



Topic 2

Algebra

THEMES

- 1 Perfect squares, difference of two squares
- 2 Factors
- 3 Quadratic formula
- 4 Equations: linear, quadratic, simultaneous
- 5 Absolute value

FORMULA TEST

- 1 $(a+b)^2 =$
- 2 $(a-b)^2 =$
- 3 $(a+b)(a-b) =$
- 4 If $ax^2+bx+c = 0$, then $x =$
- 5 If $a-b > 0$, then a b
- 6 If $a-b < 0$, then a b
- 7 If $-a > b$, then a
- 8 If $-a < b$, then a
- 9 If $\frac{1}{a} > b$, then $a < \quad$, $a > 0$
- 10 If $-\frac{1}{a} > b$, then $a > \quad$, $a > 0$
- 11 If $x^2 = a$, then $x =$
- 12 If $|x+a| = b$, then \quad or \quad
- 13 If $|x+a| > b$, then \quad or \quad

The following formulas and results are likely to be used in solving questions in this topic.

- 1 $(a+b)^2 = a^2+2ab+b^2$
- 2 $(a-b)^2 = a^2-2ab+b^2$
- 3 $(a+b)(a-b) = a^2-b^2$
- 4 If $ax^2+bx+c = 0$,
then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- 5 If $a-b > 0$, then $a > b$
- 6 If $a-b < 0$, then $a < b$
- 7 If $-a > b$, then $a < -b$
- 8 If $-a < b$, then $a > -b$
- 9 If $\frac{1}{a} > b$, then $a < \frac{1}{b}$
- 10 If $-\frac{1}{a} > b$, then $a > -\frac{1}{b}$
- 11 If $x^2 = a$, then $x = \pm\sqrt{a}$
- 12 If $|x+a| = b$,
then $x+a = b$ or $-(x+a) = b$
- 13 If $|x+a| > b$,
then $x+a > b$ or $-(x+a) > b$

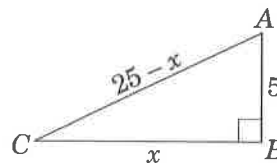
ALGEBRA

Examination questions

- 1 Simplify the expression $3x - 2(x - 4)$.
- 2 Simplify $2 - 4(x - 1)$.
- 3 Simplify $\frac{x}{2} + \frac{2x-1}{3}$.
- 4 Simplify $\frac{4}{5} - \frac{x-1}{4}$.
- 5 Factorise $4a^2 - 16$.
- 6 Factorise $25a^2 - b^2$.
- 7 Factorise $16x^2 - 9$.
- 8 Factorise $2x^2 + 7x - 15$.
- 9 Factorise $x^2 - 7x + 12$.
- 10 Factorise $3x^2 + 6x - 9$.
- 11 Solve the equation $3(x - 1) - 2 = 4$.
- 12 Solve the equation $2(3x + 1) - 3(2 - x) = 41$.
- 13 Solve the equation $2(x - 3) = x + 1$.
- 14 Solve the equation $3(x - 2) = 12 - 2(x - 1)$.
- 15 Solve $5 - (3 - x) = 4x$.
- 16 Solve $3x - 5 = 5(x + 1)$.
- 17 Solve the equation $\frac{2x}{x-7} = \frac{4}{9}$.
- 18 Solve $\frac{3x}{4} - \frac{x}{2} = 1$.
- 19 Solve the equation $3x^2 - 7x + 1 = 0$, giving each solution correct to two decimal places.

- 20 Solve the equation $v^2 - v - 1 = 0$ correct to three decimal places.

21



In the diagram, $\angle ABC$ is a right angle. Find the value of x .

- 22 Solve the simultaneous equations:

$$3x - y = 11$$

$$8x + 3y = 18$$

- 23 Solve the pair of simultaneous equations:

$$x + y = 2$$

$$3x - y = 10$$

- 24 Solve the simultaneous equations:

$$2x + y = 11$$

$$x - 2y = -2$$

- 25 Find those values of x which satisfy the inequality $3 - 2x < 6$.

- 26 Solve $5 - 3x < 8$.

- 27 Solve $|x - 2| = 3$.

- 28 Find the values of x for which $|3x - 2| \leq 4$.

- 29 Mark on a number line the values of x for which $|x - 3| < 2$.

- 30 Mark on a number line the values of x for which $|x + 2| \leq 4$.

- 31 Graph the solution of $|x + 3| \leq 2$ on a number line.

ALGEBRA

Worked solutions to examination questions

$$\begin{aligned} 1 \quad 3x - 2(x - 4) &= 3x - 2x + 8 \\ &= x + 8 \end{aligned}$$

$$\begin{aligned} 2 \quad 2 - 4(x - 1) &= 2 - 4x + 4 \\ &= 6 - 4x \end{aligned}$$

$$\begin{aligned} 3 \quad \frac{x}{2} + \frac{2x-1}{3} &= \frac{3x}{6} + \frac{4x-2}{6} \\ &= \frac{3x+4x-2}{6} \\ &= \frac{7x-2}{6} \end{aligned}$$

$$\begin{aligned} 4 \quad \frac{4}{5} - \frac{x-1}{4} &= \frac{16}{20} - \frac{5(x-1)}{20} \\ &= \frac{16 - (5x-5)}{20} \\ &= \frac{16-5x+5}{20} \\ &= \frac{21-5x}{20} \end{aligned}$$

$$\begin{aligned} 5 \quad 4a^2 - 16 &= 4(a^2 - 4) \\ &= 4(a+2)(a-2) \end{aligned}$$

$$6 \quad 25a^2 - b^2 = (5a+b)(5a-b)$$

$$7 \quad 16x^2 - 9 = (4x+3)(4x-3)$$

$$8 \quad 2x^2 + 7x - 15 = (2x-3)(x+5)$$

$$9 \quad x^2 - 7x + 12 = (x-3)(x-4)$$

$$10 \quad 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$$

$$\begin{aligned} 11 \quad 3(x-1) - 2 &= 4 \\ 3x - 3 - 2 &= 4 \\ 3x - 5 &= 4 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} 12 \quad 2(3x+1) - 3(2-x) &= 41 \\ 6x + 2 - 6 + 3x &= 41 \\ 9x - 4 &= 41 \\ 9x &= 45 \\ x &= 5 \end{aligned}$$

$$\begin{aligned} 13 \quad 2(x-3) &= x+1 \\ 2x-6 &= x+1 \\ 2x-x &= 1+6 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} 14 \quad 3(x-2) &= 12-2(x-1) \\ 3x-6 &= 12-2x+2 \\ 3x+2x &= 14+6 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

$$\begin{aligned}
 15 \quad 5 - (3 - x) &= 4x \\
 5 - 3 + x &= 4x \\
 2 &= 4x - x \\
 \therefore 3x &= 2 \\
 x &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 16 \quad 3x - 5 &= 5(x + 1) \\
 &= 5x + 5 \\
 3x - 5x &= 5 + 5 \\
 -2x &= 10 \\
 x &= \frac{10}{-2} = -5
 \end{aligned}$$

$$\begin{aligned}
 17 \quad \frac{2x}{x-7} &= \frac{4}{9} \\
 9(2x) &= 4(x-7) \\
 18x &= 4x - 28 \\
 18x - 4x &= -28 \\
 14x &= -28 \\
 x &= \frac{-28}{14} = -2
 \end{aligned}$$

$$\begin{aligned}
 18 \quad \frac{3x}{4} - \frac{x}{2} &= 1 \quad \therefore 4 \times \frac{3x}{4} - 4 \times \frac{x}{2} = 4 \times 1 \\
 3x - 2x &= 4 \\
 x &= 4
 \end{aligned}$$

$$19 \quad 3x^2 - 7x + 1 = 0$$

Using the quadratic formula:

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(1)}}{2(3)} \\
 &= \frac{7 \pm \sqrt{49 - 12}}{6} \\
 &= \frac{7 \pm \sqrt{37}}{6} \\
 &= 2.1804... \text{ or } 0.1528... \\
 &\approx 2.18 \text{ or } 0.15 \text{ (2 d.p.)}
 \end{aligned}$$

$$20 \quad v^2 - v - 1 = 0$$

Using the quadratic formula:

$$\begin{aligned}
 v &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} \\
 &= \frac{1 \pm \sqrt{1 + 4}}{2} \\
 &= \frac{1 \pm \sqrt{5}}{2} \\
 &= 1.6180... \text{ or } -0.6180... \\
 &\approx 1.618 \text{ or } -0.618 \text{ (3 d.p.)}
 \end{aligned}$$

Clear the denominators by multiplying each term by 4.

21 Using Pythagoras' theorem:

$$\begin{aligned}(25-x)^2 &= x^2 + 5^2 \\ 625 - 50x + x^2 &= x^2 + 25 \\ 625 - 50x &= 25 \\ 625 - 25 &= 50x \\ 600 &= 50x \\ x &= \frac{600}{50} = 12\end{aligned}$$

22 $3x - y = 11$ —①
 $8x + 3y = 18$ —②

Multiply ① by 3, adding with ②:

$$\begin{aligned}9x - 3y &= 33 \\ 8x + 3y &= 18 \\ \hline 17x &= 51 \\ x &= \frac{51}{17} = 3\end{aligned}$$

Substitute $x = 3$ into ①:

$$\begin{aligned}3(3) - y &= 11 \\ 9 - 11 &= y \\ \therefore y &= -2\end{aligned}$$

Solution is $x = 3, y = -2$

23 $x + y = 2$ —①
 $3x - y = 10$ —②

Adding ① and ②:

$$\begin{aligned}4x &= 12 \\ x &= 3\end{aligned}$$

Substitute $x = 3$ into ①:

$$\begin{aligned}3 + y &= 2 \\ \therefore y &= 2 - 3 = -1\end{aligned}$$

Solution is $x = 3, y = -1$

24 $2x + y = 11$ —①
 $x - 2y = -2$ —②

Multiply ① by 2 and add to ②:

$$\begin{aligned}4x + 2y &= 22 \\ x - 2y &= -2 \\ \hline 5x &= 20 \\ x &= 4\end{aligned}$$

Substitute into ①: $2(4) + y = 11$
 $8 + y = 11$
 $y = 11 - 8 = 3$

Solution is $x = 4, y = 3$

25 $3 - 2x < 6$
 $-2x < 6 - 3$
 $-2x < 3$
 $\therefore x > -\frac{3}{2}$

26 $5 - 3x < 8$
 $5 - 8 < 3x$
 $-3 < 3x$
 $-1 < x$
 i.e. $x > -1$

Remember:

Multiplication or division by a negative number reverses an inequality.

$$27 \quad |x-2| = 3 \Rightarrow x-2 = 3 \text{ or } -(x-2) = 3$$

$$x = 5 \text{ or } x-2 = -3$$

$$x = -1$$

$$\therefore x = 5 \text{ or } -1$$

$$28 \quad |3x-2| \leq 4 \Rightarrow 3x-2 \leq 4 \text{ or } -(3x-2) \leq 4$$

$$3x \leq 6 \quad 3x-2 \geq -4$$

$$x \leq 2 \quad 3x \geq -2$$

$$x \geq -\frac{2}{3}$$

$$\therefore \text{Solution is } -\frac{2}{3} \leq x \leq 2$$

$$29 \quad |x-3| < 2 \Rightarrow x-3 < 2 \text{ or } -(x-3) < 2$$

$$x < 5 \quad x-3 > -2$$

$$x > 1$$

$$\therefore \text{Solution is } 1 < x < 5$$



$$30 \quad |x+2| \leq 4 \Rightarrow x+2 \leq 4 \text{ or } -(x+2) \leq 4$$

$$x \leq 2 \quad x+2 \geq -4$$

$$x \geq -2$$

$$\therefore \text{Solution is } -2 \leq x \leq 2$$



$$31 \quad |x+3| \leq 2 \Rightarrow x+3 \leq 2 \text{ or } -(x+3) \leq 2$$

$$x \leq -1 \quad x+3 \geq -2$$

$$x \geq -5$$

$$\therefore \text{Solution is } -5 \leq x \leq -1$$

