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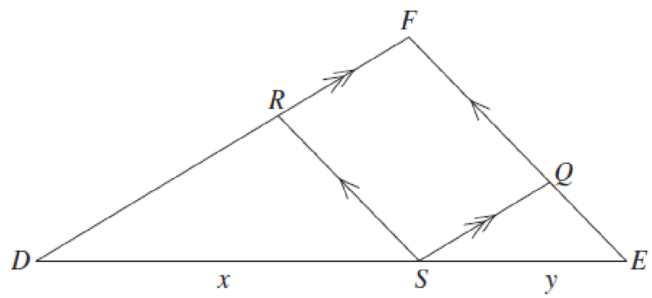
2014 15b In ΔDEF, a point S is chosen on the side DE. The length of DS is x, and the length of ES is y. The line through S parallel to DF meets EF at Q. The line through S parallel to EF meets DF at R. The area of ΔDEF is A. The areas of ΔDSR and ΔSEQ are A_1 and A_2 respectively.



(i) Show that ΔDEF is similar to ΔDSR. **2**

(ii) Explain why $\dfrac{DR}{DF} = \dfrac{x}{x+y}$. **1**

(iii) Show that $\sqrt{\dfrac{A_1}{A}} = \dfrac{x}{x+y}$. **2**

(iv) Using the result from part (iii) and a similar expression for $\sqrt{\dfrac{A_2}{A}}$, deduce that $\sqrt{A} = \sqrt{A_1} + \sqrt{A_2}$. **2**

(i) In Δs DRS, DFE:

$\angle D$ is common

$\angle DRS = \angle DFE$ (corr $\angle$s, $RS || FE$)

$\therefore \Delta DRS$ similar to ΔDFE (equiangular)

(ii) $$\frac{DR}{DF} = \frac{DS}{DE}$$

(matching sides of sim Δs are in proportion)

As $DS = x$ and $DE = x + y$,

$$\frac{DR}{DF} = \frac{x}{x+y}$$

(iii) Area $\Delta DRS = A_1$, Area $\Delta DFE = A$:

Ratio of areas $= A_1 : A$

Ratio of sides $= \sqrt{A_1} : \sqrt{A}$

$$\frac{\sqrt{A_1}}{\sqrt{A}} = \frac{x}{x+y}$$

$$\sqrt{\frac{A_1}{A}} = \frac{x}{x+y}$$

(iv) Also, it can be proved ΔDEF similar to ΔSEQ, and hence $\sqrt{\dfrac{A_2}{A}} = \dfrac{y}{x+y}$.

$$\sqrt{\frac{A_1}{A}} + \sqrt{\frac{A_2}{A}} = \frac{x}{x+y} + \frac{y}{x+y}$$

$$\frac{\sqrt{A_1}}{\sqrt{A}} + \frac{\sqrt{A_2}}{\sqrt{A}} = 1$$

$$\sqrt{A_1} + \sqrt{A_2} = \sqrt{A}$$

State Mean:
1.54
0.74
0.53
0.52

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(i) This part was generally done well, with the majority of candidates using 'equiangular' to prove similarity. A few candidates stated the intercept properties of transversals and parallel lines to show that the sides about the equal angle were in proportion.

Common problems were:

- not recognising the two pairs of corresponding angles, or indicating an equal pair of angles without appropriate reasoning;
- using tests for congruence instead of similarity;
- poor setting out with little or no reasoning, and omitting a concluding statement.

(ii) This part was generally done well. Most candidates used the result in (b)(i) to explain part (ii). Some candidates wrote a full paragraph of justification when a brief statement was all that was required for 1 mark. Most candidates knew about similar triangles having corresponding sides in the same ratio and used relevant terminology.

A common problem was:

- stating pairs of equal sides and writing $DS = DR$, $DE = DF$, then $\frac{DR}{DF} = \frac{x}{x+y}$.

(iii) Most candidates struggled with this part. Better responses were well set out with a series of logical steps. The candidates who achieved full marks for this part either performed algebraic manipulation involving $\frac{A_1}{A}$, or applied 'ratios of areas is equal to the square of the ratios of side lengths' in similar figures to achieve the result.

Common problems were:

- incorrectly using the formula Area = $\frac{1}{2}ab\sin C$, often with incorrect sides;
- assuming the triangles were right angled;
- assuming that $DR = DS = x$, $DE = DF = y$.

(iv) In the better responses candidates appreciated the link between (iii) and (iv) presenting a succinct and accurate answer. Most candidates indicated that $\sqrt{\frac{A_2}{A}} = \frac{y}{x+y}$, using their expression from (b) (iii).

However, a considerable number of candidates struggled to provide the working to deduce $\sqrt{A} = \sqrt{A_1} + \sqrt{A_2}$.

Common problems were:

- not realising how to use the result from (b) (iii), some candidates unnecessarily completed a further similar triangle proof to establish the result.
- after obtaining $\sqrt{\frac{A_1}{A}} = \frac{x}{x+y}$ and $\sqrt{\frac{A_2}{A}} = \frac{y}{x+y}$, some candidates assumed that $\sqrt{A_1} = x$, $\sqrt{A_2} = y$ and $\sqrt{A_1} + \sqrt{A_2} = x + y$.

http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths.pdf