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08	**8a**	Let $f(x) = x^4 - 8x^2$. (i) Find the coordinates of the points where the graph of $y = f(x)$ crosses the axes. (ii) Show that $f(x)$ is an even function. (iii) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. (iv) Sketch the graph of $y = f(x)$.	**2** **1** **4** **1**

(i)
$$x^4 - 8x^2 = 0$$
$$x^2(x^2 - 8) = 0$$
$$x = 0, \pm\sqrt{8} \quad \therefore \text{ points are } (0, 0), (\sqrt{8}, 0), (-\sqrt{8}, 0)$$

(ii)
$$f(x) = x^4 - 8x^2$$
$$f(-x) = (-x)^4 - 8(-x)^2$$
$$= x^4 - 8x^2$$
$$= f(x) \quad \therefore \text{ As } f(x) = f(-x), \text{ the function is even}$$

(iii)
$$f(x) = x^4 - 8x^2$$
$$f'(x) = 4x^3 - 16x = 0$$
$$4x(x^2 - 4) = 0$$
$$x = 0, 2, -2$$

At $x = 0$, $f(0) = 0^4 - 8(0)^2 = 0 \quad \therefore (0, 0)$

At $x = 2$, $f(2) = 2^4 - 8(2)^2 = -16 \quad \therefore (2, -16)$

At $x = -2$, $f(-2) = (-2)^4 - 8(-2)^2 = -16 \quad \therefore (-2, -16)$

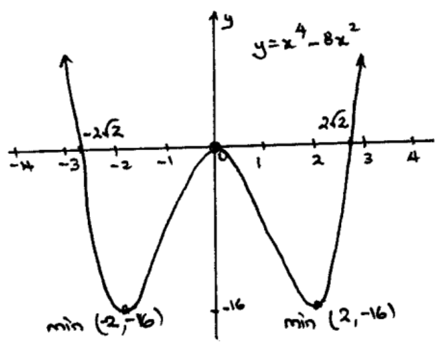
$$f''(x) = 12x^2 - 16$$

At $x = 0$, $f''(0) = 12(0)^2 - 16 < 0 \quad \therefore$ Max $(0, 0)$

At $x = 2$, $f''(2) = 12(2)^2 - 16 > 0 \quad \therefore$ Min $(2, -16)$

At $x = -2$, $f''(-2) = 12(-2)^2 - 16 < 0 \quad \therefore$ Min $(-2, -16)$

(iv)



* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) To find the x intercepts, candidates were ultimately required to solve $x^2(x^2 - 8) = 0$. Common errors included ignoring $x = 0$ or $x = \sqrt{8}$.

(ii) Better responses stated the condition for an even function, namely $f(-x) = f(x)$, and demonstrated this by successful substitution and simplification. Candidates are reminded that showing the result is true for particular integer values of x is not a proof.

(iii) Candidates were required to find all three stationary points and determine their nature. Many errors were made in solving the equation obtained by setting the derivative, $4x^3 - 16x$, equal to 0. The tests used to determine the nature of the stationary points should be clearly labelled and a conclusion explicitly drawn. Candidates who used the second derivative were generally more successful.

Only a few responses used the result of part (ii) to argue that, having found a minimum turning point at $x = 2$, another minimum must occur at $x = -2$. Candidates are also reminded that it is a waste of valuable time to find points of inflection when the question does not require it.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/