08	**5c**	Light intensity is measured in lux. The light intensity at the surface of a lake is 6000 lux. The light intensity, I lux, a distance s metres below the surface of the lake is given by $I = Ae^{-ks}$ where A and k are constants. (i) Write down the value of A. (ii) The light intensity 6 metres below the surface of the lake is 1000 lux. Find the value of k. (iii) At what rate, in lux per metre, is the light intensity decreasing 6 metres below the surface of the lake?	**1** **2** **2**

(i) When $s = 0$, $I = 6000$:

$I = Ae^{-ks}$

$6000 = Ae^{-k(0)}$

$6000 = A$

$A = 6000$

(ii) When $s = 6$, $I = 1000$:

$I = 6000\,e^{-ks}$

$1000 = 6000\,e^{-k(6)}$

$e^{-6k} = \frac{1}{6}$

$e^{6k} = 6$

Taking logs of both sides:

$6k = \log_e 6$

$k = \frac{1}{6}\log_e 6$

$= 0.298626578\ldots$

$= 0.2986$ (4 dec pl)

(iii) $I = 6000\,e^{-ks}$

$\frac{dI}{ds} = -6000k\,e^{-ks}$

When s = 6,

$\frac{dI}{ds} = -6000k\,e^{-6k}$

$= -6000 \times \frac{1}{6}\log_e 6 \times e^{-6.\frac{1}{6}\log_e 6}$

$= -298.6$ (1 dec. pl)

$\therefore$ decreasing at 298.6 lux/metre

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(ii) Candidates should be aware that the natural logarithm function $\log_e(x)$ appears as ln on most calculator keypads and not as log (which is log to base 10). Responses that presented a clear final answer in parts (ii) and (iii) in terms of natural logs and exponentials were not penalised for incorrect subsequent evaluations using the calculator.

(iii) In some better responses, candidates answered part (iii) efficiently by arguing that $I' = kI = 1000k$. An alternative for this part was to differentiate the formula for intensity with respect to distance s and then make appropriate substitutions for A, k and s. Common errors were to differentiate with respect to k rather than s, to argue that $\frac{d}{ds}e^{-ks} = -kse^{-ks-1}$ or to abandon the use of calculus altogether and calculate the average rate of change in intensity over 6 metres rather than the instantaneous rate at $s = 6$.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/