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09	7a		
		The acceleration of a particle is given by $\ddot{x} = 8e^{-2t} + 3e^{-t}$, where x is displacement in metres and t is time in seconds. Initially its velocity is -6 ms^{-1} and its displacement is 5 m. (i) Show that the displacement of the particle is given by $x = 2e^{-2t} + 3e^{-t} + t$. (ii) Find the time when the particle comes to rest. (iii) Find the displacement when the particle comes to rest.	2 3 1

(i) $\ddot{x} = 8e^{-2t} + 3e^{-t}$

$\dot{x} = v = -4e^{-2t} - 3e^{-t} + c$

When $t = 0$, $v = -6$:

$-6 = -4e^{0} - 3e^{0} + c$

$-6 = -7 + c$

$c = 1$

$\therefore v = -4e^{-2t} - 3e^{-t} + 1$

$x = 2e^{-2t} + 3e^{-t} + t + k$

When $t = 0$, $x = 5$:

$5 = 2e^{0} + 3e^{0} + 0 + k$

$5 = 5 + k$

$k = 0$

$\therefore x = 2e^{-2t} + 3e^{-t} + t$

(ii) $v = -4e^{-2t} - 3e^{-t} + 1$

When v = 0,

$0 = -4e^{-2t} - 3e^{-t} + 1$

$4e^{-2t} + 3e^{-t} - 1 = 0$

Let $m = e^{-t}$,

$4m^2 + 3m - 1 = 0$

$(4m - 1)(m + 1) = 0$

$m = \frac{1}{4}, -1$

Now, if $e^{-t} = \frac{1}{4}$ $\qquad$ $e^{-t} = -1$

Taking logs of both sides:

$-t = \log_e \frac{1}{4}$ $\qquad$ $-t = \log_e -1$

$t = -\log_e \frac{1}{4}$ $\qquad$ no solns

$t = \log_e 4$

$\therefore$ comes to rest after $\log_e 4$ seconds

(iii) $x = 2e^{-2t} + 3e^{-t} + t$

Subs $t = \log_e 4$:

$x = 2e^{-2\log 4} + 3e^{-\log 4} + \log_e 4$

$= 2.2612 \ldots$

$= 2.26$ (2 dec pl)

$\therefore$ the particle comes to rest at 2.26 m.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) In the better responses, candidates were rigorous in the process of integrating twice and testing the initial conditions to evaluate the constants of integration. Other successful candidates differentiated the expression for displacement twice to obtain acceleration as well as testing the initial conditions. Candidates are reminded to make use of the standard integral when possible.

(ii) Almost all candidates realised that the particle was at rest when the velocity was zero. Better responses saw candidates solve the resulting equation by using their knowledge of equations reducible to a quadratic and then using log laws to obtain the appropriate time. Less successful candidates failed to see that the velocity equation obtained was in fact a quadratic. Candidates are reminded that there are no solutions for t in an equation that has e^{-t} equal to a negative number.

(iii) Most students realised that they had to use their answer from (ii) in the displacement equation to answer this part. This substitution was generally well done. However, the resulting evaluation indicated that more care was needed when using a calculator.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/