07	**3b**	Heather decides to swim every day to improve her fitness level. On the first day she swims 750 metres, and on each day after that she swims 100 metres more than the previous day. That is, she swims 850 metres on the second day, 950 metres on the third day and so on.	
		(i) Write down a formula for the distance she swims on the *n*th day.	**1**
		(ii) How far does she swim on the 10^{th} day?	**1**
		(iii) What is the total distance she swims in the first 10 days?	**1**
		(iv) After how many days does the total distance she has swum equal the width of the English Channel, a distance of 34 kilometres?	**2**

i. The series is 750, 850, … with $a = 750$, $d = 100$

$$\begin{aligned} T_n &= a + (n-1)d \\ &= 750 + (n-1) \times 100 \\ &= 750 + 100n - 100 \\ &= 100n + 650 \end{aligned}$$

ii. Use $n = 10$,

$$\begin{aligned} T_n &= 100n + 650 \\ T_{10} &= 100 \times 10 + 650 \\ &= 1650 \end{aligned}$$

$\therefore$ she swims 1650m on 10^{th} day

iii. Using $S_n = \frac{n}{2}[2a + (n-1)d]$ with $a = 750$, $d = 100$ and $n = 10$

$$\begin{aligned} S_{10} &= \frac{10}{2}[2(750) + (10-1)100] \\ &= 5(1500 + 900) \\ &= 12\,000 \end{aligned}$$

$\therefore$ distance is 12 000m or 12 km

iv. Using $S_n = \frac{n}{2}[2a + (n-1)d]$ with $a = 750$, $d = 100$ and $S_n = 34\,000$

$$\begin{aligned} 34\,000 &= \frac{n}{2}[2(750) + (n-1)100] \\ 68\,000 &= n[1500 + 100n - 100] \\ 68\,000 &= n[1400 + 100n] \\ 100n^2 + 1400n - 68\,000 &= 0 \\ n^2 + 14n - 680 &= 0 \\ (n+34)(n-20) &= 0 \\ n &= -34, 20 \end{aligned}$$

As $n > 0$, then $n = 20$ $\therefore$ 20 days

- These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) In this part it was required that candidates progress to a formula for the distance in terms of the one variable n only.

(ii) and (iii) It was acceptable to use enumeration methods. However, better responses made efficient use of the theory of arithmetic progressions.

(iv) Responses showing difficulty factorising the quadratic in this part often completed the question through the careful use of the quadratic formula.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/