06 | 5

(i)

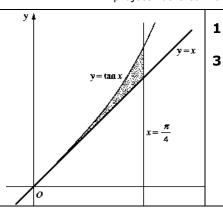
Show that $\frac{d}{dx}\log_e(\cos x) = -\tan x$.

bounded by the curve $y = \tan x$ and

(ii) The shaded region in the diagram is

the lines y = x and $x = \frac{\pi}{4}$.

Using the result of part (i), or otherwise, find the area of the shaded region.



(i)
$$\frac{d}{dx}\log_e(\cos x) = \frac{-\sin x}{\cos x}$$
$$= -\tan x$$

(ii) Area =
$$\int_{0}^{\frac{\pi}{4}} \tan x - x \, dx$$

From (i),
$$\int \tan x \ dx = -\log_e(\cos x) + c$$

$$= \left[-\log_{e}(\cos x) - \frac{x^{2}}{2} \right]_{0}^{\frac{\pi}{4}}$$

$$= -\log_{e}(\cos \frac{\pi}{4}) - ((\frac{\pi}{4})^{2}) \div 2 - (-\log_{e}(\cos 0) - 0)$$

$$= -\log_{e} \frac{1}{\sqrt{2}} - \frac{\pi^{2}}{32} - (-\log 1 - 0)$$

$$= \frac{1}{2} \log_{e} 2 - \frac{\pi^{2}}{32}$$

$$= 0.038148452 \dots$$

$$= 0.038 (3 \text{ dec pl}) \qquad \therefore \text{ are}$$

 \therefore area of 0.038 m²

Board of Studies: Notes from the Marking Centre

- (i) The majority of candidates were successful in this part.
- (ii) This part was not done well. Successful candidates recognised the connection between parts (i) and (ii) and were able to place the negative sign correctly.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/

^{*} These solutions have been provided by projectmaths and are not supplied or endorsed by the Board of Studies