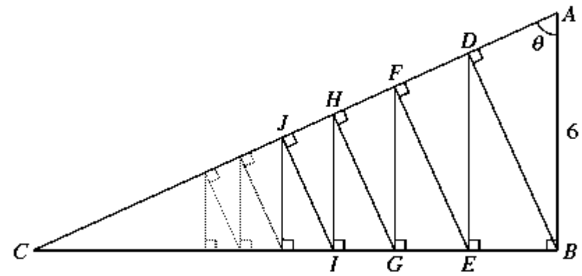
05 **9b**

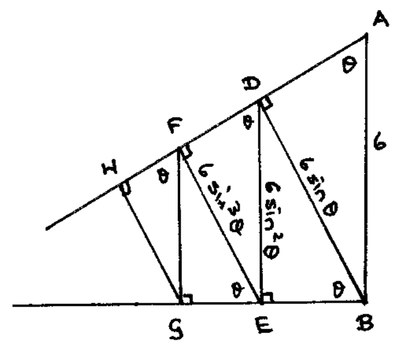
The triangle ABC has a right angle at B, $BAC = \theta$ and $AB = 6$. The line BD is drawn perpendicular to AC. The line DE is then drawn perpendicular to BC. This process continues indefinitely as shown in the diagram.



(i) Find the length of the interval BD, and hence show that the length of the interval EF is $6\sin^3\theta$. **2**

(ii) Show that the limiting sum $BD + EF + GH + \cdots$ is given by $6\sec\theta\tan\theta$. **3**

i. $\frac{BD}{6} = \sin\theta$ $\quad\therefore BD = 6\sin\theta$



$\angle DAO = \angle FDE = \angle HFG = \ldots = \theta$ (corr $\angle$s, AB || DE || FG || …)

Also, $\angle EDB = 90 - \theta$ (straight $\angle$)

$\therefore \angle EBD = 180 - (90 + 90 - \theta)$ ($\angle$ sum of triangle)

$= \theta$

Hence, $\angle FEG = \angle HGI = \ldots = \theta$

$\therefore \frac{DE}{6\sin\theta} = \sin\theta$

$DE = 6\sin^2\theta$

and, $\frac{EF}{6\sin^2\theta} = \sin\theta$

$EF = 6\sin^3\theta$

ii. $BD + EF + GH + \ldots = 6\sin\theta + 6\sin^3\theta + 6\sin^5\theta + \ldots$ which forms a geometric series

Using $S_\infty = \frac{a}{1-r}$ with $a = 6\sin\theta$ and $r = \sin^2\theta$

$$= \frac{6\sin\theta}{1-\sin^2\theta}$$

$$= \frac{6\sin\theta}{\cos^2\theta}$$

$$= 6 \times \frac{1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta}$$

$$= 6\sec\theta\tan\theta$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Many candidates correctly found the length of BD. Some then realised that the same method could be used in ΔBDE to find DE and similarly to find EF.

(ii) In better responses, candidates were able to correctly identify the terms of the series and state the correct formula. This enabled them to proceed to a correct solution. They avoided the mistakes of many who did not identify the correct terms of the series and made mistakes which could have easily been avoided. Candidates who tried to work backwards from the solution needed to show that they were evaluating the limiting sum of the series given.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/