13	**16 b**	Trout and carp are types of fish. A lake contains a number of trout. At a certain time 10 carp are introduced into the lake and start eating the trout. As a consequence, the number of trout, N, decreases according to $N = 375 - e^{0.04t}$, where t is the time in months after the carp are introduced. The population of carp, P, increases according to $\frac{dP}{dt} = 0.02P$.	
		(i) How many trout were in the lake when the carp were introduced?	**1**
		(ii) When will the population of trout be zero?	**1**
		(iii) Sketch the number of trout as a function of time.	**1**
		(iv) When is the rate of increase of carp equal to the rate of decrease of trout?	**3**
		(v) When is the number of carp equal to the number of trout?	**2**

(i) $N = 375 - e^{0.04t}$

Let $t = 0$:

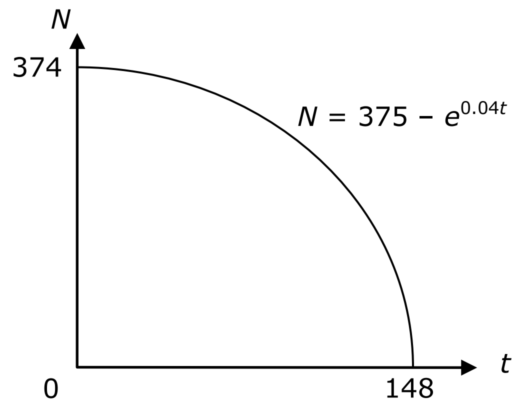
$$N = 375 - e^{0.04(0)}$$
$$= 375 - 1$$
$$= 374$$

(ii) Let $N = 0$:

$$0 = 375 - e^{0.04t}$$
$$e^{0.04t} = 375$$
$$\log_e e^{0.04t} = \log_e 375$$
$$0.04t = \log_e 375$$
$$t = \frac{\log_e 375}{0.04}$$
$$= 148.1731506 \ldots$$
$$= 148 \text{ (nearest whole)}$$

$\therefore$ after about 148 months

(iii)



(iv) $N = 375 - e^{0.04t}$

$$\frac{dN}{dt} = -0.04e^{0.04t}$$

If $\frac{dP}{dt} = 0.02P$, then

$P = Ae^{0.02t}$

Now, when $t = 0$, $P = 10$:

$\therefore 10 = Ae^{0.02(0)}$ $\quad\therefore A = 10$

$\therefore P = 10e^{0.02t}$

Now, $\frac{dP}{dt} = 0.2e^{0.02t}$

If rates equal, then:

$$0.04e^{0.04t} = 0.2e^{0.02t}$$
$$e^{0.04t} = 5e^{0.02t}$$
$$e^{0.04t} \div e^{0.02t} = 5$$
$$e^{0.02t} = 5$$
$$\log_e e^{0.02t} = \log_e 5$$
$$0.02t = \log_e 5$$
$$t = \frac{\log_e 5}{0.02}$$
$$= 80.47189562 \ldots$$
$$= 80 \text{ (nearest whole)}$$

$\therefore$ after about 80 months

(v) $N = P$

$$375 - e^{0.04t} = 10e^{0.02t}$$
$$e^{0.04t} + 10e^{0.02t} - 375 = 0$$

Let $m = e^{0.02t}$

$$m^2 + 10m - 375 = 0$$
$$(m + 25)(m - 15) = 0$$
$$m = -25, 15$$
$$e^{0.02t} = 15 \quad (\text{as } e^{0.02t} > 0)$$
$$\log_e e^{0.02t} = \log_e 15$$
$$0.02t = \log_e 15$$
$$t = \frac{\log_e 15}{0.02}$$
$$= 135.4025101 \ldots$$
$$= 135 \text{ (nearest whole)}$$

State Mean:
0.73/1
0.79/1
0.33/1
0.61/3
0.08/2

	$\therefore$ after about 135 months

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates used the substitution of $t = 0$.

Common problems were:

- not realising that $e^0 = 1$
- bald answer of 375.

(ii) Most candidates earned full marks for this part, successfully using logarithms to solve the equation.

(iii) This sketch proved to be difficult for many candidates, although some used a table of values.

Common problems were:

- the sketch of the graph not touching both horizontal and vertical axes, as was indicated in parts (i) and (ii)
- incorrect concavity of the curve.

(iv) This part was answered poorly. Some candidates were able to quote the result of $P = P_0e^{kt}$ from $\frac{dP}{dt} = kP$, where $k = 0.02$. Most candidates commenced to solve $\frac{dP}{dt} = \frac{dN}{dt}$. Some candidates understood the need to ignore the negative sign, as the questions involved the comparison of rates.

(v) This part was answered poorly, or not attempted at all, by most candidates.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/