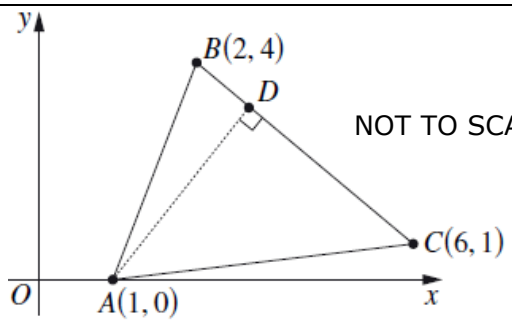
projectmaths

Linear Functions and Lines

16 **12 a** The diagram shows points $A(1, 0)$, $B(2, 4)$ and $C(6, 1)$. The point D lies on BC such that $AD \perp BC$.



NOT TO SCALE

(i) Show that the equation of BC is $3x + 4y - 22 = 0$. **2**

(ii) Find the length of AD. **2**

(iii) Hence, or otherwise, find the area of ΔABC. **2**

Solution

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15 **2** What is the slope of the line with equation $2x - 4y + 3 = 0$? **1**

(A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 2

Solution

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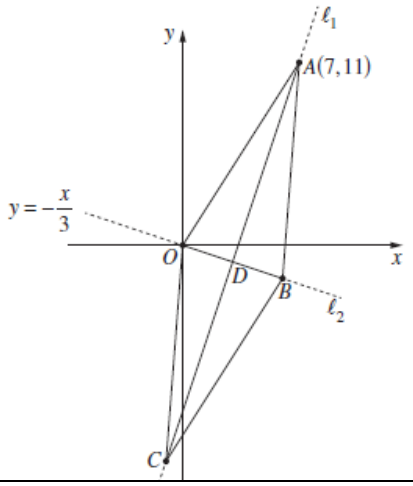
15 **12 b** The diagram shows the rhombus $OABC$. The diagonal from the point $A(7, 11)$ to the point C lies on the line ℓ_1. The other diagonal, from the origin O to the point B, lies on the line ℓ_2 which has equation $y = -\frac{x}{3}$.



Not to scale

(i) Show that the equation of the line ℓ_1 is $y = 3x - 10$. **2**

(ii) The lines ℓ_1 and ℓ_2 intersect at the point D. Find the coordinates of D. **2**

Solution

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14 **5** Which equation represents the line perpendicular to $2x - 3y = 8$, passing through the point $(2, 0)$? **1**

(A) $3x + 2y = 4$ (B) $3x + 2y = 6$ (C) $3x - 2y = -4$ (D) $3x - 2y = 6$

Solution

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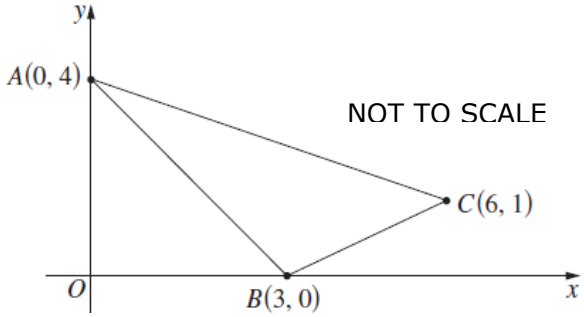
14 **12 b** The points $A(0, 4)$, $B(3, 0)$ and $C(6, 1)$ form a triangle, as shown in the diagram.



NOT TO SCALE

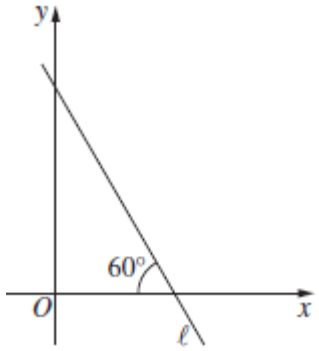
(i) Show that the equation of AC is $x + 2y - 9 = 0$. **2**

(ii) Find the perpendicular distance from B to AC. **2**

(iii) Hence, or otherwise, find the area of ΔABC. **2**

Solution

13 **2** The diagram shows the line ℓ. What is the slope of the line ℓ? **1** Solution



(A) $\sqrt{3}$

(B) $-\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) $-\frac{1}{\sqrt{3}}$

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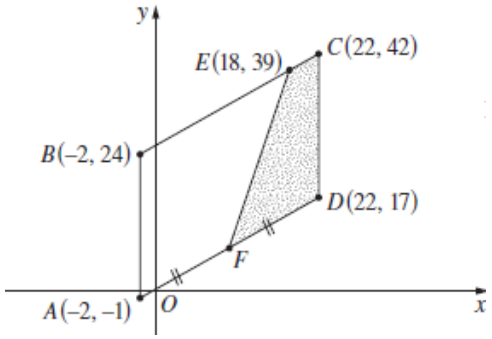
13 **12 b** The points $A(-2, -1)$, $B(-2, 24)$, $C(22, 42)$ and $D(22, 17)$ form a parallelogram as shown. The point $E(18, 39)$ lies on BC. The point F is the midpoint of AD. Solution



Not to scale

(i) Show that the equation of the line through A and D is $3x - 4y + 2 = 0$. **2**

(ii) Show that the perpendicular distance from B to the line through A and D is 20 units. **1**

(iii) Find the length of EC. **1**

(iv) Find the area of the trapezium $EFDC$. **2**

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13 **15 c** (i) Sketch the graph $y = |2x - 3|$. **1** Solution

(ii) Using the graph from part (i), or otherwise, find all values of m for which the equation $|2x - 3| = mx + 1$ has exactly one solution. **2**

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12 **5** What is the perpendicular distance of the point $(2, -1)$ from the line $y = 3x + 1$? **1** Solution

(A) $\frac{6}{\sqrt{10}}$ (B) $\frac{6}{\sqrt{5}}$ (C) $\frac{8}{\sqrt{10}}$ (D) $\frac{8}{\sqrt{5}}$

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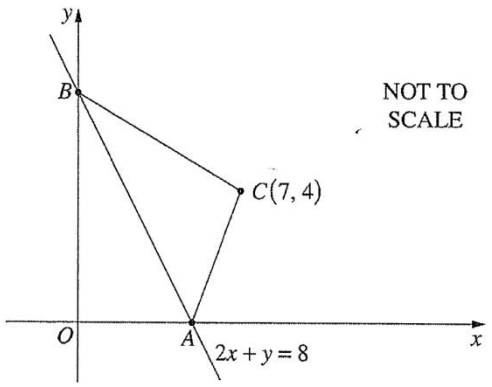
12 **13 a** The diagram shows a triangle ABC. The line $2x + y = 8$ meets the x and y axes at the points A and B respectively. The point C has coordinates $(7, 4)$. Solution

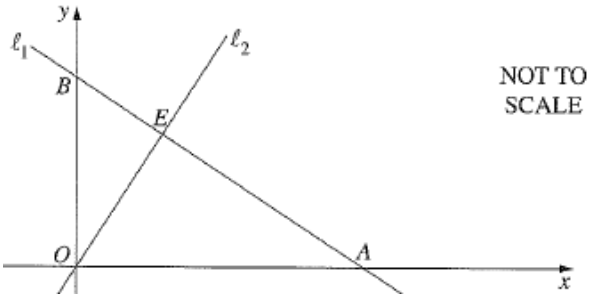


(i) Calculate the distance AB. **2**

(ii) It is known that $AC = 5$ and $BC = \sqrt{65}$. (Do NOT prove this.) Calculate the size of $\angle ABC$ to the nearest degree. **2**

(iii) The point N lies on AB such that CN is perpendicular to AB. Find the coordinates of N. **3**

11 **3c** The diagram shows a line ℓ_1, with equation $3x + 4y - 12 = 0$, which intersects the y-axis at B. A second line ℓ_2, with equation $4x - 3y = 0$, passes through the origin O and intersects ℓ_1 at E.



(i) Show that the co-ordinates of B are (0, 3) **1**

(ii) Show that ℓ_1 is perpendicular to ℓ_2. **2**

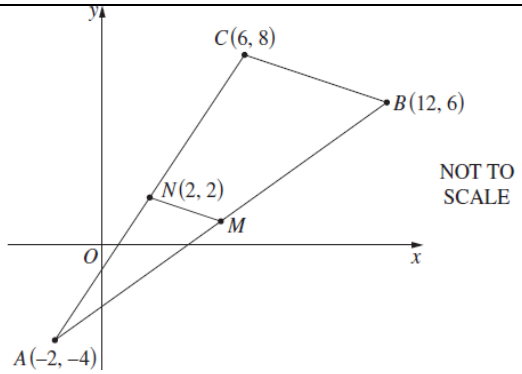
(iii) Show that the perpendicular distance from O to ℓ_1 is $\frac{12}{5}$. **1**

(iv) Using Pythagoras' theorem, or otherwise, find the length of the interval BE. **1**

(v) Hence, or otherwise, find the area of ΔBOE. **1**

Solution

10 **3a** In the diagram, A, B and C are the points (−2, −4), (12, 6) and (6, 8) respectively. The point $N(2, 2)$ is the midpoint of AC. The point M is the midpoint of AB.



(i) Find the coordinates of M. **1**

(ii) Find the gradient of BC. **1**

(iii) Prove that ΔABC is similar to ΔAMN. **2**

(iv) Find the equation of MN. **2**

(v) Find the exact length of BC. **1**

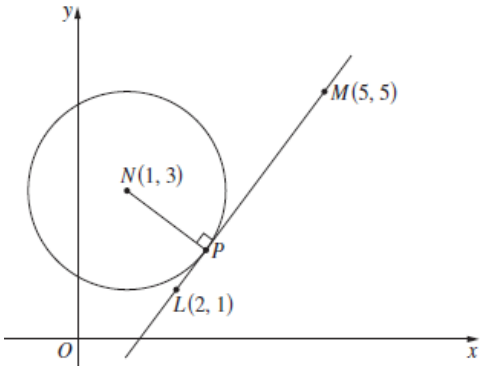
(vi) Given that the area of ΔABC is 44 square units, find the perpendicular distance from A to BC. **1**

Solution

09 **1a** Sketch the graph of $y - 2x = 3$, showing the intercepts on both axes. **2**

Solution

09 **3b** The circle in the diagram has centre N. The line LM is tangent to the circle at P.



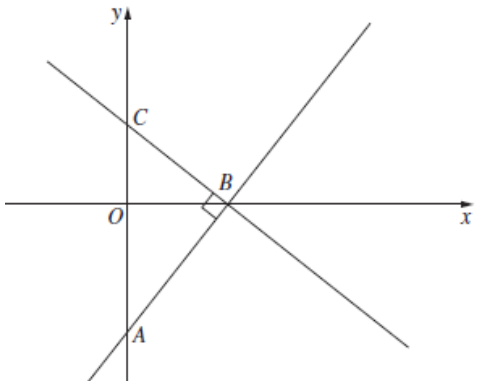
(i) Find the equation of LM is in the form $ax + by + c = 0$. **2**

(ii) Find the distance NP. **2**

(iii) Find the equation of the circle. **1**

Solution

09 **5a** In the diagram, the points A and C lie on the y-axis and the point B lies on the x-axis. The line AB has equation $y = \sqrt{3}\,x - 3$.
The line BC is perpendicular to AB.



(i) Find the equation of the line BC. **2**

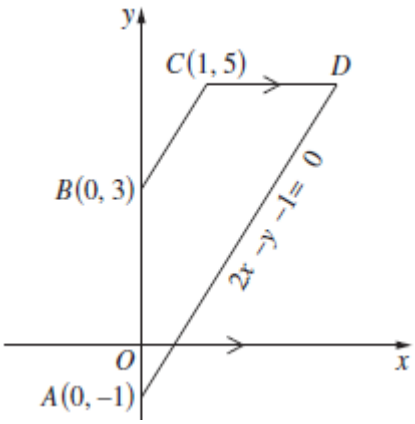
(ii) Find the area of the triangle ABC. **2**

Solution

08 **2b** Let M be the midpoint of $(-1,4)$ and $(5,8)$. **2**
Find the equation of the line through M with gradient $-\frac{1}{2}$.

Solution

08 **3a** In the diagram $ABCD$ is a quadrilateral. The equation of the line AD is $2x - y - 1 = 0$.



(i) Show that $ABCD$ is a trapezium by showing BC is parallel to AD. **2**

(ii) The line CD is parallel to the x-axis. Find the co-ordinates of D. **1**

(iii) Find the length of BC. **1**

(iv) Show that the perpendicular distance from B to AD is $\frac{4}{\sqrt{5}}$. **2**

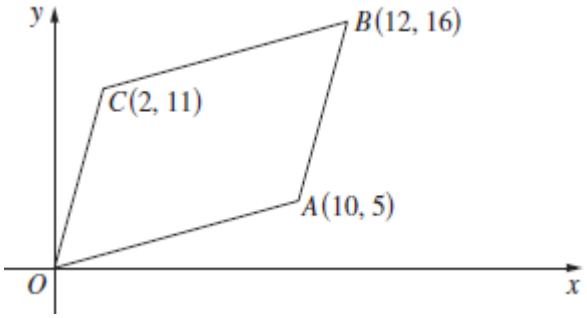
(v) Hence, or otherwise, find the area of the trapezium $ABCD$. **2**

Solution

07 **1f** Find the equation of the line that passes through the point $(-1, 3)$ and is perpendicular to $2x + y + 4 = 0$. **2**

Solution

07 **3a** In the diagram, A, B and C are the points (10, 5), (12, 16) and (2, 11) respectively. Copy or trace this diagram into your writing booklet.



(i) Find the distance AC. **1**

(ii) Find the midpoint of AC. **1**

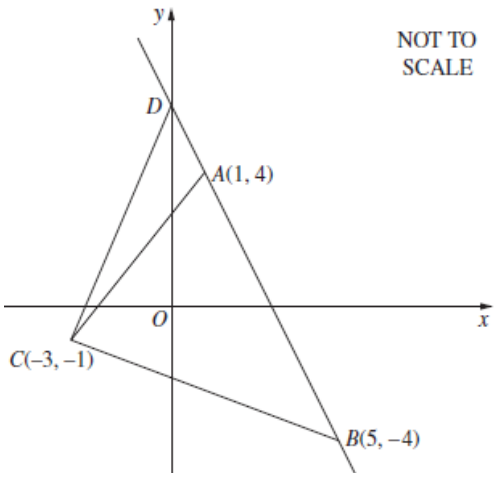
(iii) Show that $OB \perp AC$. **2**

(iv) Find the midpoint of OB and hence explain why $OABC$ is a rhombus. **2**

(v) Hence, or otherwise, find the area of $OABC$. **1**

Solution

06 **3a** In the diagram, A, B and C are the points (1, 4), (5, –4) and (–3, –1) respectively. The line AB meets the y-axis at D.



(i) Show that the equation of the line AB is $2x + y - 6 = 0$. **2**

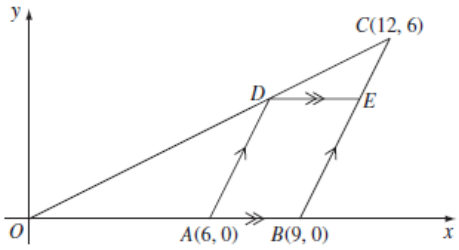
(ii) Find the coordinates of the point D. **1**

(iii) Find the perpendicular distance of the point C from the line AB. **1**

(iv) Hence, or otherwise, find the area of the triangle ADC. **2**

Solution

05 **3c** In the diagram, A, B and C are the points (6, 0), (9, 0) and (12, 6) respectively. The equation of the line OC is $x - 2y = 0$. The point D on OC is chosen so that AD is parallel to BC. The point E on BC is chosen so that DE is parallel to the x-axis.



(i) Show that the equation of the line AD is $y = 2x - 12$ **2**

(ii) Find the coordinates of the point D. **2**

(iii) Find the coordinates of the point E. **1**

(iv) Prove that $\Delta OAD \;|||\; \Delta DEC$ **2**

(v) Hence, or otherwise, find the ratio of the lengths AD and EC. **1**

Solution