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10 **3a**

In the diagram, A, B and C are the points $(-2, -4)$, $(12, 6)$ and $(6, 8)$ respectively. The point $N(2, 2)$ is the midpoint of AC. The point M is the midpoint of AB.

(i) Find the coordinates of M. **1**

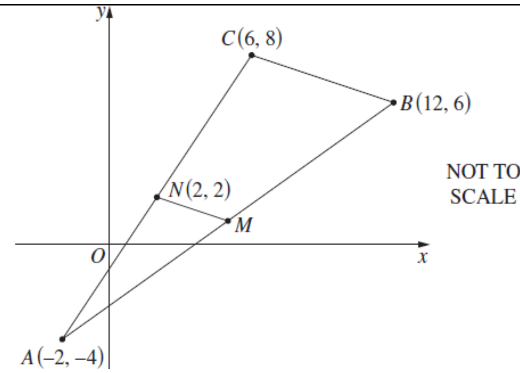
(ii) Find the gradient of BC. **1**

(iii) Prove that ΔABC is similar to ΔAMN. **2**

(iv) Find the equation of MN. **2**

(v) Find the exact length of BC. **1**

(vi) Given that the area of ΔABC is 44 square units, find the perpendicular distance from A to BC. **1**



State Mean:
0.94/1
0.91/1
1.23/2
1.79/2
0.92/1
0.71/1

(i) Using $A(-2, -4)$ and $B(12, 6)$,

$$\text{MP} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2+12}{2}, \frac{-4+6}{2}\right) = \left(\frac{10}{2}, \frac{2}{2}\right) = (5, 1) \qquad \therefore M(5, 1)$$

(ii) Using $B(12, 6)$ and $C(6, 8)$,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-6}{6-12} = \frac{2}{-6} = -\frac{1}{3}$$

(iii) In ΔAMN and ΔABC:

$\angle A$ is common

As N is midpt. of AC (given)

& M is midpt. of AB (from (i))

$$\therefore \frac{AN}{AC} = \frac{AM}{AB}$$

$\therefore \Delta AMN \ ||| \ \Delta ABC$

(2 sides in proportion and included angle equal).

(iv) Using $M(5, 1)$ and $N(2, 2)$,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{2-5} = -\frac{1}{3} \quad **$$

Using $N(2, 2)$ and $m = -\frac{1}{3}$,

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{3}(x - 2)$$

$$3y - 6 = -x + 2$$

$$\therefore \ x + 3y - 8 = 0$$

[** Or: As $\Delta AMN \ ||| \ \Delta ABC$, then MN || BC, so grad MN $= -\frac{1}{3}$]

(v) Using B(12, 6) and C(6, 8),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6-12)^2 + (8-6)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

$\therefore$ length is $2\sqrt{10}$ units

(vi) Area $= \frac{1}{2} \times$ base $\times$ height

$44 = \frac{1}{2} \times \text{BC} \times$ perp. height

$44 = \frac{1}{2} \times 2\sqrt{10} \times$ perp. height

$44 = \sqrt{10} \times$ perp. height

$$\therefore \text{perp. height} = \frac{44}{\sqrt{10}} = \frac{44}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{44\sqrt{10}}{10} = \frac{22\sqrt{10}}{5}$$

$\therefore$ perp height is $\frac{22\sqrt{10}}{5}$ units

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Board of Studies: Notes from the Marking Centre

(i) Most candidates successfully used the midpoint formula to find the coordinates of M, by averaging the x-values and the y-values.

(ii) Most candidates applied the gradient formula between 2 points with success. Some errors occurred when the negative sign was lost.

(iii) Most successful candidates followed the sequence of parts (i) and (ii) and used coordinate geometry to prove that NM is parallel to BC and hence showed that the triangles are similar. Many candidates did not successfully apply the recognised proofs for similar triangles. Many candidates also failed to supply clear and succinct reasons with their statements or clear conclusions to their proofs.

(iv) Most candidates found the appropriate equation in this part, generally using the point-gradient form of a straight line, applying their gradient and using one of the points M or N.

(v) Many candidates applied the distance formula and were explicit with their substitution to achieve the correct distance $d_{nm} = \sqrt{(6-12)^2 + (8-6)^2}$.

(vi) A significant number of responses to this part found the equation of BC and the used the perpendicular distance formula to find the required length. This elaborate method was not the most efficient one as this part of the question was only worth one mark. Many algebraic or numeric errors occurred in this process. In better responses, candidates used the area of a triangle formula, $A = \frac{1}{2} b \times h$, and substituted the relevant information of $b = \sqrt{40}$, from part (v), and the area of 44.

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