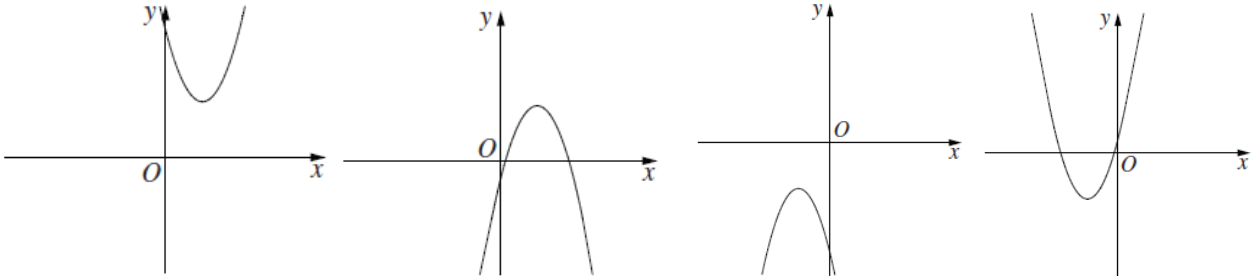
projectmaths

The Quadratic Polynomial and the Parabola

16 **3** Which diagram best shows the graph of the parabola $y = 3 - (x - 2)^2$? **1** Solution

(A) (B) (C) (D)



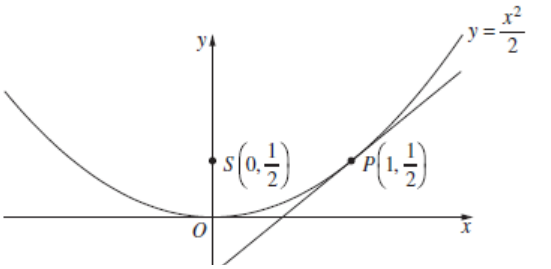
16 **13 b** Consider the parabola $x^2 - 4x = 12y + 8$. Solution

(i) By completing the square, or otherwise, find the focal length of the parabola. **2**

(ii) Find the coordinates of the focus. **1**

15 **12 d** For what values of k does the quadratic equation $x^2 - 8x + k = 0$ have real roots? **2** Solution

15 **12 e** The diagram shows the parabola $y = \frac{x^2}{2}$ with focus $S(0, \frac{1}{2})$. A tangent to the parabola is drawn at P$(1, \frac{1}{2})$. Solution



Not to scale

(i) Find the equation of the tangent at the point P. **2**

(ii) What is the equation of the directrix of the parabola? **1**

(iii) The tangent and directrix intersect at Q. Show that Q lies on the y-axis. **1**

(iv) Show that ΔPQS is isosceles. **1**

14 **14 b** The roots of the quadratic equation $2x^2 + 8x + k = 0$ are α and β. Solution

(i) Find the value of $\alpha + \beta$. **1**

(ii) Given that $\alpha^2\beta + \alpha\beta^2 = 6$, find the value of k. **2**

13 **7** A parabola has focus (5, 0) and directrix $x = 1$. **1** Solution

What is the equation of the parabola?

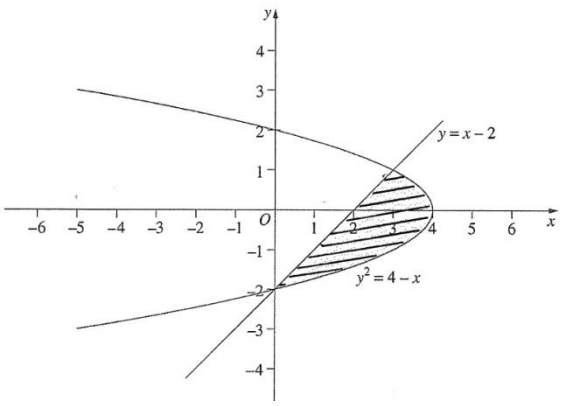
(A) $y^2 = 16(x - 5)$ (B) $y^2 = 8(x - 3)$ (C) $y^2 = -16(x - 5)$ (D) $y^2 = -8(x - 3)$

12 **3** The quadratic equation $x^2 + 3x - 1 = 0$ has roots α and β. **1** Solution

What is the value of $\alpha\beta + (\alpha + \beta)$?

(A) 4 (B) 2 (C) –4 (D) –2

12 **8** The diagram shows the region enclosed by $y = x - 2$ and $y^2 = 4 - x$.
Which of the following pairs of inequalities describes the shaded region in the diagram?



(A) $y^2 \leq 4 - x$ and $y \leq x - 2$

(B) $y^2 \leq 4 - x$ and $y \geq x - 2$

(C) $y^2 \geq 4 - x$ and $y \leq x - 2$

(D) $y^2 \geq 4 - x$ and $y \geq x - 2$

1 Solution

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12 **11 e** Find the coordinates of the focus of the parabola $x^2 = 16(y - 2)$. **2** Solution

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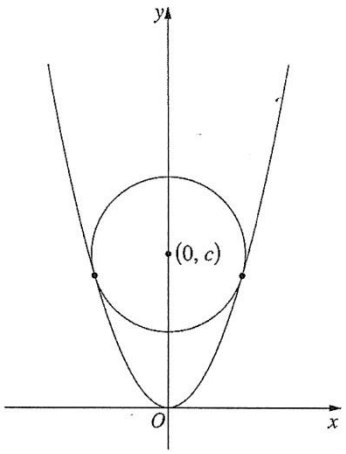
12 **16 c** The circle $x^2 + (y - c)^2 = r^2$, where $c > 0$ and $r > 0$, lies inside the parabola $y = x^2$.
The circle touches the parabola at exactly two points located symmetrically on opposite sides of the y-axis, as shown in the diagram.



(i) Show that $4c = 1 + 4r^2$. **2**

(ii) Deduce that $c > \frac{1}{2}$. **1**

Solution

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11 **2a** The quadratic equation $x^2 - 6x + 2 = 0$ has roots α and β.

(i) Find $\alpha + \beta$. **1**

(ii) Find $\alpha\beta$. **1**

(iii) Find $\frac{1}{\alpha} + \frac{1}{\beta}$. **1**

Solution

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11 **3b** A parabola has focus (3, 2) and directrix $y = -4$. **2** Solution
Find the coordinates of the vertex.

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11 **6b** A point $P(x, y)$ moves so that the sum of the square of its distance from each of the points $A(-1, 0)$ and $B(3, 0)$ is equal to 40. **3** Solution
Show that the locus of $P(x, y)$ is a circle, and state its radius and centre.

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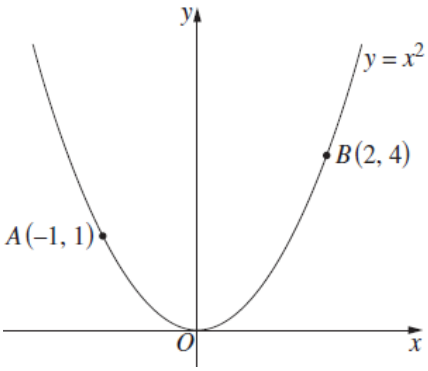
10 **2b** Solve the inequality $x^2 - x - 12 < 0$. **2** Solution

10 **7b** The parabola shown in the diagram is the graph $y = x^2$. The points $A(-1, 1)$ and $B(2, 4)$ are on the parabola.



(i) Find the equation of the tangent to the parabola at A. **2**

(ii) Let M be the midpoint of AB. There is a point C on the parabola such that the tangent at C is parallel to AB. Show that the line MC is vertical. **2**

(iii) The tangent at A meets the line MC at T. Show that the line BT is a tangent to the parabola. **2**

Solution

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09 **4b** Find the values of k for which the quadratic equation $x^2 - (k + 4)x + (k + 7) = 0$ has equal roots. **3**

Solution

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08 **4c** Consider the parabola $x^2 = 8(y - 3)$.

(i) Write down the coordinates of the vertex. **1**

(i) Find the coordinates of the focus. **1**

(iii) Sketch the parabola. **1**

Solution

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07 **7a** (i) Find the coordinates of the focus, S, of the parabola $y = x^2 + 4$. **2**

(ii) The graphs of $y = x^2 + 4$ and the line $y = x + k$ have only one point of intersection, P. Show that the x-coordinate of P satisfies $x^2 - x + 4 - k = 0$. **1**

(iii) Using the discriminant, or otherwise, find the value of k. **1**

(iv) Find the coordinates of P. **2**

(v) Show that SP is parallel to the directrix of the parabola. **1**

Solution

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06 **7a** Let α and β be the solutions of $x^2 - 3x + 1 = 0$.

(i) Find $\alpha\beta$. **1**

(ii) Hence find $\alpha + \frac{1}{\alpha}$. **1**

Solution

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06 **7c** (i) Write down the discriminant of $2x^2 + (k - 2)x + 8$, where k is a constant. **1**

(ii) Hence, or otherwise, find the values of k for which the parabola $y = 2x^2 + kx + 9$ does not intersect the line $y = 2x + 1$. **2**

Solution

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06 **9a** Find the coordinates of the focus of the parabola $12y = x^2 - 6x - 3$. **2**

Solution

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05 **1f** Find the coordinates of the focus of the parabola $x^2 = 8(y - 1)$. **2**

Solution