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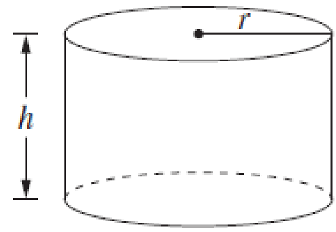
10 **5a**

A rainwater tank is to be designed in the shape of a cylinder with radius r metres and height h metres. The volume of the tank is to be 10 cubic metres. Let A be the surface area of the tank, including its top and base, in square metres.



(i) Given that $A = 2\pi r^2 + 2\pi rh$, show that $A = 2\pi r^2 + \dfrac{20}{r}$. **2**

(ii) Show that A has a minimum value and find the value of r for which the minimum occurs. **3**

(i)

$$V = \pi r^2 h$$
$$10 = \pi r^2 h$$
$$\therefore\ h = \frac{10}{\pi r^2}$$

Subs h in A:

$$\begin{aligned} A &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r^2 + 2\pi r \times \frac{10}{\pi r^2} \\ &= 2\pi r^2 + \frac{20}{r} \end{aligned}$$

(ii)

$$\begin{aligned} A &= 2\pi r^2 + \frac{20}{r} \\ &= 2\pi r^2 + 20r^{-1} \end{aligned}$$
$$\frac{dA}{dr} = 4\pi r - 20r^{-2}$$
$$\therefore\quad 4\pi r - \frac{20}{r^2} = 0$$
$$4\pi r = \frac{20}{r^2}$$
$$4\pi r^3 = 20$$
$$r^3 = \frac{20}{4\pi}$$
$$r^3 = \frac{5}{\pi}$$
$$r = \sqrt[3]{\frac{5}{\pi}}$$

As $\dfrac{dA}{dr} = 4\pi r - 20r^{-2}$,

then $\dfrac{d^2A}{dr^2} = 4\pi + 40r^{-3}$

But as $r > 0$, then $\dfrac{d^2A}{dr^2} > 0$,

so A is minimum when $r = \sqrt[3]{\dfrac{5}{\pi}}$.

[Or, use first derivative test:

r	1	$\sqrt[3]{\frac{5}{\pi}}$	2
$\frac{dA}{dr}$	< 0	0	> 0

$\therefore$ A is minimum when $r = \sqrt[3]{\dfrac{5}{\pi}}$]

State Mean:
1.22/2
1.32/3

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Board of Studies: Notes from the Marking Centre

(i) This part was done in numerous ways including straight substitution of an expression for h, equating the 2 expressions and ending up with the formula for the volume and other, more elaborate, substitutions.

Candidates who stated the correct formula $10 = \pi r^2 h$ completed the question efficiently. A significant number of candidates did not know the volume formula and tried to equate the given equations.

(ii) In this part, candidates were required to differentiate and then solve an equation involving a negative index. Those candidates who used the table method to show a minimum existed were the most successful, as many candidates struggled to correctly find the second derivative. The majority of candidates differentiated the result for surface area correctly but there were many errors made when attempting to solve for r. Some candidates also found the minimum value for A which was not required, thus wasting time that could have been used elsewhere in the paper. A small but significant number of candidates found a negative value of r and did not recognise that this must be incorrect.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/