projectmaths

Integration

16 9 What is the value of $\int_{-3}^{2} |x+1|\,dx$? **1** Solution

(A) $\frac{5}{2}$ (B) $\frac{11}{2}$ (C) $\frac{13}{2}$ (D) $\frac{17}{2}$

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16 11 d Evaluate $\int_{0}^{1} (2x+1)^3\,dx$. **2** Solution

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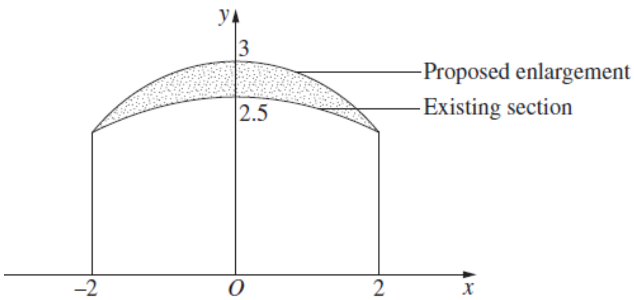
16 14 a The diagram shows the cross-section of a tunnel and a proposed enlargement. **3** Solution



The heights, in metres, of the existing section at 1 metre intervals are shown in Table *A*.

Table *A*: Existing heights

x	-2	-1	0	1	2
y	2	2.38	2.5	2.38	2

The heights, in metres, of the proposed enlargement are shown in Table *B*.

Table *B*: Proposed heights

x	-2	-1	0	1	2
y	2	2.78	3	2.78	2

Use Simpson's rule with the measurements given to calculate the approximate increase in area.

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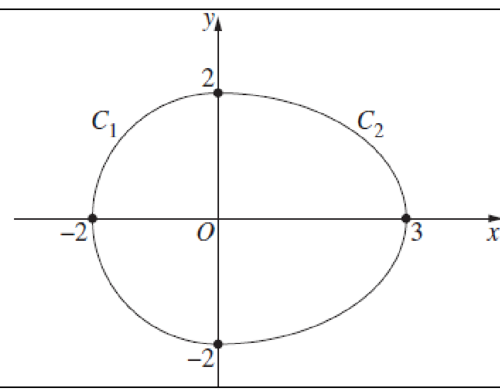
16 15 a The diagram shows two curves C_1 and C_2. The curve C_1 is the semicircle $x^2 + y^2 = 4$, $-2 \le x \le 2$. The curve C_2 has equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$, $0 \le x \le 3$. **4** Solution

An egg is modelled by rotating the curves about the x-axis to form a solid of revolution.

Find the exact value of the volume of the solid of revolution.



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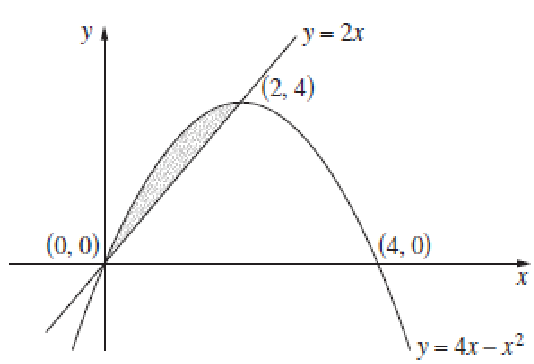
15 7 The diagram shows the parabola $y = 4x - x^2$ meeting the line $y = 2x$ at (0, 0) and (2, 4). Which expression gives the area of the shaded region bounded by the parabola and the line? **1** Solution

(A) $\int_{0}^{2} x^2 - 2x\,\,dx$ (B) $\int_{0}^{2} 2x - x^2\,\,dx$

(C) $\int_{0}^{4} x^2 - 2x\,\,dx$ (D) $\int_{0}^{4} 2x - x^2\,\,dx$



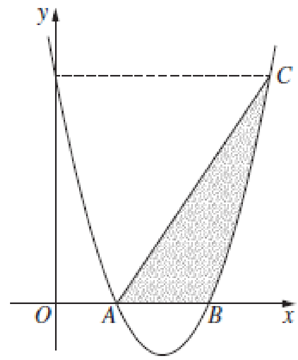
15 **16 a** The diagram shows the curve with equation $y = x^2 - 7x + 10$. The curve intersects the x-axis at points A and. The point C on the curve has the y-coordinate as the y-intercept of the curve. **Solution**

(i) Find the x-coordinates of points A and B. **1**

(ii) Write down the coordinates of C. **1**

(iii) Evaluate $\int_0^2 (x^2 - 7x + 10\ dx$. **1**

(iv) Hence, or otherwise, find the area of the shaded region. **2**



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14 **11 d** Find $\int \frac{1}{(x+3)^2}\,dx$. **2** **Solution**

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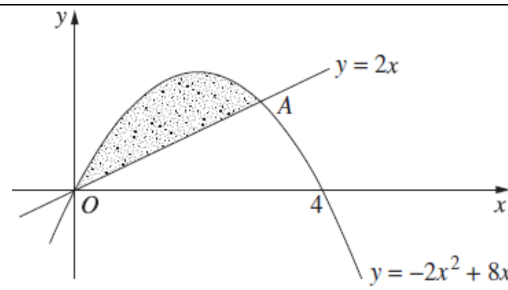
14 **12 d** The parabola $y = -2x^2 + 8x$ and the line $y = 2x$ intersect at the origin and at the point A. **Solution**

(i) Find the x-coordinate of the point A. **1**

(ii) Calculate the area enclosed by the parabola and the line. **3**



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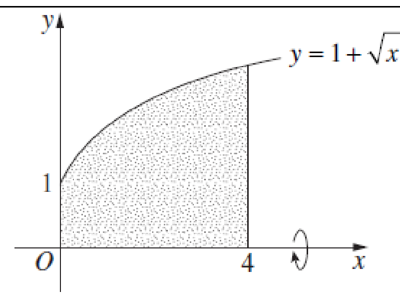
14 **14 c** The region bounded by the curve $y = 1 + \sqrt{x}$ and the x-axis between $x = 0$ and $x = 4$ is rotated about the x–axis to form a solid. Find the volume of the solid. **3** **Solution**



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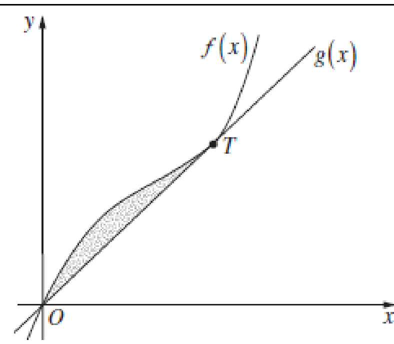
13 **13 b** The diagram shows the graphs of the functions $f(x) = 4x^3 - 4x^2 + 3x$ and $g(x) = 2x$. **Solution**

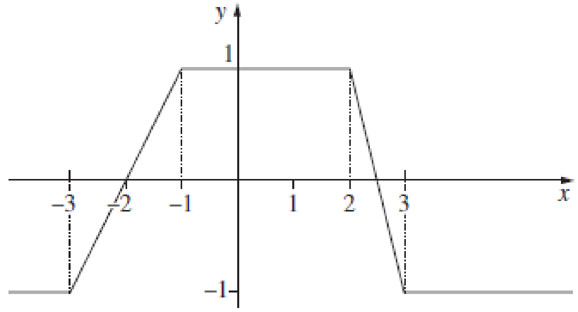
The graphs meet at O and at T.

(i) Find the x-coordinate of T. **1**

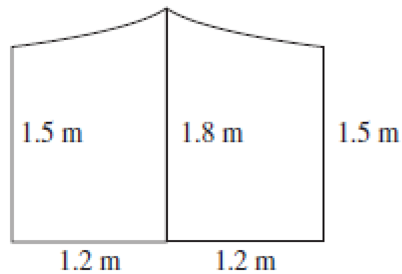
(ii) Find the area of the shaded regions between the graphs of the functions $f(x)$ and $g(x)$. **3**



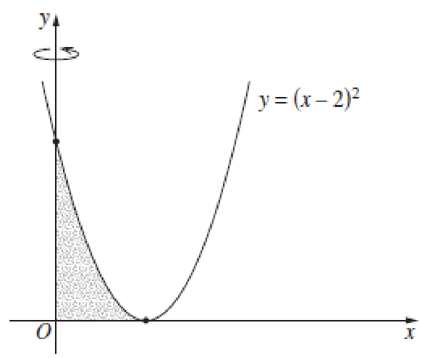
13	**14 d**	The diagram shows the graph $y = f(x)$. What is the value of a, where $a > 0$, so that $\int_{-a}^{a} f(x)\, dx = 0$.	**1**	Solution



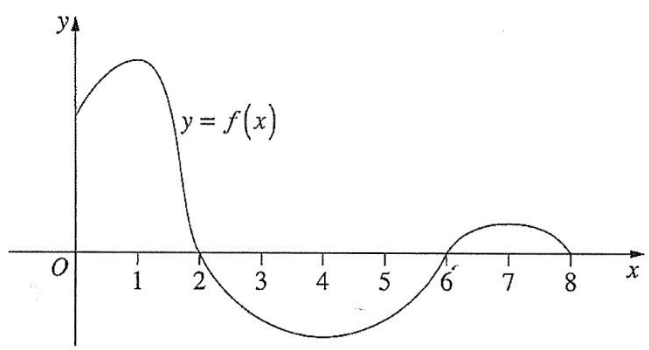
13	**15 a**	The diagram shows the front of a tent supported by three vertical poles. The poles are 1.2 m apart. The height of each outer pole is 1.5 m, and the height of the middle pole is 1.8 m. The roof hangs between the poles. The front of the tent has area A m^2. (i) Use trapezoidal rule to estimate A. (ii) Use Simpson's rule to estimate A. (iii) Explain why the trapezoidal rule gives the better estimate of A.	 **1** **1** **1**	Solution



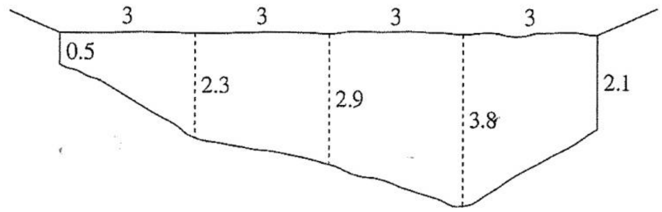
13	**15 b**	The region bounded by the x-axis, the y-axis and the parabola $y = (x - 2)^2$ is rotated about the y-axis to form a solid. Find the volume of the solid.	**4**	Solution



12	**10**	The graph of $y = f(x)$ has been drawn to scale for $0 \le x \le 8$. Which of the following integrals has the greatest value? (A) $\int_0^1 f(x)\, dx$ (B) $\int_0^2 f(x)$ (C) $\int_0^7 f(x)$ (D) $\int_0^8 f(x)$	**1**	Solution



12 **12 d** At a certain location a river is 12 metres wide. At this location the depth of the river, in metres, has been measured at 3 metre intervals. The cross-section is shown. **Solution**



(i) Use Simpson's rule with the five depth measurements to calculate the approximate area of the cross-section. **3**

(ii) The river flows at 0.4 metres per second. Calculate the approximate volume of water flowing through the cross-section in 10 seconds. **1**

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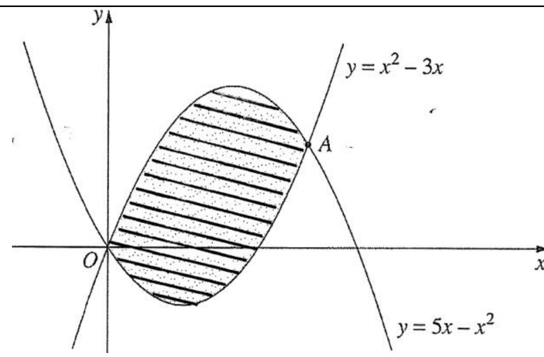
12 **13 b** The diagram shows the parabolas $y = 5x - x^2$ and $y = x^2 - 3x$. The parabolas intersect at the origin O and the point A. The region between the two parabolas is shaded. **Solution**



(i) Find the x-coordinate of the point A. **1**

(ii) Find the area of the shaded region. **3**

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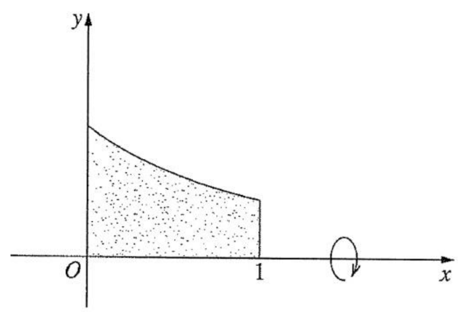
12 **14 b** The diagram shows the region bounded by $y = \dfrac{3}{(x+2)^2}$, the x-axis, the y-axis, and the line $x = 1$. **3** **Solution**

The region is rotated about the x-axis to form a solid.

Find the volume of the solid.



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11 **2e** Find $\displaystyle\int \frac{1}{3x^2}\,dx$. **2** **Solution**

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11 **4d** **Solution**

(i) Differentiate $y = \sqrt{9 - x^2}$ with respect to x. **2**

(ii) Hence, or otherwise, find $\displaystyle\int \frac{6x}{9 - x^2}\,dx$. **2**

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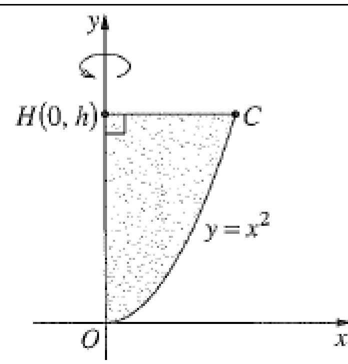
11 **5c** The table gives the speed v of a jogger at time t in minutes over a 20-minute period. The speed v is measured in metres per minute, in intervals of 5 minutes. The distance covered by the jogger over the 20-minute period is given by $\int_0^{20} v\,dt$. Use Simpson's rule and the speed at each of the five time values to find the approximate distance the jogger covers in the 20-minute period. **3** Solution

t	0	5	10	15	20
v	173	81	127	195	168

11 **8b** The diagram shows the region enclosed by the parabola $y = x^2$, the y-axis and the line $y = h$, where $h > 0$. This region is rotated about the y-axis to form a solid called a paraboloid. The point C is the intersection of $y = x^2$ and $y = h$. The point H has coordinates $(0, h)$. Solution



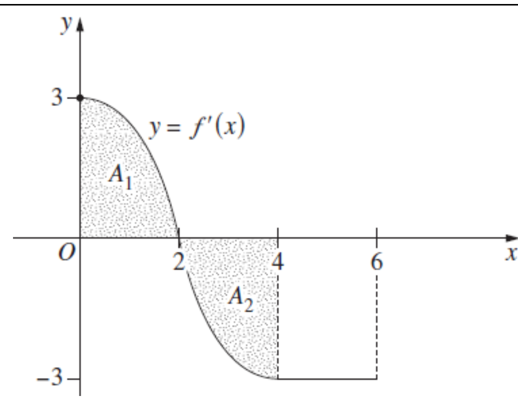
(i) Find the exact volume of the paraboloid in terms of h. **2**

(ii) A cylinder has radius HC and height h. What is the ratio of the volume of the paraboloid to the volume of the cylinder? **1**

10 **2d** (i) Find $\int \sqrt{5x+1}\,dx$. **2** Solution

10 **2e** Given that $\int_0^6 (x+k)\,dx = 30$, and k is a constant, find the value of k. **2** Solution

10 **9b** Let $y = f(x)$ be a function defined for $0 \le x \le 6$, with $f(0) = 0$. The diagram shows the graph of the derivative of f, $y = f'(x)$. The shaded region A_1 has area 4 square units. The shaded region A_2 has area 4 square units. Solution



(i) For which values of x is $f(x)$ increasing? **1**

(ii) What is the maximum value of $f(x)$? **1**

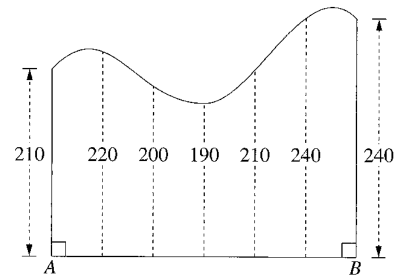
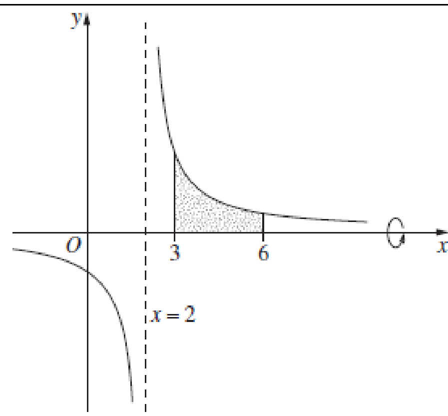
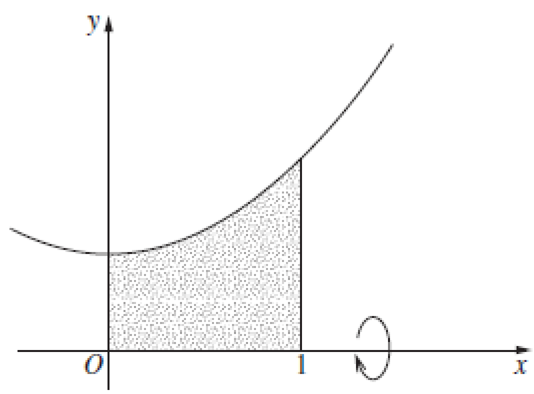
(iii) Find the value of $f(6)$. **1**

(iv) Draw a graph of $y = f(x)$ for $0 \le x \le 6$. **2**

09 **2b** (i) Find $\int 5\,dx$. **1** Solution

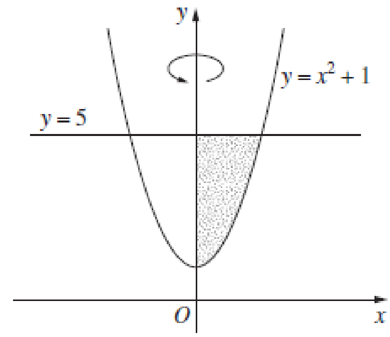
09 **2b** (ii) Find $\int \frac{3}{(x-6)^2}\,dx$. **2** Solution

09 **2b** (iii) Find $\int_1^4 x^2 + \sqrt{x}\,dx$. **3** Solution

09	**3d**	The diagram shows a block of land and its dimensions, in metres. The block of land is bounded on one side by a river. Measurements are taken perpendicular to the line *AB*, from *AB* to the river, at equal intervals of 50 m. Use Simpson's rule with six subintervals to find an approximation to the area of the block of land.  (not to scale)	**3**	Solution
08	**4c**	Consider the parabola $x^2 = 8(y - 3)$. (iv) Calculate the area bounded by the parabola and the line $y = 5$.	**3**	Solution
08	**6c**	The graph of $y = \dfrac{5}{x-2}$ is shown. The shaded region in the diagram is bounded by the curve $y = \dfrac{5}{x-2}$, the x-axis, and the lines $x = 3$ and $x = 6$. Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis. 	**3**	Solution
07	**2b**	ii. Evaluate $\displaystyle\int_1^4 \frac{8}{x^2}\,dx$.	**3**	Solution
07	**9a**	In the shaded region in the diagram is bounded by the curve $y = x^2 + 1$, the x-axis, and the lines $x = 0$ and $x = 1$. Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis. 	**3**	Solution

06 **4b** In the diagram, the shaded region is bounded by the parabola $y = x^2 + 1$, the y-axis and the line $y = 5$.

Find the volume of the solid formed when the shaded region is rotated about the y-axis.



3 Solution

05 **6a** Five values of the function $f(x)$ are shown in the table. Use Simpson's rule with the five values given in the table to estimate $\int_0^{20} f(x)\ dx$.

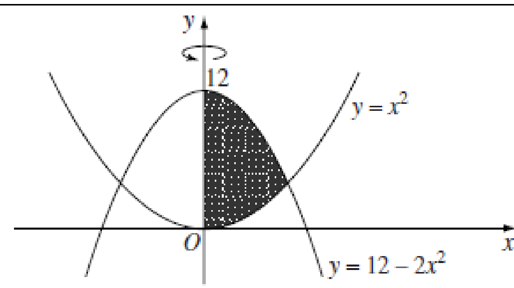
x	0	5	10	15	20
$f(x)$	15	25	22	18	10

3 Solution

05 **6c** The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram. Solution

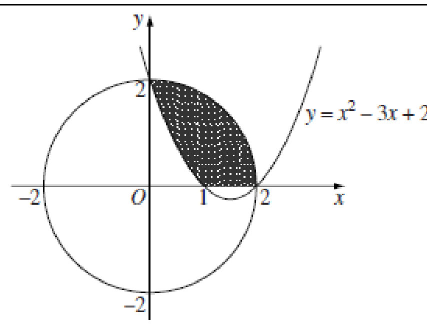
(i) Find the points of intersection of the two curves. **1**

(ii) The shaded region between the curves and the y-axis is rotated about the y-axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed. **3**



05 **8b** The shaded region in the diagram is bounded by the circle of radius 2, centred at the origin, the parabola $y = x^2 - 3x + 2$, and the x-axis.

By considering the difference of two areas, find the area of the shaded region.



Solution **3**