12	**14c**	Professor Smith has a colony of bacteria. Initially, there are 1000 bacteria. The number of bacteria, $N(t)$, after t minutes is given by $N(t) = 1000e^{kt}$.	
		(i) After 20 minutes there are 2000 bacteria. Show that $k = 0.0347$ correct to four decimal places.	**1**
		(ii) How many bacteria are there when $t = 120$?	**1**
		(iii) What is the rate of change of the number of bacteria per minute, when $t = 120$?	**1**
		(iv) How long does it take for the number of bacteria to increase from 1000 to 100 000?	**2**

(i) $N = 1000e^{kt}$, $t = 20$, $N = 2000$

$$2000 = 1000e^{20k}$$
$$e^{20k} = 2$$
$$\log_e e^{20k} = \log_e 2$$
$$20k = \log_e 2$$
$$k = \frac{\log_e 2}{20}$$
$$= 0.034657359\ \ldots$$
$$= 0.0347 \text{ (to 4 dec pl)}$$

(ii) $N = 1000e^{kt}$, $t = 120$,

$$N = 1000e^{k(120)}$$
$$= 64\,000$$

$\therefore$ 64 000 bacteria

[if using k = 0.0347 then N = 64 328]

(iii) $N = 1000e^{kt}$

$$\frac{dN}{dt} = k.\ 1000e^{kt}$$
$$= k.\ 64\,000$$
$$= 64\,000k$$
$$= 2218.070978\ \ldots$$
$$= 2218 \text{ (nearest whole)}$$

$\therefore$ 2218 per minute

[if using k = 0.0347, N = 64 328, then 2232 per min]

(iv) $N = 1000e^{kt}$, $N = 100\,000$

$$100\,000 = 1000e^{kt}$$
$$e^{kt} = 100$$
$$\log_e e^{kt} = \log_e 100$$
$$kt = \log_e 100$$
$$t = \frac{\log_e 100}{k}$$
$$= 132.8771238\ \ldots$$
$$= 133 \text{ (nearest whole)}$$

$\therefore$ 2h 13 min

State Mean:
0.93/1
0.92/1
0.56/1
1.64/2

Board of Studies: Notes from the Marking Centre

(i) In most responses, candidates provided the correct answer, and showed clear working-out. In most responses, candidates derived the value of k from the given condition. However, in some responses, candidates demonstrated that the given k could be used to produce the required conditions.

(ii) In almost all responses, candidates correctly substituted the given values to produce the correct answer. In better responses, candidates used the exact value of k, namely $k = \frac{\ln^2}{20}$, to produce a more precise solution.

(iii) This part was answered poorly, or not attempted at all, by most candidates. In better responses, candidates either correctly differentiated the function or used $N' = kN$.

(iv) The vast majority of candidates scored full marks for this part. Responses were usually well presented.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/