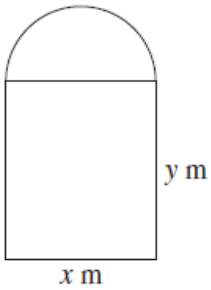
2014 16c The diagram shows a window consisting of two sections.
The top section is a semicircle of diameter x m. The bottom section is a rectangle of width x m and height y m.
The entire frame of the window, including the piece that separates the two sections, is made using 10 m of thin metal.
The semicircular section is made of coloured glass and the rectangular section is made of clear glass.
Under test conditions the amount of light coming through one square metre of the coloured glass is 1 unit and the amount of light coming through one square metre of the clear glass is 3 units. The total amount of light coming through the window under test conditions is L units.



(i) Show that $y = 5 - x\left(1 + \frac{\pi}{4}\right)$. **2**

(ii) Show that $L = 15x - x^2\left(3 + \frac{5\pi}{8}\right)$. **2**

(ii) Find the values of x and y that maximise the amount of light coming through the window under test conditions. **3**

(i) $2x + 2y + \frac{1}{2} \times \pi \times x = 10$

$4x + 4y + \pi x = 20$

$4y = 20 - x(4 + \pi)$

$y = 5 - x(1 + \frac{\pi}{4})$

(ii) $A = xy + \frac{1}{2} \times \pi \times \left(\frac{x}{2}\right)^2$

$L = 3xy + \frac{\pi x^2}{8}$

$= 3x(5 - x(1 + \frac{\pi}{4})) + \frac{\pi x^2}{8}$

$= 3x(5 - x - \frac{\pi x}{4}) + \frac{\pi x^2}{8}$

$= 15x - 3x^2 - \frac{3\pi x^2}{4} + \frac{\pi x^2}{8}$

$= 15x - 3x^2 - \frac{5\pi x^2}{8}$

$= 15x - x^2(3 + \frac{5\pi}{8})$

(iii) $L = 15x - x^2(3 + \frac{5\pi}{8})$

$\frac{dL}{dx} = 15 - 2x(3 + \frac{5\pi}{8}) = 0$

$2x(3 + \frac{5\pi}{8}) = 15$

$x = \frac{15}{2(3 + \frac{5\pi}{8})}$

$= \frac{15}{6 + \frac{5\pi}{4}}$

$= \frac{60}{24 + 5\pi}$

$\frac{d^2L}{dx^2} = -2(3 + \frac{5\pi}{8}) < 0 \quad \therefore$ maximum

$\therefore y(\frac{60}{24 + 5\pi}) = 5 - (\frac{60}{24 + 5\pi})(1 + \frac{\pi}{4})$

$= 5 - (\frac{60 + 15\pi}{24 + 5\pi})$

$= \frac{120 + 25\pi - 60 - 15\pi}{24 + 5\pi}$

$= \frac{60 + 10\pi}{24 + 5\pi}$

$\therefore x = \frac{60}{24 + 5\pi}, y = \frac{60 + 10\pi}{24 + 5\pi}$

State Mean:
1.09
0.69
1.15

Board of Studies: Notes from the Marking Centre

(i) Most candidates were able to find an expression for the length of the frame and then show appropriate logical steps to obtain the required result.

Common problems were:

- ignoring the instruction 'show' and moving directly from a correct expression for the length of the frame to the given answer;
- not being able to find the correct expression for the arc length part of the frame.

(ii) This part was often omitted or poorly attempted by candidates

Common problems were:

- incorrectly squaring the $\frac{x}{2}$ value used for the radius of the semi-circle;
- not understanding how to apply the 'units of light'
- algebraic errors when expanding $3[5x - x^2(1 + \frac{\pi}{4})]$.

(iii) Most candidates realised that this part could be attempted by using a standard calculus maxima process.

Common problems were:

- incorrectly expanding terms in $L = 15x - x^2(3 + \frac{5\pi}{8})]$;
- poor attempts at simplifying the expression for the value of x;
- finding the maximum light that came through the window rather than the dimensions of the frame that allowed for maximum light to pass through;
- omitting one of the required maxima tests or not evaluating both dimensions of the frame;
- stating a value for y (often incorrect) without showing necessary working.

http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths.pdf