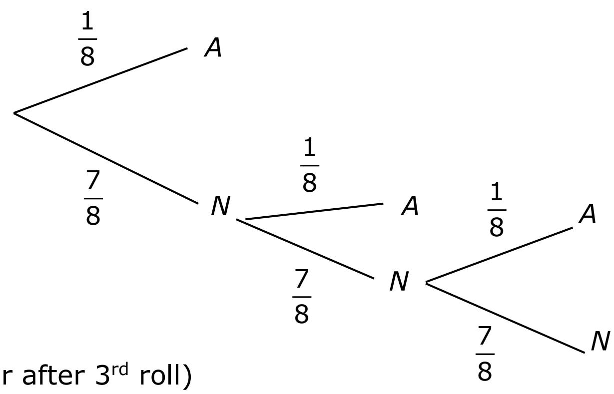
2016 15 b An eight-sided die is marked with numbers 1, 2, …, 8. A game is played by rolling the die until an 8 appears on the uppermost face. At this point the game ends.

(i) Using a tree diagram, or otherwise, explain why the probability of the game ending before the fourth roll is $\frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \left(\frac{7}{8}\right)^2 \times \frac{1}{8}$. **2**

(ii) What is the smallest value of n for which the probability of the game ending before the nth roll is more than $\frac{3}{4}$? **3**

(i) Let A = appear and N = not appear:



$\therefore$ P(ending before 4th roll)

= P(ending after 1st roll, or after 2nd roll or after 3rd roll)

= P(ending after 1st roll) + P(ending after 2nd roll) + P(ending after 3rd roll)

$$= \frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \frac{7}{8} \times \frac{7}{8} \times \frac{1}{8}$$

$$= \frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \left(\frac{7}{8}\right)^2 \times \frac{1}{8}$$

State Mean: **1.20**

(ii) P(ending before 4th roll) = $\frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \left(\frac{7}{8}\right)^2 \times \frac{1}{8}$

$\therefore$ P(ending before nth roll) = $\frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \left(\frac{7}{8}\right)^2 \times \frac{1}{8} + \ldots + \left(\frac{7}{8}\right)^{n-2} \times \frac{1}{8}$,

which is geometric sum with $a = \frac{1}{8}$, $r = \frac{7}{8}$, $n = n-1$, $S_n = \frac{a(1-r^n)}{1-r}$:

$$\frac{\frac{1}{8}\left[1-\left(\frac{7}{8}\right)^{n-1}\right]}{1-\frac{7}{8}} > \frac{3}{4}$$

$$1 - \left(\frac{7}{8}\right)^{n-1} > \frac{3}{4}$$

$$\left(\frac{7}{8}\right)^{n-1} < \frac{1}{4}$$

$$(n-1)\ln\left(\frac{7}{8}\right) < \ln\left(\frac{1}{4}\right)$$

$$n-1 > \ln\left(\frac{1}{4}\right) \div \ln\left(\frac{7}{8}\right) \quad \left(\text{as } \ln\left(\frac{7}{8}\right) < 0\right)$$

$$n-1 > 10.38178614\ldots$$

$$n > 11.38178614\ldots$$

$\therefore$ before the 12^{th} roll.

State Mean:
0.86

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

BOSTES: Notes from the Marking Centre

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