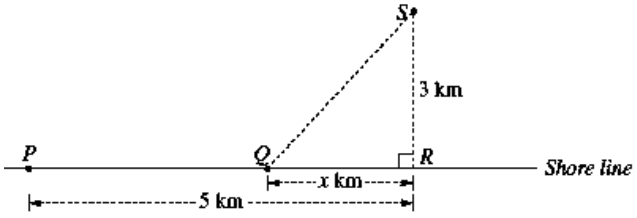
09 **9b** An oil rig, S, is 3 km offshore. A power station, P, is on the shore. A cable is to be laid from P to S. It costs \$1000 per kilometres to lay the cable along the shore and \$2600 per kilometre to lay the cable underwater from the shore to S. The point R is the point on the shore closest to S, and the distance PR is 5 km. The point Q is on the shore, at a distance of x km from R, as shown in the diagram.

(i) Find the total cost of laying the cable in a straight line from P to R and then in a straight line from R to S. **1**



(ii) Find the cost of laying the cable in a straight line from P to S. **1**

(iii) Let \$$C$ be the total cost of laying the cable in a straight line from P to Q, and then in a straight line from Q to S. **2**

Show that $C = 1000(5 - x + 2.6\sqrt{x^2+9})$.

(iv) Find the minimum cost of laying the cable. **3**

(v) New technology means that the cost of laying the cable underwater can be reduced to \$1100 per kilometre. Determine the path for laying the cable in order to minimise the cost in this case. **1**

(i) Cost $= 5 \times 1000 + 3 \times 2600$
$= 12\,800$
$\therefore$ the cost is \$12 800

(ii) $PS = \sqrt{5^2+3^2}$, by Pythag.
$= \sqrt{34}$
Cost $= \sqrt{34} \times 2600$
$= 15\,160.47493\ldots$
$= 15\,160.47$
$\therefore$ the cost is \$15 160.47

(iii) Length of $PQ = (5 - x)$ km
Cost of PQ cable $= 1000(5 - x)$
Length of $QS = \sqrt{x^2+3^2}$, by Pythag.
$= \sqrt{x^2+9}$ km
Cost of QS cable $= 2600\sqrt{x^2+9}$
$C = 1000(5 - x) + 2600\sqrt{x^2+9}$
$= 1000(5 - x + 2.6\sqrt{x^2+9})$

(iv) $C = 1000(5 - x + 2.6(x^2+9)^{\frac{1}{2}})$

$C' = 1000(-1 + 2.6.\frac{1}{2}(x^2+9)^{-\frac{1}{2}}.2x)$

$= 1000(-1 + \frac{2.6x}{\sqrt{x^2+9}}) = 0$

$\frac{2.6x}{\sqrt{x^2+9}} = 1$

$2.6x = \sqrt{x^2+9}$
$6.76x^2 = x^2 + 9$
$5.76x^2 = 9$
$x^2 = \frac{9}{5.76}$
$x = \frac{5}{4}$

Check for minimum:

x	1.2	1.25	1.3
C'	–	0	+

Subs $x = 1.25$ in C:
$C = 1000(5 - 1.25) + 2600\sqrt{1.25^2+9}$
$= 12\,200$
$\therefore$ the minimum cost is \$12 200

(iv) $C' = 1000(-1 + \frac{1.1x}{\sqrt{x^2+9}}) = 0$

$\frac{1.1x}{\sqrt{x^2+9}} = 1$

$1.1x = \sqrt{x^2+9}$
$1.21x^2 = x^2 + 9$
$0.21x^2 = 9$
$x^2 = \frac{9}{0.21}$
$x = 6.55$

But $0 < x < 5$, so minimum at $x = 5$.

$\therefore$ minimum cost when cable from P to S.

Board of Studies: Notes from the Marking Centre

(i) The most common errors in this part were: using the cost of \$1000 for both the shore and underwater component of the cost; using \$1000 for the underwater component and \$2600 for the shore component; and not adding together the two components of the cost.

(ii) Most candidates calculated the distance QS using Pythagoras' Theorem, although some successfully used the cosine rule. The most common errors in this part were arithmetic errors, for example $\sqrt{5^2+3^2}=\sqrt{36}=6$, and using \$1000 as the cost.

(iii) Better responses to this part derived the expression for C in a logical sequence, explaining each step. The most common error in this part was to begin with the given result merely expanded without any explanations.

(iv) Common errors in this part included: incorrect use of the chain rule; algebraic errors, including simplifying $\sqrt{x^2+9}$ to $x+3$; not testing the nature of the stationary point at all; not finding the second derivative correctly if using the second derivative test; and, if using the first derivative test, just writing \ _ / in the table without testing any values.

(v) Better responses to this part substituted 1.1 for 2.6 in their working for part (iv). Common errors included: not using calculus at all but rather calculating $C(0)$, $C(5)$ and $C\left(\frac{5}{4}\right)$ for the revised expression for the cost C and taking the smallest of these; successfully calculating that the new minimum occurs at $x=\frac{30}{\sqrt{21}}$ and claiming this to be the new path; or being confused that the new minimum occurred at a value greater than 5 and crossing out work thinking it to be incorrect.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/