2014 13b A quantity of radioactive material decays according to the equation $\frac{dM}{dt} = -kM$, where M is the mass of the material in kg, t is the time in years and k is a constant.

(i) Show that $M = Ae^{-kt}$ is a solution to the equation, where A is a constant. **1**

(ii) The time for half of the material to decay is 300 years. If the initial amount of material is 20 kg, find the amount remaining after 1000 years. **3**

(i) $M = Ae^{-kt}$

$\frac{dM}{dt} = -k.\ Ae^{-kt}$

$= -kM$

$\therefore\ M = Ae^{-kt}$ is a solution.

(ii) Let $A = 1$, then $M = 0.5$ and $t = 300$:

$0.5 = 1e^{-k(300)}$

$e^{-300k} = 0.5$

$-300k = \log_e 0.5$

$k = -\frac{\log_e 0.5}{300}$

Let $A = 20$, $t = 1000$:

$M = 20e^{-k(1000)}$

$= 1.984251315...$

$= 1.98$ (2 dec pl)

$\therefore$ 1.98 kg remain

State Mean:
0.59
2.27

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(i) The most successful candidates were those who started with $M = Ae^{-kt}$ and then correctly differentiated and substituted to show that the expression for M was a solution to $\frac{dM}{dt} = -kM$. Candidates who attempted to use the more complex process of integration starting with $\frac{dM}{dt}$ were generally less successful, as they often did not deal correctly with the constants or the logarithmic/exponential rearrangements.

(ii) This part was done very well with a majority of candidates earning full marks. Candidates are reminded to consider the logical correctness of their answer, as poor use of negative signs often led to a final solution larger than initial value of A.

Common problems were:

- misusing negative signs;
- careless rounding of values that were needed in subsequent working;
- incorrectly substituting for M and A, often in reverse order.

http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths.pdf