12	15b		
		The velocity of a particle is given by $\dot{x} = 1 - 2\cos t$, where x is the displacement in metres and t is the time in seconds. Initially the particle is 3 m to the right of the origin. (i) Find the initial velocity of the particle. (ii) Find the maximum velocity of the particle. (iii) Find the displacement, x, of the particle in terms of t. (iv) Find the position of the particle when it is at rest for the first time.	**1** **1** **2** **2**

State Mean:
0.83/1
0.29/1
1.21/2
1.00/2

(i) $\dot{x} = 1 - 2\cos t$

Subs $t = 0$,

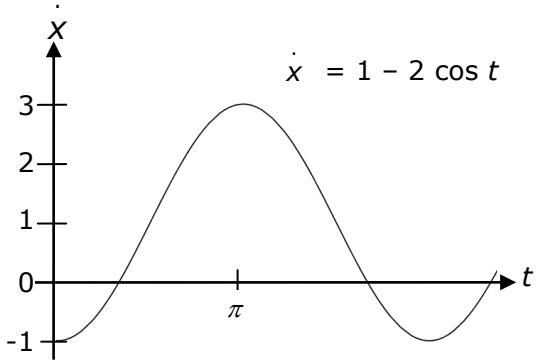
$\dot{x}(0) = 1 - 2\cos(0)$

$= 1 - 2$

$= -1$

$\therefore$ initial velocity is -1 ms^{-1}

(ii)



$\therefore$ maximum velocity is 3 ms^{-1}

OR: As $-1 \le \cos t \le 1$, then

$-2 \le 2\cos t \le 2$

$\therefore 2 \ge -2\cos t \ge -2$

$\therefore 3 \ge 1 - 2\cos t \ge -1$

$\therefore$ maximum velocity is 3 ms^{-1}

OR: Maximum velocity when $\ddot{x} = 0$

$\ddot{x} = 2\sin t = 0$

$t = 0, \pi, 2\pi, \ldots$

$\dot{x}(0) = -1$ from (i)

$\dot{x}(\pi) = 1 - 2\cos\pi$

$= 1 - 2(-1)$

$= 3$

$\therefore$ maximum velocity is 3 ms^{-1}

(iii) $\dot{x} = 1 - 2\cos t$

$x = t - 2\sin t + c$

When $t = 0$, $x = 3$,

$3 = 0 - 2\sin 0 + c$

$\therefore c = 3$

$\therefore x = t - 2\sin t + 3$

(iv) $\dot{x} = 1 - 2\cos t = 0$

$\cos t = \frac{1}{2}$

$t = \frac{\pi}{3}$

$x\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - 2\sin\frac{\pi}{3} + 3$

$= \frac{\pi}{3} - 2\left(\frac{\sqrt{3}}{2}\right) + 3$

$= \frac{\pi}{3} - \sqrt{3} + 3$

Board of Studies: Notes from the Marking Centre

(i) In the majority of responses, candidates understood that initial meant when $t = 0$ and then successfully substituted into the velocity formula to find the required velocity. A common error was an incorrect evaluation of $\cos(0)$.

(ii) In better responses, candidates approached this part by using a graph and obtained an answer quickly with efficient working. This part was only worth one mark and those candidates who employed calculus techniques had varying success but spent a lot of time to reach their, often incorrect, conclusion. In many responses, candidates gave the time when the particle was at its maximum velocity rather than the maximum velocity.

(iii) In most responses, candidates integrated the velocity equation and evaluated the resulting constant. Common errors included not finding the value of the constant and not integrating with respect to time. In many responses, candidates integrated 1 to become x rather than t.

(iv) In the majority of responses, candidates managed to get $t = \frac{\pi}{3}$ but did not substitute to find the position of the particle at this time. Candidates are reminded that when using calculus, angles are measured in radians and not degrees. Correct responses involved the substitution of $t = \frac{\pi}{3}$ rather than $t = 60$.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/