11	5a	The number of members of a new social networking site doubles every day. On Day 1 there were 27 members and on Day 2 there were 54 members.	
		(i) How many members were there on Day 12?	1
		(ii) On which day was the number of members first greater than 10 million?	2
		(iii) The site earns 0.5 cents per member per day. How much money did the site earn in the first 12 days? Give your answer to the nearest dollar.	2

A geometric series: 27, 54, 108, ...

$a = 27, r = 2$

(i) $n = 12, \ T_n = ar^{n-1}$

$$T_{12} = 27 \times 2^{11}$$

$$= 55\,296$$

∴ 55 296 members

(ii)   Let $T_n = 10\,000\,000$

$$10\,000\,000 = 27 \times 2^{n-1}$$

$$2^{n-1} = \frac{10000000}{27}$$

$$\log_e 2^{n-1} = \log_e \frac{10000000}{27}$$

$$(n-1)\log_e 2 = \log_e \frac{10000000}{27}$$

$$n - 1 = \frac{\log_e \dfrac{10000000}{27}}{\log_e 2}$$

$$n - 1 = 18.498 \ldots$$

$$n = 19.498 \ldots$$

∴ on the 20th day the number exceeds 10 million

(iii) $n = 12, \ S_n = \dfrac{a(r^n - 1)}{r - 1}$

$$S_{12} = \frac{27(2^{12} - 1)}{2 - 1}$$

$$= 110\,565$$

Total money $= 110\,565 \times \$0.005$

$$= \$552.825$$

$$= \$553 \text{ (nearest \$)}$$

State Mean:
0.86/1
1.09/2
0.87/2

Board of Studies: Notes from the Marking Centre

With the exception of part (i), this was a challenging question. Many responses used an arithmetic series. Common errors in all parts were to mix the formulae for geometric term and geometric sum, or to misquote formulae. Listing all terms was common and many successfully found all answers this way, but at a large cost in time.

(i) In better responses, the formula for T_n was used with the substitution clearly shown.

(ii) In better responses, again, the formula for T_n was used to form an equation or inequality that was successfully solved. 10 million was often incorrectly interpreted, commonly as 1 000 000. Many elementary errors were made when solving the equation and many could not correctly interpret the inequality to obtain day 20. Success from guess-and-check was common, but candidates are reminded to show the guesses and not just give the final answer.

(iii) In better responses the formula for the sum of the series was used to calculate the total membership after 12 days, followed by the calculation of the money earned. Common errors included finding the amount earned for the 12th day, using incorrect values of r (usually 0.5), calculating an answer in cents but calling it dollars or not converting to dollars at all.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/