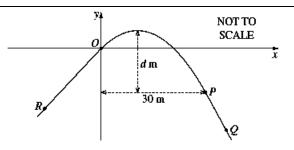
3

2

The diagram illustrates the design for part of a roller-coaster track. The section RO is a straight line with slope 1.2 and the section PQ is a straight line with slope -1.8. The section OP is a parabola $y = ax^2 + bx$. The horizontal distance from the y-axis to P is 30 m. In order that the ride is



smooth, the straight sections must be tangent to the parabola at O and at P.

- (i) Find the values of a and b so that the ride is smooth.
- (ii) Find the distance *d*, from the vertex of the parabola to the horizontal line through *P*, as shown on the diagram.

(i) $y = ax^2 + bx$ y' = 2ax + b

For section RO:

$$y' = 2ax + b = 1.2$$

Subs $x = 0$: $2a(0) + b = 1.2$
 $b = 1.2$

For section *PQ*:

For section PQ:

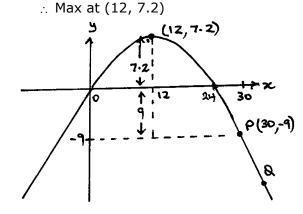
$$y' = 2ax + 1.2 = -1.8$$

Subs $x = 30$: $2a(30) + 1.2 = -1.8$
 $60a = -3$
 $a = -\frac{1}{20}$
 $= -0.05$

a = -0.05 and b = 1.2

(ii) Parabola: $y = -0.05x^2 + 1.2x$ y' = 2(-0.05)x + 1.2 = -0.1x + 1.2 = 0 for max -0.1x = -1.2x = 12

Subs
$$x = 12$$
 in $y = -0.05x^2 + 1.2x$
 $y = -0.05(12)^2 + 1.2(12)$
 $= -7.2 + 14.4$
 $= 7.2$



From diagram, at P, x = 30Subs x = 30 in $y = -0.05(30)^2 + 1.2(30)$ = -9 \therefore co-ords of P(30, -9) $\therefore d = 7.2 + |-9|$ = 16.2 $\therefore 16.2$ metres

Board of Studies: Notes from the Marking Centre

(i) This part of the question was either not attempted or was poorly done by many candidates. Many did not appear to see the relationship between the given gradients and the gradient of the curve at the point of contact of the two straight sections of track. Common errors included: finding the equation of the tangents using the given gradients and points; and creating equations which did not contain a or b, then trying to solve them using the parabola equation. Some candidates made careless mistakes with equations. For example, letting x = 0 in 2ax + b = 1.2 produced 2a + b = 1.2, or to progress from 2ax + b = 1.2 to ax + b = 0.6, or to progress from 60a = -3 to a = -20.

^{*} These solutions have been provided by projectmaths and are not supplied or endorsed by the Board of Studies

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(ii) Of the few candidates who were successful in the previous part, many were able to find the coordinates of the vertex of the parabola and the y-coordinate of the point on the parabola where x = 30 and hence calculate the required distance. Errors arose from candidates using 30 as the y-coordinate at P. Some candidates attempted to find the required distance using the distance formula or the perpendicular distance formula, but very few were successful.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/