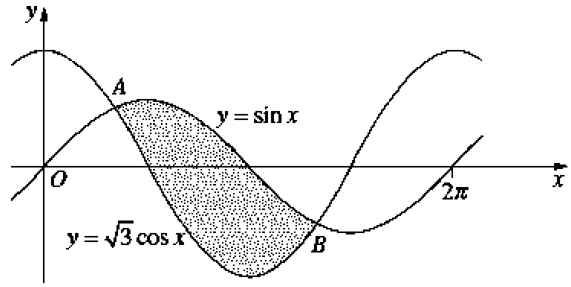
07	7b	The diagram shows the graphs of $y = \sqrt{3}\cos x$ and $y = \sin x$. The first two points of intersection to the right of the y-axis are labelled A and B. (i) Solve the equation $\sqrt{3}\cos x = \sin x$ to find the x-coordinates of A and B. (ii) Find the area of the shaded region in the diagram.		2 3

(i) $\sqrt{3}\cos x = \sin x$

$\sin x = \sqrt{3}\cos x$

$\tan x = \sqrt{3}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$

$\therefore$ x-coordinates of $\frac{\pi}{3}$ and $\frac{4\pi}{3}$

(ii) Area $= \int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \sin x - \sqrt{3}\cos x \; dx$

$= \left[-\cos x - \sqrt{3}\sin x\right]_{\frac{\pi}{3}}^{\frac{4\pi}{3}}$

$= -\cos\frac{4\pi}{3} - \sqrt{3}\sin\frac{4\pi}{3} - (-\cos\frac{\pi}{3} - \sqrt{3}\sin\frac{\pi}{3})$

$= -(-\frac{1}{2}) - \sqrt{3}(-\frac{\sqrt{3}}{2}) - (-\frac{1}{2} - \sqrt{3}(\frac{\sqrt{3}}{2}))$

$= \frac{1}{2} + \frac{3}{2} - (-\frac{1}{2} - \frac{3}{2})$

$= 2 + 2$

$= 4$ $\therefore$ 4 units^2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Better responses recognised that the equation reduced to $\tan x = \sqrt{3}$. Some candidates squared both sides and moved to expressions such as $\cos^2 x = \frac{1}{4}$ and $\cos x = \frac{1}{2}$. This squaring introduced extraneous roots and gave a solution in an incorrect quadrant. Some candidates successfully used the $R\cos(x+\alpha)$ form to solve the equation.

(ii) Most successful responses went directly to the area by using a single integral $\int \sin x - \sqrt{3}\cos x dx$ and proceeded from there. Most candidates were able to find the correct primitives. Incorrect answers from part (i) still allowed a sensible calculation for this part. Exact values were well handled by most candidates. When candidates reversed the functions in the integrand they sometimes failed to take the absolute value of the answer or did not realise that they were finding an area in square units. Those who chose to split the area into multiple regions generally had trouble obtaining full marks. Their regions often overlapped and the number of places where errors could occur increased.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/