06	**6b**	A rare species of bird lives only on a remote island. A mathematical model predicts that the bird population, P, is given by $P = 150 + 300e^{-0.05t}$ where t is the number of years after observations began. (i) According to the model, how many birds were there when observations began? (ii) According to the model, what will be the rate of change in the bird population ten years after observations began? (iii) What does the model predict will be the limiting value of the bird population? (iv) The species will become eligible for inclusion in the endangered species list when the population falls below 200. When does the model predict that this will occur?	**1** **2** **1** **2**

(i) $P = 150 + 300e^{-0.05t}$

When $t = 0$:

$P = 150 + 300e^{0}$

$= 150 + 300$

$= 450$

$\therefore$ 450 birds

(ii) $P = 150 + 300e^{-0.05t}$

$\frac{dP}{dt} = -15e^{-0.05t}$

When $t = 10$:

$\frac{dP}{dt} = -15e^{-0.5}$

$= -9.097959896 \ldots$

$= -9.098$ (3 dec places)

(iii) $P = 150 + 300e^{-0.05t}$

$= 150 + \frac{300}{e^{0.05t}}$

If $t \to \infty$, then $0.05t \to \infty$,

and then $\frac{300}{e^{0.05t}} \to 0$.

This means P $\to$ 150.

The limiting value is 150 birds.

(iv) $P = 150 + 300e^{-0.05t}$

When $P = 200$:

$200 = 150 + 300e^{-0.05t}$

$50 = 300e^{-0.05t}$

$e^{-0.05t} = \frac{1}{6}$

Taking logs of both sides:

$-0.05t = \log_e \frac{1}{6}$

$t = \log_e \frac{1}{6} \div -0.05$

$= 35.83518938 \ldots$

$\therefore$ Birds placed on endangered list in the 36th year.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates substituted $t = 0$ into the mathematical model $P = 150 + 300e^{-0.05t}$ and obtained the correct answer of 450 birds.

(ii) A significant number of candidates did not calculate $\frac{dP}{dt}$ to determine the rate of change.

Most candidates who determined the rate of change obtained the correct answer. Candidates who omitted the negative sign or did not indicate a decreasing rate were awarded one mark.

(iii) Many candidates attempted to use the limiting sum of a geometrical progression to answer this question. Candidates providing better responses understood that $e^{-0.05t} \to 0$ when $t \to \infty$ and obtained the correct answer of 150 birds.

(iv) A pleasing number of candidates obtained full marks for this question. Candidates solved the exponential equation $50 = 300e^{-0.05t}$ using logarithms or obtained the correct answer using a trial-and-error method and their calculator.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/