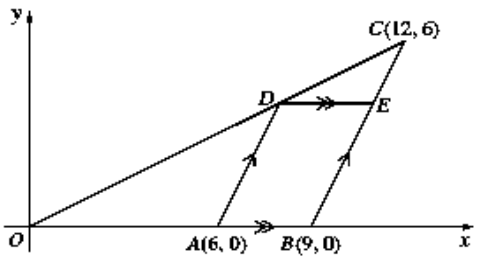
05 **3c** In the diagram, A, B and C are the points (6, 0), (9, 0) and (12, 6) respectively. The equation of the line OC is $x - 2y = 0$. The point D on OC is chosen so that AD is parallel to BC. The point E on BC is chosen so that DE is parallel to the x-axis.



NOT TO SCALE

(i) Show that the equation of the line AD is $y = 2x - 12$. **2**

(ii) Find the coordinates of the point D. **2**

(iii) Find the coordinates of the point E. **1**

(iv) Prove that $\Delta OAD \,|||\, \Delta DEC$ **2**

(v) Hence, or otherwise, find the ratio of the lengths AD and EC. **1**

i. AD is parallel to BC.

First find the gradient of BC:

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}, \; B(9, 0) \text{ and } C(12, 6)$$
$$= \frac{6 - 0}{12 - 9}$$
$$= \frac{6}{3}$$
$$= 2$$

$y - y_1 = m(x - x_1)$, with (6, 0), grad 2

$y - 0 = 2(x - 6)$

$y - 0 = 2x - 12$

$y = 2x - 12$

ii. Solve the equations for OC and AD simult:

$x - 2y = 0$ **1**

$y = 2x - 12$ **2**

Subs equation **1** into **2**:

$x - 2(2x - 12) = 0$

$x - 4x + 24 = 0$

$-3x = -24$

$x = 8$

Subs $x = 8$ into $y = 2x - 12$

$= 2(8) - 12$

$= 4$ $\therefore D(8, 4)$

iii. As ADEB is parallelogram and D is 2 to the right and 4 up from A, then E is (11, 4) as it too is '2 to the right and 4 up from' B.

$\therefore E(11, 4)$

Or, find the equation of BC using (9, 0) and gradient 2:

$y - y_1 = m(x - x_1)$

$y - 0 = 2(x - 9)$

$y - 0 = 2x - 18$

$y = 2x - 18$

Now substitute y = 4 in $y = 2x - 18$

$4 = 2x - 18$

$2x = 22$

$x = 11$

$\therefore E(11, 4)$

iv. In Δs OAD and DEC,

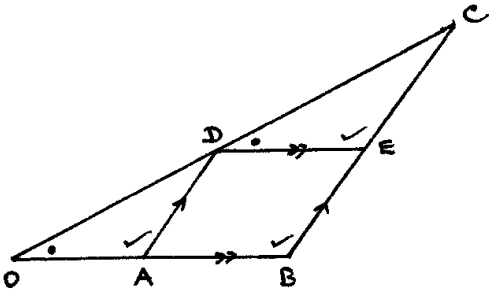
$\angle DOA = \angle CDE$ (corr $\angle$s, $DE \,||\, AB$)

Also $\angle OAD = \angle OBC$ (corr $\angle$s, $AD \,||\, BC$)

And $\angle DEC = \angle OBC$ (corr $\angle$s, $AB \,||\, DE$)

$\therefore \angle OAD = \angle DEC$

$\therefore \Delta OAD \,|||\, \Delta DEC$ (2 angles equal)



v. As OA = 6 units & DE = AB = 3 units, then OA : DE = 2:1

$\therefore$ AD:DC = 2:1

(matching sides of sim Δs in proportion)

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates calculated the gradient or the equation of BC and used the point-gradient formula to determine the equation of AD.

(ii) This question was successfully answered by the majority of candidates. However, some candidates made errors in solving the two simultaneous equations.

(iii) A number of candidates did not use the opposite sides of a parallelogram to determine point E. The majority of candidates calculated the equation of BC and then substituted $y = 4$ to determine point E (11, 4).

(iv) The better responses to this question presented a logical argument with appropriate theorems to justify their statements. However, common errors included not specifying the parallel lines, incorrectly stating the angle, or not determining accurately the ratio of the sides. Candidates who attempted to show the sides are in proportion needed to show three pairs of corresponding sides in proportion.

(v) Some candidates did not present their answer as a ratio or fraction. It was also common for candidates to incorrectly calculate the lengths using the distance formula. A number of candidates followed $\frac{AD}{EC} = \frac{2}{1}$ with remarks such as the ratio is 1:2 or $EC = 2 \times AD$.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/