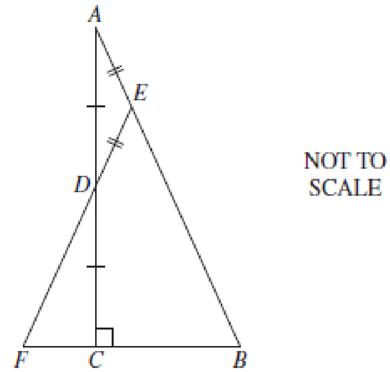
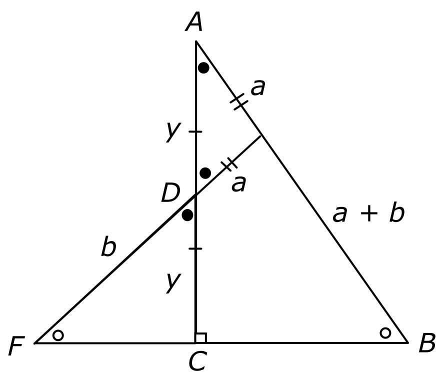
2015 15 b The diagram shows ΔABC which has a right angle at C. The point D is the midpoint of the side AC. The point E is chosen on AB such that $AE = ED$. The line segment ED is produced to meet the line BC at F. Copy or trace the diagram into your writing booklet.



(i) Prove that ΔACB is similar to ΔDCF. **2**

(ii) Explain why ΔEFB is isosceles. **1**

(iii) Show that $EB = 3AE$. **2**



(i) $\angle BAC = \angle ADE$ (base $\angle$s of isos Δ)

$\angle ADE = \angle FDC$ (vertically opposite $\angle$s)

$\therefore \angle BAC = \angle FDC$

Also, $\angle BCA = \angle FCD$ (given)

$\therefore \Delta ACB$ is similar to ΔDCF (2 $\angle$s equal)

State Mean: **1.33**

(ii) $\angle ABC = \angle DFC$

(matching $\angle$s of similar Δs)

$\therefore \Delta EFB$ is isosceles (two $\angle$s equal)

State Mean: **0.45**

(iii) Let $AE = ED = a$, $AD = DC = y$.

Also, let $DF = b$, then $EF = a + b$,

and hence $EB = a + b$.

Now, $\dfrac{2a+b}{b} = \dfrac{2y}{y}$

(matching sides of similar Δs in proportion)

$\dfrac{2a+b}{b} = 2$

$2a + b = 2b$

$b = 2a$

But $EB = a + b$

$= a + 2a$

$= 3a$

$\therefore EB = 3AE$

State Mean: **0.46**

Board of Studies: Notes from the Marking Centre

(b)(i) This similarity proof was found to be quite challenging. Most candidates were able to identify $\angle ACB = \angle DCF$ and provide a correct reason. Showing $\angle BAC = \angle ADE = \angle CDF$ proved to be difficult.

Common problems were:

- writing incorrect reasons; for example, stating that angle C was a common angle or stating that a pair of angles were alternate when they were vertically opposite
- labelling angles incorrectly
- using an incorrect test for similarity
- poor setting out with little or no reasoning
- using congruency tests to prove similarity.

(b)(ii) Most candidates recognised the need to use the similar triangle result from (b)(i) to identify the pair of corresponding equal angles.

Common problems were:

- using incorrect reasoning or no reasoning
- assuming all angles are equal in similar triangles.

(b)(iii) This part was found to be quite challenging. A popular method was to prove $AB = 2FD$ and then use the result of (b)(ii) to find $EF = EB$. Other successful approaches included constructions and trigonometry.

Common problems were:

- using incorrect reasoning or no reasoning
- using incorrect proportion statements.