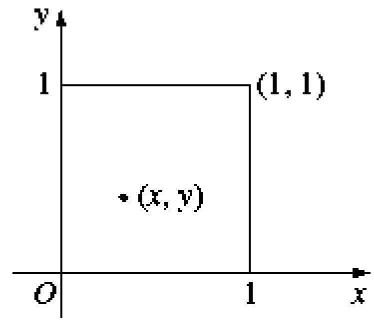
05 **10 b**

Xuan and Yvette would like to meet at a cafe on Monday. They each agree to come to the cafe sometime between 12 noon and 1 pm, wait for 15 minutes, and then leave if they have not seen the other person. Their arrival times can be represented by the point (x, y) in the Cartesian plane, where x represents the fraction of an hour after 12 noon that Xuan arrives, and y represents the fraction of an hour after 12 noon that Yvette arrives.



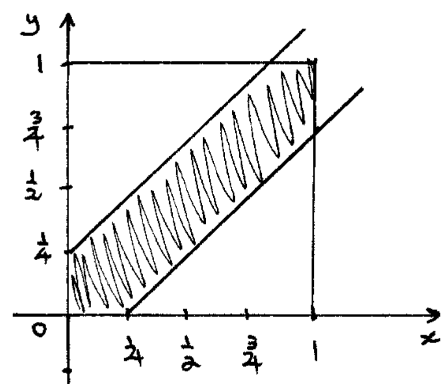
Thus $\left(\frac{1}{3}, \frac{2}{5}\right)$ represents Xuan arriving at 12:20 pm and Yvette arriving at 12:24 pm. Note that the point (x, y) lies somewhere in the unit square $0 \le x \le 1$ and $0 \le y \le 1$ as shown in the diagram.

(i) Explain why Xuan and Yvette will meet if $x - y \le \frac{1}{4}$ or $y - x \le \frac{1}{4}$. **1**

(ii) The probability that they will meet is equal to the area of the part of the region given by the inequalities in part (i) that lies within the unit square $0 \le x \le 1$ and $0 \le y \le 1$. Find the probability that they will meet. **2**

(iii) Xuan and Yvette agree to try to meet again on Tuesday. They agree to arrive between 12 noon and 1 pm, but on this occasion they agree to wait for t minutes before leaving. For what value of t do they have a 50% chance of meeting? **2**

i. $x - y \le \frac{1}{4}$ or $y - x \le \frac{1}{4}$ represent the regions where the difference in the arrival times is less that 15 minutes (or quarter of an hour)



ii. P(the girls meet) = Area of shaded region

$$= 1 \times 1 - 2 \times \frac{1}{2} \times \frac{3}{4} \times \frac{3}{4}$$

$$= 1 - \frac{9}{16}$$

$$= \frac{7}{16}$$

iii. P(the girls meet on Tuesday)

$$= 1 \times 1 - 2 \times \frac{1}{2} \times (1 - \frac{t}{60}) \times (1 - \frac{t}{60})$$

$$= 1 - (1 - \frac{t}{60})^2$$

As 50% $= \frac{1}{2}$,

$$1 - (1 - \frac{t}{60})^2 = \frac{1}{2}$$

$$1 - 1 + \frac{t}{30} - \frac{t^2}{3600} = \frac{1}{2}$$

$$t^2 - 120t + 1800 = 0$$

$$t = \frac{120 \pm \sqrt{(-120)^2 - 4(1)(1800)}}{2}$$

$$= \frac{120 \pm \sqrt{7200}}{2}$$

$$= \frac{120 \pm 60\sqrt{2}}{2}$$

$$= 60 \pm 30\sqrt{2}$$

$$= 30(2 \pm \sqrt{2})$$

As $t < 60$, $t = 30(2 - \sqrt{2})$
$= 17.57$ minutes

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) This part had the highest attempt rate in this question but generally was done poorly as most candidates had difficulty in explaining their answer in a logical way. Instead of focusing on the difference in arrival times, candidates were more concerned with waiting times. Many candidates worked through the given numerical values and stated that X and Y met without realising they were working with a given example and did not explain the inequalities.

(ii) Candidates who attempted this question with a diagram were normally successful. Some candidates were successful in working out the area of the triangles and went on to find the correct probability. Some candidates tried to work out the overlapped area directly, but typically did not find the correct dimensions.

(iii) Very few candidates attempted this part. Successful candidates used the same approach as in part (ii), but with an introduced pronumeral to establish a probability equation to solve for the time.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/