10	**9a**	(i)	When Chris started a new job, \$500 was deposited into his superannuation fund at the beginning of each month. The money was invested at 0.5% per month, compounded monthly. Let \$$P$ be the value of the investment after 240 months, when Chris retires. Show that P = 232 175.55	**2**
		(ii)	After retirement, Chris withdraws \$2000 from the account at the end of each month, without making any further deposits. The account continues to earn interest at 0.5% per month. Let \$$A_n$ be the amount left in the account n months after Chris's retirement. (1) Show that $A_n = (P - 400\,000) \times 1.005^n + 400\,000$. (2) For how many months after retirement will there be money left in the account?	 **3** **2**

State Mean:
1.19/2
0.91/3
0.81/2

(i) $P = 500 \times 1.005^{240} + 500 \times 1.005^{239} + 500 \times 1.005^{238} + \ldots + 500 \times 1.005$

$= 500[1.005 + 1.005^2 + \ldots\ldots + 1.005^{240}]$

Using $a = 1.005$, $r = 1.005$, $n = 240$, $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$\therefore P = 500 \times \dfrac{1.005(1.005^{240} - 1)}{1.005 - 1}$

$= 232175.5498 \ldots$

$= 232\,175.55$ (to 2 dec pl)

(ii) (1) $A_1 = P \times 1.005 - 2000$

$A_2 = A_1 \times 1.005 - 2000$

$= (P \times 1.005 - 2000) \times 1.005 - 2000$

$= P \times 1.005^2 - 2000(1 + 1.005)$

Hence, $A_n = P \times 1.005^n - 2000(1 + 1.005 + \ldots + 1.005^{n-1})$

Using $a = 1$, $r = 1.005$, $n = n$, $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$A_n = P \times 1.005^n - 2000 \times \dfrac{1(1.005^n - 1)}{1.005 - 1}$

$= P \times 1.005^n - 400\,000 \times [1.005^n - 1\,]$

$= (P - 400\,000) \times 1.005^n + 400\,000$

(2) Let $A_n = 0$

$(P - 400\,000) \times 1.005^n + 400\,000 = 0$

$(P - 400\,000) \times 1.005^n = -400\,000$

$1.005^n = \dfrac{-400000}{232175.55 - 400000}$

$= 2.383442937 \ldots.$

$\log_e(1.005)^n = \log_e 2.383\ldots.$

$n = \dfrac{\log_e 2.383\ldots}{\log_e 1.005}$

$= 174.14312\ldots$ $\therefore$ Money for 175 months

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates were aware that the solution involved compound interest and they used an appropriate formula to correctly find at least one term in the series. However many were unable to continue the pattern to form a geometric series. The most common errors were using $r = 1.5$ or 1.05 instead of the correct 1.005, or not realising that the last term included interest, hence substituting incorrectly into the formula for the sum of a geometric series using 500×1 instead of 500×1.005.

(ii) (1) Some candidates did not recognise this as a time-payment type question. Others had difficulty interpreting the question, with many responses stating incorrectly that $A_1 = (P - 2000)1.005$ rather than $A_1 = P \times 1.005 - 2000$. Other candidates found correct expressions for A_1 and A_2 but were then unable to develop a correct expression for A_n (or even A_3 in some cases) or developing the series for $n + 1$ terms instead of n terms. Incorrect use of brackets also caused the incorrect meaning to be implied in working.

(2) Many candidates attempted this question by stating correctly that $A_n = 0$ and most knew to use the answer provided in the previous part. Most correct solutions involved logarithms to solve the exponential equation and a small percentage of candidates used a trial and error approach to complete their solution.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/