12	**14a**	A function is given by $f(x) = 3x^4 + 4x^3 - 12x^2$. (i) Find the nature of the stationary points of $f(x)$ and determine their nature. (ii) Hence, sketch the graph of $y = f(x)$ showing the stationary points. (iii) For what values of x is the function increasing? (iv) For what values of k will $3x^4 + 4x^3 - 12x^2 + k = 0$ have no solution?	 **3** **2** **1** **1**

(i) $f'(x) = 12x^3 + 12x^2 - 24x = 0$

$12x(x^2 + x - 2) = 0$

$12x(x + 2)(x - 1) = 0$

$\therefore$ Stat points at $x = 0, -2, 1$

$f(0) = 3(0)^4 + 4(0)^3 - 12(0)^2 = 0 \quad \therefore (0, 0)$

$f(-2) = 3(-2)^4 + 4(-2)^3 - 12(-2)^2 = -32 \quad \therefore (-2, -32)$

$f(1) = 3(1)^4 + 4(1)^3 - 12(1)^2 = -5 \quad \therefore (1, -5)$

$\therefore$ Stat points at $(0, 0), (-2, -32), (1, -5)$

$f''(x) = 36x^2 + 24x - 24$

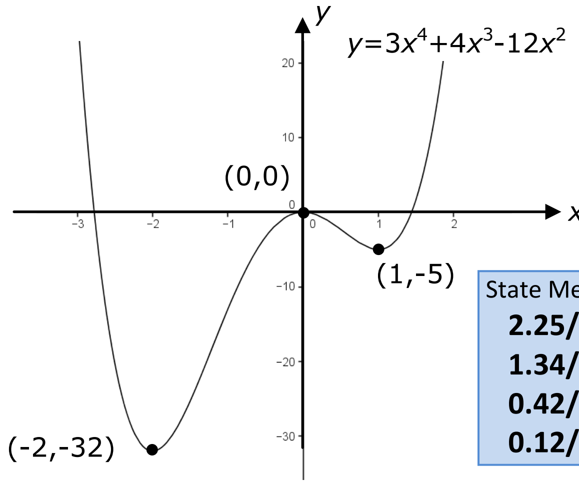
For $(0, 0)$:
$f''(0) = 36(0)^2 + 24(0) - 24 < 0 \quad \therefore$ Max $(0, 0)$

For $(-2, -32)$:
$f''(-2) = 36(-2)^2 + 24(-2) - 24 > 0 \quad \therefore$ Min $(-2, -32)$

For $(1, -5)$:
$f''(1) = 36(1)^2 + 24(1) - 24 > 0 \quad \therefore$ Min $(1, -5)$

$\therefore$ Max $(0, 0)$, Min $(-2, -32)$, Min $(1, -5)$

(ii)



State Mean:
2.25/3
1.34/2
0.42/1
0.12/1

(iii) From (ii), function increasing
$-2 < x < 0$ and $x > 1$

(iv) $3x^4 + 4x^3 - 12x^2 + k = 0$
$3x^4 + 4x^3 - 12x^2 = -k$
Solution is point(s) of intersection of $y = 3x^4 + 4x^3 - 12x^2$ and $y = -k$.
As $-k = -32$, therefore $k = 32$.
No solution when $k > 32$.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most responses were well set out, allowing part marks to be awarded when small errors were made. In most responses, candidates specified that $y' = 0$ but a significant number failed to correctly factorise to find the solutions. A common error was to recognise $12x$ as a common factor, but leave the constant term as 12 in the remaining factor. This led to three stationary points that could not be graphed consistently.
In most responses, candidates used the sign of y'' to ascertain the nature of the stationary points. However, in some responses candidates used the gradient to the left of and right of the stationary points to determine their nature.

(ii) Most candidates who correctly answered the first part produced suitable sketches in their response. It was necessary to either label the stationary points, or to clearly label the axes to identify their location.

Candidates who made errors in the first part were presented with data that could not be graphed consistently. In many responses, candidates, rather than finding and correcting these error(s), chose to draw extremely elaborate relations in an attempt to satisfy the information they had found.

(iii) Candidates who did not solve (i) but had made an attempt at sketching (ii) could obtain the mark, subject to use of correct inequalities.

(iv) Few candidates apprehended the effect of the constant k. Despite the function being a quartic equation, a majority of candidates attempted to use quadratic discriminant to resolve the question. In many responses, candidates achieved the mark by finding a correct k for their incorrect graph from part (ii).

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/