projectmaths

The Trigonometric Functions

16 **6** What is the period of the function $f(x) = \tan(3x)$? **1** [Solution]

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) 3π (D) 6π

---

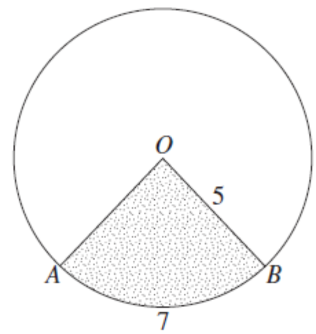
16 **7** The circle centred at O has radius 5. Arc AB has length 7 as shown in the diagram.
What is the area of the shaded sector OAB? **1** [Solution]

(A) $\frac{35}{2}$

(B) $\frac{35}{2}\pi$

(C) $\frac{125}{14}$

(D) $\frac{125}{14}\pi$



---

16 **8** How many solutions does the equation $|\cos(2x)| = 1$ have for $0 \le x \le 2\pi$? **1** [Solution]

(A) 1 (B) 3 (C) 4 (D) 5

---

16 **11 f** Find the gradient of the tangent to the curve $y = \tan x$ at the point where $x = \frac{\pi}{8}$. **2** [Solution]

Give your answer correct to 3 significant figures.

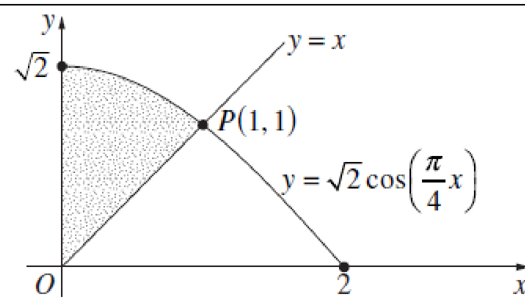
---

16 **11 g** Solve $\sin\left(\frac{x}{2}\right) = \frac{1}{2}$ for $0 \le x \le 2\pi$? **2** [Solution]

---

16 **13 d** The curve $y = \sqrt{2}\cos\left(\frac{\pi}{4}x\right)$ meets the line $y = x$ at $P(1, 1)$, as shown in the diagram. **3** [Solution]

Find the exact value of the shaded area.



---

15 **6** What is the value of the derivative of $y = 2\sin 3x - 3\tan x$ at $x = 0$? **1** [Solution]

(A) −1 (B) 0 (C) 3 (D) −9

---

15 **11 g** Evaluate $\int_0^{\frac{\pi}{4}} \cos 2x \; dx$. **2** [Solution]

---

15 **12 a** Find the solutions of $2\sin\theta = 1$ for $0 \le \theta \le 2\pi$. **2** [Solution]

---

14 **7** How many solutions of the equation $(\sin x - 1)(\tan x + 2) = 0$ lie between 0 and 2π? **1** [Solution]

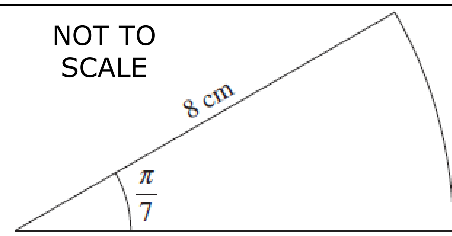
(A) 1 (B) 2 (C) 3 (D) 4

---

14 **11 e** Evaluate $\int_0^{\frac{\pi}{2}} \sin\frac{x}{2} \; dx$. **3** [Solution]

14 **11 g** The angle of a sector in a circle of radius 8 cm is $\frac{\pi}{7}$ radians, as shown in the diagram. **2** Solution

Find the exact value of the perimeter of the sector.



---

14 **13 a** (i) Differentiate $3 + \sin 2x$. **1** Solution

(ii) Hence, or otherwise, find $\int \frac{\cos 2x}{3 + \sin 2x}\, dx$. **2**

---

14 **15 a** Find all solutions of $2\sin^2 x + \cos x - 2 = 0$, where $0 \le x \le 2\pi$. **3** Solution

---

14 **16 a** Use Simpson's Rule with five function values to show that $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec dx \approx \frac{\pi}{9}\left(3 + \frac{8}{\sqrt{3}}\right)$. **3** Solution

---

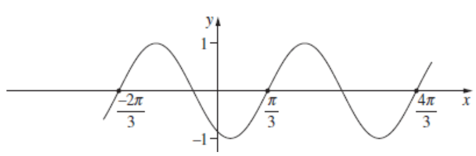
13 **4** What is the derivative of $\frac{x}{\cos x}$? **1** Solution

(A) $\frac{\cos x + x\sin x}{\cos^2 x}$ (B) $\frac{\cos x - x\sin x}{\cos^2 x}$ (C) $\frac{x\sin x - \cos x}{\cos^2 x}$ (D) $\frac{-x\sin x - \cos x}{\cos^2 x}$

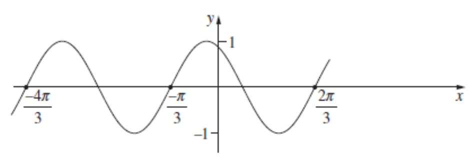
---

13 **6** Which diagram shows the graph $y = \sin(2x + \frac{\pi}{3})$? **1** Solution

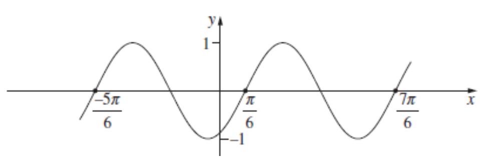
(A)



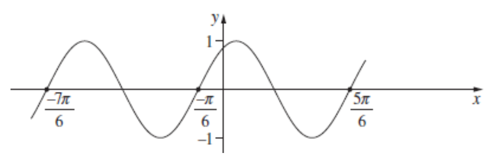
(B)



(C)



(D)



---

13 **11 c** Differentiate $(\sin x - 1)^8$. **2** Solution

---

13 **13 a** The population of a herd of wild horses is given by $P(t) = 400 + 50\cos\left(\frac{\pi}{6}t\right)$, where t is time in months. Solution

(i) Find all times during the first 12 months when the population equals 375 horses. **2**

(ii) Sketch the graph of $P(t)$ for $0 \le t \le 12$. **2**

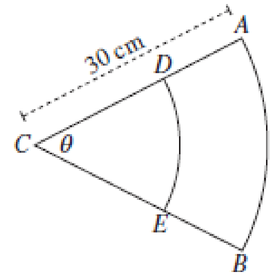
---

13 **13 c**

The region ABC is a sector of a circle with radius 30 cm, centred at C. The angle of the sector is θ. The arc DE lies on a circle also centred at C, as shown in the diagram.

The arc DE divides the sector ABC into two regions of equal area.

Find the exact length of the interval CD.



2 Solution

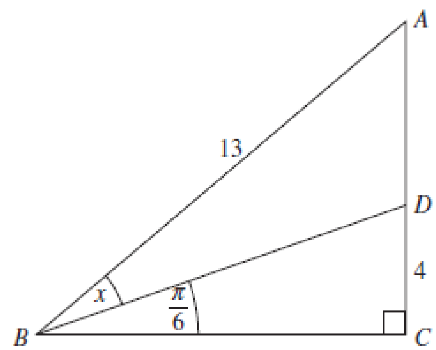
---

13 **14 c**

The right-angled triangle ABC has hypotenuse $AB = 13$.

The point D is on AC such that $DC = 4$, $\angle DBC = \frac{\pi}{6}$ and $\angle DBC = x$.

Using the sine rule, or otherwise, find the exact value of $\sin x$.



Not to scale

3 Solution

---

12 **6**

What are the solutions of $\sqrt{3}\tan x = -1$ for $0 \le x \le 2\pi$?

(A) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ (B) $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$ (C) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$ (D) $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

1 Solution

---

12 **11 f**

The area of a sector of a circle of radius 6 cm is 50 cm^2. Find the length of the arc of the sector.

2 Solution

---

12 **11 g**

Find $\displaystyle\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2}\, dx$.

3 Solution

---

12 **12 a**

Differentiate with respect to x:

(ii) $\dfrac{\cos x}{x^2}$

2 Solution

---

11 **2b**

Find the exact values of x such that $2\sin x = -\sqrt{3}$, where $0 \le x \le 2\pi$.

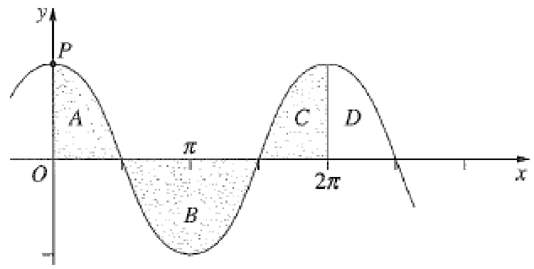
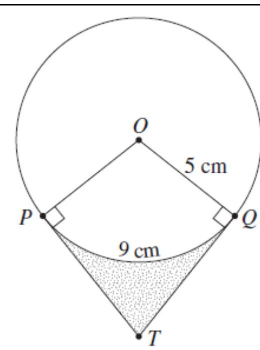
2 Solution

---

11 **4a**

Differentiate $\dfrac{x}{\sin x}$ with respect to x.

2 Solution

11	**6c**	The diagram shows the graph $y = 2\cos x$. 		Solution
		(i) State the coordinates of P.	**1**	
		(ii) Evaluate the integral $\int_0^{\frac{\pi}{2}} 2\cos x\, dx$.	**2**	
		(iii) Indicate which area in the diagram, A, B, C or D, is represented by the integral $\int_{\frac{3\pi}{2}}^{2\pi} 2\cos x \,\mathrm{dx}$.	**1**	
		(iii) Using parts (ii) and (iii), or otherwise, find the area of the region bounded by the curve $y = 2\cos x$ and the x-axis, between $x = 0$ and $x = 2\pi$.	**1**	
		(v) Using the parts above, write down the value of $\int_{\frac{\pi}{2}}^{2\pi} 2\cos x\, dx$.	**1**	
10	**1e**	Differentiate $x^2 \tan x$ with respect to x.	**2**	Solution
10	**2a**	Differentiate $\dfrac{\cos x}{x}$ with respect to x.	**2**	Solution
10	**5b**	(i) Prove that $\sec^2 x + \sec x \tan x = \dfrac{1+\sin x}{\cos^2 x}$.	**1**	Solution
		(ii) Hence prove that $\sec^2 x + \sec x \tan x = \dfrac{1}{1-\sin x}$.	**1**	
		(iii) Hence, use the table of standard integrals to find the exact value of $\int_0^{\frac{\pi}{4}} \dfrac{1}{1-\sin x}\, dx$.	**2**	
10	**6b**	The diagram shows a circle with centre O and radius 5 cm. The length of the arc PQ is 9 cm. Lines drawn perpendicular to OP and OQ at P and Q respectively meet at T.  (Not to scale)		Solution
		(i) Find $\angle POQ$ in radians.	**1**	
		(ii) Prove that ΔOPT is congruent to ΔOQT.	**2**	
		(iii) Find the length of PT.	**1**	
		(iv) Find the area of the shaded region.	**2**	

---

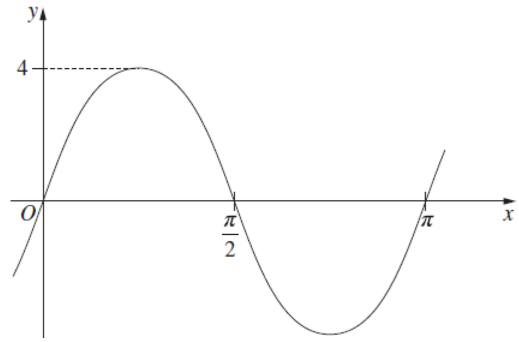
10 **8c** Solution

The graph shown is $y = A\sin bx$.

(i) Write down the value of A. **1**

(ii) Find the value of b. **1**

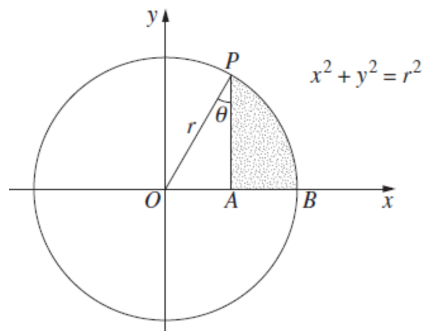
(iii) Copy or trace the graph into your writing booklet. On the same set of axes, draw the graph $y = 3\sin x + 1$, for $0 \le x \le \pi$. **2**



---

10 **10 b** Solution

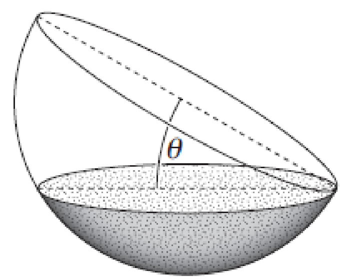
The circle $x^2 + y^2 = r^2$ has radius r and centre O. The circle meets the positive x-axis at B. The point A is on the interval OB. A vertical line through A meets the circle at P. Let $\theta = \angle OPA$.



(i) The shaded region bounded by the arc PB and the intervals AB and AP is rotated about the x-axis. Show that the volume, V, formed is given by **3**

$$V = \frac{\pi r^3}{3}(2 - 3\sin\theta + \sin^3\theta).$$

(ii) A container is in the shape of a hemisphere of radius r metres. The container is initially horizontal and full of water. The container is then tilted at an angle of θ to the horizontal so that some water spills out.



(1) Find θ so that the depth of water remaining is one half of the original depth. **1**

(2) What fraction of the original volume is left in the container? **2**

---

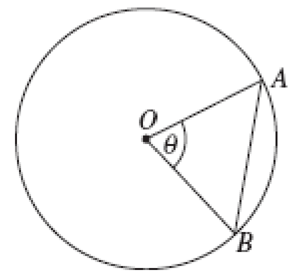
09 **1e** Find the exact value of θ such that $2\cos\theta = 1$, where $0 \le \theta \le \frac{\pi}{2}$. **2** Solution

---

09 **2a** (i) Differentiate with respect to x: $x \sin x$ **2** Solution

---

09 **5c** The diagram shows a circle with centre O and radius 2 centimetres. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$. Solution



(Not to scale)

(i) There are two possible values of θ for which the area of Δ AOB is $\sqrt{3}$ square centimetres. One value is $\frac{\pi}{3}$. Find the other value. **2**

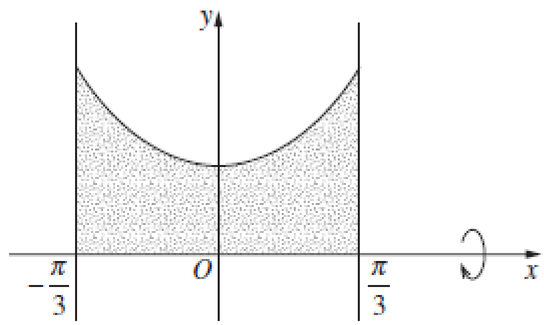
(ii) Suppose that $\theta = \frac{\pi}{3}$.

(1) Find the area of the sector AOB. **1**

(2) Find the exact length of the perimeter of the minor segment bounded by the chord AB and the arc AB. **2**

---

09 **6a** The diagram shows the region bounded by the curve $y = \sec x$, the lines $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$, and the x-axis. Solution



The region is rotated about the x-axis. Find the volume of the solid of revolution formed. **2**

---

09 **7b** Between 5 am and 5 pm on 3 March 2009, the height, h, of the tide in a harbour was given by $h = 1 + 0.7 \sin \frac{\pi}{6} t$ for $0 \leq t \leq 12$, where h is in minutes and t is in hours, with $t = 0$ at 5 am. Solution

(i) What is the period of the function h? **1**

(ii) What was the value of h at low tide, and at what time did low tide occur? **2**

(iii) A ship is able to enter the harbour only if the height of the tide is at least 1.35 m. Find all times between 5 am and 5 pm on 3 March 2009 during which the ship was able to enter the harbour. **3**

---

08 **1a** Evaluate $2 \cos \frac{\pi}{5}$ correct to three significant figures. **2** Solution

---

08 **2a** (iii) Differentiate with respect to x: $\frac{\sin x}{x + 4}$ **2** Solution

---

08 **2c** (ii) Evaluate $\int_0^{\frac{\pi}{12}} \sec^2 3x \, dx$. **3** Solution

---

08 **3b** (i) Differentiate $\log_e(\cos x)$ with respect to x. **2** Solution

(ii) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{4}} \tan x \, dx$. **2**

08 5a The gradient of a curve is given by $\dfrac{dy}{dx} = 1 - 6\sin 3x$. **3** [Solution]

The curve passes through the point (0, 7).
What is the equation of the curve?

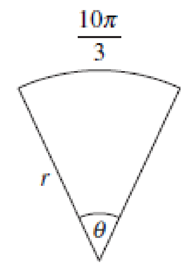
---

08 6a Solve $2\sin^2 \dfrac{x}{3} = 1$ for $-\pi \le x \le \pi$. **3** [Solution]

---

08 7b The diagram shows a sector with radius r and angle θ where $0 \le \theta \le 2\pi$. [Solution]

The arc length is $\dfrac{10\pi}{3}$.



(i) Show that $r \ge \dfrac{5}{3}$. **2**

(ii) Calculate the area of the sector when $r = 4$. **2**

---

07 2a (ii) Differentiate with respect to x: $(1 + \tan x)^{10}$. **2** [Solution]

---

07 2b (i) Find $\int (1 + \cos 3x)\ dx$. **2** [Solution]

---

07 2c The point $P(\pi, 0)$ lies on the curve $y = x \sin x$. Find the equation of the tangent to the curve at P. **3** [Solution]

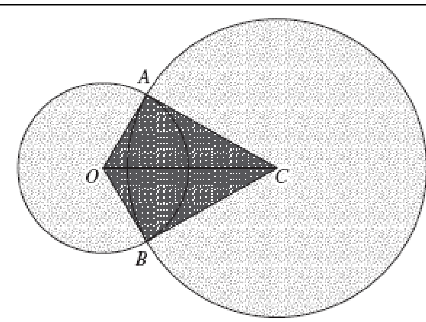
---

07 4a Solve $\sqrt{2}\sin x = 1$ for $0 \le x \le 2\pi$. **2** [Solution]

---

07 4c An advertising logo is formed from two circles, which intersect as shown in the diagram. The circles intersect at A and B and have centres at O and C. [Solution]

The radius of the circle centred at O is 1 metre and the radius of the circle centred at C is $\sqrt{3}$ metres. The length of OC is 2 metres.



Not to scale

(i) Use Pythagoras' theorem to show that $\angle OAC = \dfrac{\pi}{2}$. **1**

(ii) Find $\angle ACO$ and $\angle AOC$. **2**

(iii) Find the area of the quadrilateral AOBC. **1**

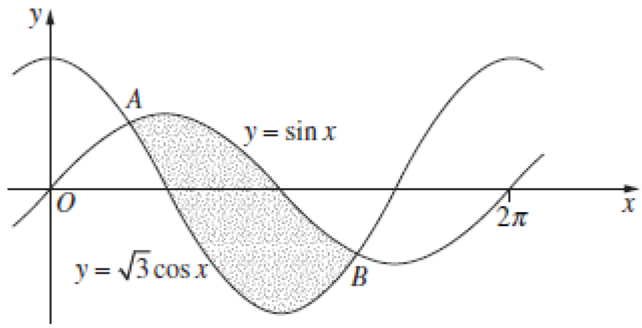
(iv) Find the area of the major sector ACB. **1**

(v) Find the total area of the logo (the sum of all the shaded areas). **2**

07 7b The diagram shows the graphs of $y = \sqrt{3}\cos x$ and $y = \sin x$. The first two points of intersection to the right of the y-axis are labelled A and B.

(i) Solve the equation $\sqrt{3}\cos x = \sin x$ to find the x-coordinates of A and B. **2**

(ii) Find the area of the shaded region in the diagram. **3**



Solution

---

06 2a Differentiate with respect to x:

(i) $x \tan x$ **2**

(ii) $\dfrac{\sin x}{x+1}$. **2**

Solution

---

06 2c Find the equation of the tangent to the curve $y = \cos 2x$ at the point whose x-coordinate is $\frac{\pi}{6}$. **3**

Solution

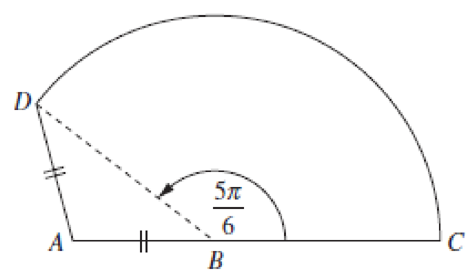
---

06 4a In the diagram, $ABCD$ represents a garden. The sector BCD has centre B and $\angle DBC = \frac{5\pi}{6}$. The points A, B and C lie on a straight line and $AB = AD = 3$ metres. Copy or trace the diagram into your writing booklet.

(i) Show that $\angle DAB = \frac{2\pi}{3}$. **1**

(ii) Find the length of BD. **2**

(iii) Find the area of the garden $ABCD$. **2**



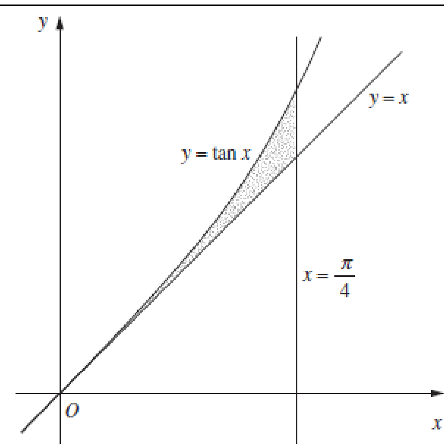
Solution

---

06 5b (i) Show that $\frac{d}{dx}\log_e(\cos x) = -\tan x$. **1**

(ii) The shaded region in the diagram is bounded by the curve $y = \tan x$ and the lines $y = x$ and $x = \frac{\pi}{4}$.

Using the result of part (i), or otherwise, find the area of the shaded region. **3**



Solution

---

06 7b A function $f(x)$ is defined by $f(x) = 1 + 2\cos x$.

(i) Show that the graph of $y = f(x)$ cuts the x-axis at $x = \frac{2\pi}{3}$. **1**

(ii) Sketch the graph of $y = f(x)$ for $-\pi \le x \le \pi$ showing where the graph cuts each of the axes. **3**

(iii) Find the area under the curve $y = f(x)$ between $x = -\frac{\pi}{2}$ and $x = \frac{2\pi}{3}$. **3**

Solution

05	**1c**	Find a primitive of $4 + \sec^2 x$.	**2**	Solution
05	**2a**	Solve $\cos\theta = \dfrac{1}{\sqrt{2}}$ for $0 \le \theta \le 2\pi$.	**2**	Solution
05	**2b**	Differentiate with respect to x: (i) $x \sin x$	**2**	Solution
05	**2c**	(ii) Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \cos 3x \; dx$.	**2**	Solution
05	**4a**	A pendulum is 90 cm long and swings through an angle of 0.6 radians. The extreme positions of the pendulum are indicated by the points A and B in the diagram. (i) Find the length of the arc AB. (ii) Find the straight-line distance between the extreme positions of the pendulum. (iii) Find the area of the sector swept out by the pendulum.	 **1** **2** **2**	Solution