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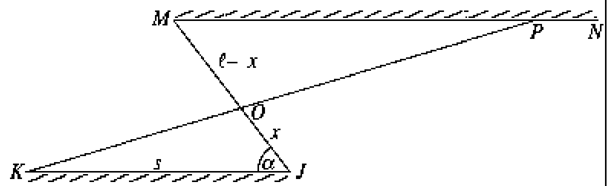
08 | **10 b**

The diagram shows two parallel brick walls KJ and MN joined by a fence from J to M. The wall KJ is s metres long and $\angle KJM = \alpha$. The fence JM is l metres long.



A new fence is to be built from K to a point P somewhere on MN. The new fence KP will cross the original fence JM at O.
Let $OJ = x$ metres, where $0 < x < l$.

(i) Show that the total area, A square metres, enclosed by ΔOKJ and ΔOMP is given by $A = s(x - l + \dfrac{l^2}{2x})\sin\alpha$. **3**

(ii) Find the value of x that makes A as small as possible. Justify the fact that this value of x gives the minimum value for A. **3**

(iii) Hence, find the length of MP when A is as small as possible. **1**

i.

$\angle KJO = \angle PMO$ (alt $\angle$s, $KJ \parallel MP$)

$\angle JKO = \angle MPO$ (alt $\angle$s, $KJ \parallel MP$)

$\therefore \Delta KJO \,|||\, \Delta PMO$ (2 $\angle$s equal)

$\therefore \dfrac{MP}{KJ} = \dfrac{MO}{JO}$

(matching sides of sim Δs in prop)

$\therefore \dfrac{MP}{s} = \dfrac{l-x}{x}$

$MP = \dfrac{s(l-x)}{x}$

Also, $\angle PMO = \alpha$ (alt $\angle$s, $KJ \parallel MP$)

$$\therefore A = \frac{1}{2} \times s \times x \times \sin\alpha + \frac{1}{2} \times (l-x) \times \frac{s(l-x)}{x} \times \sin\alpha$$

$$= \frac{s}{2}\sin\alpha\,(x + \frac{l^2 - 2lx + x^2}{x})$$

$$= \frac{s}{2}\sin\alpha\,(\frac{x^2 + l^2 - 2lx + x^2}{x})$$

$$= \frac{s}{2}\sin\alpha\,(2x - 2l + \frac{l^2}{x})$$

$$= s(x - l + \frac{l^2}{2x})\sin\alpha.$$

ii.

$$A = s(x - l + \frac{l^2}{2x})\sin\alpha$$

$$= s(x - l + \frac{l^2}{2}x^{-1})\sin\alpha$$

$$\frac{dA}{dx} = s(1 - \frac{l^2}{2}x^{-2})\sin\alpha$$

$$= s(1 - \frac{l^2}{2x^2})\sin\alpha = 0$$

$$1 - \frac{l^2}{2x^2} = 0\text{, as } s \text{ and } \alpha \text{ const.}$$

$$1 = \frac{l^2}{2x^2}$$

$$x^2 = \frac{l^2}{2}$$

$$x = \frac{l}{\sqrt{2}}\text{, as } x > 0$$

$$\frac{d^2A}{dx^2} = s(l^2x^{-3})\sin\alpha$$

$$\frac{d^2A}{dx^2}\left(\frac{l}{\sqrt{2}}\right) > 0$$

$\therefore$ maximum value of A

iii.

$$MP = \frac{s(l - \frac{1}{\sqrt{2}})}{\frac{1}{\sqrt{2}}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= s(\sqrt{2}\,l - 1)$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Many candidates correctly found an expression for the area of triangle OKJ. Most found it much more difficult to find an expression for the area of triangle OMP as they were not able to use similar triangles to find an expression for the length of side MP. A number of responses that provided a correct expression for the area of each triangle did not simplify the sum of these areas to reach a given expression. This was mostly due to errors involving algebraic manipulation.

(ii) Candidates are reminded that they did not need to find the given expression in the first part in order to use it successfully in this part. Many candidates who could not complete part (i) were able to gain marks in this part by successfully differentiating the expression, although a significant number did not treat l, and α as constants in this expression. Again algebraic errors caused problems. Many who attempted this part did endeavour to find the values of x for which their derivative was equal to zero, and then use either the first or second derivative to justify that their x value would give a minimum value of A. Many responses would have earned additional marks if evidence of working had been provided.

(iii) For those few better responses that had correct expression for MP and the correct value of x, this final substitution and simplification were usually well done.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/