projectmaths

Applications of Calculus to the Physical World – Kinematics (x, v, a)

16 **16 a** A particle moves in a straight line. Solution

Its velocity v ms^{-1} at time t seconds is given by $v = 2 - \dfrac{4}{t+1}$.

(i) Find the initial velocity. **1**

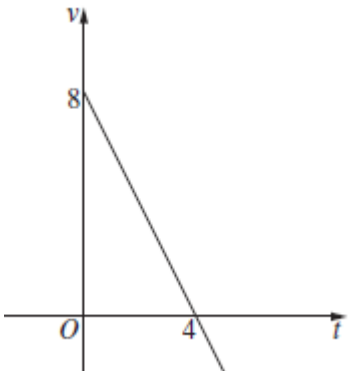
(ii) Find the acceleration of the particle when the particle is stationary. **2**

(iii) By considering the behavior of v for large t, sketch a graph of v against t for $t \geq 0$, showing any intercepts. **2**

(iv) Find the exact distance travelled by the particle in the first 7 seconds. **3**

---

15 **9** A particle is moving along the x-axis. The graph shows its velocity v metres per second at time t seconds. When $t = 0$ the displacement x is equal to 2 metres. What is the maximum value of the displacement x? **1** Solution

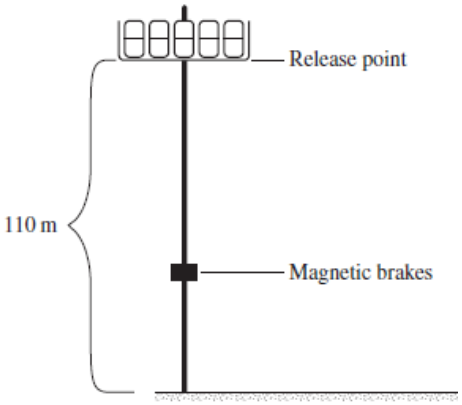


(A) 8 m (B) 14 m

(C) 16 m (D) 18 m

---

15 **14 a** In a theme park ride, a chair is released from a height of 110 metres and falls vertically. Magnetic brakes are applied when the velocity of the chair reaches −37 metres per second. The height of the chair at time t seconds is x metres. The acceleration of the chair is given by $\ddot{x} = -10$. At the release point, $t = 0$, $x = 110$ and $\dot{x} = 0$. Solution



Not to scale

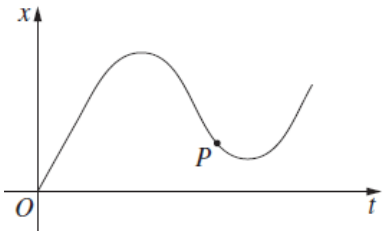
(i) Using calculus, show that $x = -5t^2 + 110$. **2**

(ii) How far has the chair fallen when the magnetic brakes are applied? **2**

---

14 **9** The graph shows the displacement x of a particle moving along a straight line as a function of time t. **1** Solution

Which statement describes the motion of the particle at the point P?



(A) The velocity is negative and the acceleration is positive.

(B) The velocity is negative and the acceleration is negative.

(C) The velocity is positive and the acceleration is positive.

(D) The velocity is positive and the acceleration is negative.

---

14 **13 c** Solution

The displacement of a particle moving along the x-axis is given by $x = t - \frac{1}{1+t}$, where x is the displacement from the origin in metres, t is the time in seconds, and $t \geq 0$.

(i) Show that the acceleration of the particle is always negative. **2**

(ii) What value does the velocity approach as t increases indefinitely? **1**

---

13 **14 a** Solution

The velocity of a particle moving along the x-axis is given by $\dot{x} = 10 - 2t$, where x is the displacement from the origin in metres and t is the time in seconds. Initially the particle is 5 metres to the right of the origin.

(i) Show that the acceleration of the particle is constant. **1**

(ii) Find the time when the particle is at rest. **1**

(iii) Show that the position of the particle after 7 seconds is 26 metres to the right of the origin. **2**

(iv) Find the distance travelled by the particle during the first 7 seconds. **2**

---

12 **15 b** Solution

The velocity of a particle is given by $\dot{x} = 1 - 2\cos t$, where x is the displacement in metres and t is the time in seconds. Initially the particle is 3 m to the right of the origin.

(i) Find the initial velocity of the particle. **1**

(ii) Find the maximum velocity of the particle. **1**

(iii) Find the displacement, x, of the particle in terms of t. **2**

(iv) Find the position of the particle when it is at rest for the first time. **2**

---

11 **7b** Solution

The velocity of a particle moving along the x-axis is given by $\dot{x} = 8 - 8e^{-2t}$, where t is the time in seconds and x is the displacement in metres.

(i) Show that the particle is initially at rest. **1**

(ii) Show that the acceleration of the particle is always positive. **1**

(iii) Explain why the particle is moving in the positive direction for all $t > 0$. **2**

(iv) As $t \to \infty$, the velocity of the particle approaches a constant. Find the value of this constant. **1**

(v) Sketch the graph of the particle's velocity as a function of time. **2**

---

10 **7a** Solution

The acceleration of a particle is given by $\ddot{x} = 4\cos 2t$, where x is displacement in metres and t is time in seconds. Initially the particle is at the origin with a velocity of 1 m s^{-1}.

(i) Show that the velocity of the particle is given by $\dot{x} = 2\sin 2t + 1$. **2**

(ii) Find the time when the particle first comes to rest. **2**

(iii) Find the displacement, x, of the particle in terms of t. **2**

---

09 **7a** Solution

The acceleration of a particle is given by $\ddot{x} = 8e^{-2t} + 3e^{-t}$, where x is displacement in metres and t is time in seconds.
Initially its velocity is -6 ms^{-1} and its displacement is 5 m.

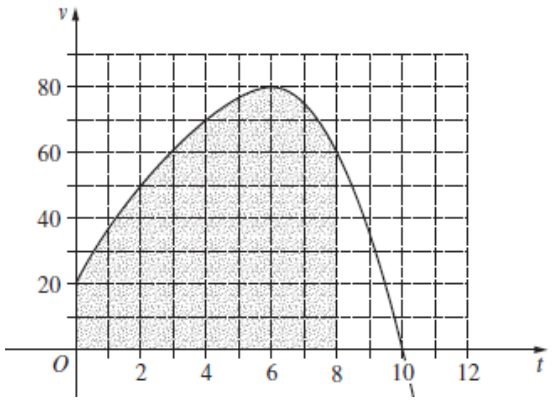
(i) Show that the displacement of the particle is given by $x = 2e^{-2t} + 3e^{-t} + t$. **2**

(ii) Find the time when the particle comes to rest. **3**

(iii) Find the displacement when the particle comes to rest. **1**

---

08 **6b** The graph shows the velocity of a particle, v metres per second, as a function of time, t seconds.



(i) What is the initial velocity of the particle? **1**

(ii) When is the velocity of the particle equal to zero? **1**

(iii) When is the acceleration of the particle equal to zero? **1**

(iv) By using Simpson's Rule with five function values, estimate the distance travelled by the particle between $t = 0$ and $t = 8$. **3**

Solution

---

07 **5b** A particle is moving on the x-axis and is initially at the origin. Its velocity, v metres per second, at time t seconds is given by $v = \dfrac{2t}{16+t^2}$.

(i) What is the initial velocity of the particle? **1**

(ii) Find an expression for the acceleration of the particle. **2**

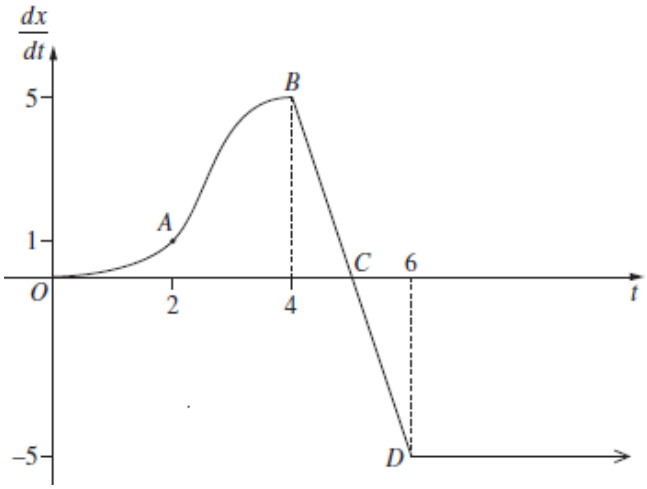
(iii) Find the time when the acceleration of the particle is zero. **1**

(iv) Find the position of the particle when $t = 4$. **3**

Solution

---

07 **10 a** An object is moving on the x-axis. The graph shows the velocity, $\dfrac{dx}{dt}$, of the object, as a function of time, t. The coordinates of the points shown on the graph are A(2, 1), B(4, 5), C(5, 0) and D(6, -5). The velocity is constant for $t \geq 6$.



(Not to scale)

(i) Using Simpson's rule, estimate the distance travelled between $t = 0$ and $t = 4$. **2**

(ii) The object is initially at the origin. During which time(s) is the displacement of the object decreasing? **1**

(iii) Estimate the time at which the object returns to the origin. Justify your answer. **2**

(iv) Sketch the displacement, x, as a function of time. **1**

Solution

---

06 **8a** A particle is moving in a straight line. Its displacement, x metres, from the origin, O, at time t seconds, where $t \geq 0$, is given by $x = 1 - \dfrac{7}{t+4}$.

(i) Find the initial displacement of the particle. **1**

(ii) Find the velocity of the particle as it passes through the origin. **3**

(iii) Show that the acceleration of the particle is always negative. **1**

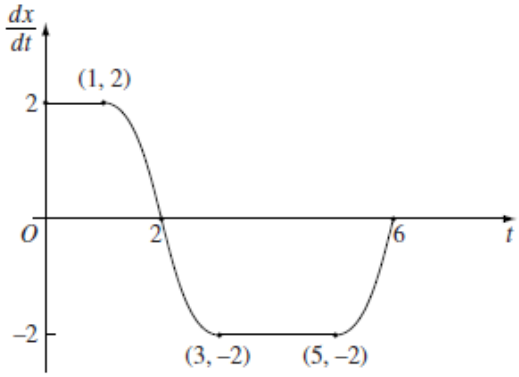
(iv) Sketch the graph of the displacement of the particle as a function of time. **2**

Solution

---

05 7b Solution

The graph shows the velocity, $\frac{dx}{dt}$, of a particle as a function of time. Initially the particle is at the origin.



(i) At what time is the displacement, x, from the origin a maximum? **1**

(ii) At what time does the particle return to the origin? Justify your answer. **2**

(iii) Draw a sketch of the acceleration, $\frac{d^2x}{dt^2}$, as a function of time for $0 \le t \le 6$. **2**

05 9a Solution

A particle is initially at rest at the origin. Its acceleration as a function of time, t, is given by $\ddot{x} = 4\sin 2t$.

(i) Show that the velocity of the particle is given by $\dot{x} = 2 - 2\cos 2t$. **2**

(ii) Sketch the graph of the velocity for $0 \le t \le 2\pi$ AND determine the time at which the particle first comes to rest after $t = 0$. **3**

(iii) Find the distance travelled by the particle between $t = 0$ and the time at which the particle first comes to rest after $t = 0$. **2**