1

2

1

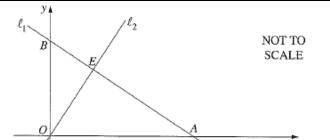
1

State Mean: **0.92/1** 

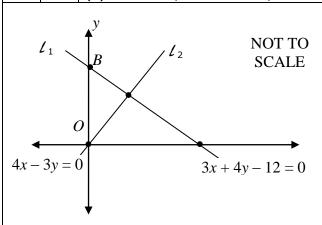
1.58/2

0.81/1 0.73/1 0.77/1

11	3с	The diagram shows a line $\ell_1$ , with
		equation $3x + 4y - 12 = 0$ , which intersects the <i>y</i> -axis at <i>B</i> . A second line $\ell_2$ , with equation $4x - 3y = 0$ ,
		passes through the origin $O$ and intersects $\ell_1$ at $E$ .



- (i) Show that the coordinates of B are (0, 3)
- (ii) Show that  $\ell_1$  is perpendicular to  $\ell_2$ .
- (iii) Show that the perpendicular distance from O to  $\ell_1$  is  $\frac{12}{5}$ .
- (iv) Using Pythagoras' theorem, or otherwise, find the length of the interval *BE*.
- (v) Hence, or otherwise, find the area of  $\triangle BOE$ .



- (i) B is on y-axis has x-value of 0. Now, subs (0, 3) in 3x + 4y - 12: 3(0) + 4(3) - 12 = 0.  $\therefore$  B(0, 3)
- (ii) gradient  $l_1: \frac{-3}{4}$  gradient  $l_2: \frac{4}{3}$ 
  - As  $\frac{-3}{4} \times \frac{4}{3} = -1$ , : perpendicular

As 
$$l_1 \perp l_2$$
,  
use  $(0, 0)$ ,  $3x + 4y - 12 = 0$ ,  

$$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$= \left| \frac{0 + 0 + (-12)}{\sqrt{3^2 + 4^2}} \right|$$

$$= \frac{12}{5}$$

(iv) As 
$$OB = 3$$
,  $OE = \frac{12}{5} = 2.4$ , then
$$BE^{2} = OB^{2} - OE^{2}$$

$$= 3^{2} - 2.4^{2}$$

$$= 3.24$$

$$BE = \sqrt{3.24}$$

$$= 1.8$$
(v) Area =  $\frac{1}{-} \times OE \times BE$ 

(v) Area = 
$$\frac{1}{2} \times OE \times BE$$
  
=  $\frac{1}{2} \times 2.4 \times 1.8$   
= 2.16  
 $\therefore$  area is 2.16 units<sup>2</sup>

## **Board of Studies: Notes from the Marking Centre**

<sup>\*</sup> These solutions have been provided by projectmaths and are not supplied or endorsed by the Board of Studies

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(i) This part was answered well with the majority of responses having the substitution of x = 0 into  $l_1$  then solving for y. Some expressed  $l_1$  in the form y = mx + b and stated that the y-intercept was 3.

- (ii) In better responses, candidates rearranged the equations for l<sub>1</sub> and l<sub>2</sub> into the gradient-intercept form, stated the gradients and showed that their product was -1. Many who attempted to use the general form of the equation and the fact that the gradient is given by m = -\frac{a}{b}, made errors in stating this formula and hence obtained incorrect gradients. A time-consuming strategy used by many candidates was to find the point of intersection E then the gradients of OE and BE. Candidates should use and be familiar with correct mathematical terminology, particularly that the reciprocal is not an 'inverse' or 'flip'.
- (iii) Most responses included the perpendicular distance formula with correct substitutions to obtain the required result. The most common error was to substitute +12 instead of −12 into the formula. Many who found the coordinates of E then the distance from E to the origin, made some errors.
- (iv) The better responses used the lengths of two sides known from parts (i) and (iii) and applied Pythagoras' Theorem to correctly to find BE. Several unnecessarily recalculated OE. Many used the distance formula and the points B and E to calculate the length of BE.
- (v) This part was done well. In most responses, answers to parts (iii) and (iv) were used to find the area of the triangle.

Source: http://www.boardofstudies.nsw.edu.au/hsc\_exams/