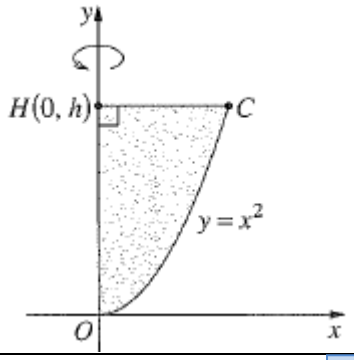
11	**8b**	The diagram shows the region enclosed by the parabola $y = x^2$, the y-axis and the line $y = h$, where $h > 0$. This region is rotated about the y-axis to form a solid called a paraboloid. The point C is the intersection of $y = x^2$ and $y = h$. The point H has coordinates $(0, h)$. (i) Find the exact volume of the paraboloid in terms of h. (ii) A cylinder has radius HC and height h. What is the ratio of the volume of the paraboloid to the volume of the cylinder?	**2** **1**



(i) $V = \pi \int x^2 \, dy$

$$= \pi \int_0^h y \, dy$$

$$= \pi \left[\frac{y^2}{2} \right]_0^h$$

$$= \pi \left[\frac{h^2}{2} - 0 \right]$$

$$= \frac{\pi h^2}{2}$$

$\therefore$ the volume is $\frac{\pi h^2}{2}$ units^3

State Mean:
1.29/2
0.24/1

(ii) If $y = h$, then $x^2 = h$

$x = \sqrt{h}$

$\therefore C(\sqrt{h}, h)$

V of cylinder $= \pi \times (\sqrt{h})^2 \times h$

$= \pi h^2$

Ratio $= \frac{\pi h^2}{2} : \pi h^2$

$= 1:2$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Common errors included the application of an incorrect volume formula (commonly confusing the axis of rotation) and incorrect evaluation of the definite integral (commonly using incorrect limits).

(ii) Many candidates were not able to complete all the steps required to correctly determine the ratio. Common errors included not being able to determine the radius of the cylinder, and not correctly stating the formula of the volume of a cylinder.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/