07	**5b**	A particle is moving on the x-axis and is initially at the origin. Its velocity, v metres per second, at time t seconds is given by $v = \frac{2t}{16+t^2}$. (i) What is the initial velocity of the particle? (ii) Find an expression for the acceleration of the particle. (iii) Find the time when the acceleration of the particle is zero. (iv) Find the position of the particle when $t = 4$.	 **1** **2** **1** **3**

(i) Initial means $t = 0$:

$$v = \frac{2t}{16+t^2} = \frac{2(0)}{16+0^2} = 0$$

$\therefore$ the particle initially at rest.

(ii) Acceleration is $\frac{dV}{dt}$.

$v = \frac{2t}{16+t^2}$, and using quotient rule

$$\frac{dV}{dt} = \frac{(16+t^2).2-2t(2t)}{(16+t^2)^2} = \frac{32+2t^2-4t^2}{(16+t^2)^2} = \frac{32-2t^2}{(16+t^2)^2}$$

(iii) $\frac{dV}{dt} = \frac{32-2t^2}{(16+t^2)^2}$

When $\frac{dV}{dt} = 0$,

$$\frac{32-2t^2}{(16+t^2)^2} = 0$$
$$32 - 2t^2 = 0$$
$$2t^2 = 32$$
$$t^2 = 16$$
$$t = 4 \quad (\text{as } t \geq 0)$$

$\therefore$ Acceleration is zero after 4 seconds

(iv) $x = \int \frac{2t}{16+t^2}\, dx$

$= \log_e(16 + t^2) + c$

Now 'initially at origin': $t = 0$, $x = 0$

$0 = \log_e(16 + 0) + c$

$c = -\log_e 16$

$x = \log_e(16 + t^2) - \log_e 16$

When $t = 4$,

$x = \log_e(16 + 16) - \log_e 16$

$= \log_e 32 - \log_e 16$

$= \log_e \frac{32}{16}$

$= \log_e 2$

$\therefore$ The particle is at $\log_e 2$ metres

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Responses which showed full working and the substitution step achieved best results. These responses minimised errors and when mistakes were made, marks were still able to be awarded.

(i) Candidates were awarded the mark for showing they understood how to find the initial velocity by substituting t = 0 into the velocity function.

(ii) The quotient formula was generally very well known and was the most successful way of finding the derivative (rather than the product rule). Candidates were expected to show that they knew the acceleration was found by finding the derivative of the velocity function.

(iii) To find when the acceleration was zero it was necessary to solve an equation involving an algebraic fraction. Better responses solved the equation resulting from equating the numerator to zero.

(iv) Marks were awarded if the primitive was found to be $\ln(16 + t^2) + C$, then the initial conditions used to find $C = -\ln 16$ and hence the displacement at $t = 4$. It was a common mistake to use the initial conditions and then ignore working, to state $C = 0$.
Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/