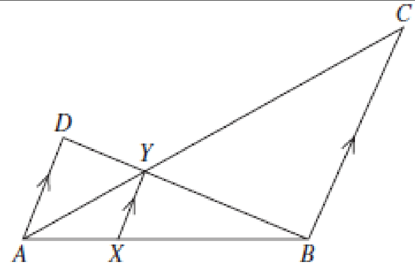
13 | **16 c**

The diagram shows triangles ABC and ABD with AD parallel to BC. The sides AC and BD intersect at Y. The point X lies on AB such that XY is parallel to AD and BC.

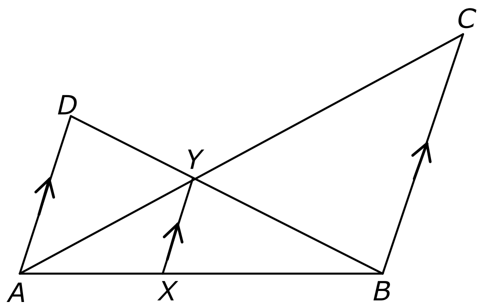
(i) Prove that ΔABC is similar to ΔAXY. **2**

(ii) Hence, or otherwise, prove that **2**

$$\frac{1}{XY} = \frac{1}{AD} + \frac{1}{BC}.$$



(i)



In Δs ABC, AXY:

$\angle A$ is common

$\angle AXY = \angle ABC$ (corr $\angle$s, $XY \parallel BC$)

$\therefore \Delta ABC$ similar to ΔAXY

(equiangular)

(ii) As ΔABC similar to ΔAXY,

State Mean: **1.44/2** **0.25/2**

$$\frac{XY}{BC} = \frac{AX}{AB}$$

$$\therefore AX = \frac{XY.AB}{BC} \quad \text{...........} \ \mathbf{1}$$

(matching sides of sim Δs in prop^n)

Similarly, ΔBXY is similar to ΔBAD (equiangular)

$$\therefore \frac{XY}{AD} = \frac{XB}{AB}$$

$$\therefore XB = \frac{XY.AB}{AD} \quad \text{...........} \ \mathbf{2}$$

Now as $AX + XB = AB$,

2 + **1**:

$$AB = \frac{XY.AB}{AD} + \frac{XY.AB}{BC}$$

Divide through by $XY.AB$:

$$\therefore \frac{1}{XY} = \frac{1}{AD} + \frac{1}{BC}.$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates gained full marks.

A common problem was not giving a final reason (ie the test) for the similarity.

(ii) Many candidates identified the correct ratios of sides for similar triangles.

A common problem was not seeing connection with the ratio of sides involving XY in both triangles ABD and ABC.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/