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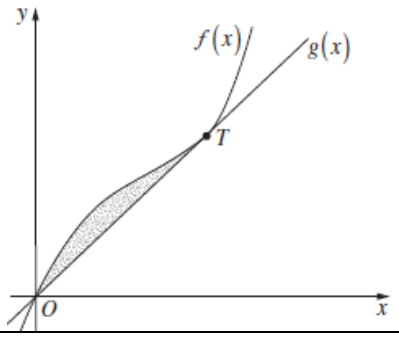
|---|---|---|---|
| **13** | **13 b** | The diagram shows the graphs of the functions $f(x) = 4x^3 - 4x^2 + 3x$ and $g(x) = 2x$.<br><br>The graphs meet at $O$ and at $T$.<br><br>(i) Find the $x$-coordinate of $T$.<br><br>(ii) Find the area of the shaded regions between the graphs of the functions $f(x)$ and $g(x)$.<br><br>  <br> | **1**<br><br>**3** |

(i) $4x^3 - 4x^2 + 3x = 2x$

$$4x^3 - 4x^2 + x = 0$$
$$x(4x^2 - 4x + 1) = 0$$
$$x(2x - 1)^2 = 0$$
$$x = 0, \frac{1}{2}$$

$\therefore$ $T$ has x-coordinate of $\frac{1}{2}$

(ii) $$\text{Area} = \int_0^{\frac{1}{2}} 4x^3 - 4x^2 + 3x - 2x \; dx$$
$$= \int_0^{\frac{1}{2}} 4x^3 - 4x^2 + x \; dx$$
$$= \left[x^4 - \frac{4x^3}{3} + \frac{x^2}{2}\right]_0^{\frac{1}{2}}$$
$$= \left(\frac{1}{2}\right)^4 - \frac{4\left(\frac{1}{2}\right)^3}{3} + \frac{\left(\frac{1}{2}\right)^2}{2} - 0$$
$$= \frac{1}{16} - \frac{1}{6} + \frac{1}{8}$$
$$= \frac{1}{48}$$

$\therefore$ area is $\frac{1}{48}$ units$^2$

State Mean:
**0.63/1**
**2.15/3**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

**Board of Studies: Notes from the Marking Centre**

(i) Most candidates could equate the two functions $f(x)$ and $g(x)$ and set up a quadratic equation, leading to a correct answer $x = \frac{1}{2}$.

Common problems were:

- not factorising and solving correctly
- differentiating the given function.

(ii) Most candidates integrated and used their $x$ value from part (b) (i) to achieve full marks for this part of the question. A small number subtracted $f(x)$ from $g(x)$, hence requiring the use of absolute values. Most candidates realised that the area had to be positive.

Common problems were:

- using the $y$-value in part (i) as their upper limit for integration (ie $x = 1$)
- differentiating instead of integrating.

**Source:** [http://www.boardofstudies.nsw.edu.au/hsc_exams/](http://www.boardofstudies.nsw.edu.au/hsc_exams/)