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| **08** | **5b** | Consider the geometric series $5 + 10x + 20x^2 + 40x^3 + \ldots$<br>(i) For what values of $x$ does this series have a limiting sum?<br>(ii) The limiting sum of this series is 100. Find the value of $x$. | <br>**2**<br>**2** |

The series is $5 + 10x + 20x^2 + 40x^3 + \ldots$ with $a = 5$, $r = 2x$

i. Limiting sum occurs if $|r| < 1$

$$|2x| < 1$$
$$-1 < 2x < 1$$
$$\frac{-1}{2} < x < \frac{1}{2}$$

ii. Using $S_\infty = \frac{a}{1-r}$ with $a = 5$, $r = 2x$ and $S_\infty = 100$

$$100 = \frac{5}{1-2x}$$
$$100(1 - 2x) = 5$$
$$100 - 200x = 5$$
$$-200x = 5 - 100$$
$$-200x = -95$$
$$\frac{-200x}{-200} = \frac{-95}{-200}$$
$$x = \frac{19}{40}$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

**Board of Studies: Notes from the Marking Centre**

(i) Although many candidates correctly recognised that the geometric series had a common ratio of $r = 2x$, many did not then proceed to argue that the limiting sum only existed when $|r| < 1$. Responses which reached $|2x| < 1$ often then failed to solve the absolute value inequality correctly, with common errors being to conclude that $x < \frac{1}{2}$, $x < \pm\frac{1}{2}$ or even $x < \left|\frac{1}{2}\right|$. Candidates should be aware that $\leq$ and $<$ are not interchangeable symbols and that in the context of this part all inequalities needed to be strict.

(ii) In better responses, candidates correctly implemented the formula $S_\infty = \frac{a}{1-r} = \frac{5}{1-2x} = 100$.

**Source:** **http://www.boardofstudies.nsw.edu.au/hsc_exams/**