2015 14 c Sam borrows \$100 000 to be repaid at a reducible interest rate of 0.6% per month. Let $\$A_n$ be the amount owing at the end of n months and $\$M$ be the monthly repayment.

(i) Show that $A_2 = 100\,000(1.006)^2 - M(1 + 1.006)$. **1**

(ii) Show that $A_n = 100\,000(1.006)^n - M\left(\dfrac{(1.006)^n - 1}{0.006}\right)$. **2**

(iii) Sam makes monthly repayments of \$780. Show that after making 120 monthly repayments the amount owing is \$68 500 to the nearest \$100. **1**

(iv) Immediately after making the 120th repayment, Sam makes a one-off payment, reducing the amount owing to \$48 500. The interest rate and monthly repayment remain unchanged. After how many more months will the amount owing be completely repaid? **3**

(i)

$$A_1 = 100\,000 \times 1.006 - M$$
$$A_2 = [100\,000 \times 1.006 - M] \times 1.006 - M$$
$$= 100\,000 \times 1.006^2 - 1.006M - M$$
$$= 100\,000 \times 1.006^2 - M(1 + 1.006)$$

State Mean: **0.67**

(ii)

$$A_n = 100\,000 \times 1.006^n - M(1 + 1.006 + 1.006^2 + \ldots + 1.006^{n-1})$$

Considering the geometric series with $a = 1$, $r = 1.006$, $n = n$, and using $S_n = \dfrac{a(r^n - 1)}{r - 1}$:

$$A_n = 100\,000 \times 1.006^n - M\left[\frac{1(1.006^n - 1)}{1.006 - 1}\right]$$
$$= 100\,000 \times 1.006^n - M\left[\frac{1.006^n - 1}{0.006}\right]$$

State Mean: **1.05**

(iii) Let $M = 780$, $n = 120$:

$$A_n = 100\,000 \times 1.006^{120} - 780\left[\frac{1.006^{120} - 1}{0.006}\right]$$
$$= 68\,499.4583\ldots$$
$$= 68\,500 \text{ (nearest 500)}$$

$\therefore$ the amount owing is \$68 500.

State Mean: **0.81**

(iv) Let amount be 48 500 and $A_n = 0$:

$$0 = 48\,500 \times 1.006^n - 780\left[\frac{1.006^n - 1}{0.006}\right]$$
$$48\,500 \times 1.006^n = 130\,000(1.006^n - 1)$$
$$48\,500 \times 1.006^n = 130\,000 \times 1.006^n - 130\,000$$
$$81\,500 \times 1.006^n = 130\,000$$
$$1.006^n = \frac{260}{163}$$
$$\log_e 1.006^n = \log_e \frac{260}{163}$$
$$n = \frac{\log_e \frac{260}{163}}{\log_e 1.006}$$
$$= 78.05513798\ldots$$

$\therefore$ Sam will need 79 repayments.

State Mean: **1.55**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(c)(i) This part was generally done well, with many candidates clearly writing the expression for A_1 and then showing the working and steps needed to generate the expression for A_2. Candidates are reminded that all 'show' questions require working with logical steps which lead to the required answer and writing the last step before the given answer is crucial.

Common problems were:

- finding an expression for A_1 and then going straight to the given expression for A_2 without showing how A_2 is actually obtained
- incorrect use or omission of brackets when writing the expressions for A_1 and A_2
- omitting zeros with 1.006 changing to 1.06 at some stage in the solution
- attempting to work backwards from the given expression for A_2.

(c)(ii) In the better responses, candidates showed the expression for A_3 and then generalised the pattern to show $A_n = 100\,000(1.006)^n - M[1 + (1.006) + (1.006)^2 + \cdots + (1.006)^{n-1}]$. Using the first three terms of their series, they deduced that it was geometric. Correct substitution into the GP sum formula and simplification, led to the given result.

Common problems were:

- using a rote-learned formula instead of deriving A_n
- only showing two terms in the series
- incorrect use of brackets
- writing the last term of the series to the power n instead of $n - 1$
- not showing the substitution of values into the GP sum formula and merely stating the given answer.

(c)(iii) This part was done very well, with most candidates substituting $M = 780$ and $n = 120$ into the result provided in (c)(ii) to arrive at the answer \$68 499.46 which could then be shown to be equal to \$68 500 correct to the nearest \$100.

Common problems were:

- making the substitution but not writing the calculator display in their working
- performing their calculation in sections, rounding off each result and combining them to obtain an incorrect answer.

(c)(iv) Candidates who removed the denominator of 0.006 by dividing 780 by 0.006 were usually the most successful at achieving a correct solution. Many candidates did not link the equation given in (c)(ii) to part (c)(iv) and so spent considerable time and effort re-establishing a pattern and a formula to use.

Common problems were:

- using incorrect values for A_n , P and M
- being unable to perform the algebraic steps necessary to isolate $(1.006)^n$ on one side of the equation
- inability to use logs to solve an exponential equation
- using a trial and error method to solve the exponential equation but not showing any or sufficient evidence of the values of n tested and the associated answers.