projectmaths

Applications of Calculus to the Physical World – Rates of Change

16	**16 b**	Some yabbies are introduced into a small dam. The size of the population, y, of yabbies can be modelled by the function $y = \dfrac{200}{1+19e^{-0.5t}}$, where t is the time in months after the yabbies are introduced into the dam. (i) Show that the rate of growth of the size of the population is $\dfrac{1900e^{-0.5t}}{(1+19e^{-0.5t})^2}$. (ii) Find the range of the function y, justifying your answer. (iii) Show that the rate of growth of the size of the population can be rewritten as $\dfrac{y}{400}(200 - y)$. (iv) Hence, find the size of the population when it is growing at its fastest rate.	 **2** **2** **1** **2**	Solution
15	**15 c**	Water is flowing in and out of a rock pool. The volume of water in the pool at time t hours is V litres. The rate of change of the volume is given by $\dfrac{dV}{dt} = 80\sin(0.5t)$. At time $t = 0$, the volume of water in the pool is 1200 litres and is increasing. (i) After what time does the volume of water first start to decrease? (ii) Find the volume of water in the pool when $t = 3$. (iii) What is the greatest volume of water in the pool?	 **2** **2** **1**	Solution
11	**9b**	A tap releases liquid A into a tank at the rate of $\left(2 + \dfrac{t^2}{t+1}\right)$ litres per minute, where t is time in minutes. A second tap releases liquid B into the same tank at the rate of $\left(1 + \dfrac{1}{t+1}\right)$ litres per minute. The taps are opened at the same time and release the liquids into an empty tank. (i) Show that the rate of liquid A is greater than the rate of flow of liquid B by t litres per minute. (ii) The taps are closed after 4 minutes. By how many litres is the volume of liquid A greater than the volume of liquid B in the tank when the taps are closed?	 **1** **2**	Solution
06	**9b**	During a storm, water flows into a 7000-litre tank at a rate of $\dfrac{dV}{dt}$ litres per minute, where $\dfrac{dV}{dt} = 120 + 26t - t^2$ and t is the time in minutes since the storm began. (i) At what times is the tank filling at twice the initial rate? (ii) Find the volume of water that has flowed into the tank since the start of the storm as a function of t. (iii) Initially, the tank contains 1500 litres of water. When the storm finishes, 30 minutes after it began, the tank is overflowing. How many litres of water have been lost?	 **2** **1** **2**	Solution

05	**6b**	A tank initially holds 3600 litres of water. The water drains from the bottom of the tank. The tank takes 60 minutes to empty. A mathematical model predicts that the volume, V litres, of water that will remain in the tank after t minutes is given by $V = 3600(1 - \frac{t}{60})^2$, where $0 \leq t \leq 60$.	Solution
		(i) What volume does the model predict will remain after ten minutes?	**1**
		(ii) At what rate does the model predict that the water will drain from the tank after twenty minutes?	**2**
		(iii) At what time does the model predict that the water will drain from the tank at its fastest rate?	**2**