projectmaths

Apps of Calculus to Phys World – Exponential Growth & Decay

16 **13 c** A radioactive isotope of Curium has a half-life of 163 days. Solution
Initially there are 10 mg of Curium in a container.
The mass $M(t)$ in milligrams of Curium, after t days, is given by $M(t) = Ae^{-kt}$, where A and k are constants.

(i) State the value of A. **1**

(ii) Given that after 163 days only 5 mg of Curium remain, find the value of k. **3**

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15 **15 a** The amount of caffeine, C, in the human body decreases according to the equation Solution
$\frac{dC}{dt} = -0.14C$, where C is measured in mg and t is the time in hours.

(i) Show that $C = Ae^{-0.14t}$ is a solution to $\frac{dC}{dt} = -0.14C$, where A is a constant. **1**

When $t = 0$, there are 130 mg of caffeine in Lee's body.

(ii) Find the value of A. **1**

(iii) What is the amount of caffeine in Lee's body after 7 hours? **1**

(iv) What is the time taken for the amount of caffeine in Lee's body to halve? **2**

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14 **13 b** Solution
A quantity of radioactive material decays according to the equation $\frac{dM}{dt} = -kM$,
where M is the mass of the material in kg, t is the time in years and k is a constant.

(i) Show that $M = Ae^{-kt}$ is a solution to the equation, where A is a constant. **1**

(ii) The time for half of the material to decay is 300 years. If the initial amount of material is 20 kg, find the amount remaining after 1000 years. **3**

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13 **16 b** Trout and carp are types of fish. A lake contains a number of trout. At a certain time 10 carp are introduced into the lake and start eating the trout. As a consequence, the number of trout, N, decreases according to $N = 375 - e^{0.04t}$, where t is the time in months after the carp are introduced. Solution

The population of carp, P, increases according to $\frac{dP}{dt} = 0.02P$.

(i) How many trout were in the lake when the carp were introduced? **1**

(ii) When will the population of trout be zero? **1**

(iii) Sketch the number of trout as a function of time. **1**

(iv) When is the rate of increase of carp equal to the rate of decrease of trout? **3**

(v) When is the number of carp equal to the number of trout? **2**

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12 **14 c** Professor Smith has a colony of bacteria. Initially, there are 1000 bacteria. The number of bacteria, $N(t)$, after t minutes is given by $N(t) = 1000e^{kt}$. Solution

(i) After 20 minutes there are 2000 bacteria. **1**
Show that $k = 0.0347$ correct to four decimal places.

(ii) How many bacteria are there when $t = 120$? **1**

(iii) What is the rate of change of the number of bacteria per minute, when $t = 120$? **1**

(iv) How long does it take for the number of bacteria to increase from 1000 to 100 000? **2**

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11 **10 a** The intensity I, measured in watt/m^2, of a sound is given by $I = 10^{-12} \times e^{0.1L}$, where L is the loudness of the sound in decibels. Solution

(i) If the loudness of a sound at a concert is 110 decibels, find the intensity of the sound. Give your answer in scientific notation. **1**

(ii) Ear damage occurs if the intensity of a sound is greater than 8.1×10^{-9} watt/m^2. What is the maximum loudness of a sound so that no ear damage occurs? **2**

(iii) By how much will the loudness of a sound have increased if its intensity has doubled? **2**

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10 **8a** Assume that the population, P, of cane toads in Australia has been growing at a rate proportional to P. That is, $\frac{dP}{dt} = kP$, where k is a positive constant. **4** Solution

There were 102 cane toads brought to Australia from Hawaii in 1935.
Seventy-five years later, in 2010, it is estimated that there are 200 million cane toads in Australia.
If the population continues to grow at this rate, how many cane toads will there be in Australia in 2035?

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09 **6b** Radium decays at a rate proportional to the amount of radium present. That is, if $Q(t)$ is the amount of radium present at time t, then $Q = Ae^{-kt}$, where k is a positive constant and A is the amount present at $t = 0$. It takes 1600 years for an amount of radium to reduce by half. Solution

(i) Find the value of k. **2**

(ii) A factory site is contaminated with radium. The amount of radium on the site is currently three times the safe level. How many years will it be before the amount of radium reaches the safe level? **2**

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08 **5c** Light intensity is measured in lux. The light intensity at the surface of a lake is 6000 lux. The light intensity, I lux, a distance s metres below the surface of the lake is given by $I = Ae^{-ks}$ where A and k are constants. Solution

(i) Write down the value of A. **1**

(ii) The light intensity 6 metres below the surface of the lake is 1000 lux. Find the value of k. **2**

(iii) At what rate, in lux per metre, is the light intensity decreasing 6 metres below the surface of the lake? **2**

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07 **8a** One model for the number of mobile phones in use worldwide is the exponential growth model, $N = Ae^{kt}$, where N is the estimate for the number of mobile phones in use (in millions), and t is the time in years after 1 January 2008. Solution

(i) It is estimated that at the start of 2009, when $t = 1$, there will be 1600 million mobile phones in use, while at the start of 2010, when $t = 2$, there will be 2600 million. Find A and k. **3**

(ii) According to the model, during which month and year will the number of mobile phones in use first exceed 4000 million? **2**

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06 **6b** A rare species of bird lives only on a remote island. A mathematical model predicts that the bird population, P, is given by $P = 150 + 300e^{-0.05t}$ where t is the number of years after observations began. **Solution**

(i) According to the model, how many birds were there when observations began? **1**

(ii) According to the model, what will be the rate of change in the bird population ten years after observations began? **2**

(iii) What does the model predict will be the limiting value of the bird population? **1**

(iv) The species will become eligible for inclusion in the endangered species list when the population falls below 200. When does the model predict that this will occur? **2**