12	12c	Jay is making a pattern using triangular tiles. The pattern has 3 tiles in the first row, 5 tiles in the second row, and each successive row has 2 more tiles than the previous row. (i) How many tiles would Jay use in row 20? (ii) How many tiles would Jay use altogether to make the first 20 rows? (iii) Jay has only 200 tiles. How many complete rows of the pattern can Jay make?	Row 1 Row 2 Row 3	2 1 2
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Arithmetic series: 3 + 5 + 7 + ...a = 3, d = 2

(i) Use
$$n = 20$$
,
 $T_n = a + (n - 1)d$
 $T_{20} = 3 + (20 - 1) \times 2$
 $= 3 + 38$
 $= 41$

(ii)
$$S_n = \frac{n}{2}[a + \ell]$$
$$= \frac{20}{2}[3 + 41]$$
$$= 440$$

(iii)
$$S_n = \frac{n}{2}[2a + (n-1)d] = 200$$

$$\frac{n}{2}[2 \times 3 + (n-1) \times 2] = 200$$

$$n[6 + 2n - 2] = 400$$

$$n[2n + 4] = 400$$

$$2n^2 + 4n - 400 = 0$$

$$n^2 + 2n - 200 = 0$$

$$\therefore n = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-200)}}{2 \times 1}$$

$$= \frac{-2 \pm \sqrt{804}}{2}$$

$$= 13.18, -15.18 \quad \text{(corr. 2 dec pl)}$$

.. only 13 complete rows could be made

Board of Studies: Notes from the Marking Centre

- (i) In most responses, candidates recognised that this question involved an arithmetic series. In better responses, candidates either substituted into the correct formula or listed the sequence of numbers. In a small number of responses, candidates used an incorrect formula but provided the correct values for the first term and the common difference.
- (ii) In better responses, candidates stated the correct formula, either $S_n = \frac{n}{2}(a+l)$ or

 $S_n = \frac{n}{2}(2a + (n-1)d)$ and showed their substitution before any calculation was

performed. This allowed markers to allocate marks to those candidates who made numerical errors. In some responses, candidates wrote down the arithmetic series and calculated the correct sum. In a small number of weaker responses, candidates attempted to use an incorrect formula.

^{*} These solutions have been provided by projectmaths and are not supplied or endorsed by the Board of Studies

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(iii) In most responses, candidates recognised the need to equate the 200 tiles to their formula for the sum of n terms, although there were some who equated this to the formula for individual terms. In many responses, candidates arrived at the correct quadratic equation or inequation, and then attempted to factorise to find the solution. If unsuccessful, they then attempted, often several times, to apply the quadratic formula to obtain a solution. In many responses, elementary errors were made when attempting to solve the quadratic equation or inequation, but most candidates correctly interpreted their decimal solution to obtain an integer answer. After unsuccessfully attempting to solve a quadratic equation or inequation many candidates used a numerical approach and were usually successful using this method, probably at the expense of time. Success from guess and check was common; however candidates who use this approach should show any workings in their response.

Source: http://www.boardofstudies.nsw.edu.au/hsc exams/