07	**6b**	Let $f(x) = x^4 - 4x^3$. (i) Find the coordinates of the points where the curve crosses the axes. (ii) Find the coordinates of the stationary points and determine their nature. (iii) Find the coordinates of the points of inflexion. (iv) Sketch the graph of $y = f(x)$, indicating clearly the intercepts, stationary points and points of inflexion.	 **2** **4** **1** **3**

i. $x^4 - 4x^3 = 0$

$x^3(x - 4) = 0$

$x = 0, 4$

$\therefore$ points are (0, 0), (4, 0)

ii. $f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2 = 0$

$4x^2(x - 3) = 0$

$x = 0, 3$

At $x = 0$, $f(0) = 0^4 - 4(0)^3$

$= 0 \quad \therefore (0, 0)$

At $x = 3$, $f(3) = 3^4 - 4(3)^3$

$= -27 \quad \therefore (3, -27)$

$f''(x) = 12x^2 - 24x$

At $x = 0$, $f''(0) = 12(0)^2 - 24(0) = 0$

$\therefore$ possible hor. point of inflexion

Check neighbourhood of $f'(x)$:

$f'(-1) < 0$ and $f'(1) < 0$

$\therefore$ hor. point of inflexion at (0, 0)

At $x = 3$, $f''(3) = 12(3)^2 - 24(3) > 0$

$\therefore$ minimum at (3, -27)

iii. $f''(x) = 12x^2 - 24x = 0$

$12x(x - 2) = 0$

$x = 0, 2$

At $x = 0$, $f(0) = 0 \quad \therefore (0, 0)$

At $x = 2$, $f(2) = 2^4 - 4(2)^3$

$= 16 - 32$

$= -16 \quad \therefore (2, -16)$

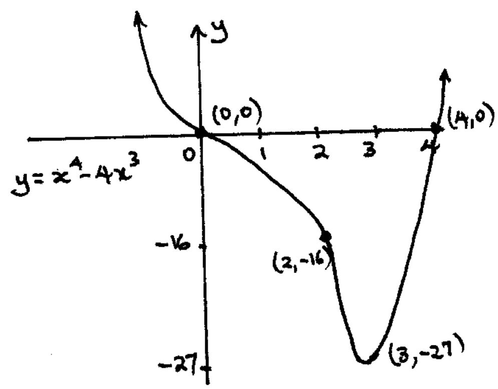
Already (0, 0) is hor. Point of inflexion.

Check neighbourhood of $f''(x)$ for $x = 2$:

$f''(1) < 0$ and $f''(3) > 0$

$\therefore$ pts of inflexion at (2, -16) and (0, 0)

iv.



* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) A significant percentage of candidates failed to note the difference between finding 'coordinates' and determining x and y coordinates, choosing simply to solve with the substitution of $y = 0$ and/or $x = 0$. Also, candidates are advised to show at least a simple step of working regardless of the answers being 'obvious'. For example, $y = x^4 - 4x^3 = x^3(x - 4)$. Therefore the intercepts are obviously (0, 0) and (4, 0).

(ii) Candidates should be reminded of the fact that horizontal inflexion points are also stationary points, as many appeared to omit the (0, 0) stationary point from their discussion once they had determined that it was neither a maximum nor a minimum, leaving it to part (iii). Where tables were used it was often left to the marker to determine what the values represented, with no y, y' or y'' being indicated. A common error was the use of 'gradient' sketches under a table for the second derivative, leading to the incorrect conclusion that there was a maximum turning point at (0, 0).

(iii) Candidates are advised to check their working where a conflict arises in their solution. Many who had indicated (0, 0) as a maximum turning point in part (ii), now said that it was an inflexion in part (iii). A significant number of candidates divided by x and hence lost $x = 0$ as a possible solution. Second derivative errors were also common throughout, eg $y' = 4x^3 - 12x^2$, followed by $y' = 12x^2 - 12x$.

(iv) Candidates are advised to practise sketching different curves given different circumstances. Most responses failed to show the concept of a horizontal, let alone an oblique inflexion. Smooth curves were few and far between with lots of 'feathering'. Labelling and scale on axes were almost non-existent.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/