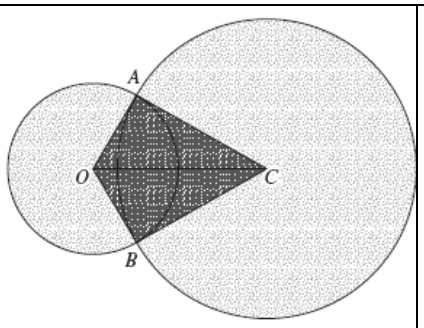
07 **4c**

An advertising logo is formed from two circles, which intersect as shown in the diagram. The circles intersect at *A* and *B* and have centres at *O* and *C*.

The radius of the circle centred at *O* is 1 metre and the radius of the circle centred at *C* is $\sqrt{3}$ metres. The length of *OC* is 2 metres.



Not to scale

(i) Use Pythagoras' theorem to show that $\angle OAC = \frac{\pi}{2}$. **1**

(ii) Find $\angle ACO$ and $\angle AOC$. **2**

(iii) Find the area of the quadrilateral *AOBC.* **1**

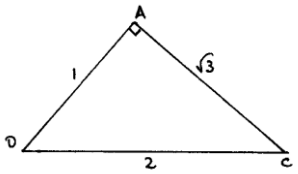
(iv) Find the area of the major sector *ACB*. **1**

(v) Find the total area of the logo (the sum of all the shaded areas). **2**

(i) Pythag theorem: $1^2 + (\sqrt{3})^2 = 1 + 3$
$= 4$
$= 2^2$

As Pythag Theorem holds,

then $\angle OAC = 90^\circ$ or $\frac{\pi}{2}$.



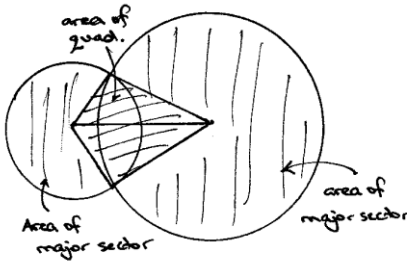
(ii) By trig, $\sin C = \frac{1}{2}$

$C = 30^\circ$ or $\frac{\pi}{6}$

$\therefore \angle ACO = 30^\circ$ or $\frac{\pi}{6}$

$\therefore \angle AOC = 60^\circ$ or $\frac{\pi}{3}$

(iii) Area = $2 \times$ Area of ΔAOC
$= 2 \times \frac{1}{2} \times 1 \times \sqrt{3}$
$= \sqrt{3}$ $\therefore$ area is $\sqrt{3}$ m^2



(iv) As $\angle ACO = \frac{\pi}{6}$

$\therefore \angle ACB = 2 \times \frac{\pi}{6} = \frac{\pi}{3}$

$\therefore$ reflex $\angle ACB = 2\pi - \frac{\pi}{3}$
$= \frac{5\pi}{3}$

Now, area of sector $= \frac{1}{2} r^2 \theta$
$= \frac{1}{2} \times (\sqrt{3})^2 \times \frac{5\pi}{3}$
$= \frac{5\pi}{2}$

$\therefore$ area of major sector *ACB* is $\frac{5\pi}{2}$ m^2

(v) Similarly, find the area of major sector *AOB*:

$\therefore$ reflex $\angle ACB = 2\pi - \frac{2\pi}{3}$
$= \frac{4\pi}{3}$

Now, area of sector $= \frac{1}{2} \times 1^2 \times \frac{4\pi}{3}$
$= \frac{2\pi}{3}$

$\therefore$ area of major sector *ACB* is $\frac{2\pi}{3}$ m^2

Area of logo = Area of quad + areas of major sectors
$= \sqrt{3} + \frac{5\pi}{2} + \frac{2\pi}{3}$
$= \sqrt{3} + \frac{19\pi}{6}$

$\therefore$ area of logo is $(\sqrt{3} + \frac{19\pi}{6})$ m^2

Board of Studies: Notes from the Marking Centre

(i) This caused problems because weaker responses were fixated on finding the angle, often using the cosine rule. There were many poor attempts at a mathematically correct Pythagoras proof with problems using surds. In the poorer responses, candidates had difficulty applying Pythagoras's theorem and in using surds.

(iii) Weaker responses omitted this part.

(iv) Many candidates did not find the major sector but then they often found it correctly in part (v). Both the names and formulas for parts of the circle seemed to be interchangeable and some invented their own terms such as 'slither'.

(v) Many candidates did not realise that the earlier parts were leading to this final answer. Concept of logo or 'shaded' area was lost on many candidates. Those responses that included a diagram showing the parts that needed to be found enjoyed a high level of success.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/