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2016 14 b A gardener develops an eco-friendly spray that will kill harmful insects on fruit trees without contaminating the fruit. A trial is to be conducted with 100 000 insects. The gardener expects the spray to kill 35% of the insects each day and exactly 5000 new insects will be produced each day.

The number of insects expected at the end of the nth day of the trial is A_n.

(i) Show that $A_2 = 0.65(0.65 \times 100\,000 + 5000) + 5000$. **2**

(ii) Show that $A_n = 0.65^n \times 100\,000 + 5000\dfrac{(1-0.65^n)}{0.35}$. **1**

(iii) Find the expected insect population at the end of the fourteenth day, correct to the nearest 100. **1**

(i) As 35% of insects die, then 65% of insects survive.

$\therefore\ A_1 = 0.65 \times 100\,000 + 5000$

$A_2 = 0.65 \times A_1 + 5000$

$= 0.65(0.65 \times 100\,000 + 5000) + 5000$

State Mean: **1.65**

(ii) $A_2 = 0.65^2 \times 100\,000 + 0.65 \times 5000 + 5000$

$= 0.65^2 \times 100\,000 + 5000(1 + 0.65)$

$\therefore\ A_n = 0.65^n \times 100\,000 + 5000(1 + 0.65 + 0.65^2 + \ldots + 0.65^{n-1})$

Using a geometric sum with $a = 1$, $r = 0.65$, $n = n$, $S_n = \dfrac{a(1-r^n)}{1-r}$:

$$A_n = 0.65^n \times 100\,000 + 5000\left[\frac{1(1-0.65^n)}{1-0.65}\right]$$

$$= 0.65^n \times 100\,000 + 5000\frac{(1-0.65^n)}{0.35}$$

State Mean: **0.60**

(iii) Let $n = 14$:

$$A_{14} = 0.65^{14} \times 100\,000 + 5000\frac{(1-0.65^{14})}{0.35}$$

$= 14\,491.70147\ldots$

$= 14\,500$ (nearest hundred)

$\therefore$ the expected population of 14 500.

State Mean: **0.84**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

BOSTES: Notes from the Marking Centre

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