13	**13 d**	A family borrows \$500 000 to buy a house. The loan is to be repaid in equal monthly instalments. The interest, which is charged at 6% per annum, is reducible and calculated monthly. The amount owing after n months, $\$A_n$, is given by $A_n = Pr^n - M(1 + r + r^2 + \ldots + r^{n-1})$, (Do NOT prove this) where $\$P$ is the amount borrowed, $r = 1.005$ and $\$M$ is the monthly repayment.	
		(i) The loan is to be repaid over 30 years. Show that the monthly repayment is \$2998 to the nearest dollar.	**2**
		(ii) Show that the balance owing after 20 years is \$270 000 to the nearest thousand dollars.	**1**
		(iii) After 20 years the family borrows an extra amount, so that the family then owes a total of \$370 000. The monthly repayment remains \$2998, and the interest rate remains the same. How long will it take to repay the \$370 000?	**2**

State Mean:
1.17/2
0.52/1
0.66/2

(i) $A_n = Pr^n - M(1 + r + r^2 + \ldots + r^{n-1})$ $n = 30 \times 12 = 360$

$A_{360} = 500\,000 \times 1.005^{360} - M(1 + 1.005 + \ldots + 1.005^{359})$

Now, $1 + 1.005 + \ldots + 1.005^{359}$ is Geom, with $a = 1$, $r = 1.005$, $n = 360$, using $S_n = \dfrac{a(r^n - 1)}{r - 1}$,

$$A_{360} = 500\,000 \times 1.005^{360} - M \times \frac{1(1.005^{360} - 1)}{1.005 - 1}$$

Let $A_{360} = 0$:

$$M = 500\,000 \times 1.005^{360} \div \frac{1(1.005^{360} - 1)}{1.005 - 1}$$

$= 2997.752626 \ldots$

$= 2998$ (nearest whole) $\therefore$ \$2998

(ii) Let $n = 240$, $M = 2998$:

$$A_{240} = 500\,000 \times 1.005^{240} - 2998 \times \frac{1(1.005^{240} - 1)}{1.005 - 1}$$

$= 269\,903.6342 \ldots$

$= 270\,000$ (nearest thousand) $\therefore$ \$270 000

(iii) Let $P = 370\,000$, $M = 2998$:

$$A_n = 370\,000 \times 1.005^n - 2998 \times \frac{1(1.005^n - 1)}{1.005 - 1}$$

$$A_n = 370\,000 \times 1.005^n - 599\,600(1.005^n - 1) = 0$$

$$370\,000 \times 1.005^n - 599\,600 \times 1.005^n + 599\,600 = 0$$

$$229\,600 \times 1.005^n = 599\,600$$

$$1.005^n = \frac{599600}{229600}$$

$$n\ log_e 1.005^n = \log_e \frac{5996}{2296}$$

$$n = \log_e \frac{5996}{2296} \div \log_e 1.005$$

$= 192.4643834 \ldots$	$\therefore$ will take 193 repayments

Board of Studies: Notes from the Marking Centre

(i) Most candidates scored full marks for this part. Candidates who substituted the correct values of n and r into the correct geometric series formula and equated to zero were able to access the first mark. Candidates who went on to evaluate the correct value of M were awarded full marks.

Common problems were:

- using $n = 30$ instead of 360 months (not converting 30 years into months)
- not writing the correct decimal answer $M = \$2\,997.75$ before rounding to the required accuracy of \$2998, as was given in the question.

(ii) Most candidates substituted, summed up the resulting geometric progression series and evaluated correctly to get \$269 903.63.

A common problem was using $n = 20$ instead of 240 months.

(iii) Many candidates had difficulty with this part. Candidates were able to access one mark by successfully substituting and equating to zero. Candidates who went on to introduce logarithms eventually found the value of $n = 192.46$.

Common problems were:

- not solving accurately for n
- solving for $370000 = 500000(1.005)^n - 599600(1.005^n - 1)$
- beginning the question using incorrect amounts owed
- writing the initial equation as an expression but going no further.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/