11 **10 a** The intensity I, measured in watt/m^2, of a sound is given by $I = 10^{-12} \times e^{0.1L}$, where L is the loudness of the sound in decibels.

(i) If the loudness of a sound at a concert is 110 decibels, find the intensity of the sound. Give your answer in scientific notation. **1**

(ii) Ear damage occurs if the intensity of a sound is greater than 8.1×10^{-9} watt/m^2. What is the maximum loudness of a sound so that no ear damage occurs? **2**

(iii) By how much will the loudness of a sound have increased if its intensity has doubled? **2**

(i) $I = 10^{-12} \times e^{0.1L}$

Subs $L = 110$,

$I = 10^{-12} \times e^{0.1(110)}$

$= 0.000\ 000\ 059$

$= 5.9 \times 10^{-8}$

(ii) Let $I = 8.1 \times 10^{-9}$

$8.1 \times 10^{-9} = 10^{-12} \times e^{0.1L}$

$e^{0.1L} = (8.1 \times 10^{-9}) \div 10^{-12}$

$= 8100$

$0.1L \log_e e = \log_e 8100$

$L = \dfrac{\log_e 8100}{0.1}$

$= 89.99619 \ldots$

$= 90$ (nearest whole)

$\therefore$ max loudness is 90 decibels

(iii) *Method 1:* From (i),

$I = 5.9 \times 10^{-8}$, when $L = 110$.

We can double the value of I:

Let $I = 11.8 \times 10^{-8}$

$= 1.18 \times 10^{-7}$

$1.18 \times 10^{-7} = 10^{-12} \times e^{0.1L}$

$e^{0.1L} = (1.18 \times 10^{-7}) \div 10^{-12}$

$= 118\ 000$

$0.1L \log_e e = \log_e 118\ 000$

$L = \dfrac{\log_e 118000}{0.1}$

$= 116.7843 \ldots$

$= 117$ (nearest whole)

$\therefore$ As $117 - 110 = 7$, the loudness has increased by 7 decibels.

State Mean:
0.73/1
1.36/2
0.64/2

Method 2:

Using exponential growth & decay:

$I = I_0 e^{0.1L}$

Let $I = 2I_0$

$\therefore 2I_0 = I_0 e^{0.1L}$

$e^{0.1L} = 2$

$0.1L = \log_e 2$

$L = 10 \log_e 2$

$= 6.93147 \ldots$

$= 7$ (nearest whole)

increased by 7 decibels.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Common errors included incorrect calculator use and assuming $e^{11} \times 10^{-12}$ to be correctly expressed in scientific notation.

(ii) Most candidates showed a clear understanding of this question and correctly substituted into the given equation. The common errors involved the incorrect manipulation of indices and the misuse of logarithms. A number of candidates misinterpreted the meaning of the word 'maximum' and attempted to solve a differential equation.

(iii) This part was challenging. The popular methods used to find the increase in loudness involved either an algebraic approach or the use of results from parts (i) and (ii). Some candidates introduced other values for I, doubled the intensity and compared the resulting expressions for loudness. A simple solution involved the use of the exponential growth model $I = I_0 e^{0.1 \times L}$ with $I = 2I_0$. In many responses, candidates incorrectly doubled both sides of the given equation rather than only doubling the intensity. A few responses used squaring instead of doubling and others doubled the loudness rather than the intensity. A number of candidates misinterpreted the question and found an expression for the increased loudness.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/