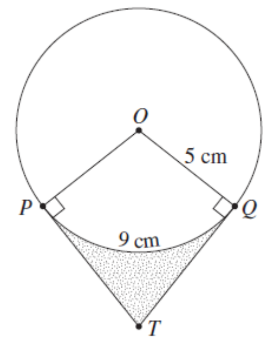
10	**6b**	The diagram shows a circle with centre O and radius 5 cm. The length of the arc PQ is 9 cm. Lines drawn perpendicular to OP and OQ at P and Q respectively meet at T. (i) Find $\angle POQ$ in radians. (ii) Prove that ΔOPT is congruent to ΔOQT. (iii) Find the length of PT. (iv) Find the area of the shaded region.	**1** **2** **1** **2**



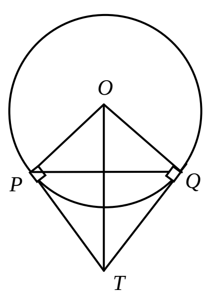
(Not to scale)

State Mean:
0.53/1
1.01/2
0.29/1
0.70/2

(i) Using $l = 9$, $r = 5$,

$$l = r\theta$$
$$9 = 5\theta$$
$$\theta = \frac{9}{5}$$
$$\therefore \angle POQ \text{ is } \frac{9}{5} \text{ radians}$$

(ii)



In ΔOPT and ΔOQT,
OT is common
$\angle OPT = \angle OQT$ (given)
$OP = OQ$ (equal radii)
$\therefore \Delta OPT \equiv \Delta OQT$ (RHS test)

(iii) From (ii) $\angle POT = \angle QOT$
(matching $\angle$s of cong Δs)

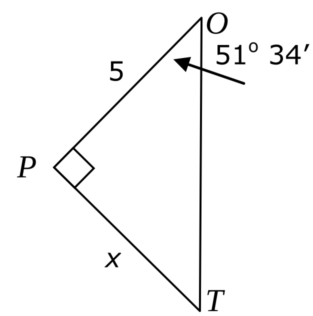
Also, $\angle POQ = \frac{9}{5}$ radians (from (i)),

then $\angle POT = \frac{1}{2} \times \frac{9}{5}$ radians

$= \frac{9}{10}$ radians

$= \frac{9}{10} \times \frac{\pi}{180}$ degrees

$= 51^\circ\, 34'$



$$\frac{x}{5} = \tan 51^\circ\, 34'$$
$$x = 5 \times \tan 51^\circ\, 34'$$
$$= 6.300\ \ldots$$
$$= 6.3 \text{ (corr to 1 dec pl)}$$

$\therefore$ length of PT is 6.3 cm

(iv) Area $OPTQ = 2 \times$ Area of ΔOPT

$= 2 \times \frac{1}{2} \times 5 \times 6.300..$

$= 31.503...$

$= 31.5$ (corr to 1 dec pl)

$\therefore$ area of $OPTQ$ is 31.5 cm^2

Shaded area $=$ Area $OPTQ$ – Area sector

$= 31.503... - \frac{1}{2} \times 5^2 \times \frac{9}{5}$

$= 9.003...$

$= 9$ (nearest whole)

$\therefore$ shaded area is 9 cm^2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i)	Many candidates struggled to work in radians. Many attempted to convert the correct answer of 1.8 radians into degrees or radians by multiplying by $\frac{\pi}{180}$ or $\frac{180}{\pi}$. Many candidates were not familiar with radian measure being used in right-angled triangles and so converted back to degrees.
(ii)	The congruency test of RHS was not well recognised by candidates. Many identified the correct statements and reasons but then declared the test SAS. A number of candidates used circle geometry but the reason for $PT = QT$ was not reasoned well. Many candidates attempted to use the result in their proof of congruency; for example many used $\angle POT = \angle TOQ$.
(iii)	Many candidates confused the letters used on the diagram; for example, $PO = QQ$. Many candidates attempted to use a side length of 9 in their calculations. A large number of candidates assumed figure $OPTQ$ was a square as it had two right angles and so let $PT = 5$. Many did, however, use this length successfully in part (iv).
(iv)	It is worth noting that candidates who found an incorrect value for $\angle POT$ were still able to complete the remainder of the question and so earn marks. Despite candidates being able to state the arc length and area of a sector formulae, the rules were often not used properly or appropriately; for example, the size of the angle was not correct or was used in degrees.
Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/	