proje*c*tmaths   **Logarithmic and Exponential Functions**

| 16 | 5 | What is the derivative of $\ln(\cos x)$? | 1 | **Solution** |

(A) $-\sec x$ (B) $-\tan x$ (C) $\sec x$ (D) $\tan x$

| 16 | 10 | Which expression is equivalent to $4 + \log_2 x$? | 1 | **Solution** |

(A) $\log_2 (2x)$ (B) $\log_2 (16 + x)$ (C) $4\log_2 (2x)$ (D) $\log_2 (16x)$

| 16 | 12 d | (i)   Differentiate $y = xe^{3x}$. | 1 | **Solution** |

(ii)   Hence find the exact value of $\displaystyle\int_0^2 e^{3x}(3 + 9x)\,dx$.   **2**

| 16 | 14 e | Write $\log 2 + \log 4 + \log 8 + … + \log 512$ in the form of $a \log b$ where a and b are integers greater than 1. | 2 | **Solution** |

| 15 | 5 | Using the trapezoidal rule with 4 subintervals, which expression gives the approximate area under the curve $y = xe^x$ between $x = 1$ and $x = 3$? | 1 | **Solution** |

(A) $\dfrac{1}{4}(e^1+6e^{1.5}+4e^2+10e^{2.5}+3e^3)$ (B) $\dfrac{1}{4}(e^1+3e^{1.5}+4e^2+5e^{2.5}+3e^3)$

(C) $\dfrac{1}{2}(e^1+6e^{1.5}+4e^2+10e^{2.5}+3e^3)$ (D) $\dfrac{1}{2}(e^1+3e^{1.5}+4e^2+5e^{2.5}+3e^3)$

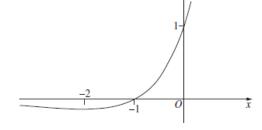| 15 | 8 | The diagram shows the graph of $y = e^x(1 + x)$. | 1 | **Solution** |

How many solutions are there to the equation $e^x(1 + x) = 1 - x^2$?
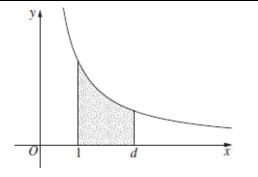
(A) 0   (B) 1

(C) 2   (D) 3

| 15 | 10 | The diagram shows the area under the curve $y = \dfrac{2}{x}$ from $x = 1$ to $x = d$. What value of d makes the shaded area equal to 2? | 1 | **Solution** |

(A) e (B) $e + 1$

(C) $2e$ (D) e^3

15	11 e	Differentiate $(e^x + x)^5$.	2	**Solution**
15	11 f	Differentiate $y = (x + 4)\ln x$.	2	**Solution**
15	11 h	Find $\displaystyle\int \dfrac{x}{x^2 - 3}\,dx$.	2	**Solution**

| 15 | 16 b | A bowl is formed by rotating the curve $y = 8\log_e(x - 1)$ about the y-axis for $0 \le y \le 6$.

Find the volume of the bowl.
Give your answer correct to 1 decimal place.

Not to scale | 3 | <u>Solution</u> |

| 14 | 3 | What is the solution to the equation $\log_2(x - 1) = 8$?
(A) 4   (B) 17   (C) 65   (D) 257 | 1 | <u>Solution</u> |

| 14 | 4 | Which expression is equal to $\int e^{2x}\, dx$?

(A) $e^{2x} + c$ (B) $2e^{2x} + c$ (C) $\dfrac{e^{2x}}{2} + c$ (D) $\dfrac{e^{2x+1}}{2x+1} + c$ | 1 | <u>Solution</u> |

| 14 | 14 a | Find the coordinates of the stationary point on the graph $y = e^x - ex$ and determine its nature. | 3 | <u>Solution</u> |

| 14 | 15 c | The line $y = mx$ is a tangent to the curve $y = e^{2x}$ at a point P.
(i)   Sketch the line and the curve on one diagram.
(ii)   Find the coordinates of P.
(iii)   Find the value of m. | 1
3
1 | <u>Solution</u> |

| 13 | 9 | What is the solution of $5^x = 4$?

(A) $x = \dfrac{\log_e 4}{5}$ (B) $x = \dfrac{4}{\log_e 5}$ (C) $x = \dfrac{\log_e 4}{\log_e 5}$ (D) $x = \log_e\left(\dfrac{4}{5}\right)$ | 1 | <u>Solution</u> |

| 13 | 11 a | Evaluate $\ln 3$ correct to three significant figures. | 1 | <u>Solution</u> |

| 13 | 11 d | Differentiate $x^2 e^x$. | 2 | <u>Solution</u> |

| 13 | 11 e | Find $\int e^{4x+1}\, dx$. | 2 | <u>Solution</u> |

| 13 | 11 f | Evaluate $\displaystyle\int_0^1 \dfrac{x^2}{x^3 + 1}\, dx$. | 3 | <u>Solution</u> |

| 12 | 7 | Let $a = e^x$.
Which expression is equal to $\log_e(a^2)$?
(A) e^{2x} (B) e^{x^2} (C) $2x$ (D) x^2 | 1 | <u>Solution</u> |

| 12 | 11 d | Differentiate $(3 + e^{2x})^5$. | 2 | <u>Solution</u> |

| 12 | 12 a | Differentiate with respect to x:
(i) $(x - 1)\log_e x$ | 2 | <u>Solution</u> |

12	**12 b**	Find $\int \dfrac{4x}{x^2+6}\ dx$.	**2**	

11	**1d**	Differentiate $\ln(5x+2)$ with respect to x.	**2**	

11	**2d**	Find the derivative of $y = x^2e^x$ with respect to x.	**2**	

11	**4b**	Evaluate $\displaystyle\int_{e}^{e^3} \dfrac{5}{x}\ dx$.	**2**	

10	**2c**	Find the gradient of the tangent to the curve $y = \ln(3x)$ at the point where $x = 2$.	**2**	

10	**2d**	(ii) Find $\int \dfrac{x}{4+x^2}\ dx$.	**2**	

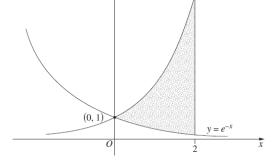|				
10	**3b**	(i) Sketch the curve $y = \ln x$.	**1**	
		(ii) Use the trapezoidal rule with three function values to find an approximation to $\displaystyle\int_{1}^{3} \ln x\ dx$.	**2**	
		(iii) State whether the approximation found in (ii) is greater than or less than the exact value of $\displaystyle\int_{1}^{3} \ln x\ dx$. Justify your answer.	**1**	

10	**4b**	The curves $y = e^{2x}$ and $y = e^{-x}$ intersect at the point $(0, 1)$ as shown in the diagram.	**3**	
		Find the exact area enclosed by the curves and the line $x = 2$.		

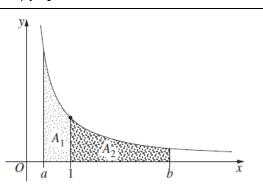|				
10	**4d**	Let $f(x) = 1 + e^x$. Show that $f(x) \times f(-x) = f(x) + f(-x)$.	**2**	

10 5c   The diagram shows the curve $y = \dfrac{1}{x}$,

for $x > 0$.
The area under the curve between
$x = a$ and $x = 1$ is A_1. The area under the
curve between $x = 1$ and $x = b$ is A_2.
The area A_1 and A_2 are each equal to 1
square unit.
Find the values of a and b.

3 <u>Solution</u>

09 1f   Solve the equation ln $x = 2$. Give your answer correct to four decimal places.   2 <u>Solution</u>

09 2a   Differentiate with respect to x: (ii) $(e^x + 1)^2$.   2 <u>Solution</u>

08 2a   (ii)   Differentiate with respect to x: $x^2\log_e x$ 2 <u>Solution</u>

08 2c   (i)   Find $\int \dfrac{dx}{x+5}$ 1 <u>Solution</u>

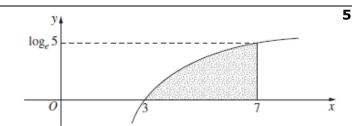**08 7a**   Solve $\log_e x - \dfrac{3}{\log_e x} = 2$.   3 <u>Solution</u>

08 10 a   In the diagram, the shaded region is bounded by $y = \log_e(x - 2)$, the x-axis and the line $x = 7$. Find the exact value of the area of the shaded region.   5 <u>Solution</u>

07 2a   (i)   Differentiate with respect to x: $\dfrac{2x}{e^x + 1}$.   2 <u>Solution</u>

07 6a   Solve the following equation for x: $2e^{2x} - e^x = 0$.   2 <u>Solution</u>

06 1a   Evaluate $e^{-0.5}$ correct to three decimal places.   2 <u>Solution</u>

06 2b   (i)   Find $\int 1 + e^{7x}\, dx$ 2   <u>Solution</u>

(ii)   Evaluate $\int_0^3 \dfrac{8x}{1 + x^2}\, dx$.   3

06 10 a   Use Simpson's rule with three function values to find an approximation to the value of $\int_{0.5}^{1.5} (\log_e x)^3\, dx$. Give your answer correct to three decimal places.   2 <u>Solution</u>

05 2c   (i)   Find $\int \dfrac{6x^2}{x^3 + 1}\, dx$ 2 <u>Solution</u>

05	**2d**	Find the equation of the tangent to $y = \log_e x$ at the point $(e, 1)$.	**2**	
05	**5a**	Use the change of base formula to evaluate $\log_3 7$, correct to two decimal places.	**2**	
05	**5c**	Find the coordinates of the point P on the curve $y = 2e^x + 3x$ at which the tangent to the curve is parallel to the line $y = 5x - 3$.	**3**	