09	**8b**	One year ago Daniel borrowed \$350 000 to buy a house. The interest rate was 9% per annum, compounded monthly. He agreed to repay the loan in 25 years with equal monthly repayments of \$2937.	
		(i) Calculate how much Daniel owed after his monthly repayment.	**1**
		(ii) Daniel has just made his 12th monthly repayment. He now owes \$346 095. The interest rate now decreases to 6% per annum, compounded monthly. The amount $\$A_n$, owing on the loan after the nth monthly repayment is now calculated using the formula $A_n = 346\,095 \times 1.005^n - 1.005^{n-1}M - \ldots - 1.005M - M$ where $\$M$ is the monthly repayment and $n = 1, 2, \ldots, 288$. (Do NOT prove this formula.) Calculate the monthly repayment if the loan is to be repaid over the remaining 24 years (288 months).	**3**
		(iii) Daniel chooses to keep his monthly repayments at \$2937. Use the formula in part (ii) to calculate how long it will take him to repay the \$346 095.	**3**
		(iv) How much will Daniel save over the term of the loan by keeping his monthly repayments at \$2937, rather than reducing his repayments to the amount calculated in part (ii)?	**1**

(i) Let $B_1 = 350\,000 \times 1.0075 - 2937$ as 9% pa, then 0.75% = 0.0075 per month

$= 349\,688$ $\therefore$ Daniel owes \$349 688 after his repayment.

(ii) Let $A_n = 346\,095 \times 1.005^n - 1.005^{n-1}M - \ldots - 1.005M - M$

$= 346\,095 \times 1.005^n - M(1 + 1.005 + \ldots + 1.005^{n-1})$

$A_{288} = 346\,095 \times 1.005^{288} - M(1 + 1.005 + \ldots + 1.005^{287}) = 0$

Now, for $1 + 1.005 + \ldots + 1.005^{287}$, $a = 1$, $r = 1.005$ and $n = 288$ and use

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1(1.005^{288} - 1)}{1.005 - 1}$$

$$\therefore M = 346\,095 \times 1.005^{288} \div \frac{1(1.005^{288} - 1)}{1.005 - 1}$$

$$= \frac{346095 \times 1.005^{288} \times 0.005}{1.005^{288} - 1}$$

$= 2270.307289 \ldots$

$= 2270.31$ (2 dec pl) $\therefore$ repayment is \$2270.31

(iii)

$$2937 = 346\,095 \times 1.005^n \div \frac{1(1.005^n - 1)}{1.005 - 1}$$

$$2937 = 346\,095 \times 1.005^n \times \frac{0.005}{1.005^n - 1}$$

$2937(1.005^n - 1) = 1730.475 \times 1.005^n$

$1206.525 \times 1.005^n = 2937$

$$1.005^n = \frac{2937}{1206525}$$

Take logs of both sides:

$$n \log_e 1.005 = \log_e \frac{2937}{1206525}$$

$$n = \log_e \frac{2937}{1206525} \div \log_e 1.005$$

$= 178.3733175 \ldots$

$= 179$ $\therefore$ there will be 179 repayments, or 14 years 11 months.

(iv) Difference in payments $= 2270.31 \times 288 - 2937 \times 179$

$= 128\,126.26$ $\therefore$ he will save \$128 126.26

Board of Studies: Notes from the Marking Centre

The magnitude of the numbers that candidates needed to manipulate in this part proved a difficulty. Many of the errors could be explained by careless setting-out of working and very poor handwriting, especially where indices were involved.

(i) This part was well attempted by most candidates. Common errors were the poor understanding of the process of time payment (Amount owing = Balance + interest – payment) and incorrect calculation of the monthly interest rate.

(ii) Many candidates failed to set A_{288} to zero to set up the equation. Those that went on to solve the equation for M were able to score 2 or 3 marks depending on their computational accuracy. Many candidates showed inaccuracy in their use of algebraic skills, in particular when expanding brackets preceded by a negative sign. Incorrect geometric sum formula for the sum of n terms caused confusion for many. Candidates misinterpreted the value for n, which left a wide variety of answers that showed that many students did not really understand the concept that was being addressed.

(iii) Many candidates set up the equation correctly but did not go on with the algebraic manipulation to find n. Some candidates who found an expression for n used logarithms to find the correct answer. Most candidates did not proceed any further, showing a lack of knowledge in handling this type of evaluation involving factorising with powers and logarithms. Some candidates used trial and error to arrive at a solution.

(iv) Candidates who had perfect solutions in (ii) and (iii) made poor attempts here as they either misunderstood the question or used incorrect values in their calculations. Most candidates did not attempt this part as they had not completed part (iii). Some candidates, who had little idea as to what was required to get the correct answer, went back to incorrectly work through the geometric series again.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/