2014 13c The displacement of a particle moving along the x-axis is given by $x = t - \dfrac{1}{1+t}$,
where x is the displacement from the origin in metres, t is the time in seconds, and $t \geq 0$.

(i) Show that the acceleration of the particle is always negative. **2**

(ii) What value does the velocity approach as t increases indefinitely? **1**

(i)

$$x = t - (1+t)^{-1}$$

$$\dot{x} = 1 + 1(1+t)^{-2}$$

$$= 1 + \frac{1}{(1+t)^2}$$

$$\ddot{x} = -2(1+t)^{-3}$$

$$= \frac{-2}{(1+t)^3}$$

As $t > 0$, then $(1+t)^3 > 0$,

then $\ddot{x} < 0$ for all t.

(ii) Use $\dot{x} = 1 + \dfrac{1}{(1+t)^2}$.

As $t \to \infty$, $(1+t)^2 \to \infty$, and

$\dfrac{1}{(1+t)^2} \to 0$.

Hence, $1 + \dfrac{1}{(1+t)^2} \to 1$.

$\therefore$ velocity approaches 1 m/s.

State Mean:
1.16
0.56

Board of Studies: Notes from the Marking Centre

(i) A significant number of candidates found this part quite challenging. Some made the question more difficult by first expressing displacement as a single fraction and then differentiating twice using the quotient rule. Poor algebraic skills and incorrect use of the quotient rule made this a time consuming and inefficient method to use.

Common problems were:

- mislabelling expressions for velocity and acceleration;
- integrating to obtain $v = \ln(1+t)$, and then differentiating to return to the original expression with $a = \dfrac{1}{1+t}$;
- stating the derivative of t to be 0 instead of 1;
- not verifying that $\dfrac{d^2x}{dt^2} < 0$.

(ii) A significant number of candidates also had difficulty with this part and there was a clear lack of familiarity with limits and the concept of infinity.

Common problems were:

- substituting $t = 0$ into an expression for velocity;
- attempting to solve $v = 0$.

http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths.pdf