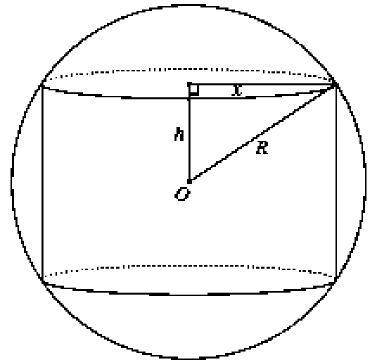
05 **8a**

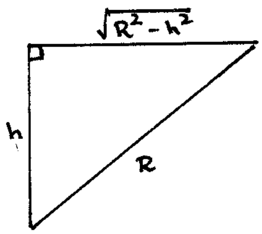
A cylinder of radius x and height $2h$ is to be inscribed in a sphere of radius R centred at O as shown.

(i) Show that the volume of the cylinder is given by $V = 2\pi h(R^2 - h^2)$. **1**

(ii) Hence, or otherwise, show that the cylinder has a maximum volume when $h = \dfrac{R}{\sqrt{3}}$. **3**



i. Vol of cylinder = $\pi \times \text{radius}^2 \times$ height with height = $2h$ and radius = x



$x^2 = R^2 - h^2$ by Pythagoras

$\therefore V = \pi(R^2 - h^2).2h$

$= 2\pi h(R^2 - h^2)$

ii. $V = 2\pi h(R^2 - h^2)$

$= 2\pi R^2 h - 2\pi h^3$

$V' = 2\pi R^2 - 6\pi h^2 = 0$

$6\pi h^2 = 2\pi R^2$

$6h^2 = 2R^2$

$h^2 = \dfrac{2R^2}{6}$

$= \dfrac{R^2}{3}$

$h = \pm\dfrac{R}{\sqrt{3}}$

But $h > 0$, then $h = \dfrac{R}{\sqrt{3}}$

$V'' = -12\pi h^2$

$V''\left(\dfrac{R}{\sqrt{3}}\right) = -12\pi\left(\dfrac{R}{\sqrt{3}}\right)^2 < 0$

$\therefore$ maximum volume when $\dfrac{R}{\sqrt{3}}$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Better responses to this part used clear setting out and explained the steps that were being attempted.

(i) Showing that the given expression for the volume was correct entailed beginning with the correct formula for the volume of a cylinder and then recognising that the radius was x, the height was $2h$ and then eliminating x via the use of Pythagoras' theorem. Better responses to this part clearly stated these substitutions.

(ii) The most common error in this part was in differentiating with respect to h. Many candidates did not treat R^2 as a constant. Candidates who expanded to find $V = 2\pi R^2 h - 2\pi h^3$ generally had more success than those who tried to use the product rule. Amongst those candidates who did differentiate successfully the next most common error was in determining the nature of the stationary point. Some candidates did not attempt to determine its nature while others did not show that this particular turning point satisfied the conditions stated in the first derivative or second derivative test.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/