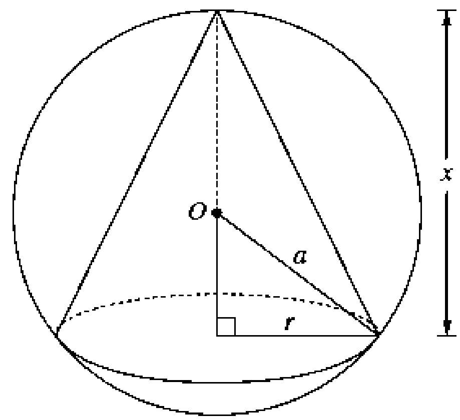
06 **9c**

A cone is inscribed in a sphere of radius a, centred at O. The height of the cone is x and the radius of the base is r, as shown in the diagram.

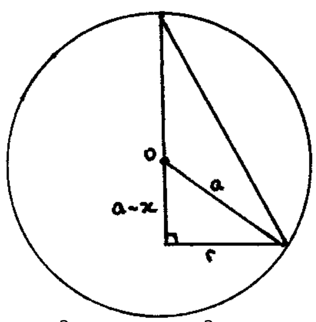
(i) Show that the volume, V, of the cone is given by **2**

$$V = \frac{1}{3}\pi(2ax^2 - x^3).$$

(ii) Find the value of x for which the volume of the cone is a maximum. You must give reasons why your value of x gives the maximum volume. **3**



i. Vol of cone $= \frac{1}{3}\pi r^2 h$ with $h = x$



$$\begin{aligned} r^2 &= a^2 - (x-a)^2 \\ &= a^2 - (x^2 - 2ax + a^2) \\ &= a^2 - x^2 + 2ax - a^2 \\ &= 2ax - x^2 \end{aligned}$$

$\therefore$ V $= \frac{1}{3}\pi(2ax - x^2).x$

$\therefore$ $V = \frac{1}{3}\pi(2ax^2 - x^3)$

ii. $V = \frac{1}{3}\pi(2ax^2 - x^3)$

$$\begin{aligned} V' &= \frac{1}{3}\pi(4ax - 3x^2) = 0 \\ 4ax - 3x^2 &= 0 \\ x(4a - 3x) &= 0 \\ x &= 0, \frac{4a}{3} \end{aligned}$$

But $x \neq 0$, then $x = \frac{4a}{3}$

$V'' = \frac{1}{3}\pi(4a - 6x)$

$V''(\frac{4a}{3}) = \frac{1}{3}\pi(4a - 6(\frac{4a}{3})) < 0$ as $a > 0$.

$\therefore$ maximum if $x = \frac{4a}{3}$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) This part was not done well. Candidates who were most successful drew a diagram and correctly applied Pythagoras' theorem to give $r^2 = a^2 - (x-a)^2$. They were then able to arrive at the correct expression for V.

(ii) Candidates who attempted this part usually did well, demonstrating that they have been well taught in maximum/minimum problems. A significant number gained full marks even though they were unable to do part (i). Candidates are reminded of the key steps in such problems, viz. finding the derivative and setting it to zero, solving this equation, testing the values so found in a clear and well-labelled manner and, finally, drawing a conclusion. The more successful candidates left V, V', V'' in factored form and used the second derivative to test. Common errors included dropping the $\frac{\pi}{3}$ or a, failing to do a test correctly, and not referring to a general principle to make a concluding statement.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/