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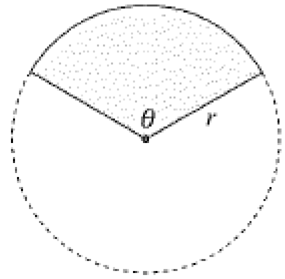
11	**10 b**	A farmer is fencing a paddock using P metres of fencing. The paddock is to be in the shape of a sector of a circle with radius r and sector θ in radians, as shown in the diagram.  (i) Show that the length of fencing required to fence the perimeter of the paddock is $P = r(\theta + 2)$. (ii) Show that the area of the sector is $A = \frac{1}{2}Pr - r^2$. (iii) Find the radius of the sector, in terms of P, that will maximize the area of the paddock. (iv) Find the angle θ that gives the maximum area of the paddock. (v) Explain why it is only possible to construct a paddock in the shape of a sector if $\frac{P}{2(\pi+1)} < r < \frac{P}{2}$.	**1** **1** **2** **1** **2**

(i) Using arc length $= r\theta$

$$P = r + r + r\theta$$
$$= 2r + r\theta$$
$$= r(\theta + 2)$$

(ii) $P = r(\theta + 2)$ from (i)

$P = r\theta + 2r$

$r\theta = P - 2r$

$\theta = \frac{P-2r}{r}$

Using area of sector $= \frac{1}{2}r^2\theta$

$$A = \frac{1}{2}r^2 \times \frac{P-2r}{r}$$
$$= \frac{1}{2}Pr - r^2$$

$\therefore A = \frac{1}{2}Pr - r^2$

(iii) $A = \frac{1}{2}Pr - r^2$

$$\frac{dA}{dr} = \frac{1}{2}P - 2r = 0$$
$$2r = \frac{P}{2}$$
$$r = \frac{P}{4}$$
$$\frac{d^2A}{dr^2} = -2 < 0, \ \therefore \text{ maximum}$$

$\therefore$ max when $r = \frac{P}{4}$

(iv) Subs $r = \frac{P}{4}$ in $P = r(\theta + 2)$

$$P = \frac{P}{4}(\theta + 2)$$
$$4 = \theta + 2$$
$$\theta = 2$$

$\therefore$ angle is 2 radians

(v) When $\theta > 0$, and from (i),

as $P = r(\theta + 2)$

$\therefore P > 2r$

$2r < P$

$r < \frac{P}{2}$ .... (1)

Also, $\theta < 2\pi$, and again from (i),

as $P = r(\theta + 2)$

$P > r(2\pi + 2)$

$P > 2r(\pi + 1)$

$r > \frac{P}{2(\pi+1)}$ .... (2)

From (1) and (2),

$$\frac{P}{2(\pi+1)} < r < \frac{P}{2}$$

State Mean:
0.73/1
0.41/1
0.57/2
0.20/1
0.08/2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Candidates are reminded that when a question asks to 'show' a result, they are required to demonstrate clear and logical working. When asked to 'explain', candidates should support their answer with a mathematical argument.

(i) The majority of responses included the correct formula.

(ii) The most popular and succinct approach involved stating the formula for the area of a sector and substituting θ in terms of *r* and *P* from part (i). Those who started with the given result $A = \frac{1}{2}\mathrm{Pr} - r^2$ and substituted the result from part (i) quite often found the resulting algebraic manipulation difficult.

(iii) The majority of candidates recognised the need to use calculus in this question. The most common and successful method was to solve $\frac{dA}{dr} = 0$ for r, then test by the second derivative. Candidates who used the first derivative test often omitted or struggled to find first derivative values for $r < \frac{P}{4}$ and $r > \frac{P}{4}$. A number of candidates appeared to ignore the given direction involving *r* and *P* and tried to maximise an area expressed in terms of θ and either *P* or *r*. This involved rigorous algebra.

(iv) Many candidates struggled to determine θ correctly. A significant number used calculus for a second time and maximised the expression for area in terms of θ.

(v) This was a challenging question, with very few responses demonstrating a quality argument. Many candidates manipulated the given result and produced equations or inequations to support the situation. However, they often did not validate their findings. Those who commenced with a restriction on θ, *A* or *P* were generally much more successful. A common and succinct method was to state the domain $0 < \theta < 2\pi$ for a sector to exist and use the expression for θ from part (i). Many explanations lacked reasoning and some candidates presented only a circular argument involving a suggested constraint.

Source: **http://www.boardofstudies.nsw.edu.au/hsc_exams/**