11	**7b**	The velocity of a particle moving along the x-axis is given by $\dot{x} = 8 - 8e^{-2t}$, where t is the time in seconds and x is the displacement in metres.	
		(i) Show that the particle is initially at rest.	**1**
		(ii) Show that the acceleration of the particle is always positive.	**1**
		(iii) Explain why the particle is moving in the positive direction for all $t > 0$.	**2**
		(iv) As $t \to \infty$, the velocity of the particle approaches a constant. Find the value of this constant.	**1**
		(v) Sketch the graph of the particle's velocity as a function of time.	**2**

(i) $\dot{x} = 8 - 8e^{-2t}$

Subs $t = 0$,

$$\begin{aligned} \dot{x} &= 8 - 8e^{-2(0)} \\ &= 8 - 8e^{0} \\ &= 8 - 8 \\ &= 0 \end{aligned}$$

$\therefore$ particle initially at rest.

(ii) $\ddot{x} = 16e^{-2t}$

As $e^{-2t} > 0$, for all values of t,

then $\ddot{x}$ is always positive.

(iii) Since particle is initially at rest (from (i)) and always a positive acceleration applied (from (ii)), then the velocity must be positive for all $t > 0$. This means that it is moving in the positive direction.

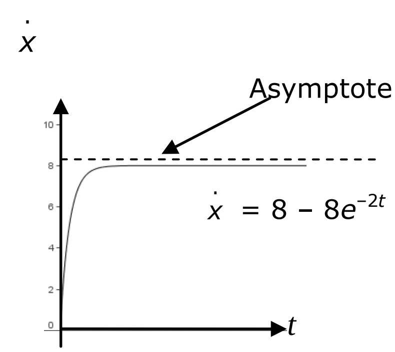
(iv) $\dot{x} = 8 - 8e^{-2t}$

As t $\to \infty$, $e^{-2t} \to 0$,

$\therefore$ $\dot{x}$ approaches 8.

The constant is 8.

(v)



State Mean:
0.88/1
0.68/1
0.44/2
0.59/1
1.00/2

Board of Studies: Notes from the Marking Centre

In better responses, candidates linked the particle's initial velocity of 0 with its positive acceleration to explain the movement in the positive direction then linked the limiting velocity to sketch velocity against time.

(i) Most candidates substituted $t = 0$ to show that $\dot{x} = 0$. Some solved $\dot{x} = 0$ to show that $t = 0$ and then used the fact that the particle does not stop again to help answer part (iii).

(ii) In many responses candidates differentiated incorrectly, giving $-16e^{-2t}$ and crossing out the negative sign in order to answer the question. Others gave e^{-3t} or e^{-2t-1}.

(iii) In better responses, candidates described the motion by linking the concepts in parts (i) and (ii). Others successfully solved $8 - 8e^{-2t} > 0$ to show that $t > 0$, while some struggled with negative expressions and negative powers to show that $t < 0$. In many responses candidates claimed that positive acceleration indicates positive velocity. Many candidates integrated to find x and claimed that positive displacement indicated movement in the positive direction.

(iv) This part was done well especially when $8-8e^{-2t}$ was written as $8-\frac{8}{e^{2t}}$.

A significant number relied on substituting values of t and observing the trend.

(v) In better responses, candidates clearly showed the curve increasing from the origin, concave down and approaching a clearly-drawn asymptote. Others resorted to completing a table of values and plotting points, rather than using their previous responses.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/