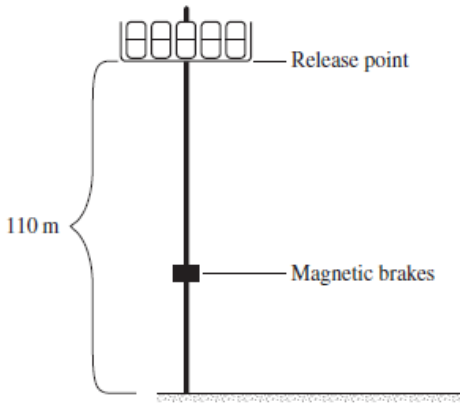
2015 14 a In a theme park ride, a chair is released from a height of 110 metres and falls vertically. Magnetic brakes are applied when the velocity of the chair reaches −37 metres per second. The height of the chair at time t seconds is x metres. The acceleration of the chair is given by $\ddot{x} = -10$. At the release point, $t = 0$, $x = 110$ and $\dot{x} = 0$.

(i) Using calculus, show that $x = -5t^2 + 110$. **2**

(ii) How far has the chair fallen when the magnetic brakes are applied? **2**



Not to scale

(i) $\ddot{x} = -10$

$\dot{x} = -10t + c$

Substitute $\dot{x} = 0$ and $t = 0$:

$0 = -10(0) + c$

$c = 0$

$\therefore \dot{x} = -10t$

$x = -5t^2 + k$

Substitute $x = 110$ and $t = 0$:

$110 = -5(0)^2 + k$

$k = 110$

$\therefore x = -5t^2 + 110$

State Mean: **1.56**

(ii) Substitute $\dot{x} = -37$ in $\dot{x} = -10t$:

$-37 = -10t$

$\therefore t = 3.7$

Substitute $t = 3.7$ in $x = -5t^2 + 110$:

$x = -5(3.7)^2 + 110$

$= 41.55$

$\therefore$ the brakes are applied at a height of 41.55 m.

As 110 − 41.55 = 68.45, the chair has fallen 68.45 metres.

State Mean: **1.05**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(a)(i) This part was generally done well, with most candidates correctly integrating and making the appropriate substitutions to arrive at the required equation for displacement.

Common problems were:

- not showing the required substitutions to find the constants of integration
- integrating with respect to x instead of t
- very poor use of notation, especially $\int$ and dx
- differentiating the given equation to show that acceleration is $\ddot{x} = -10$.

(a)(ii) The vast majority of candidates found the correct time but did not realise that they needed to find the distance that the chair had fallen and so merely found its height at that time.

Common problems were:

- not subtracting the height of the chair from the original height of 110 m to find the distance fallen
- using a velocity of 37 ms^{-1} when the brakes were applied which resulted in a negative value for time.