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10	**5b**	(i) Prove that $\sec^2 x + \sec x \tan x = \dfrac{1+\sin x}{\cos^2 x}$.	**1**
		(ii) Hence prove that $\sec^2 x + \sec x \tan x = \dfrac{1}{1-\sin x}$.	**1**
		(iii) Hence, use the table of standard integrals to find the exact value of $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x}\,dx$.	**2**

State Mean:
0.58/1
0.31/1
0.74/2

(i)

$$\begin{aligned} \text{LHS} &= \sec^2 x + \sec x \tan x \\ &= \frac{1}{\cos^2 x} + \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} \\ &= \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \\ &= \frac{1+\sin x}{\cos^2 x} \\ &= \text{RHS} \end{aligned}$$

(ii) First, show $\dfrac{1+\sin x}{\cos^2 x} = \dfrac{1}{1-\sin x}$

$$\begin{aligned} \text{LHS} &= \frac{1+\sin x}{\cos^2 x} \\ &= \frac{1+\sin x}{1-\sin^2 x} \\ &= \frac{1+\sin x}{(1-\sin x)(1+\sin x)} \\ &= \frac{1}{1-\sin x} \\ &= \text{RHS} \end{aligned}$$

$\therefore\ \sec^2 x + \sec x \tan x = \dfrac{1}{1-\sin x}$

(iii)

$$\begin{aligned} &\int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x}\,dx \\ &= \int_0^{\frac{\pi}{4}} \sec^2 x + \sec x \tan x\,dx \\ &= [\tan x + \sec x]_0^{\frac{\pi}{4}} \quad *** \\ &= \left[\tan\frac{\pi}{4} + \sec\frac{\pi}{4} - (\tan 0 + \sec 0)\right] \\ &= 1 + \sqrt{2} - (0 + 1) \\ &= \sqrt{2} \end{aligned}$$

[*** from table of standard integrals:

$\int \sec ax \tan ax\,dx = \sec ax + c$,

where $a = 1$]

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) In this part, most candidates demonstrated their knowledge of trigonometric identities. Most began with the left-hand side but a significant number worked from right to left. Some responses were far too complicated given that this part was only worth one mark.

(ii) A number of candidates equated the required expression with the one in part (i) and then cross multiplied. Many candidates realised that $\cos^2 x = 1 - \sin^2 x$ but did not then factorise the denominator to simplify.

(iii) Far too many candidates went astray by not referring to the table of standard integrals, as suggested in the question. Most candidates who did not use the table did not succeed in correctly integrating. A significant number of candidates who integrated correctly, confused signs in their evaluation of the limits of integration and found the solution to be $2+\sqrt{2}$ instead of just $\sqrt{2}$.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/