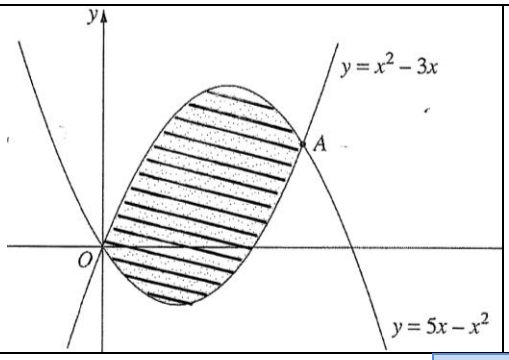
12 **13b** The diagram shows the parabolas $y = 5x - x^2$ and $y = x^2 - 3x$. The parabolas intersect at the origin O and the point A. The region between the two parabolas is shaded.

(i) Find the x-coordinate of the point A.

(ii) Find the area of the shaded region.



1 3

(i) Solve simultaneously:

$$x^2 - 3x = 5x - x^2$$
$$2x^2 - 8x = 0$$
$$2x(x - 4) = 0$$
$$x = 0, 4$$

$\therefore$ A has x-coordinate of 4

State Mean: **0.71/1 2.13/3**

(ii) Area $= \int_0^4 5x - x^2 - (x^2 - 3x)\ dx$

$$= \int_0^4 8x - 2x^2\ dx$$
$$= \left[4x^2 - \frac{2x^3}{3}\right]_0^4$$
$$= 4 \times 4^2 - \frac{2(4)^3}{3} - 0$$
$$= 64 - \frac{128}{3}$$
$$= \frac{64}{3}$$

$\therefore$ area is $21\frac{1}{3}$ units^2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) In the majority of responses, candidates equated the quadratic equations to find the x-coordinate of A. In weaker responses, candidates divided through by x thus finding only one of the two solutions for a quadratic equation.

(ii) In better responses, candidates recognised that this question involved the area enclosed between two curves and simplified the definite integral of the form $\int_0^4 \left[\left(5x - x^2\right) - \left(x^2 - 3x\right)\right] dx$ before evaluating. In many weaker responses, candidates made the question more difficult by dividing the area into several parts and then evaluating them separately. Other common errors included incorrect limits and the use of absolute value signs. In weaker responses, candidates differentiated instead of integrating.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/