Want more revision exercises? Get **MathsFit** - *New from projectmaths.*

10	8a	Assume that the population, P, of cane toads in Australia has been growing at a rate proportional to P. That is, $\frac{dP}{dt} = kP$, where k is a positive constant. There were 102 cane toads brought to Australia from Hawaii in 1935. Seventy-five years later, in 2010, it is estimated that there are 200 million cane toads in Australia. If the population continues to grow at this rate, how many cane toads will there be in Australia in 2035?	4

$\frac{dP}{dt} = kP$

Let $P = P_0e^{kt}$,

where P_0 = initial population

When $t = 0$, $P_0 = 102$

$\therefore P = 102e^{kt}$,

From 1935 to 2010 is 75 years.

Let $P = 200\,000\,000$, $t = 75$:

$P = 102e^{kt}$

$200\,000\,000 = 102e^{75k}$

$e^{75k} = \frac{200000000}{102}$

$75k = \log_e\left[\frac{200000000}{102}\right]$

$k = \frac{\log_e\left[\frac{200000000}{102}\right]}{75}$

$= 0.193184734 \ldots$

From 2010 to 2035 is another 25 yrs:

$\therefore$ let $t = 100$:

$P = 102e^{100k}$

$= 2.503 \ldots \times 10^{10}$

$\therefore$ the population will be approximately

2.5×10^{10} cane toads

State Mean: **2.30/4**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Graphs were often too small, and should be drawn around a third to a half page in size.

Candidates used a variety of methods to answer this question and a significant number scored full marks. Some of the methods used include:

(i) Using $P = P_0e^{kt}$, the solution of the differential equation. Candidates who used this method made the following errors: not being able to write 200 million as a number (writing it as 20000000, 2000000, and so on); writing P_0 as 200 million (in its variety of forms) and P as 102 when solving for k; inability to solve for k correctly, including using $\log_{10}(\ldots)$ not $\log_e(\ldots)$; using $t = 25$ instead of $t = 100$ when evaluating the population in 2035.

(ii) Solving for P using a geometric series $= 102r^n$. This method made the solution much easier because logarithms need not be used to solve this equation. However, some candidates who used this method incorrectly used $n = 74$ to determine r.

(iii) Some candidates solved the differential equation $\frac{dp}{dt} = kp$ by using the relationship $\frac{dp}{dt} = \frac{1}{\frac{dt}{dp}}$ and solving $\int \frac{dt}{dp} dp = \int \frac{1}{kp} dp$. This method was quite lengthy and involved more mathematical processes than the other methods.

This question differs from those in previous years in that the candidates were not asked to verify that $P = P_0 e^{kt}$ is a solution to $\frac{dp}{dt} = kp$. Because of this, many candidates chose inappropriate methods of solution.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/