2016 13 a Consider the function $y = 4x^3 - x^4$.

(i) Find the two stationary points and determine their nature. **4**

(ii) Sketch the graph of the function, clearly showing the stationary points and the x and y intercepts. **2**

(i)
$$y = 4x^3 - x^4$$
$$y' = 12x^2 - 4x^3 = 0$$
$$4x^2(3 - x) = 0$$
$$x = 0 \text{ or } 3$$
$$y(0) = 4(0)^3 - (0)^4 = 0$$
$$y(3) = 4(3)^3 - (3)^4 = 27$$

$\therefore$ stationary points at (0, 0) and (3, 27).

$$y'' = 24x - 12x^2$$
$$y''(0) = 24(0) - 12(0)^2 = 0$$

$\therefore$ possible horizontal point of inflexion.

Now, consider the slope of curve near $x = 0$:

x	−1	0	1
y'	>0	0	>0

$\therefore$ horizontal point of inflexion at (0, 0).

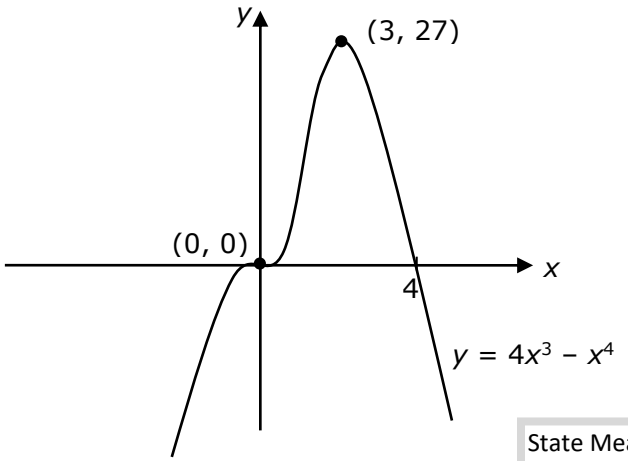
Also, $y''(3) = 24(3) - 12(3)^2 < 0$

$\therefore$ maximum at (3, 27).

$\therefore$ horizontal point of inflexion at (0, 0) and maximum at (3, 27).

State Mean: **3.01**

(ii)
$$y = 4x^3 - x^4$$
$$= x^3(4 - x) = 0$$
$$x = 0, 4$$



State Mean: **1.44**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

BOSTES: Notes from the Marking Centre

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