05	**9a**	A particle is initially at rest at the origin. Its acceleration as a function of time, t, is given by $\ddot{x} = 4\sin 2t$.	
		(i) Show that the velocity of the particle is given by $\dot{x} = 2 - 2\cos 2t$.	**2**
		(ii) Sketch the graph of the velocity for $0 \le t \le 2\pi$ AND determine the time at which the particle first comes to rest after $t = 0$.	**3**
		(iii) Find the distance travelled by the particle between $t = 0$ and the time at which the particle first comes to rest after $t = 0$.	**2**

(i) $\ddot{x} = 4\sin 2t$.
By integration,

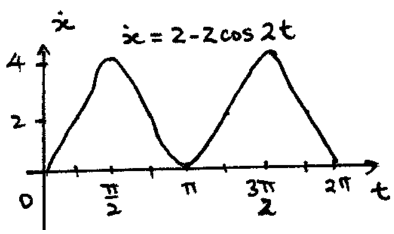
$\dot{x} = -2\cos 2t + c$

When $t = 0$, $v = \dot{x} = 0$:
$0 = -2\cos 2(0) + c$
$0 = -2 + c$
$c = 2$

$\dot{x} = -2\cos 2t + 2$

$\dot{x} = 2 - 2\cos 2t$

(ii)



Comes to rest when $\dot{x} = 0$.
This means it first comes to rest $t = \pi$.

(iii) Distance $= \int_0^{\pi} 2 - 2\cos 2t \; dt$

$= [2t - \sin 2t]_0^{\pi}$
$= 2(\pi) - \sin 2\pi - (2(0) - \sin 0)$
$= 2\pi - 0$
$= 2\pi$

The particle has travelled 2π units.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

When asked to 'Show that …', candidates need to show the reasons and working that allowed them to arrive at the conclusion.

(a) (i) Nearly all candidates showed that they understood the primitive of $4\sin 2t$ is $-2\cos 2t$. The better responses correctly included the constant of integration and the substitution of the initial conditions to show that the constant was 2.

(ii) A majority of candidates who attempted this question showed that they understood the key feature of this graph was a cosine curve with a wavelength of π. Better responses were given by candidates who understood this and then plotted some key points to correctly place the graph on the number plane. Common errors included graphing a sine curve or $2\cos 2t$ or $-2\cos 2t$ and then not translating it correctly. Many candidates correctly identified that the velocity was zero when the curve crossed the horizontal axis.

(iii) Many candidates demonstrated that they understood that the required distance was found by finding the area under the curve from $t = 0$ to $t = \pi$ and then correctly evaluating their definite integral. A common alternative method was to integrate $2 - 2\cos 2t$ to find the displacement function. Again candidates were expected to show the use of the initial conditions to find the constant and proceed with the required substitution.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/