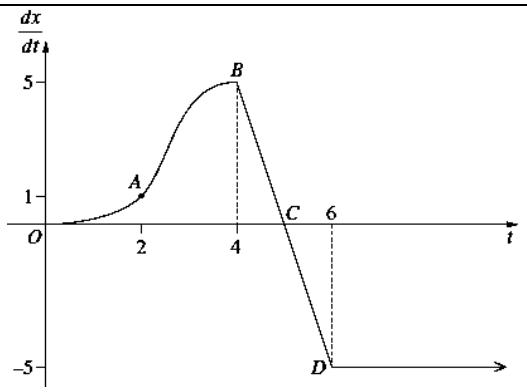
07 **10 a** An object is moving on the x-axis. The graph shows the velocity, $\frac{dx}{dt}$, of the object, as a function of time, t. The coordinates of the points shown on the graph are A(2, 1), B(4, 5), C(5, 0) and D(6, -5). The velocity is constant for $t \geq 6$.



(Not to scale)

(i) Using Simpson's rule, estimate the distance travelled between $t = 0$ and $t = 4$. **2**

(ii) The object is initially at the origin. During which time(s) is the displacement of the object decreasing? **1**

(iii) Estimate the time at which the object returns to the origin. Justify your answer. **2**

(iv) Sketch the displacement, x, as a function of time. **1**

(i) Simpson's Rule:

Area = $\frac{h}{3}$[first + last + 2 × odd + 4 × even]

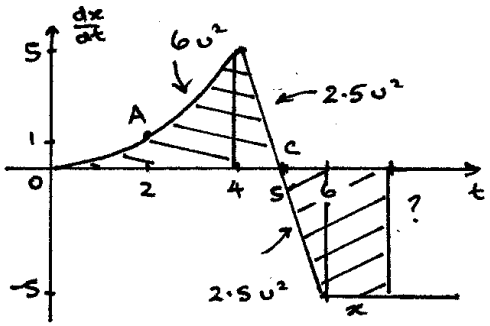
t	0	2	4
$\frac{dx}{dt}$	0	1	5

$= \frac{2}{3}[0 + 5 + 4 \times 1]$

$= 6$ $\therefore$ distance travelled is 6 units

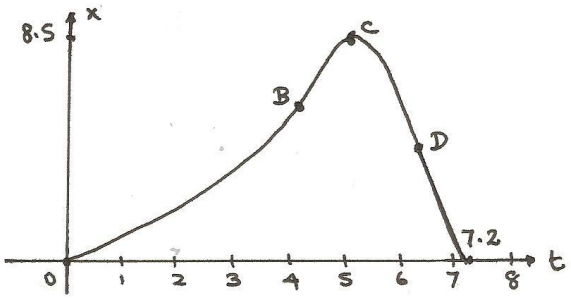
(ii) If particle starts at 0 and travels with a positive velocity then it travels in a positive direction until $t = 5$. The displacement of the particle is decreasing when $t > 5$.

(iii) Need to consider when area above x-axis is equal to area under x-axis.



Area above x-axis
= 6 (from (i)) + area of triangle
$= 6 + \frac{1}{2} \times 1 \times 5$
$= 8.5$
Area below x-axis
= area of triangle + area of rectangle
$= \frac{1}{2} \times 1 \times 5 + 5 \times x$
$= 2.5 + 5x$
This means $5x + 2.5 = 8.5$
$5x = 6$
$x = 1.2$
As 6 + 1.2 = 7.2, then after 7.2 seconds the particle returns to origin.

(iv)



* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) This part was attempted by the majority of candidates. In successful responses, candidates recognised that the function values were to be read from the graph and applied the $A = \frac{h}{3}\left\{ f(a) + 4f(\frac{a+b)}{2}) + f(b) \right\}$ version of Simpson's Rule. A number of candidates, perhaps assuming Question 10 was meant to be difficult, assumed the curve to be a sine function. Better responses made the link with the area under the curve and the displacement of the object. Candidates who attempted to use five function values increased the level of difficulty by not being able to read off clear function values.

(ii) The question asked when was the displacement of the object decreasing, ie $t > 5$. Many candidates included $t = 5$ or assumed the object was returning when the line had a negative slope, ie when the velocity was decreasing from $t = 4$ to $t = 6$. A number of candidates gave multiple answers, perhaps misinterpreting the question which asked 'during which time(s)' and assuming this implied there was more than one answer.

(iii) The instruction 'estimate' was misunderstood by candidates who did not determine a precise time. Responses to 'justify your answer' varied greatly from candidates roughly estimating a time, to speaking of changes in velocity and acceleration, to the correct responses which included either distances travelled or areas above and below the t-axis being equal. Many responses included the conclusion that the object would not return to the origin, due to the horizontal line starting at $t = 6$, despite the question asking when the object did return. Better responses linked the answer to part (i) to the area under the axis for $t > 6$.

(iv) Candidates are reminded to use a ruler when drawing the axes of graphs. The scales used on axes need to be clear and consistent. Candidates are advised not to place multiple attempts on the same axes. Sketches require important features of the curve such as turning points, points of inflexion, intercepts and straight lines to be clearly identified. Many candidates attempted unsuccessfully to include a point of inflexion at $t = 2$. Better responses correctly applied the information from part (ii) that 'the object is initially at the origin' in their sketch.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/