06 **9b**

During a storm, water flows into a 7000-litre tank at a rate of $\frac{dV}{dt}$ litres per minute, where $\frac{dV}{dt} = 120 + 26t - t^2$ and t is the time in minutes since the storm began.

(i) At what times is the tank filling at twice the initial rate? **2**

(ii) Find the volume of water that has flowed into the tank since the start of the storm as a function of t. **1**

(iii) Initially, the tank contains 1500 litres of water. When the storm finishes, 30 minutes after it began, the tank is overflowing. How many litres of water have been lost? **2**

(i) Initial means $t = 0$:

$\frac{dV}{dt} = 120 + 26t - t^2$

$= 120 + 26(0) - (0)^2$

$= 120$

$\therefore$ initial rate is 120 litres/minute

Twice initial rate is 240 litres/minute

$\therefore \quad 120 + 26t - t^2 = 240$

$t^2 - 26t + 120 = 0$

$(t - 6)(t - 20) = 0$

$t = 6$ and 20

$\therefore$ after 6 minutes and after 20 minutes

(ii) $\frac{dV}{dt} = 120 + 26t - t^2$

$V = 120t + 13t^2 - \frac{t^3}{3} + c$

where c = initial volume

(iii) $V = 120t + 13t^2 - \frac{t^3}{3} + c$

$t = 0$, $V = 1500$

$V = 120t + 13t^2 - \frac{t^3}{3} + 1500$

Let $t = 30$,

$V = 120(30) + 13(30)^2 - \frac{(30)^3}{3} + 1500$

$= 3600 + 11\,700 - 9000 + 1500$

$= 7800$

Difference $= 7800 - 7000$

$= 800$

$\therefore$ 800 litres lost

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates were able to find the initial rate and then write $240 = 120 + 26t - t^2$. Unfortunately many then made careless errors and were unable to factorise correctly. In better responses candidates changed the equation so that the coefficient of t^2 was positive. Candidates are reminded of the importance of basic algebra skills.

(ii) This part was very poorly done. Many candidates correctly integrated and gave $V = 120t + 13t^2 - \frac{t^3}{3} + c$, but then either did not bother to evaluate c or incorrectly said that c was 1500. They did not appear to understand that V was the 'volume of water that has flowed into the tank since the start of the storm'.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/