2015 15 a The amount of caffeine, C, in the human body decreases according to the equation $\frac{dC}{dt} = -0.14C$, where C is measured in mg and t is the time in hours.

(i) Show that $C = Ae^{-0.14t}$ is a solution to $\frac{dC}{dt} = -0.14C$, where A is a constant. **1**

When $t = 0$, there are 130 mg of caffeine in Lee's body.

(ii) Find the value of A. **1**

(iii) What is the amount of caffeine in Lee's body after 7 hours? **1**

(iv) What is the time taken for the amount of caffeine in Lee's body to halve? **2**

(i) $C = Ae^{-0.14t}$

$\frac{dC}{dt} = -0.14 \times Ae^{-0.14t}$

$= -0.14C$

$\therefore \quad C = Ae^{-0.14t}$ is a solution.

State Mean: **0.63**

(ii) Substitute $C = 130$, $t = 0$ in $C = Ae^{-0.14t}$

$130 = Ae^{-0.14(0)}$

$130 = A$

$\therefore \quad A = 130$

State Mean: **0.93**

(iii) Substitute $t = 7$ in $C = 130e^{-0.14t}$

$C = 130e^{-0.14(7)}$

$= 48.79044285...$

$= 48.79$ (2 dec pl)

$\therefore$ there is 48.79 mg of caffeine in Lee's body.

State Mean: **0.91**

(iv) Substitute $C = 65$ in $C = 130e^{-0.14t}$

$65 = 130e^{-0.14t}$

$e^{-0.14t} = 0.5$

$\log_e e^{-0.14t} = \log_e 0.5$

$-0.14t = \log_e 0.5$

$t = \frac{\log_e 0.5}{-0.14}$

$= 4.95105129...$

$= 4.95$ (2 dec pl)

$\therefore$ it will take 4.95 hours.

State Mean: **1.54**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(a)(i) The most successful approach was to start with $C = Ae^{-0 \cdot 14t}$, correctly differentiate and substitute to show that the expression for C was a solution to $\frac{dC}{dt} = -0 \cdot 14C$. Candidates who used the more complex process of integration starting with $\frac{dt}{dC}$, were generally less successful as they often did not deal correctly with the constants or logarithmic/exponential rearrangements.

(a)(ii) This part was answered extremely well.

Common problems were:

- substituting incorrectly
- assuming e^0 was e^1.

(a)(iii) Most of the responses were correct, with $A = 130$ and $t = 7$ being substituted into the formula given in (a) (i).

(a)(iv) This part was challenging, especially in using logarithms to solve an exponential equation.

Common problems were:

- using half of the answer from (a)(iii)
- using 70 or 75 as half of 130
- making algebraic errors when solving the equation
- using logarithms in base 10 instead of e in the calculation.