| 12 | 15c | Ari takes out a loan of \$360 000. The loan is to be repaid in equal monthly repayments, \$$M$, at the end of each month, over 25 years (300 months). Reducible interest is charged at 6% per annum, calculated monthly. Let \$$A_n$ be the amount owing after the $n^{th}$ repayment.<br>(i) Write down an expression for the amount owing after two months, \$$A_2$. **1**<br>(ii) Show that the monthly repayment is approximately \$2319.50. **2**<br>(iii) After how many months will the amount owing, \$$A_n$, become less than \$180 000? **3** |
|---|---|---|

State Mean:
**0.57/1**
**1.11/2**
**1.14/3**

(i) 6% pa = 0.5% per month. As 0.5% = 0.005,

$A_1 = 360\,000 \times 1.005 - M$

$A_2 = A_1 \times 1.005 - M$

$= (360\,000 \times 1.005 - M) \times 1.005 - M$

$= 360\,000 \times 1.005^2 - M(1 + 1.005)$

(ii) $A_{300} = 360\,000 \times 1.005^{300} - M(1 + 1.005 + 1.005^2 + \ldots + 1.005^{299})$

Now, $A_{300} = 0$. Using geometric series with $a = 1$, $r = 1.005$, $n = 300$

$$M = 360\,000 \times 1.005^{300} \div \frac{1(1.005^{300} - 1)}{1.005 - 1}$$

$= 2319.485045$ $\therefore$ approximately \$2319.50

(iii) Let $A_n = 180\,000$

$360\,000 \times 1.005^n - 2319.50\,(1 + 1.005 + \ldots + 1.005^{n-1}) = 180\,000$

$$360\,000 \times 1.005^n - 2319.50 \times \frac{1(1.005^n - 1)}{0.005} = 180\,000$$

$1800 \times 1.005^n - 2319.5 \times (1.005^n - 1) = 900$

$1800 \times 1.005^n - 2319.5 \times 1.005^n + 2319.5 = 900$

$519.5 \times 1.005^n = 1419.5$

$$1.005^n = \frac{1419.5}{519.5}$$

$$\log_e 1.005^n = \log_e\left[\frac{1419.5}{519.5}\right]$$

$$n \log_e 1.005 = \log_e\left[\frac{1419.5}{519.5}\right]$$

$$n = \frac{\log_e\left[\frac{1419.5}{519.5}\right]}{\log_e 1.005}$$

$= 201.5408119 \ldots$

$\therefore$ after 202 months the amount will be less than \$180 000

**Board of Studies: Notes from the Marking Centre**

(i) In most responses, candidates could form a correct expression for $A_2$. In some responses, candidates evaluated the rate incorrectly.

(ii) In a considerable number of responses, candidates found the correct expression for $A_{300}$ and equated it to 0. In many weaker responses, candidates who did not gain the answer \$2319.50 used backtracking through their formula and working to try to get the correct answer – which often led to introduced errors. In particular, a number of candidates continued their sum to include $1.005^{300}$, instead of finishing at $1.005^{299}$.

(iii) In most responses, candidates substituted their value of $M$ into $A_n$ to form an inequality. However, a considerable number of candidates made mistakes throughout their subsequent working. Many candidates realised that they needed to use logarithms in this question but struggled to rearrange the formula $A_n < 1800$. In many responses, candidates applied the trial and error method on the inequality. However, candidates are reminded that they should show their guesses and checks rather than just the final answer. In a small number of responses, candidates failed to interpret their final answer correctly and rounded to 201 months rather than 202 months.

**Source:** http://www.boardofstudies.nsw.edu.au/hsc_exams/