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09	**7b**	Between 5 am and 5 pm on 3 March 2009, the height, h, of the tide in a harbour was given by $h = 1 + 0.7\sin\frac{\pi}{6}t$ for $0 \le t \le 12$, where h is in metres and t is in hours, with $t = 0$ at 5 am. (i) What is the period of the function h? (ii) What was the value of h at low tide, and at what time did low tide occur? (iii) A ship is able to enter the harbour only if the height of the tide is at least 1.35 m. Find all times between 5 am and 5 pm on 3 March 2009 during which the ship was able to enter the harbour.	 **1** **2** **3**

(i) Period $= \frac{2\pi}{n}$

Now, $n = \frac{\pi}{6}$,

$$\text{Period} = 2\pi \div \frac{\pi}{6}$$
$$= 2\pi \times \frac{6}{\pi}$$
$$= 12$$

(ii) As $-1 \le \sin f(x) \le 1$, then minimum value of $\sin\frac{\pi}{6}t = -1$.

$$\therefore h = 1 + 0.7 \times -1$$
$$= 1 - 0.7$$
$$= 0.3$$

At low tide, $h = 0.3$

Also, $\sin\frac{\pi}{6}t = -1$, $\frac{\pi}{6}t = \frac{3\pi}{2}$

$$t = \frac{3\pi}{2} \div \frac{\pi}{6}$$
$$= \frac{3\pi}{2} \times \frac{6}{\pi}$$
$$= 9$$

After 9 hours, the low tide occurs at 2 pm.

(iii)
$$h = 1 + 0.7\sin\frac{\pi}{6}t$$
$$1.35 = 1 + 0.7\sin\frac{\pi}{6}t$$
$$0.7\sin\frac{\pi}{6}t = 0.35$$
$$\sin\frac{\pi}{6}t = \frac{0.35}{0.7}$$
$$\sin\frac{\pi}{6}t = 0.5$$
$$\frac{\pi}{6}t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6},$$
$$t = 1, 5, 13, 17, \ldots$$

But as $0 \le t \le 12$, at $t = 1$ and $t = 5$ the height is 1.35.
Now, add 1 hour on to 5 am gives 6 am and add 5 hours on to 5 am gives 10 am.

This means at between 6 am and 10 am the water is at least 1.35 metres.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) This part was not well done. Better responses resulted when candidates used the fact that the period was $\frac{2\pi}{n}$ or when they used a graph correctly.

(ii) Candidates with the most elegant solutions realised that the least value of $\sin\frac{\pi}{6}t$ was -1 and this occurred when $\frac{\pi}{6}t = \frac{3\pi}{2}$. Candidates who used a graph also quickly found the correct answers. Other candidates achieved success by using calculus to find the stationary points and then tested them to identify the appropriate minimum and its associated value. Many candidates who adopted this approach failed to test the stationary points to locate the minimum and used $t = 3$ instead of $t = 9$. A large number of candidates thought that low tide was at $t = 0$.

(iii) Most candidates were able to arrive at a point where they obtained $\sin\frac{\pi}{6}t \geq \frac{1}{2}$. From this point they often found only one solution, neglecting angles in other quadrants. Many candidates who correctly obtained $t = 1$ and $t = 5$ failed to convert this to an answer between 6 am and 10 am. Some candidates mistakenly interpreted the equation as linear.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/