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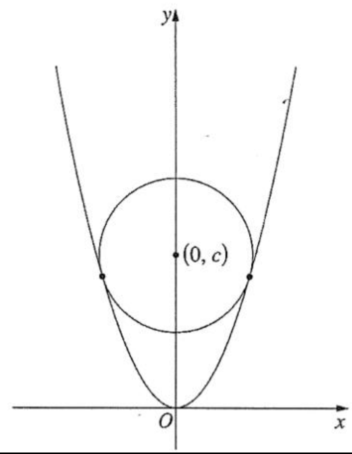
12 **16c**

The circle $x^2 + (y - c)^2 = r^2$, where $c > 0$ and $r > 0$, lies inside the parabola $y = x^2$.
The circle touches the parabola at exactly two points located symmetrically on opposite sides of the y-axis, as shown in the diagram.

(i) Show that $4c = 1 + 4r^2$. **2**

(ii) Deduce that $c > \frac{1}{2}$. **1**



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(i) $y = x^2$ 1

$x^2 + (y - c)^2 = r^2$ 2

$\therefore$ Subs 1 in 2 :

$$y + (y - c)^2 = r^2$$

$$y + y^2 - 2cy + c^2 = r^2$$

$$y^2 + (1 - 2c)y + c^2 - r^2 = 0 \text{ .... } 3$$

$\Delta = 0$ for equal roots:

$$\therefore \quad \Delta = (1 - 2c)^2 - 4 \times 1 \times (c^2 - r^2)$$

$$= 1 - 4c + 4c^2 - 4c^2 + 4r^2 = 0$$

$$4r^2 - 4c + 1 = 0$$

$$4c = 1 + 4r^2$$

(ii) From 3

$$y = \frac{-(1-2c) \pm \sqrt{(1-2c)^2 - 4(1)(c^2 - r^2)}}{2}$$

But, $\Delta = 0$, $\therefore \quad y = \dfrac{-(1-2c)}{2}$

Now, $y > 0$, $\quad \therefore \quad y = \dfrac{-(1-2c)}{2} > 0$

$$\frac{-1+2c}{2} > 0$$

$$-1 + 2c > 0$$

$$c > \frac{1}{2}$$

Alternatively,

From diagram, $c > r$

$$\therefore \quad 4c > 4r$$

$$16c^2 > 16r^2$$

$$16c^2 > 4(4r^2)$$

$$16c^2 > 4(4c - 1) \quad \text{(using part i)}$$

$$16c^2 > 16c - 4$$

$$16c^2 - 16c + 4 > 0$$

$$4c^2 - 4c + 1 > 0$$

$$(2c - 1)^2 > 0$$

$$2c - 1 > 0 \quad \text{(as } c > 0\text{)}$$

$$2c > 1$$

$$c > \frac{1}{2}$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Those candidates who attempted this part realised the need to solve the two equations simultaneously. In many responses, candidates successfully substituted in for either y or x, but very few simplified correctly or used the discriminant to derive the required result. In a number of responses, candidates tried various substitutions in an attempt to gain the required result. In some responses, candidates tried to substitute x or y into the given result or work their way backwards from this given result.

(ii) In many responses, candidates attempted to manipulate the result given in part (i) with no success. The most common incorrect answer was $c > \frac{1}{4}$, obtained by using the result from part (i) and the fact that $r > 0$. Those candidates who wrote down the formula solution in part (i), that is $y = \dfrac{-(1-2c) \pm \sqrt{(1-2c)^2 - 4 \times 1 \times (c^2 - r^2)}}{2}$ then realising that $y > 0$ and $\Delta = 0$, obtained the desired result.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/