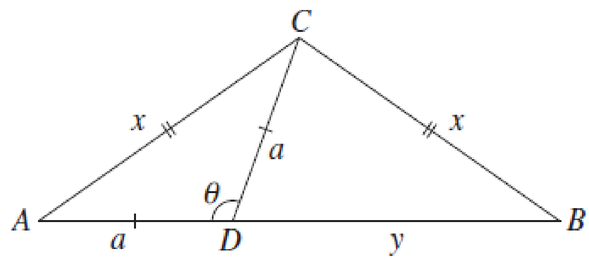
Combined Topics

10 **10 a** In the diagram, ABC is an isosceles triangle $AC = BC = x$. The point D on the interval AB is chosen so that $AD = CD$. Let $AD = a$, $DB = y$ and $\angle ADC = \theta$. [Solution]



(i) Show that ΔABC is similar to ΔACD. **2**

(ii) Show that $x^2 = a^2 + ay$ **1**

(iii) Show that $y = a(1 - 2\cos\theta)$ **2**

(iv) Deduce that $y \le 3a$. **1**

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09 **10** Let $f(x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3}$. [Solution]

(a) Show that the graph of $y = f(x)$ has no turning points. **2**

(b) Find the point of inflexion of $y = f(x)$. **1**

(c) (i) Show that $1 - x + x^2 - \dfrac{1}{1+x} = \dfrac{x^3}{1+x}$ for $x \neq -1$. **1**

(ii) Let $g(x) = \ln(1 + x)$. **2**
Use the result in part (c) (i) to show that $f'(x) \ge g'(x)$ for all $x \ge 0$.

(d) On the same set of axes, sketch the graphs of $y = f(x)$ and $y = g(x)$ for $x \ge 0$. **2**

(e) Show that $\dfrac{d}{dx}[(1 + x)\ln(1 + x) - (1 + x)] = \ln(1 + x)$. **2**

(f) Find the area enclosed by the graphs of $y = f(x)$ and $y = g(x)$, and the straight line $x = 1$. **2**

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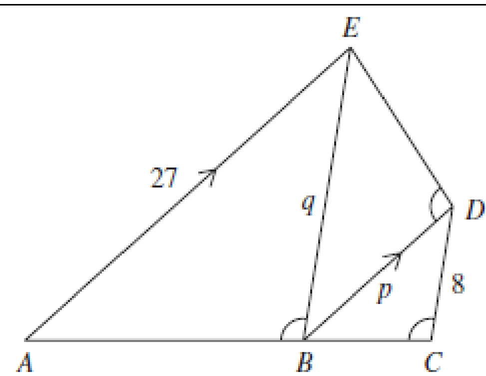
07 **8b** In the diagram, AE is parallel to BD, $AE = 27$, $CD = 8$, $BD = p$, $BE = q$ and $\angle ABE$, $\angle BCD$ and $\angle BDE$ are equal. Copy or trace this diagram into your writing booklet. [Solution]

(i) Prove that $\Delta ABE \;|||\; \Delta BCD$. **2**

(ii) Prove that $\Delta EDB \;|||\; \Delta BCD$. **2**

(iii) Show that 8, p, q, 27 are the first four terms of a geometric series. **1**

(iv) Hence find the values of p and q. **2**

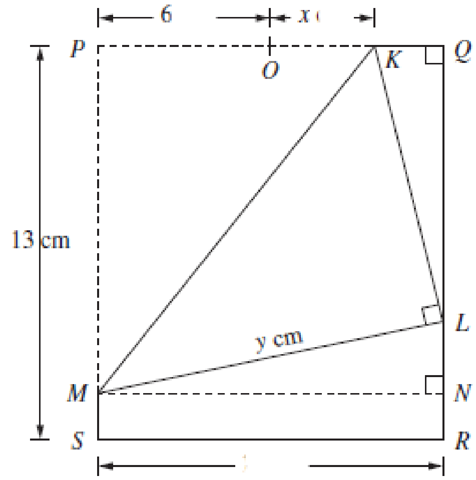


(Not to scale)

06 **10 b** Solution

A rectangular piece of paper $PQRS$ has sides PQ = 12 cm and PS = 13 cm. The point O is the midpoint of PQ. The points K and M are to be chosen on OQ and PS respectively, so that when the paper is folded along KM, the corner that was at P lands on the edge QR at L. Let $OK = x$ cm and $LM = y$ cm.

Copy or trace the diagram into your writing booklet.



(i) Show that $QL^2 = 24x$. **1**

(ii) Let N be the point on QR for which MN is perpendicular to QR. By showing that $\Delta QKL \;|||\; \Delta NLM$, deduce that **3**

$$y = \frac{\sqrt{6}(6+x)}{\sqrt{x}}$$

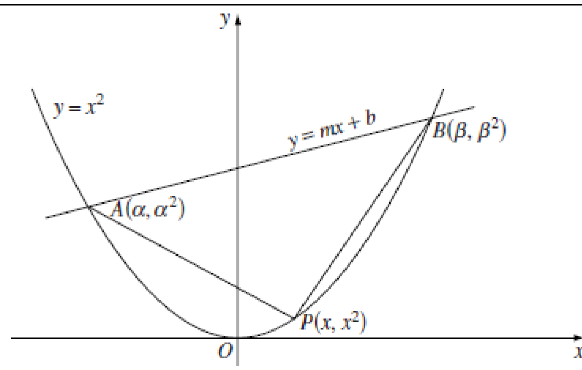
(iii) Show that the area, A, of ΔKLM is given by $A = \dfrac{\sqrt{6}(6+x)^2}{2\sqrt{x}}$. **1**

(iv) Use the fact that $12 \le y \le 13$ to find the possible values of x. **2**

(v) Find the minimum possible area of ΔKLM. **3**

05 **10 a** Solution

The parabola $y = x^2$ and the line $y = mx + b$ intersect at the points $A(\alpha, \alpha^2)$ and $B(\beta, \beta^2)$ as shown in the diagram.



(i) Explain why $\alpha + \beta = m$ and $\alpha\beta = -b$. **1**

(ii) Given that **2**

$$(\alpha - \beta)^2 + (\alpha^2 - \beta^2)^2 = (\alpha - \beta)^2\,[1 + (\alpha + \beta)^2]$$

show that the distance

$$AB = \sqrt{(m^2 + 4b)(1 + m^2)}\,.$$

(iii) The point $P(x, x^2)$ lies on the parabola between A and B. Show that the area of the triangle ABP is given by $\dfrac{1}{2}(mx - x^2 + b)\sqrt{m^2 + 4b}$. **2**

(iv) The point P in part (iii) is chosen so that the area of the triangle ABP is a maximum. Find the coordinates of P in terms of m. **2**