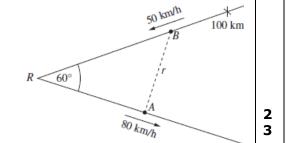
13 14 Two straight roads meet at R at an angle of 60° .

At time t = 0 car A leaves R on one road, and car B is 100 km from R on the other road. Car A travels away from R at a speed of 80 km/h, and car B travels towards R at a speed of 50 km/h. The distance between the cars at time t hours is t km.



(i) Show that
$$r^2 = 12\ 900t^2 - 18\ 000t + 10\ 000$$
.

(ii) Find the minimum distance between the cars.

(i) As
$$D = ST$$
, then:

Car A travels 80t km; car B travels 50t km.

$$\therefore AR = 80t$$

$$BR = 100 - 50t$$

Using cosine rule:

$$r^{2} = (100 - 50t)^{2} + (80t)^{2}$$
$$- 2(100 - 50t)(80t) \times \cos 60^{\circ}$$
$$= 10\ 000 - 10\ 000t + 2500t^{2}$$
$$+ 6400t^{2} - 8000t + 4000t^{2}$$
$$\therefore r^{2} = 12\ 900t^{2} - 18\ 000t + 10\ 000$$

(ii) Min when
$$\frac{d(r^2)}{dt} = 0$$

$$\frac{d(r^2)}{dt} = 25\ 800t - 18\ 000 = 0$$
$$t = \frac{18000}{25800}$$

$$=\frac{30}{43}$$

Consider nature:

$$\frac{d^2(r^2)}{dt^2} = 25\ 800 > 0$$
 :: Minimum

Subs in r^2 :

$$r^2 = 12\ 900\ (\frac{30}{43})^2 - 18\ 000(\frac{30}{43}) + 10\ 000$$

 $r = 60.99942813 \dots$

= 61 (nearest whole)

.. minimum distance is 61 km

Board of Studies: Notes from the Marking Centre

(i) A significant number of candidates could not establish that RB = 100 - 50t and RA = 80t. Most candidates correctly identified the cosine rule.

Common problems were:

- · not substituting correct values into the cosine rule
- making algebraic and simplification errors, as more than one algebraic step was required to prove the given result.
- (ii) Finding the derivative of r^2 (rather than making r the subject) made the algebra much easier.

Common problems were:

- when making r the subject, finding the second derivative posed a problem
- not testing the value found to justify a minimum.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/

^{*} These solutions have been provided by <u>projectmaths</u> and are not supplied or endorsed by the Board of Studies

