13	**12 c**	Kim and Alex start jobs at the beginning of the same year. Kim's annual salary in the first year is \$30 000, and increases by 5% at the beginning of each subsequent year. Alex's annual salary in the first year is \$33 000, and increases by \$1500 at the beginning of each subsequent year.	
		(i) Show that in the 10^{th} year Kim's annual salary is higher than Alex's annual salary.	**2**
		(ii) In the first 10 years how much, in total, does Kim earn?	**2**
		(iii) Every year, Alex saves $\frac{1}{3}$ of her annual salary. How many years does it take her to save \$87 500?	**3**

(i) Kim: 30 000 + 31 500 + …

$a = 30\,000,\ r = 1.05,\ n = 10$

$T_n = ar^{n-1}$

$T_{10} = 30\,000 \times (1.05)^{10-1}$

$= 30\,000 \times (1.05)^9$

$= 46\,539.84648 \ldots$

$= 46\,540$ (nearest whole)

Alex: 33 000 + 34 500 + …

$a = 33\,000,\ d = 1500,\ n = 10$

$T_n = a + (n - 1)d$

$T_{10} = 33\,000 + (10 - 1) \times 1500$

$= 46\,500$

$\therefore$ Kim's salary is \$40 more than Alex

(ii) $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$S_{10} = \dfrac{30000(1.05^{10} - 1)}{1.05 - 1}$

$= 377\,336.7761 \ldots$

$= 377\,336.78$ (2 dec pl)

$\therefore$ Kim earns \$377 336.78

(iii) Total Salary = \$87 500 × 3

= \$262 500

State Mean:
1.46/2
1.35/2
1.35/3

$S_n = \dfrac{n}{2}[2a + (n - 1)d]$

$= \dfrac{n}{2}[2(33\,000) + (n - 1)1500] = 262\,500$

$n[33\,000 + (n - 1)750] = 262\,500$

$33\,000n + 750n^2 - 750n - 262\,500 = 0$

$750n^2 + 32\,250n - 262\,500 = 0$

$n^2 + 43n - 350 = 0$

$(n - 7)(n + 50) = 0$

$= 7, -50$

(but, $n > 0$)

$\therefore$ takes 7 years

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Candidates substituted into the correct T_n formula: for Alex the arithmetic progression formula $T_n = a + (n - 1)d$ and for Kim the geometric progression formula $T_n = ar^{n-1}$.

Some candidates listed the sequence for each salary and were generally successful with this method.

Common problems were:

- using $n - 1 = 10$, rather than the correct $n - 1 = 9$
- using an incorrect formula, but could state that $d = 1500$ for Alex and $r = 1.05$ for Kim
- using T_2 as a for both Kim and Alex
- mixing up the starting salary for either Kim or Alex
- using the incorrect value for r.

(ii) Most candidates used the correct formula for the sum of a geometric progression. Those candidates who listed Kim's salary for the first 10 years were generally successful in calculating the correct total.

Common problems were:

- using the incorrect starting salary
- using the incorrect value for n or r
- using the formula for the sum of an arithmetic progression.

(iii) Many candidates recognised the sum as an arithmetic progression. Some candidates stated the correct S_n formula and showed the substitution before any calculation was made. This allowed markers to allocate marks to those candidates who made numerical errors.

Candidates who arrived at the correct quadratic equation were successful in either factorising or using the quadratic formula to find the correct solution. Some candidates listed the series and were generally successful using this method.

Common problems were:

- using the incorrect combination of $S_n = 262\ 500$ with $a = 33\ 000$ and $d = 1500$
- using Kim's salary of \$30 000, arriving at a quadratic equation that did not factorise
- using the geometric progression formula, resulting in an equation involving a log function
- using the method applying to loan repayments to answer the question.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/