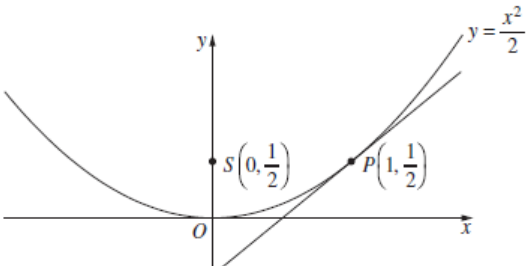
2015 12 e The diagram shows the parabola $y = \frac{x^2}{2}$ with focus $S(0, \frac{1}{2})$. A tangent to the parabola is drawn at $P(1, \frac{1}{2})$.



Not to scale

(i) Find the equation of the tangent at the point P. **2**

(ii) What is the equation of the directrix of the parabola? **1**

(iii) The tangent and directrix intersect at Q. Show that Q lies on the y-axis. **1**

(iv) Show that ΔPQS is isosceles. **1**

(i) $y = \frac{x^2}{2}$

$y' = \frac{2x}{2}$

$= x$

$y'(1) = 1$

Using $(1, \frac{1}{2})$, $m = 1$:

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = 1(x - 1)$

$y - \frac{1}{2} = x - 1$

$y = x - \frac{1}{2}$

State Mean: **1.63**

$\therefore$ the equation of tangent is $y = x - \frac{1}{2}$.

(ii) For $y = \frac{x^2}{2}$, vertex (0, 0) and focus is $(0, \frac{1}{2})$.

$\therefore$ focal length: $a = \frac{1}{2}$.

$\therefore$ equation of the directrix is $y = -\frac{1}{2}$.

State Mean: **0.79**

(iii) Subs $y = -\frac{1}{2}$ in $y = x - \frac{1}{2}$:

$-\frac{1}{2} = x - \frac{1}{2}$

$\therefore \quad x = 0$

$\therefore$ $Q(0, -\frac{1}{2})$, which lies on y-axis.

State Mean: **0.61**

(iv) $PS = 1$

$QS = \frac{1}{2} + \frac{1}{2}$

$= 1$

$\therefore$ $PS = QS$

$\therefore$ ΔPQS is isosceles (2 sides equal).

State Mean: **0.73**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(e)(i) The majority of candidates were able to correctly find the derivative and use the point-gradient form to find the equation of the tangent. Some candidates complicated this part by unsuccessfully attempting to use the quotient rule to find the derivative.

Common problems were:

- finding an incorrect derivative of $y = \frac{x^2}{2}$
- not substituting the x coordinate of P into the derivative to find the gradient.

(e)(ii) The majority of candidates scored full marks for this part.

Common problems were:

- incorrectly labelling the directrix as $x = -\frac{1}{2}$ or $d = -\frac{1}{2}$
- incorrectly stating that the point $Q(0, -\frac{1}{2})$ was the equation of the directrix.

(e)(iii) Most candidates realised that they needed to solve simultaneously the equations found in (e)(i) and (e)(ii). Those candidates who were not successful in finding the correct equation of the tangent in (e)(i) generally struggled to complete this part. Some candidates did not make the link between their coordinates for Q and the fact that Q lies on the y-axis.

Common problems were:

- finding the y-intercept of the tangent without linking it to the directrix
- showing that $Q(0, -\frac{1}{2})$ lies on the tangent instead of the y-axis.

(e)(iv) Most candidates were able to use their diagram to establish that $PS = QS = 1$.

Common problems were:

- finding the lengths of PQ and PS and stating that they were equal.
- making arithmetic errors when using the distance formula.

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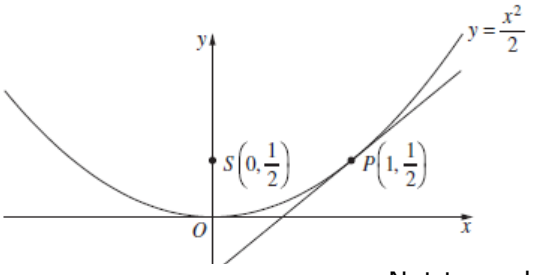

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Solution