10	6a	Let $f(x) =$	$(x+2)(x^2+4).$
			\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\

(i) Show that the graph of y = f(x) has no stationary points.

2

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- (ii) Find the values of x for which the graph y = f(x) is concave down, and the values for which it is concave up.
- (iii) Sketch the graph of y = f(x), indicating the values of the x and y intercepts.

(iv)

(i) Let
$$y = (x + 2)(x^2 + 4)$$

$$= x^3 + 4x + 2x^2 + 8$$

$$= x^3 + 2x^2 + 4x + 8$$

$$\frac{dy}{dx} = 3x^2 + 4x + 4$$

For $3x^2 + 4x + 4$, check discriminant:

$$\Delta = b^{2} - 4ac$$

$$= 4^{2} - 4(3)(4)$$

$$= -32 < 0$$

- \therefore no real roots for $\frac{dy}{dx} = 0$
- .. no stationary points

(ii)
$$\frac{dy}{dx} = 3x^2 + 4x + 4$$
$$\frac{d^2y}{dx^2} = 6x + 4$$

Now, concave down when $\frac{d^2y}{dx^2} < 0$,

$$6x + 4 < 0$$

$$6x < -4$$

$$x < \frac{-2}{3}$$

$$\therefore$$
 concave down when $x < \frac{-2}{3}$.

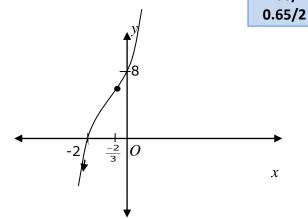
Similarly, concave up when $x > \frac{-2}{3}$.

(iii)
$$y = (x + 2)(x^2 + 4)$$

x-intercept: let y = 0, then x = -2 *y*-intercept: let x = 0, then y = 8

Also, at
$$x = \frac{-2}{3}$$
,
then $y = (\frac{-2}{3} + 2)((\frac{-2}{3})^2 + 4)$
= 5.925925 State Mean:

1.49/2 1.06/2



^{*} These solutions have been provided by projectmaths and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Most candidates stated correctly that f'(x) = 0 is a stationary point. Most candidates successfully showed, using the discriminate or the quadratic formula, that this was not possible for the given function. In poorer responses, candidates stated that the equation could not be solved as it was not possible to factorise.

Many candidates did not appreciate the connection between parts (i), (ii) and (iii). Responses from part (ii) often did not correspond with the graph drawn.

Most candidates correctly determined the x and y intercepts. Some did state that $x = \pm 2$ after incorrectly factorising $x^2 + 4$.

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Many candidates determined concavity by substituting values into the second derivative.

The concept that $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$ and the significance of the sign of the second derivative

were not well understood.

Candidates are advised to use a ruler and to consider the scale on each axis carefully for all graph work. The better sketches were achieved by candidates who chose appropriate scales on the axes. Candidates are to take care when sketching that curves do not result in turning points at the extremities of the curves.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/