06	8a	A particle is moving in a straight line. Its displacement, x metres, from the origin, O, at time t seconds, where $t \geq 0$, is given by $x = 1 - \frac{7}{t+4}$.	
		(i) Find the initial displacement of the particle.	**1**
		(ii) Find the velocity of the particle as it passes through the origin.	**3**
		(iii) Show that the acceleration of the particle is always negative.	**1**
		(iv) Sketch the graph of the displacement of the particle as a function of time.	**2**

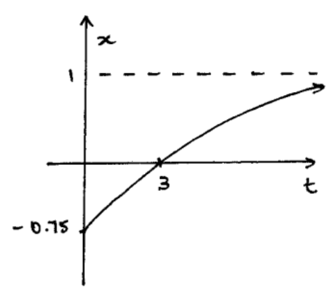
(i) When $t = 0$,
$$x = 1 - \frac{7}{t+4} = 1 - \frac{7}{0+4} = 1 - 1.75 = -0.75$$
Initially, at 0.75 m to the left of origin.

(ii) First, find when particle passes through origin, $x = 0$.
$$x = 1 - \frac{7}{t+4}$$
$$0 = 1 - \frac{7}{t+4}$$
$$\frac{7}{t+4} = 1$$
$$7 = t + 4$$
$$t = 3$$
After 3 seconds, particle passes through O.
$$x = 1 - \frac{7}{t+4} = 1 - 7(t+4)^{-1}$$
$$\frac{dx}{dt} = v = 7(t+4)^{-2}.1 = \frac{7}{(t+4)^2}$$

At $t = 3$,
$$v = \frac{7}{(3+4)^2} = \frac{7}{49} = \frac{1}{7}$$
Particle moving at $\frac{1}{7}$ m/s when passing O.

(iii)
$$v = \frac{7}{(t+4)^2} = 7(t+4)^{-2}$$
$$a = -14(t+4)^{-3}.1 = \frac{-14}{(t+4)^3}$$
As $a < 0$, for any value of t. This means that the acceleration is always negative.

(iv)



* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Candidates are reminded that the concepts of distance and displacement are not interchangeable and marks were lost in part (i) for answers which were positive rather than negative.

(ii) It should be stressed in relation to parts (i) and (ii) that the term 'initial' demands that $t = 0$ (not $x = 0$) whereas 'passing through the origin' implies that $x = 0$ (as opposed to $t = 0$). A common error in this part was to integrate rather than differentiate when producing a velocity formula. Candidates who differentiated correctly often then substituted $t = 0$ instead of the correct $t = 3$.

(iii) Generally, this part was handled well if the correct velocity formula was obtained in part (ii).

(iv) In this part many candidates incorrectly sketched a straight line. Better responses not only displayed a hyperbolic shape but also fully addressed the issue of the horizontal asymptote.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/