06	**5a**	A function $f(x)$ is defined by $f(x) = 2x^2(3 - x)$. (i) Find the coordinates of the turning points of $y = f(x)$ and determine their nature. (ii) Find the coordinates of the point of inflexion. (iii) Hence sketch the graph of $y = f(x)$, showing the turning points, the point of inflexion and the points where the curve meets the x-axis. (iv) What is the minimum value of $f(x)$ for $-1 \le x \le 4$?	**3** **1** **3** **1**

i. $f(x) = 2x^2(3 - x)$
$= 6x^2 - 2x^3$
$f'(x) = 12x - 6x^2 = 0$
$6x(2 - x) = 0$
$x = 0, 2$
At $x = 0$, $f(0) = 2(0)^2[3 - 0]$
$= 0 \quad \therefore (0, 0)$
At $x = 2$, $f(2) = 2(2)^2[3 - 2]$
$= 8 \quad \therefore (2, 8)$

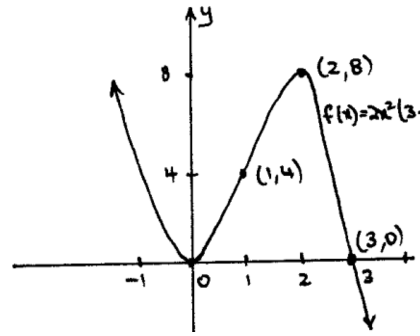
$f''(x) = 12 - 12x$

At $x = 0$, $f''(0) = 12 - 12(0) > 0$
$\therefore$ minimum at (0, 0)
At $x = 2$, $f''(0) = 12 - 12(2) < 0$
$\therefore$ maximum at (2, 8)

ii $f''(x) = 12 - 12x = 0$
$12x = 12$
$x = 1$
At $x = 1$, $f(1) = 2(1)^2[3 - 1]$
$= 4 \quad \therefore (1, 4)$

Check neighbourhood of $f''(x)$ for $x = 1$:
$f''(0) > 0$ and $f''(2) < 0$
$\therefore$ pt of inflexion at (1, 4)

iii. $f(x) = 2x^2(3 - x) = 0$
$x = 0, 3$
$\therefore$ x intercepts at (0, 0) and (3, 0)



iv. Check $x = -1$ and $x = 4$:
At $x = -1$, $f(-1) = 2(-1)^2[3 - (-1)]$
$= 2 \times 4$
$= 8 \quad \therefore (-1, 8)$
At $x = 4$, $f(2) = 2(4)^2[3 - 4]$
$= 32 \times -1$
$= -32 \quad \therefore (4, -32)$
$\therefore$ minimum value of -32

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates understood that $\frac{dy}{dx}$ was required and that stationary points occurred when $\frac{dy}{dx} = 0$. Candidates who attempted the differentiation using the product rule were more likely to make an error than those who wrote the function as a simple polynomial before differentiating. Poor factorising and substitution skills were often evident.
(ii) This part was done well. Almost all candidates knew to equate the second derivative to zero and most successfully tested for the change in concavity.
(iii) The graph did not pose a problem to candidates who had successfully completed parts (i) and (ii). Many did not continue their graph far enough to show the x-intercept of 3. Algebraic errors often made it impossible to draw a graph to fit the candidate's incorrect stationary points. Candidates are reminded that the graph of a cubic function is smooth and continuous.
(iv) This part was answered well. Some candidates substituted successfully but were unsure whether the minimum value was the x- or y-coordinate of the lowest point.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/