2015 13 c Consider the curve $y = x^3 - x^2 - x + 3$.

(i) Find the stationary points and determine their nature. **4**

(ii) Given that the point $P(\frac{1}{3}, \frac{70}{27})$ lies on the curve, prove that there is a point of inflexion at P. **2**

(iii) Sketch the curve, labelling the stationary points, point of inflexion and y-intercept. **2**

(i) $y' = 3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$x = -\frac{1}{3}$ or 1

$y(-\frac{1}{3}) = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 - (-\frac{1}{3}) + 3$

$= 3\frac{5}{27}$

$y(1) = 1^3 - 1^2 - 1 + 3$

$= 2$

$\therefore$ stationary points at $(-\frac{1}{3}, 3\frac{5}{27})$ and $(1, 2)$.

$y'' = 6x - 2$

$y''(-\frac{1}{3}) = 6(-\frac{1}{3}) - 2 < 0$ $\therefore$ max $(-\frac{1}{3}, 3\frac{5}{27})$.

$y''(1) = 6(1) - 2 > 0$ $\therefore$ min at $(1, 2)$.

State Mean: **3.25**

(ii) $y'' = 6x - 2 = 0$

$6x = 2$

$x = \frac{1}{3}$

$\therefore$ possible point of inflexion at $(\frac{1}{3}, \frac{70}{27})$.

Consider neighbourhood:

x	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$
y''	< 0	0	> 0

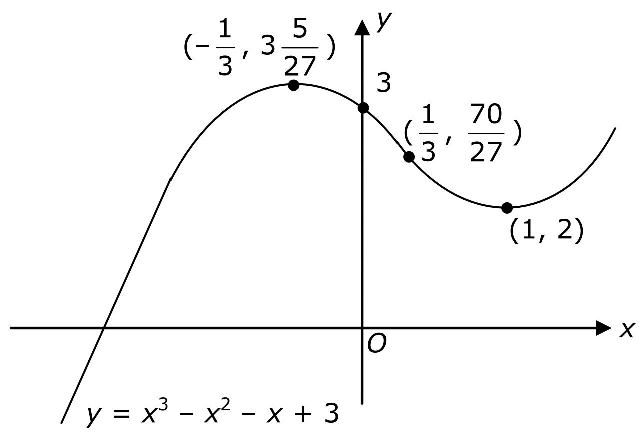
As change in concavity, then the point of inflexion is at $(\frac{1}{3}, \frac{70}{27})$.

State Mean: **1.23**

(iii) Substitute $x = 0$ in $y = x^3 - x^2 - x + 3$:

$\therefore y = 3$.

$\therefore$ y-intercept of 3.



State Mean: **1.31**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(c)(i) Most candidates answered this part very well, setting out their work in clear, logical steps. Responses in which the second derivative test was used to determine concavity were generally more successful than those in which a table and the first derivative test were used.

Common problems were:

- inability to correctly factorise the quadratic derivative
- finding an incorrect y value
- attempting to determine the nature of the stationary points using a table but not indicating whether they were using y, y' or y'' and often not showing the resulting value
- using the second derivative test to determine the nature but incorrectly identifying the condition for a maximum or minimum turning point.

(c)(ii) Most candidates successfully solved $y'' = 0$ to find the point of inflection and then used the second derivative to show a change in concavity.

(c)(iii) In the majority of responses, candidates used their previous results to sketch the cubic curve. Candidates are again reminded to draw large diagrams of at least one-third of a page and clearly label the required features.

Common problems were:

- not finding the y-intercept
- not labelling the stationary points, point of inflexion and y-intercept as required
- not graphing their correct points with relative position
- graphing the point of inflexion as a horizontal point of inflexion.