07	3a	In the diagram, A, B and C are the points (10, 5), (12, 16) and (2, 11) respectively. Copy or trace this diagram into your writing booklet. (i) Find the distance AC. (ii) Find the midpoint of AC. (iii) Show that OB ⊥ AC. (iv) Find the midpoint of OB and hence explain why OABC is a rhombus. (v) Hence, or otherwise, find the area of OABC.	C(2, 11) $A(10, 5)$	1 1 2 2

i.
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
 and $A(10, 5)$ and $C(2, 11)$
$$= \sqrt{(2 - 10)^2 + (11 - 5)^2}$$
$$= \sqrt{64 + 36}$$
$$= \sqrt{100}$$
$$= 10$$
 \therefore length of AC is 10 units

ii. Midpoint
$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
 and $A(10, 5)$ and $C(2, 11)$

$$= \left(\frac{10 + 2}{2}, \frac{5 + 11}{2}\right)$$

$$= \left(\frac{12}{2}, \frac{16}{2}\right)$$

$$= (6, 8)$$
 \therefore midpoint of AC is $(6, 8)$

iii. Gradient =
$$\frac{Rise}{Run}$$
 Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$ Gradient of $OB = \frac{16}{12}$ Gradient of $AC = \frac{11 - 5}{2 - 10}$ = $\frac{6}{-8}$ = $\frac{3}{-4}$

As
$$\frac{4}{3} \times \frac{3}{-4} = -1$$
, then OB $\perp AC$

iv. Midpoint
$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
 and $O(0, 0)$ and $B(12, 16)$

$$= \left(\frac{0 + 12}{2}, \frac{0 + 16}{2}\right)$$

$$= (6, 8)$$

As midpoints of AC and OB is (6, 8), then OABC is rhombus as diagonals bisect each other.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
 and $O(0, 0)$ and $B(12, 16)$
= $\sqrt{(12 - 0)^2 + (16 - 0)^2}$
= $\sqrt{144 + 256}$

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$$= \sqrt{400}$$

$$= 20$$

$$\therefore \text{ length of } OB \text{ is } 20 \text{ units}$$
Area of rhombus
$$= \frac{1}{2} \times \text{product of diagonals}$$

$$= \frac{1}{2} \times 10 \times 20$$

$$= 100$$

$$\therefore \text{ area is } 100 \text{ units}^2$$

Board of Studies: Notes from the Marking Centre

Typical responses answered (i) and (ii) correctly, with occasional errors stemming from the use of incorrect formulas, particularly errors in sign.

- (iii) The most common line of attack was to argue in terms of properties of gradients $(m_1m_2 = -1)$, however geometric constructions and approaches involving the judicious use of Pythagoras's theorem were also accepted.
- (iv) Responses exhibited a lack of precision when explaining why *OABC* is a rhombus. Some candidates confused the term *dissect* with bisect. It is inappropriate in this context to refer to unnamed diagonals simply as *lines*.
- (v) Weaker responses omitted this part completely, presumably a result of not being able to recall the formula for the area of a rhombus. It was certainly acceptable however to consider instead the areas of the various minor triangles on display. Many responses achieved a correct area with this elementary approach.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/

^{*} These solutions have been provided by projectmaths and are not supplied or endorsed by the Board of Studies