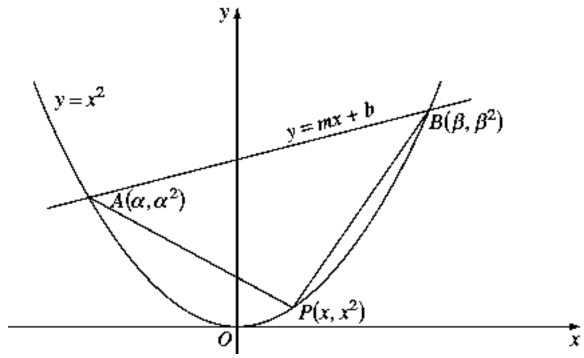
05	10 a	The parabola $y = x^2$ and the line $y = mx + b$ intersect at the points $A(\alpha, \alpha^2)$ and $B(\beta, \beta^2)$ as shown in the diagram. 	
		(i) Explain why $\alpha + \beta = m$ and $\alpha\beta = -b$.	1
		(ii) Given that $(\alpha - \beta)^2 + (\alpha^2 - \beta^2)^2 = (\alpha - \beta)^2\,[1 + (\alpha + \beta)^2]$ show that the distance $AB = \sqrt{(m^2 + 4b)(1 + m^2)}$.	2
		(iii) The point $P(x, x^2)$ lies on the parabola between A and B. Show that the area of the triangle ABP is given by $\frac{1}{2}(mx - x^2 + b)\sqrt{m^2 + 4b}$.	2
		(iv) The point P in part (iii) is chosen so that the area of the triangle ABP is a maximum. Find the coordinates of P in terms of m.	2

(i) Points of intersection of parabola and line:

$x^2 = mx + b$

$x^2 - mx - b = 0$

Now roots are α and β, then

sum of roots: $\alpha + \beta = m$ and

product of roots: $\alpha\beta = -b$.

(ii)
$$AB = \sqrt{(\alpha-\beta)^2 + (\alpha^2-\beta^2)^2}$$
$$= \sqrt{(\alpha-\beta)^2[1+(\alpha+\beta)^2]}$$
$$= \sqrt{(\alpha^2 - 2\alpha\beta + \beta^2)[1+(\alpha+\beta)^2]}$$
$$= \sqrt{((\alpha+\beta)^2 - 4\alpha\beta)[1+(\alpha+\beta)^2]}$$
$$= \sqrt{(m^2 - 4(-b))[1+m^2]}$$
$$= \sqrt{(m^2+4b)(1+m^2)}$$

(iii) Perp. distance from P(x, x^2) and $mx - y + b = 0$.
$$d = \left|\frac{mx - 1(x^2) + b}{\sqrt{m^2+1}}\right|$$
$$\therefore A = \frac{1}{2} \times \frac{mx - x^2 + b}{\sqrt{m^2+1}} \times \sqrt{(m^2+4b)}.\sqrt{1+m^2}$$
$$= \frac{1}{2}\left(mx - x^2 + b\right)\sqrt{m^2+4b}$$

(iv)
$$A = \frac{1}{2}\left(mx - x^2 + b\right)\sqrt{m^2+4b}$$
$$\frac{dA}{dx} = \frac{1}{2}(m - 2x)\sqrt{m^2+4b} = 0$$
$$m - 2x = 0$$
$$2x = m$$
$$x = \frac{m}{2}$$
$$\frac{d^2A}{dx^2} = \frac{1}{2}(-2)\sqrt{m^2+4b} < 0,$$
$\therefore$ maximum for all x.

As $P(x, x^2)$, then $P(\frac{m}{2}, \frac{m^2}{4})$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) This question was done poorly as most candidates did not recognise the simultaneous solution required to form the correct quadratic equation $x^2 - mx - b = 0$. Other candidates used the gradient formula to show $\alpha + \beta = m$ but could not then show $\alpha\beta = -b$. Many candidates tried to use $y - mx - b = 0$ as the quadratic.

(ii) Most candidates attempted this question and the majority made the link to the distance formula but were unable to manipulate their expression to get to the required answer. The majority did not establish the result $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$, which led to the required result. Many candidates did not correctly quote the distance formula or, having correctly quoted it, did not use it correctly. Some candidates were careless in the use of the square root symbol, leaving it only half-complete or absent in lines of working. Other candidates who tried working backwards were again unsuccessful due to the increasing complexity of the algebra involved.

(iii) Some candidates were able to quote the perpendicular distance formula and use it correctly. Some candidates were able to quote the correct formula but substituted incorrectly or were unable to complete the algebraic manipulation correctly.

(iv) This part had a low attempt rate. Those who did attempt the question had difficulty with the differentiation, treating the constants as variables. This generally led to pages of meaningless algebra. Candidates who differentiated correctly went on to find the correct point, but more than half of them did not establish the nature of the stationary point.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/