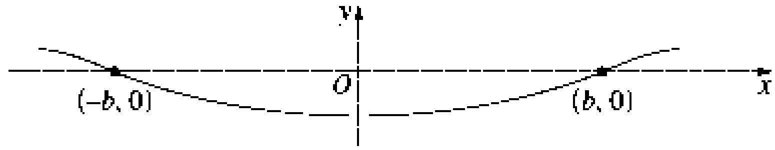
08 **9c** A beam is supported at $(-b, 0)$ and $(b, 0)$ as shown in the diagram.



It is known that the shape formed by the beam has equation $y = f(x)$, where $f(x)$ satisfies $f''(x) = k(b^2 - x^2)$ (k is a positive constant) and $f'(-b) = -f'(b)$.

(i) Show that $f'(x) = k(b^2x - \frac{x^3}{3})$ **2**

(ii) How far is the beam below the x-axis at $x = 0$? **2**

i. $f''(x) = k(b^2 - x^2)$

By integration: $f'(x) = k(b^2x - \frac{x^3}{3}) + c_1$

Now, as minimum at $x = 0$, then $f'(0) = 0$.

$\therefore k(b^2(0) - \frac{0^3}{3}) + c_1 = 0$

$\therefore c_1 = 0$

$\therefore f'(x) = k(b^2x - \frac{x^3}{3})$ ... **1**

ii. By integration: $f(x) = k(b^2\frac{x^2}{2} - \frac{x^4}{12}) + c_2$

Now, as curve passes through $(b, 0)$:

$k(b^2\frac{b^2}{2} - \frac{b^4}{12}) + c_2 = 0$

$k(\frac{b^4}{2} - \frac{b^4}{12}) + c_2 = 0$

$k(\frac{6b^4}{12} - \frac{b^4}{12}) + c_2 = 0$

$c_2 = \frac{5b^4k}{12}$

$\therefore f(x) = k(b^2\frac{x^2}{2} - \frac{x^4}{12}) + \frac{5b^4k}{12}$,

which has a y-intercept of $\frac{5b^4k}{12}$

$\therefore$ the beam is $\frac{5b^4k}{12}$ units below the x-axis at $x = 0$.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) In better responses, candidates recognised that integration was required to find the given result. Few candidates were awarded full marks due to the absence of the constant of integration in their solution or failing to clearly find the value of that constant. A common mistake was attempting to integrate b^2 as if it was a variable. The constant of integration could be evaluated by stating that a stationary point occurred at $x = 0$ or by noting that $f'(x)$ was an odd function, although that approach was not common. Responses that attempted to evaluate the constant by considering $f'(-b)$ and $-f'(b)$ were less successful. A small number of responses attempted to complete this question by differentiation of the given result, with limited success.

(ii) Better responses found a primitive with a constant of integration that could be evaluated by considering the point $(b, 0)$, the most common error being in the integral of $\frac{x^3}{3}$. A significant number of responses attempted to set $f'(x) = 0$, without integrating. This approach did not lead to the correct solution.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/