06	**8b**	Joe borrows \$200 000 which is to be repaid in equal monthly instalments. The interest rate is 7.2% per annum reducible, calculated monthly. It can be shown that the amount, $\$A_n$, owing after the nth repayment is given by the formula: $A_n = 200\,000r^n - M(1 + r + r^2 + \cdots + r^{n-1})$, where $r = 1.006$ and $\$M$ is the monthly repayment. (Do NOT show this.) (i) The minimum monthly repayment is the amount required to repay the loan in 300 instalments. Find the minimum monthly repayment. (ii) Joe decides to make repayments of \$2800 each month from the start of the loan. How many months will it take for Joe to repay the loan?	**3** **2**

i. As 7.2% pa, and 7.2 ÷ 12 = 0.6, then 0.6% per month = 0.006 per month

$$A_n = 200\,000r^n - M(1 + r + r^2 + \cdots + r^{n-1}),$$
$$A_{300} = 200\,000 \times 1.006^{300} - M(1 + 1.006 + 1.006^2 + \cdots + 1.006^{299}) = 0$$

$$\therefore M = 200\,000 \times 1.006^{300} \div (1 + 1.006 + \ldots + 1.006^{299})$$

Now, for $1 + 1.006 + \ldots + 1.006^{299}$, $a = 1$, $r = 1.006$ and $n = 300$ and use

$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$= \frac{1(1.006^{300} - 1)}{1.006 - 1}$$

$$\therefore M = 200\,000 \times 1.006^{300} \div \frac{1(1.006^{300} - 1)}{1.006 - 1}$$
$$= 1439.177383$$
$$= 1439.18 \quad \text{(2 dec pl)}$$

$\therefore$ the repayment is \$1439.18

ii. Use $M = 200\,000 \times 1.006^n \div \frac{1(1.006^n - 1)}{1.006 - 1}$ and $M = 2800$

$$2800 = 200\,000 \times 1.006^n \times \frac{0.006}{1.006^n - 1}$$
$$2800(1.006^n - 1) = 1200 \times 1.006^n$$
$$2800 \times 1.006^n - 2800 = 1200 \times 1.006^n$$
$$1600 \times 1.006^n = 2800$$
$$1.006^n = \frac{2800}{1600}$$
$$1.006^n = 1.75$$
$$\log 1.006^n = \log 1.75$$
$$n \log 1.006 = \log 1.75$$
$$n = \frac{\log 1.75}{\log 1.006}$$
$$= 93.54882691$$

$\therefore$ repaid after 94 months

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Responses to this part were generally strong. Common errors were to put $A_n = 300$ rather than $A_n = 0$, to use $n = 299$ rather than 300, and finally to claim that $200\,000\,(1.06)^n = 212\,000^n$. Many students carefully reconstructed (without reward) the supplied expression for A_n.

(ii) It was common for candidates to use trial and error rather than an algebraic approach in part (ii) and carefully presented responses were accepted. Despite significant algebraic demands, many candidates solved the relevant exponential equation in part (ii) to achieve full marks in part (b).

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/