05	**4b**	A function $f(x)$ is defined by $f(x) = (x + 3)(x^2 - 9)$.	
		(i) Find all solutions of $f(x) = 0$.	**2**
		(ii) Find the coordinates of the turning points of the graph $y = f(x)$, and determine their nature.	**3**
		(iii) Hence sketch the graph of $y = f(x)$, showing the turning points and the points where the curve meets the x-axis.	**2**
		(iv) For what values of x is the graph of $y = f(x)$ concave down?	**1**

(i)
$$(x + 3)(x^2 - 9) = 0$$
$$x = -3, 3$$

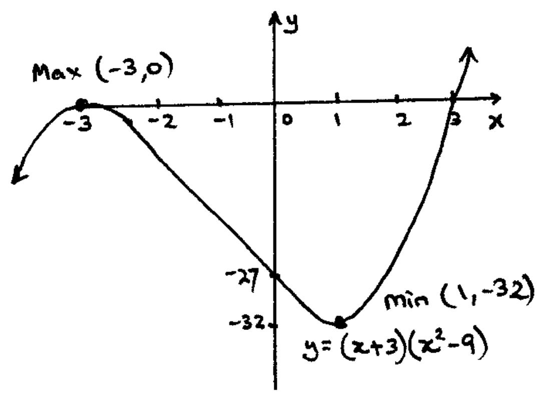
(ii)
$$f(x) = (x + 3)(x^2 - 9)$$
$$= x^3 - 9x + 3x^2 - 27$$
$$f'(x) = 3x^2 - 9 + 6x = 0$$
$$3x^2 + 6x - 9 = 0$$
$$x^2 + 2x - 3 = 0$$
$$(x + 3)(x - 1) = 0$$
$$x = -3, 1$$
At $x = -3$, $f(-3) = (-3 + 3)((-3)^2 - 9)$
$= 0 \quad \therefore (-3, 0)$
At $x = 1$, $f(1) = (1 + 3)(1^2 - 9)$
$= -32 \quad \therefore (1, -32)$

$$f''(x) = 6x + 6$$

At $x = -3$, $f''(-3) = 6(-3) + 6 < 0$
$\therefore$ max $(-3, 0)$
At $x = 1$, $f''(1) = 6(1) + 6 > 0$
$\therefore$ min $(1, -32)$

$\therefore$ max $(-3, 0)$ and min $(1, -32)$

(iii)



(iv) Concave down when $f''(x) < -1$
$$f''(x) = 6x + 6 < 0$$
$$6x < -6$$
$$x < -1$$
$\therefore$ concave down when $x < -1$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Most candidates successfully demonstrated knowledge of differentiation and stationary points. However, poor algebraic skills in expansion, factorisation and substitution were evident.

(i) This part was done well. Common errors included finding $f(0)$, discovering an extra solution and attempts at expanding and refactorising.

(ii) Almost all candidates knew to differentiate and solve $f'(x) = 0$, but many made algebraic errors. Candidates are reminded of a number of points:

- substitute into the given function to find y values
- the test being used should be clearly indicated and a conclusion drawn about the stationary point
- when using the first derivative test it is advisable to use a table, which must be clearly labelled
- $f''(x) = 0$ is a necessary but not sufficient condition for a point of inflexion.

(iii) The graph caused no problem to candidates who had successfully completed part (ii). Some, however, did not continue their graph far enough to show the x-intercept of 3. Others found points of inflexion when the question did not require this.
Algebraic errors meant that it was often impossible to draw a curve to fit the candidate's stationary points. Candidates are reminded that the graph of a cubic function is continuous and smooth. Care should be taken with curve sketching and graphs should be large, neat and roughly to scale.

(iv) In the better responses candidates stated the general principle for a curve to be concave down and then solved the inequality. A number of candidates could not correctly solve $6x + 6 < 0$.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/