07	**7a**	(i)	Find the coordinates of the focus, S, of the parabola $y = x^2 + 4$.	**2**
		(ii)	The graphs of $y = x^2 + 4$ and the line $y = x + k$ have only one point of intersection, P. Show that the x-coordinate of P satisfies $x^2 - x + 4 - k = 0$.	**1**
		(iii)	Using the discriminant, or otherwise, find the value of k.	**1**
		(iv)	Find the coordinates of P.	**2**
		(v)	Show that SP is parallel to the directrix of the parabola.	**1**

i.

$$y = x^2 + 4$$
$$x^2 = y - 4$$
$$x^2 = 4.\frac{1}{4}(y - 4) \quad \text{which is of the form } (x - h)^2 = 4a(y - k)$$

$\therefore$ Vertex (0, 4), and $a = \frac{1}{4}$, then focus $S(0, 4\frac{1}{4})$

ii.

$y = x^2 + 4$ **1**

$y = x + k$ **2**

Let **1** = **2**: $x^2 + 4 = x + k$

$x^2 - x + 4 - k = 0$

As P is point of intersection of the two lines, then it will be the solution of the equation $x^2 - x + 4 - k = 0$

iii.

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (-1)^2 - 4(1)(4 - k) \\ &= 1 - 16 + 4k \\ &= 4k - 15 \end{aligned}$$

As $\Delta = 0$ when equal roots: $4k - 15 = 0$

$$4k = 15$$
$$k = 3\frac{3}{4}$$

iv.

Subs $k = 3\frac{3}{4}$ into $x^2 - x + 4 - k = 0$

$$x^2 - x + 4 - 3\frac{3}{4} = 0$$
$$x^2 - x + \frac{1}{4} = 0$$
$$(x - \frac{1}{2})^2 = 0$$
$$x = \frac{1}{2}$$

Subs $x = \frac{1}{2}$ into $y = x^2 + 4$

$$y = (\frac{1}{2})^2 + 4$$
$$= 4\frac{1}{4} \qquad \therefore P(\frac{1}{2}, 4\frac{1}{4})$$

v.

As $S(0, 4\frac{1}{4})$ and $P(\frac{1}{2}, 4\frac{1}{4})$ then gradient $SP = 0$, and equation of directrix is $y = 3\frac{3}{4}$ which has a gradient of 0. This means it is parallel.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) The correct focal length was often obtained but then used with an incorrect vertex and either added or subtracted from the y value. Candidates who used the formula $(x-h)^2 = 4a(y-k)$ were able to make good progress with the question.

(ii) This mark was easily obtained by indicating use of simultaneous equations. Some candidates went on trying to solve the quadratic for x, which was not required.

(iii) Most candidates were familiar with the discriminant, ie $\Delta = b^2 - 4ac$. However, a significant number of candidates were not aware that for one root, $\Delta = 0$. Other common errors were incorrect formula, incorrect substitution into formula and inability to solve the equation that arises from substitution.

(iv) The value of k obtained in the previous part affected the success of candidates in this part. Some candidates were able to make progress towards solving the quadratic but were faced with no solution or two solutions. Better responses multiplied through the quadratic expression by 4 so that it did not involve fractions and then were able to factorise to $(2x-1)^2$.

(v) There were many occasions where this part indicated to candidates that the focus and P needed to have the same y value.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/