05	**6b**	A tank initially holds 3600 litres of water. The water drains from the bottom of the tank. The tank takes 60 minutes to empty. A mathematical model predicts that the volume, V litres, of water that will remain in the tank after t minutes is given by $V = 3600(1 - \frac{t}{60})^2$, where $0 \le t \le 60$. (i) What volume does the model predict will remain after ten minutes? (ii) At what rate does the model predict that the water will drain from the tank after twenty minutes? (iii) At what time does the model predict that the water will drain from the tank at its fastest rate?	 **1** **2** **2**

(i) $V = 3600(1 - \frac{t}{60})^2$

Subs $t = 10$,

$$V = 3600(1 - \frac{t}{60})^2$$
$$= 3600(1 - \frac{10}{60})^2$$
$$= 2500$$

$\therefore$ 2500 litres after 10 minutes

(ii) $\frac{dV}{dt} = 7200(1 - \frac{t}{60})^1 \times \frac{-1}{60}$

(by function of function rule)

$$= -120(1 - \frac{t}{60})$$

Subs $t = 20$,

$$\frac{dV}{dt} = -120(1 - \frac{20}{60})$$
$$= -80$$

$\therefore$ water draining at 80 litres/minute

(iii) $\frac{dV}{dt} = -120(1 - \frac{t}{60})$

Now, fastest rate is when $\frac{dV}{dt}$ is maximum.

As $0 \le t \le 60$, maximum occurs when $t = 0$.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) The majority of candidates were successful in this part.

(iii) Most candidates understood that $\frac{dV}{dt}$ was required. Some candidates struggled with this derivative, often leaving out the negative sign. Some common variations included trying to use an average rate and introducing exponentials.

(iii) This part required an interpretation of a physical situation and when $\frac{d^2V}{dt^2} = 2$. A large number of candidates could not understand what to do next (many just took $t = 2$ from this). Many candidates incorrectly solved $\frac{dV}{dt} = 0$ to get $t = 60$, and a few used graphs of V or V' to help justify their answer.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/