^2}$

e m^7n^4 **f** $\frac{-8}{a^2b^2}$ **g** $\frac{2}{xy^4}$ **h** $\frac{9}{m^{11}}$

i $\frac{1}{p^5q^3}$ **j** $\frac{-2}{h^5}$ **k** $\frac{1}{a^4k^{10}}$ **l** $\frac{5}{2x^5y^4}$

m $\frac{20t}{r}$ **n** $\frac{a}{2b^6}$ **o** $\frac{1}{4h}$ **p** $\frac{4y^{13}}{w^3}$

q $\frac{1}{a^{11}}$ **r** $\frac{c}{d^{14}}$

10 **a** $\frac{1}{2}$ **b** 27 **c** $\frac{2}{5}$ **d** $\frac{4}{3}$

e $\frac{1}{4}$ **f** $\frac{1}{9}$