

11	9b	<p>A tap releases liquid A into a tank at the rate of $\left(2 + \frac{t^2}{t+1}\right)$ litres per minute, where t is time in minutes. A second tap releases liquid B into the same tank at the rate of $\left(1 + \frac{1}{t+1}\right)$ litres per minute. The taps are opened at the same time and release the liquids into an empty tank.</p> <p>(i) Show that the rate of liquid A is greater than the rate of flow of liquid B by t litres per minute.</p> <p>(ii) The taps are closed after 4 minutes. By how many litres is the volume of liquid A greater than the volume of liquid B in the tank when the taps are closed?</p>	1 2
<p>(i) Difference = $2 + \frac{t^2}{t+1} - \left(1 + \frac{1}{t+1}\right)$</p> $= 1 + \frac{t^2}{t+1} - \frac{1}{t+1}$ $= 1 + \frac{t^2 - 1}{t+1}$ $= 1 + \frac{(t-1)(t+1)}{t+1}$ $= 1 + t - 1$ $= t$ <p>\therefore A is greater by t litres per minute.</p>		<p>(ii) Volume difference</p> $= \int_0^4 \left(2 + \frac{t^2}{t+1}\right) - \left(1 + \frac{1}{t+1}\right) dt$ $= \int_0^4 t \, dt \quad \text{(from (i))}$ $= \left[\frac{t^2}{2}\right]_0^4$ $= 8 - 0$ $= 8$ <p>\therefore A has 8 L more than B.</p>	

State Mean
0.16/1
0.29/2

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0.16/1
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Board of Studies: Notes from the Marking Centre

- (i) A significant number of candidates misunderstood and did not show that the difference between the two rates was equal to t . Many differentiated the given rates (not realising that they were rates) or even substituted values into each rate to show that rate A was greater than rate B, but not showing that the difference was always t . When the general case of the difference of the two rates was attempted, there were many errors made in the algebra. Candidates who proved that $\text{Rate}_A - \text{Rate}_B = t$ were in general also able to complete part (ii). A number of the candidates interpreted 'is greater than ... by t litres per minute' as 'is multiplied by t '.
- (ii) A small number of candidates integrated t , then easily calculated the correct answer of 8 litres. However, many attempted to integrate the given rates for A and B (most being unsuccessful in integrating A) rather than using the result from part (i) to answer part (ii). The majority of candidates substituted $t = 4$ into the original rates.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/