10	4a	Susannah is training for a fun run by running every week for 26 weeks. She runs 1
		km in the first week and each week after that she runs 750 m more than the previous
		week, until she reaches 10 km in a week. She then continues to run 10 km each
		week.
		(i) How far does Susannah run in the 9 <sup>th</sup> week?
		(ii) In which week does she first run 10 km?

What is the total distance that Susannah runs in 26 weeks?

1, 1.75, 2.5, ..., 10, then 10, 10, ...

(i) Using 
$$a = 1$$
,  $d = 0.75$ ,  $n = 9$ ,
$$T_n = a + (n - 1)d$$

$$= 1 + (9 - 1) \times 0.75$$

$$= 7$$

∴ she runs 7 km in 9<sup>th</sup> week

(ii) Using 
$$a = 1$$
,  $d = 0.75$ ,  $T_n = 10$ ,
$$T_n = a + (n - 1)d$$

$$10 = 1 + (n - 1) \times 0.75$$

$$9 = 0.75(n - 1)$$

$$n - 1 = \frac{9}{0.75}$$

$$n - 1 = 12$$

$$n = 13$$

$$\therefore \text{ in the } 13^{\text{th}} \text{ week}$$

(iii) First find her total distance in the first 13 weeks:

Using 
$$a = 1$$
,  $d = 0.75$ ,  $n = 13$ ,

State Mean:

0.89/1

0.84/1

1.29/2

Using  $a = 1$ ,  $d = 0.75$ ,  $n = 13$ ,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{13}{2} [2 + 12 \times 0.75]$$

$$= 71.5$$

.. She runs 71.5 km in the first 13 weeks.

1

2

For the following 13 weeks she runs 10 km each week.

As 71.5 + 130 = 201.5, Susannah runs a total of 201.5 km in the 26 weeks.

## **Board of Studies: Notes from the Marking Centre**

This part was generally well done. A number of candidates used km and m in the same calculation or misinterpreted the question and so gave unrealistic answers like 6000 km being run in the 9th week. Listing all 26 distances was not uncommon and many successfully found all answers this way, but at a large cost in time.

- (i) Better responses used the formula for the nth term with the substitution clearly shown. Common errors included misquoting the formula, using the formula for the sum of n terms, or using the formula for a geometric series.
- (ii) Better responses again used the formula for the n th term with the substitutions clearly made and the resulting equation solved. Many errors were made when solving the equation, including deducing that n = 11 from the equation n 1 = 12.
- (iii) Many candidates recognised that the arithmetic series stopped at the 13th (or 12th) term and then added the correct multiple of 10 km for the remaining weeks. But many summed 26 terms of a series to reach the common but incorrect result of 269.75 km. Although

candidates were well practised in using both formulae, those using  $S_n = \frac{n}{2(a+1)}$  tended to

make fewer errors. Other candidates quoted the correct formula but did not make the correct substitutions. Some candidates incorrectly identified 12 terms, from the 14th to the 26th.

Source: http://www.boardofstudies.nsw.edu.au/hsc\_exams/

<sup>\*</sup> These solutions have been provided by projectmaths and are not supplied or endorsed by the Board of Studies

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