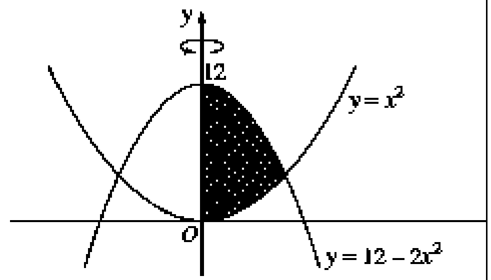
05 **6c**

The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram.

(i) Find the points of intersection of the two curves. **1**

(ii) The shaded region between the curves and the y-axis is rotated about the y-axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed. **3**



i.

$$\begin{aligned} x^2 &= 12 - 2x^2 \\ x^2 + 2x^2 &= 12 \\ 3x^2 &= 12 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

Subs $x = 2$ in $y = x^2$ $\quad \therefore\ y = 4 \quad \therefore\ (2, 4)$

Subs $x = -2$ in $y = x^2$ $\quad \therefore\ y = 4 \quad \therefore\ (-2, 4)$

ii. As $y = x^2$ and $y = 12 - 2x^2$

then $x^2 = y$ then $2x^2 = 12 - y$

$x^2 = 6 - \dfrac{y}{2}$

Using Volume $= \pi \int x^2 \; dy$,

$$\begin{aligned} \text{Total volume} &= \pi \int_4^{12} 6 - \frac{y}{2} \; dy + \pi \int_0^4 y \; dy \\ &= \pi \left[6y - \frac{y^2}{4} \right]_4^{12} + \pi \left[\frac{y^2}{2} \right]_0^4 \\ &= \pi \left[72 - \frac{144}{4} - (24 - \frac{16}{4}) \right] + \pi \left[\frac{16}{2} - 0 \right] \\ &= \pi [72 - 36 - 20 + 8] \\ &= 24\pi \qquad \therefore \text{ volume of } 24\pi \text{ unit}^3 \end{aligned}$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates attempted this part and many were successful in getting $x = \pm 2$. However, not all of these answered the question of finding the points of intersection as shown on the diagram.

(ii) It was challenging for most candidates to understand the process needed and the link to part (i). Many tried to use x functions, while many others could not come up with the correct split of regions and/or wanted to take a difference of volumes, or they doubled an expression because of the symmetry. Many candidates also used the wrong limits, such as using $y = 2$ rather than 4 for the middle value.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/

HSC examination papers © Board of Studies NSW for and on behalf of the Crown in right of State of New South Wales