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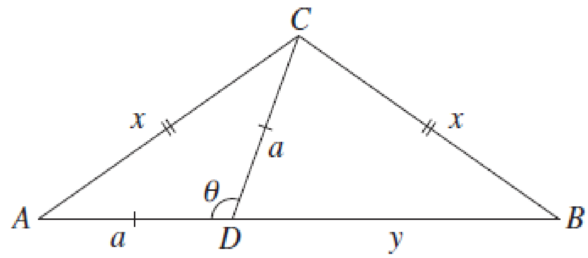
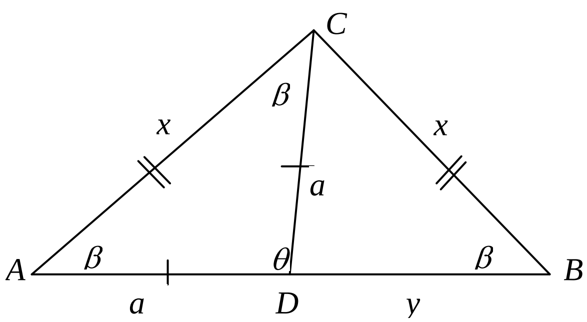
10 **10 a** In the diagram, ABC is an isosceles triangle $AC = BC = x$. The point D on the interval AB is chosen so that $AD = CD$. Let $AD = a$, $DB = y$ and $\angle ADC = \theta$.



		Marks
(i)	Show that ΔABC is similar to ΔACD.	**2**
(ii)	Show that $x^2 = a^2 + ay$	**1**
(iii)	Show that $y = a(1 - 2\cos\theta)$	**2**
(iv)	Deduce that $y \le 3a$.	**1**



(i) In ΔABC,

Let $\angle CAB = \beta$

$\therefore \angle CBA = \beta$ (base $\angle$s of isos Δs)

$\therefore \angle CAB = \angle CBA = \beta$

Also, in ΔACD:

$\angle ACD = \beta$ (base $\angle$s of isos Δs)

$\therefore \angle CAD = \angle ACD = \beta$

$\therefore \Delta ABC$ and ΔACD are similar (2 $\angle$s equal)

(ii) $\dfrac{AB}{AC} = \dfrac{AC}{CD}$ (matching sides of similar Δs in proportion)

$$\frac{a+y}{x} = \frac{x}{a}$$

$$\therefore x^2 = a(a+y) \quad \ldots\ldots (1)$$

(iii) In ΔACD, use cosine rule:

$$x^2 = a^2 + a^2 - 2 \times a \times a \times \cos\theta$$

$$\therefore x^2 = 2a^2 - 2a^2\cos\theta$$

$$x^2 = 2a^2(1 - \cos\theta) \quad \ldots. (2)$$

Let (1) = (2)

$$a(a + y) = 2a^2(1 - \cos\theta)$$

$$a^2 + ay = 2a^2 - 2a^2\cos\theta$$

$$ay = a^2 - 2a^2\cos\theta$$

$$y = a - 2a\cos\theta$$

$$\therefore y = a(1 - 2\cos\theta)$$

State Mean:
0.77/2
0.25/1
0.36/2
0.04/1

(iv) *METHOD 1:* As $\cos\theta \ge -1$

$$2\cos\theta \ge -2$$

$$-2\cos\theta \le 2$$

$$1 - 2\cos\theta \le 3$$

$$a(1 - 2\cos\theta) \le 3a$$

But, $y = a(1 - 2\cos\theta)$ from (iii)

$$\therefore y \le 3a$$

METHOD 2: From ΔACD, $x \le a + a$

$$\therefore x \le 2a$$

$$\therefore 2x \le 4a \quad \ldots\ldots (3)$$

From ΔABC, $a + y \le x + x$

$$a + y \le 2x \quad \ldots\ldots (4)$$

From (3) and (4):

$$\therefore a + y \le 4a$$

$$y \le 4a - a$$

$$y \le 3a$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Many candidates attempted (i); however, in most cases imprecise reasoning and presentation restricted the marks that could be obtained. Candidates are reminded that when dealing with proofs involving multiple overlapping triangles, it is essential to unambiguously identify all angles and objects of interest at each stage of the argument. In many responses, the connection between parts (i) and (ii) was not used, with many candidates incorrectly trying to implement Pythagoras' theorem in part (ii) rather than simply taking ratios of corresponding sides of similar triangles.

Interestingly some candidates achieved full marks in part (ii) by considering instead the areas of the three triangles.

Part (iii) could be answered by applying part (ii) and using the cosine rule on any of the three triangles in the diagram. Although there was some confusion as to the exact nature of the cosine rule, most responses failed to successfully complete the resulting algebra.

Responses in part (iv) were awarded full marks for correctly identifying the amplitude of the cosine function in part (iii). A common error was to claim that $\cos\theta < -1$. Strategies which abandoned the previous results and used a fresh algebraic attack seldom made any progress

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/