2015 **15 c** Water is flowing in and out of a rock pool. The volume of water in the pool at time t hours is V litres. The rate of change of the volume is given by $\frac{dV}{dt} = 80\sin(0.5t)$.

At time $t = 0$, the volume of water in the pool is 1200 litres and is increasing.

(i) After what time does the volume of water first start to decrease? **2**

(ii) Find the volume of water in the pool when $t = 3$. **2**

(iii) What is the greatest volume of water in the pool? **1**

(i) 'First starts to decrease' when $\frac{dV}{dt} = 0$:

$$80\sin(0.5t) = 0$$
$$\sin(0.5t) = 0$$
$$0.5t = 0,\ \pi,\ 2\pi,\ \ldots$$
$$t = 0,\ 2\pi,\ 4\pi,\ \ldots$$

As $t > 0$, then $t = 2\pi,\ 4\pi,\ \ldots$

$$\frac{d^2V}{dt^2} = 40\cos(0.5t)$$
$$\frac{d^2V}{dt^2}(2\pi) = 40\cos(0.5(2\pi)) < 0$$

$\therefore$ maximum volume when $t = 2\pi$.

$\therefore$ the volume starts to decrease after 2π hours.

State Mean: **0.73**

(ii) $$\frac{dV}{dt} = 80\sin(0.5t)$$
$$V = -160\cos(0.5t) + c$$

Substitute $V = 1200$ and $t = 0$:

$$1200 = -160\cos(0.5(0)) + c$$
$$1200 = -160 + c$$
$$c = 1360$$
$$\therefore\ V = 1360 - 160\cos(0.5t)$$

Substitute $t = 3$:

$$V = 1360 - 160\cos(0.5(3))$$
$$= 1348.682048\ldots$$
$$= 1349 \text{ (nearest whole)}$$

$\therefore$ the volume is 1349 L.

State Mean: **1.13**

(iii) Greatest volume when $t = 2\pi$ (from part (i)).

$$V = 1360 - 160\cos(0.5(2\pi))$$
$$= 1360 - 160\cos\pi$$
$$= 1520$$

$\therefore$ the greatest volume is 1520 L.

State Mean: **0.27**

Board of Studies: Notes from the Marking Centre

(c)(i) In many responses, candidates started by attempting to integrate $\frac{dV}{dt}$. For many, this resulted in them using V rather than $\frac{dV}{dt}$ to find when the water started to decrease. The better responses included a sketch of the function $\frac{dV}{dt}$ and this was used to determine the solution.

Common problems were:
- incorrectly solving $\sin(0 \cdot 5t) < 0$; for example finding $t < 0, \; t < 2\pi$
- misreading the solution when using the graphical approach; for example finding $t = \pi$
- using the solution for (c) (ii)
- writing the answer as $t = 360$ hours
- integrating $80 \sin(0 \cdot 5t)$.

(c)(ii) In many responses, candidates obtained the correct primitive function and used $V = 1200$ when $t = 0$ to find the correct expression for $V(t)$.

Common problems were:
- differentiating instead of integrating
- not using a constant of integration
- using the calculator in degree mode instead of radian mode
- not realising $\cos 0 = 1$ and hence not correctly evaluating the constant of integration
- poor setting out, misuse of brackets.

(c)(iii) Many candidates found it difficult to link (c)(i) and(c)(ii) and completed the same working twice. In the better responses, candidates stated and used the fact that $-1 \le \cos t \le 1$. Only a small percentage of candidates used a graph to identify the maximum volume.

Common problems were:
- substituting/calculating in degrees when t was given in radians
- unnecessarily calculating the second derivative.