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09	10	Let $f(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$.	
		(a) Show that the graph of $y = f(x)$ has no turning points.	**2**
		(b) Find the point of inflexion of $y = f(x)$.	**1**
		(c) (i) Show that $1 - x + x^2 - \frac{1}{1+x} = \frac{x^3}{1+x}$ for $x \neq -1$.	**1**
		(ii) Let $g(x) = \ln(1 + x)$. Use the result in part (c) (i) to show that $f'(x) \geq g'(x)$ for all $x \geq 0$.	**2**
		(d) On the same set of axes, sketch the graphs of $y = f(x)$ and $y = g(x)$ for $x \geq 0$.	**2**
		(e) Show that $\frac{d}{dx}[(1 + x)\ln(1 + x) - (1 + x)] = \ln(1 + x)$.	**2**
		(f) Find the area enclosed by the graphs of $y = f(x)$ and $y = g(x)$, and the straight line $x = 1$.	**2**

(a) $f(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$.

$f'(x) = 1 - x + x^2$

But as $\Delta = b^2 - 4ac$

$= (-1)^2 - 4(1)(1) < 0$

$\therefore$ no real roots

$\therefore$ $f'(x) \neq 0$, and hence no turning points

(b) $f'(x) = 1 - x + x^2$

$f''(x) = -1 + 2x = 0$

$2x = 1$

$x = \frac{1}{2}$

$f(\frac{1}{2}) = \frac{1}{2} - \frac{(\frac{1}{2})^2}{2} + \frac{(\frac{1}{2})^3}{3}$

$= \frac{5}{12}$

Possible pt of infl. at $(\frac{1}{2}, \frac{5}{12})$

Check concavity:

x	0	$\frac{1}{2}$	1
$f''(x)$	–	0	+

$\therefore$ pt of infl. at $(\frac{1}{2}, \frac{5}{12})$

(c) (i) To show $1 - x + x^2 - \frac{1}{1+x} = \frac{x^3}{1+x}$

LHS $= 1 - x + x^2 - \frac{1}{1+x}$

$= \frac{1 + x - x(1 + x) + x^2(1 + x) - 1}{1 + x}$

$= \frac{1 + x - x - x^2 + x^2 + x^3 - 1}{1 + x}$

$= \frac{x^3}{1 + x}$

= RHS

$\therefore 1 - x + x^2 - \frac{1}{1+x} = \frac{x^3}{1+x}$

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(ii) $f'(x) = 1 - x + x^2$ from (a).

$g(x) = \ln(1 + x)$.

$g'(x) = \frac{1}{1+x}$

Now, $\frac{x^3}{1+x} \geq 0$ for $x \geq 0$

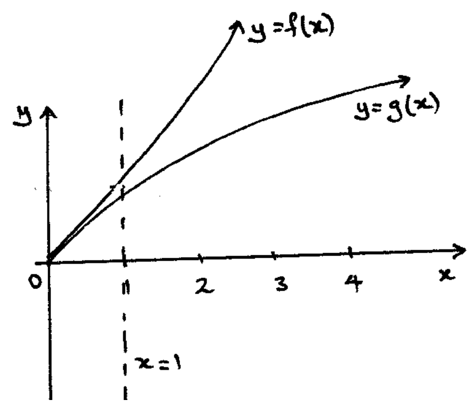
and using $1 - x + x^2 - \frac{1}{1+x} = \frac{x^3}{1+x}$ (from (i))

$\therefore 1 - x + x^2 - \frac{1}{1+x} \geq 0$ for $x \geq 0$

then $f'(x) - g'(x) \geq 0$ for $x \geq 0$.

$f'(x) \geq g'(x)$ for $x \geq 0$

(d)



(e) $\frac{d}{dx}[(1+x)\ln(1+x) - (1+x)]$ $= 1(\ln(1+x) + \frac{1}{1+x}.(1+x) - 1$ $= \ln(1+x) + 1 - 1$ $= \ln(1+x)$	(f) Area $= \int_0^1 x - \frac{x^2}{2} + \frac{x^3}{3}\ dx - \int_0^1 \ln(1+x)\ dx$ $= \left[\frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12}\right]_0^1 - [(1+x)\ln(1+x) - (1+x)]_0^1$ $= \frac{1}{2} - \frac{1}{6} + \frac{1}{12} - 0 - (2\ln 2 - 2 - (\ln 1 - 1))$ $= \frac{5}{12} - (2\ln 2 - 1)$ $= \frac{17}{12} - 2\ln 2$ $= 0.030372305 \ldots$ $= 0.03$ (2 dec pl) $\therefore$ 0.03 unit^2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(a) Candidates are reminded that when a question asks them to show or prove something, then it requires that clear and logical working be shown. Candidates whose conclusions lead to a contradiction are reminded that this signals that an error has been made and that returning to check working could locate the error. Some candidates had a problem working with fractions.

Many candidates obtained the correct first derivative $f'(x) = 1 - x + x^2$, but in an attempt to rewrite the expression and start with the term x^2 the incorrect use of the minus sign led to many mistakes.

Candidates are reminded that even if a quadratic equation cannot be factorised it may still have real roots.

Most of the correct responses involved the use of the discriminant to argue that the quadratic equation $f'(x) = 0$ has no real solutions when the discriminant is negative.

A few candidates wrote the derivative in the form $\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$ and then successfully argued that $f'(x) \neq 0$.

(b) Most candidates found the point of inflection by setting $f''(x) = 0$.

(c) When cancelling terms in an algebraic expression, they can be crossed out but this should not be done so heavily that the terms are no longer legible.

The statements in parts (i) and (ii) cannot be proved by the substitution of several values of x into both sides.

The fact that 'Let $f(x) = \ldots$' occurs before the separate parts of this question implies that this function will apply throughout the entire question. A common error was to take $f(x) = \frac{x^3}{1+x}$ in part (c) (ii) and also later in part (f).

Many candidates differentiated $\ln(1+x)$ correctly.

(d) Notice should be taken of what has been developed in previous parts of a question, particularly the significance of the fact that $f'(x) > g'(x)$. Also it was important to realise that both graphs pass through the origin. Many incorrect responses did not have the graph of $f(x)$ above the graph of $g(x)$ for $x > 0$.

If more than one graph is shown on the same set of axes then each must be labelled. Diagrams need to be large and clear.

(e) Responses to this part became messy when some candidates developed expressions with incorrect use of brackets. Ironically, many candidates had more success in differentiating $(1+x)\ln(1+x)$ by using the product rule than in differentiating the simpler expression $-(1+x)$.

(f) Candidates are encouraged to make full use of the table of standard integrals included at the end of the examination paper. Many candidates made an error by writing $\int \ln(1-x)\,dx = \dfrac{1}{1+x}$ instead of using the link in part (e).

Some candidates correctly managed to evaluate $\int_0^1 \ln(1-x)\,dx$ by using the area between the curve and the y-axis.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/