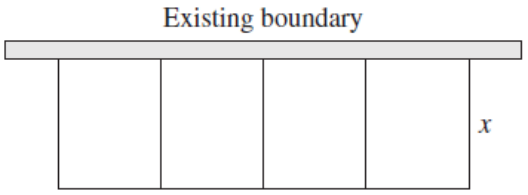
2016 14 c A farmer wishes to make a rectangular enclosure of area 720 m^2. She uses an existing straight boundary as one side of the enclosure. She uses wire fencing for the remaining three sides and also to divide the enclosure into four equal rectangular areas of width x m as shown.



1

(i) Show that the total length, ℓ m, of the wire fencing is given by $\ell = 5x + \frac{720}{x}$. **3**

(ii) Find the minimum length of wire fencing required, showing why this is the minimum length.

(i) Let the length of the enclosure be y m.

$\therefore$ Area $= xy = 720$

$$\therefore y = \frac{720}{x}$$

$\therefore \ \ell = 5x + y$

$$= 5x + \frac{720}{x}$$

State Mean: **0.67**

(ii) $\ell = 5x + 720x^{-1}$

$$\frac{d\ell}{dx} = 5 - 720x^{-2}$$

$$= 5 - \frac{720}{x^2} = 0$$

$$5x^2 = 720$$

$$x^2 = 144$$

$$x = 12 \ \ (\text{as } x > 0)$$

$$\frac{d^2\ell}{dx^2} = 1440x^{-3}$$

As $\frac{d^2\ell}{dx^2} > 0$, for all positive x, then minimum.

Subs $x = 12$:

$$\ell = 5(12) + \frac{720}{12}$$

$$= 120$$

$\therefore$ the minimum length is 120 metres.

State Mean: **1.62**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

BOSTES: Notes from the Marking Centre

This information is released by BOSTES in late Term 1 2017.