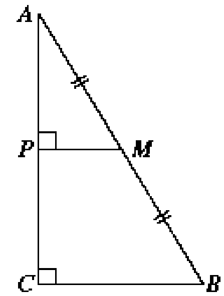
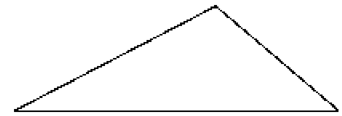
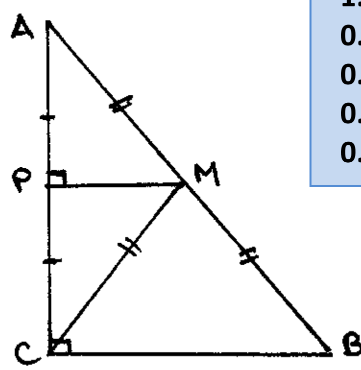
09	**4c**	In the diagram, ΔABC is a right-angled triangle, with the right angle at C. The midpoint of AB is M, and $MP \perp AC$. (i) Prove that ΔAMP is similar to ΔABC. (ii) What is the ratio of AP to AC? (iii) Prove that ΔAMC is isosceles. (iv) Show that ΔABC can be divided into two isosceles triangles.  (v) Copy or trace this triangle into your writing booklet and show how to divide it into four isosceles triangles. 	**2** **1** **2** **1** **1**

State Mean:
1.56/2
0.75/1
0.91/2
0.80/1
0.25/1

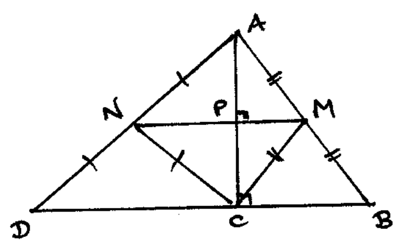
(i) $\angle A$ is common
$\angle APM = \angle ACB$ (given)
$\therefore \Delta AMP$ is similar to ΔABC (2 $\angle$s equal)



(ii) $\dfrac{AM}{AB} = \dfrac{1}{2}$ (M is midpoint of AB)
$\therefore \dfrac{AP}{AC} = \dfrac{1}{2}$ (matching sides of sim Δs in proportion)
$\therefore AP:AC = 1:2$

(iii) $AP = CP$ (from (ii))
$\angle APM = \angle CPM$ (given)
PM is common
$\therefore \angle PAM = \angle PCM$ (matching $\angle$s of congruent Δs)
$\therefore \Delta AMC$ is isosceles (base $\angle$s of isos. Δs equal)

(iv) $\therefore AM = CM$ (matching sides of congruent Δs)
$\therefore \Delta MCB$ is isosceles (two sides equal)
$\therefore \Delta ABC$ is divided into isosceles Δs ΔAMC and ΔMCB



(v) Name vertices of triangles A, B, D. Perpendiculars from A meets DB at C. M and N are midpoints of AB, AD respectively. Isosceles triangles are ΔAMC, ΔCMB, ΔANC and ΔCND .

Board of Studies: Notes from the Marking Centre

Most candidates were able to achieve some success in this part, but only a small percentage achieved full marks. This was often due to candidates not realising that there was a clear and intended connection between successive parts of the question. In better responses, candidates:

- drew a clear diagram at the top of the page
- used full and correct lettering notation for angles and sides
- included only the facts that were relevant to the question
- supplied clear and succinct reasons with their statements
- supplied clear conclusions to their proofs.

(i) The most direct way of proving similarity was by using equi-angularity and many candidates successfully completed this proof. For some responses taking this path, a common error was to state a connection between one pair of corresponding sides and then claim that one condition for similarity was two angles equal and one side in proportion. Candidates who opted to use the proportionality of two pairs of sides and equality of one angle were generally less successful.

(ii) Well answered by most candidates, but a significant number of successful responses were written as a connection between AP and AC, rather than as a ratio.

(iii) Successful responses hinged on stating that $AP = AC$, a direct consequence from part (ii), and candidates were rewarded for making this inference. Those candidates who then continued and proved the congruence of the two right-angled triangles that make up triangle AMC were generally successful. Other successful responses included the use of Pythagoras' Theorem and the property of the perpendicular bisector of the base of an isosceles triangle passing through the apex. Unsuccessful responses often involved circular arguments assuming the triangle was isosceles or trying to use the equality of angles.

(iv) A significant number of responses contained elaborate proofs that were not required of a part allocated one mark.

(v) Very few candidates were successful in this part. Responses that recognised the connection of this question to part (iv) and started by drawing a perpendicular from the base were often successful. Responses that started with an isosceles triangle were often unsuccessful as they indicated an ambiguity in the intent of the first dissection. Candidates are encouraged to use a ruler to draw diagrams and to take care in the way they indicate equal sides and angles.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/