



projectmaths

HSC Questions by Topic

Mathematics

2016 – 2005

Preliminary Course

[Basic Arithmetic and Algebra](#)

[Real Functions](#)

[Trigonometric Ratios](#)

[Linear Functions and Lines*](#)

[Quadratic Polynomial and Parabola](#)

[Plane Geometry**](#)

[Tangent to Curve and Derivative](#)

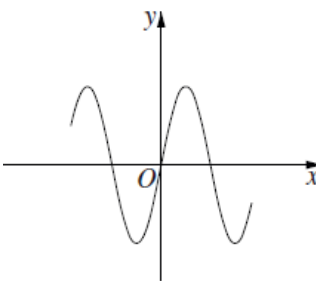
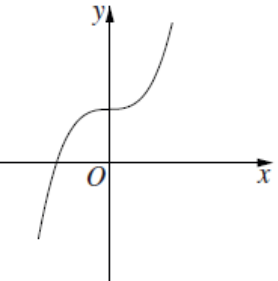
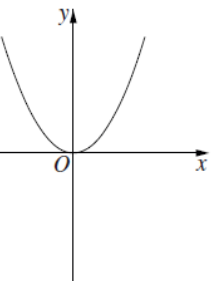
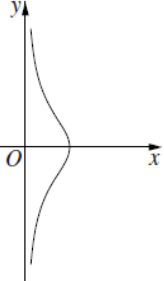
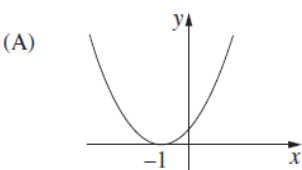
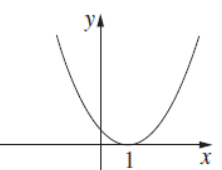
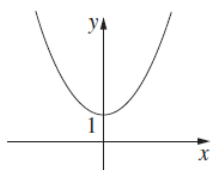
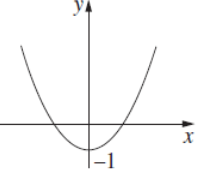
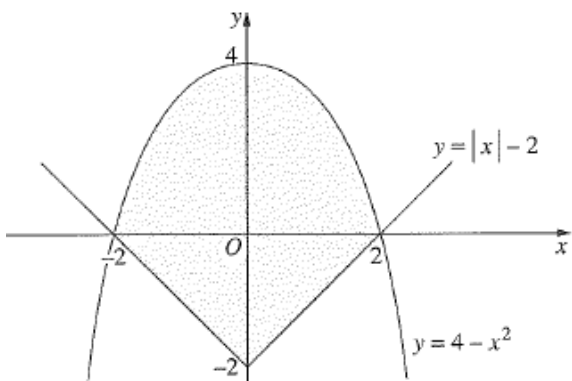
Includes HSC 'Coordinate methods in geometry'

** Includes HSC 'Applications of geometrical properties'

16	11 c	Solve $ x - 2 \leq 3$.	2	Solution
16	11 e	Find the points of intersection of $y = -5 - 4x$ and $y = 3 - 2x - x^2$.	3	Solution
15	1	What is 0.00523359 written in scientific notation, correct to 4 significant figures? (A) 5.2336×10^{-2} (B) 5.234×10^{-2} (C) 5.2336×10^{-3} (D) 5.234×10^{-3}	1	Solution
15	11 a	Simplify $4x - (8 - 6x)$	1	Solution
15	11 b	Factorise fully $3x^2 - 27$	2	Solution
15	11 c	Express $\frac{8}{2 + \sqrt{7}}$ with a rational denominator.	2	Solution
14	1	What is the value of $\frac{\pi^2}{6}$, correct to 3 significant figures? (A) 1.64 (B) 1.65 (C) 1.644 (D) 1.645	1	Solution
14	6	Which expression is a factorisation of $8x^3 + 27$? (A) $(2x - 3)(4x^2 + 12x - 9)$ (B) $(2x + 3)(4x^2 - 12x + 9)$ (C) $(2x - 3)(4x^2 + 6x - 9)$ (D) $(2x + 3)(4x^2 - 6x + 9)$	1	Solution
14	11 a	Rationalise the denominator of $\frac{1}{\sqrt{5} - 2}$.	2	Solution
14	11 b	Factorise $3x^2 + x - 2$.	2	Solution
13	1	What are the solutions of $2x^2 - 5x - 1 = 0$? (A) $x = \frac{-5 \pm \sqrt{17}}{4}$ (B) $x = \frac{5 \pm \sqrt{17}}{4}$ (C) $x = \frac{-5 \pm \sqrt{33}}{4}$ (D) $x = \frac{5 \pm \sqrt{33}}{4}$	1	Solution
12	1	What is 4.097 84 correct to three significant figures? (A) 4.09 (B) 4.10 (C) 4.097 (D) 4.098	1	Solution
12	2	Which of the following is equal to $\frac{1}{2\sqrt{5} - \sqrt{3}}$? (A) $\frac{2\sqrt{5} - \sqrt{3}}{7}$ (B) $\frac{2\sqrt{5} + \sqrt{3}}{7}$ (C) $\frac{2\sqrt{5} - \sqrt{3}}{17}$ (D) $\frac{2\sqrt{5} + \sqrt{3}}{17}$	1	Solution
12	11 a	Factorise $2x^2 - 7x + 3$.	2	Solution
12	11 b	Solve $ 3x - 1 < 2$.	2	Solution
11	1a	Evaluate $\sqrt[3]{\frac{651}{4\pi}}$ correct to four significant figures.	2	Solution
11	1b	Simplify $\frac{n^2 - 25}{n - 5}$.	1	Solution

11	1c	Solve $2^{2x+1} = 32$.	2	Solution
11	1e	Solve $2 - 3x \leq 8$.	2	Solution
11	1f	Rationalise the denominator of $\frac{4}{\sqrt{5} - \sqrt{3}}$. Give your answer in the simplest form.	2	Solution
11	9d	(i) Rationalise the denominator in the expression $\frac{1}{\sqrt{n} + \sqrt{n+1}}$, where n is an integer and $n \geq 1$.	1	Solution
		(ii) Using your result from part (i), or otherwise, find the value of the sum $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{99} + \sqrt{100}}$.	2	
10	1a	Solve $x^2 = 4x$.	2	Solution
10	1b	Find integers a and b such that $\frac{1}{\sqrt{5} - 2} = a + b\sqrt{5}$.	2	Solution
10	1d	Solve $ 2x + 3 = 9$.	2	Solution
09	1b	Solve $\frac{5x-4}{x} = 2$.	2	Solution
09	1c	Solve $ x + 1 = 5$.	2	Solution
08	1b	Factorise $3x^2 + x - 2$.	2	Solution
08	1c	Simplify $\frac{2}{n} - \frac{1}{n+1}$.	2	Solution
08	1d	Solve $ 4x - 3 = 7$.	2	Solution
08	1e	Expand and simplify $(\sqrt{3} - 1)(2\sqrt{3} + 5)$.	2	Solution
07	1a	Evaluate $\sqrt{\pi^2 + 5}$ correct to two decimal places.	2	Solution
07	1b	Solve $2x - 5 > -3$ and graph the solution on a number line.	2	Solution
07	1c	Rationalise the denominator of $\frac{1}{\sqrt{3} - 1}$.	2	Solution
07	1e	Factorise $2x^2 + 5x - 12$.	2	Solution
06	1b	Factorise $2x^2 + 5x - 3$.	2	Solution
06	1e	Solve $3 - 5x \leq 2$.	2	Solution

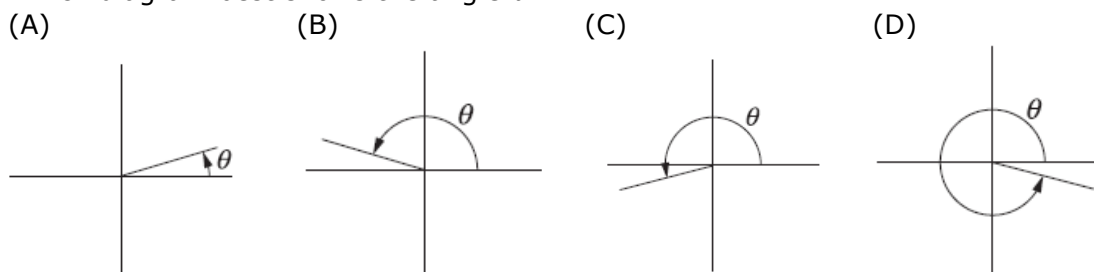
05	1a	Evaluate $\sqrt{\frac{275.4}{5.2 \times 3.9}}$ correct to two significant figures.	2	Solution
<hr/>				
05	1b	Factorise $x^3 - 27$.	2	Solution
<hr/>				
05	1d	Express $\frac{(2x-3)}{2} - \frac{(x-1)}{5}$ as a single fraction in its simplest form.	2	Solution
<hr/>				
05	1e	Find the values of x for which $ x - 3 \leq 1$.	2	Solution

16	4	Which diagram best shows the graph of an odd function? (A)  (B)  (C)  (D) 	1	Solution
16	11	Sketch the graph of $(x - 3)^2 + (y + 2)^2 = 4$.	2	Solution
15	13	(i) Find the domain and range for the function $f(x) = \sqrt{9 - x^2}$.	2	Solution
	b	(ii) On a number plane, shade the region where the points (x, y) satisfy both of the inequalities $y \leq \sqrt{9 - x^2}$ and $y \geq x$.	2	
14	2	Which graph best represents $y = (x - 1)^2$?	1	Solution
		(A)  (B)  (C)  (D) 		
13	3	Which inequality defines the domain of the function $f(x) = \frac{1}{\sqrt{x + 3}}$?	1	Solution
		(A) $x > -3$ (B) $x \geq -3$ (C) $x < -3$ (D) $x \leq -3$		
13	11	Sketch the region defined by $(x - 2)^2 + (y - 3)^2 \geq 4$.	3	Solution
11	4e	The diagram shows the graphs of $y = x - 2$ and $y = 4 - x^2$. Write down the inequalities that together describe the shaded region. Not to scale	2	Solution
				
10	1c	Write down the equation of the circle with centre $(-1, 2)$ and radius 5.	1	Solution
10	1g	Let $f(x) = \sqrt{x - 8}$. What is the domain of $f(x)$?	1	Solution
09	3c	Shade the region in the plane defined by $y \geq 0$ and $y \leq 4 - x^2$.	2	Solution
06	1c	Sketch the graph of $y = x + 4 $.	2	Solution

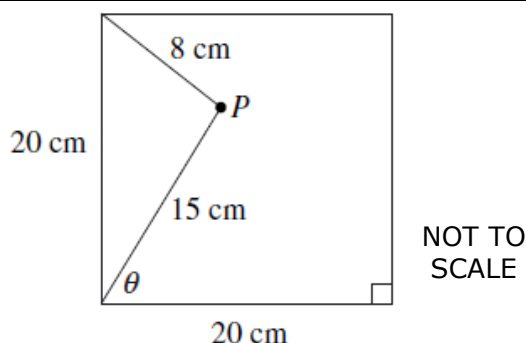
- 16 1** For the angle θ , $\sin \theta = \frac{7}{25}$ and $\cos \theta = -\frac{24}{25}$.

1 [Solution](#)

Which diagram best shows the angle θ ?



- 16 12 c** Square tiles of side length 20 cm are being used to tile a bathroom. The tiler needs to drill a hole in one of the tiles at a point P which is 8 cm from one corner and 15 cm from an adjacent corner. To locate the point P the tiler needs to know the size of the angle θ shown in the diagram. Find the size of the angle θ to the nearest degree.

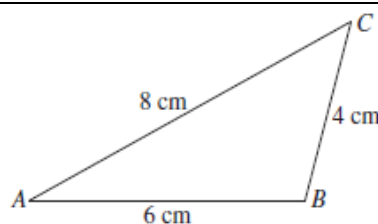


3 [Solution](#)

- 15 13 a** The diagram shows $\triangle ABC$ with sides $AB = 6$ cm, $BC = 4$ cm and $AC = 8$ cm.

[Solution](#)

- (i) Show that $\cos A = \frac{7}{8}$.
- (ii) By finding the exact value of $\sin A$, determine the exact value of the area of $\triangle ABC$.

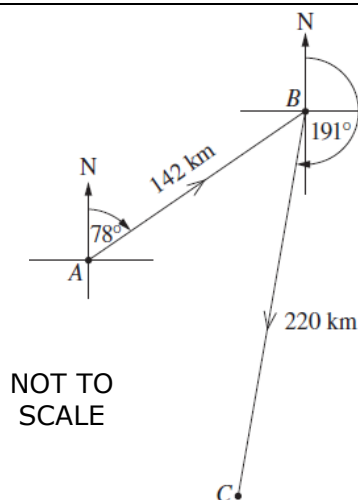


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Not to scale

- 14 13 d** Chris leaves island A in a boat and sails 142 km on a bearing of 078° to island B . Chris then sails on a bearing of 191° for 220 km to island C , as shown in the diagram.
- (i) Show that the distance from island C to island A is approximately 210 km.
- (ii) Chris wants to sail from island C directly to island A . On what bearing should Chris sail? Give your answer correct to the nearest degree.

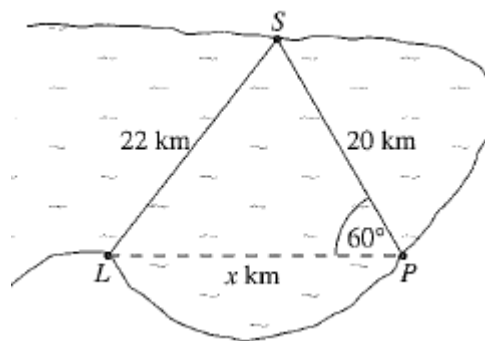
[Solution](#)



2
3

- 11 8a** In the diagram, the shop at S is 20 kilometres across the bay from the post office at P . The distance from the shop to the lighthouse at L is 22 kilometres and $\angle SPL$ is 60° . Let the distance PL be x kilometres.

- (i) Use the cosine rule to show that $x^2 - 20x - 84 = 0$.
(ii) Hence, find the distance from the post office to the lighthouse. Give your answer correct to the nearest kilometre.

[Solution](#)**1****2**

- 06 1d** Find the value of θ in the diagram. Give your answer to the nearest degree.

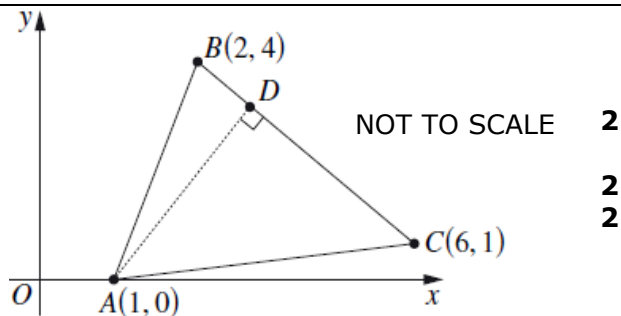
**2**[Solution](#)

- 05 3b** The lengths of the sides of a triangle are 7 cm, 8 cm and 13 cm.

- (i) Find the size of the angle opposite the longest side.
(ii) Find the area of the triangle.

2**1**[Solution](#)

- 16 12** The diagram shows points $A(1, 0)$, $B(2, 4)$ and $C(6, 1)$. The point D lies on BC such that $AD \perp BC$.
- a**
- Show that the equation of BC is $3x + 4y - 22 = 0$.
 - Find the length of AD .
 - Hence, or otherwise, find the area of $\triangle ABC$.



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2

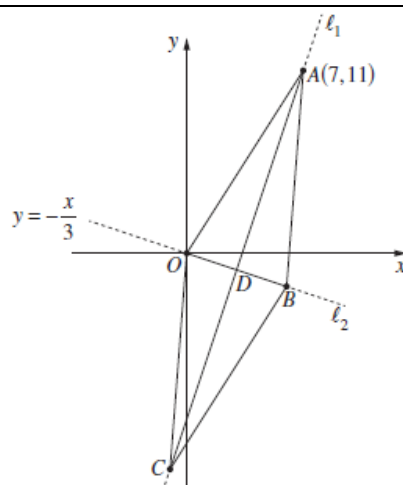
- 15 2** What is the slope of the line with equation $2x - 4y + 3 = 0$?

(A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 2

1 [Solution](#)

- 15 12** The diagram shows the rhombus $OABC$. The diagonal from the point $A(7, 11)$ to the point C lies on the line ℓ_1 . The other diagonal, from the origin O to the point B , lies on the line ℓ_2 which has equation $y = -\frac{x}{3}$.

- Show that the equation of the line ℓ_1 is $y = 3x - 10$.
- The lines ℓ_1 and ℓ_2 intersect at the point D . Find the coordinates of D .



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Not to scale

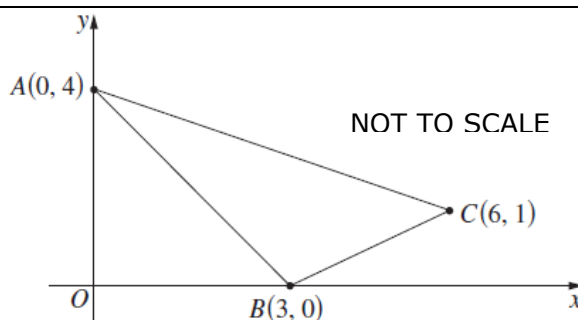
- 14 5** Which equation represents the line perpendicular to $2x - 3y = 8$, passing through the point $(2, 0)$?

(A) $3x + 2y = 4$ (B) $3x + 2y = 6$ (C) $3x - 2y = -4$ (D) $3x - 2y = 6$

1 [Solution](#)

- 14 12** The points $A(0, 4)$, $B(3, 0)$ and $C(6, 1)$ form a triangle, as shown in the diagram.

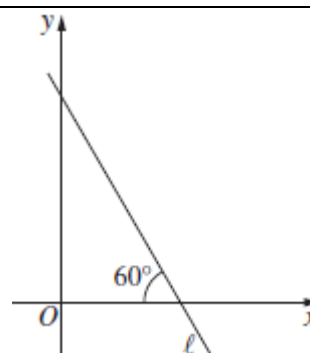
- Show that the equation of AC is $x + 2y - 9 = 0$.
- Find the perpendicular distance from B to AC .
- Hence, or otherwise, find the area of $\triangle ABC$.



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- 13 2** The diagram shows the line ℓ . What is the slope of the line ℓ ?

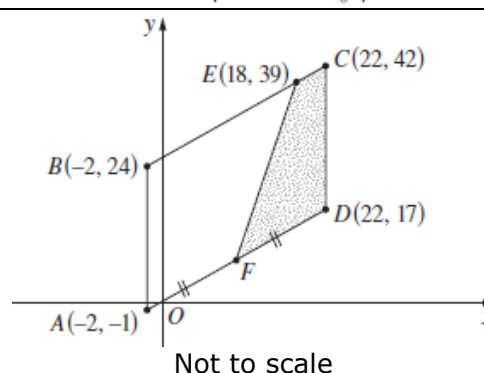
- (A) $\sqrt{3}$
 (B) $-\sqrt{3}$
 (C) $\frac{1}{\sqrt{3}}$
 (D) $-\frac{1}{\sqrt{3}}$



1 [Solution](#)

- 13 12 b** The points $A(-2, -1)$, $B(-2, 24)$, $C(22, 42)$ and $D(22, 17)$ form a parallelogram as shown. The point $E(18, 39)$ lies on BC . The point F is the midpoint of AD .

- (i) Show that the equation of the line through A and D is $3x - 4y + 2 = 0$.
 (ii) Show that the perpendicular distance from B to the line through A and D is 20 units.
 (iii) Find the length of EC .
 (iv) Find the area of the trapezium $EFDC$.



[Solution](#)

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- 13 15 c** (i) Sketch the graph $y = |2x - 3|$.
 (ii) Using the graph from part (i), or otherwise, find all values of m for which the equation $|2x - 3| = mx + 1$ has exactly one solution.

1 [Solution](#)

2

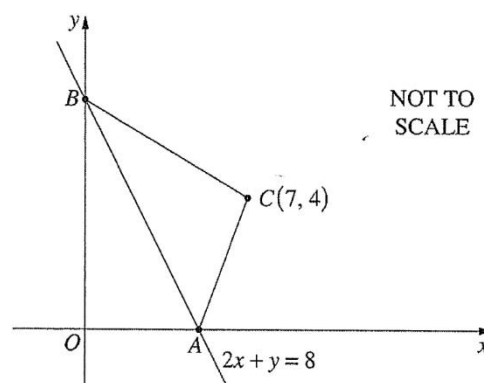
- 12 5** What is the perpendicular distance of the point $(2, -1)$ from the line $y = 3x + 1$?

- (A) $\frac{6}{\sqrt{10}}$ (B) $\frac{6}{\sqrt{5}}$ (C) $\frac{8}{\sqrt{10}}$ (D) $\frac{8}{\sqrt{5}}$

1 [Solution](#)

- 12 13 a** The diagram shows a triangle ABC . The line $2x + y = 8$ meets the x and y axes at the points A and B respectively. The point C has coordinates $(7, 4)$.

- (i) Calculate the distance AB .
 (ii) It is known that $AC = 5$ and $BC = \sqrt{65}$. (Do NOT prove this.) Calculate the size of $\angle ABC$ to the nearest degree.
 (iii) The point N lies on AB such that CN is perpendicular to AB . Find the coordinates of N .



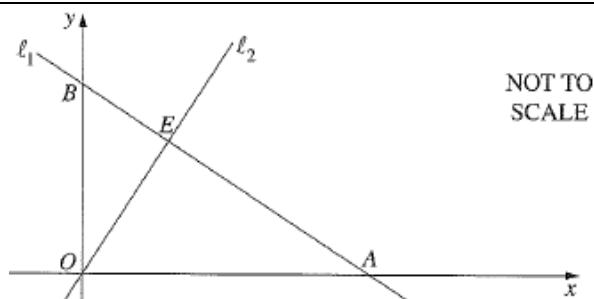
[Solution](#)

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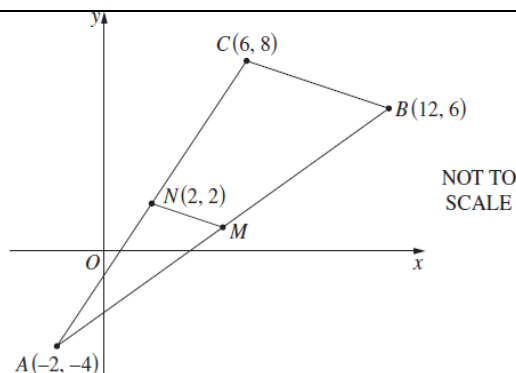
3

- 11 3c** The diagram shows a line ℓ_1 , with equation $3x + 4y - 12 = 0$, which intersects the y -axis at B . A second line ℓ_2 , with equation $4x - 3y = 0$, passes through the origin O and intersects ℓ_1 at E .

[Solution](#)

- (i) Show that the co-ordinates of B are $(0, 3)$ **1**
- (ii) Show that ℓ_1 is perpendicular to ℓ_2 . **2**
- (iii) Show that the perpendicular distance from O to ℓ_1 is $\frac{12}{5}$. **1**
- (iv) Using Pythagoras' theorem, or otherwise, find the length of the interval BE . **1**
- (v) Hence, or otherwise, find the area of $\triangle BOE$. **1**

- 10 3a** In the diagram, A , B and C are the points $(-2, -4)$, $(12, 6)$ and $(6, 8)$ respectively. The point $N(2, 2)$ is the midpoint of AC . The point M is the midpoint of AB .

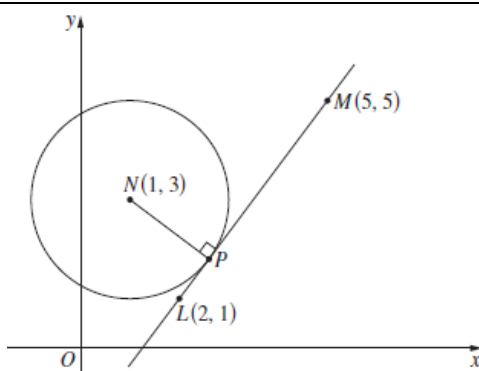
[Solution](#)

- (i) Find the coordinates of M . **1**
- (ii) Find the gradient of BC . **1**
- (iii) Prove that $\triangle ABC$ is similar to $\triangle AMN$. **2**
- (iv) Find the equation of MN . **1**
- (v) Find the exact length of BC . **1**
- (vi) Given that the area of $\triangle ABC$ is 44 square units, find the perpendicular distance from A to BC . **1**

- 09 1a** Sketch the graph of $y - 2x = 3$, showing the intercepts on both axes. **2**

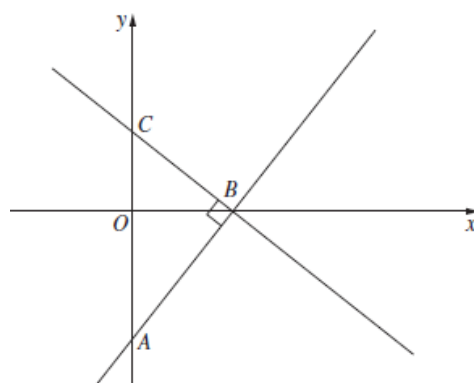
[Solution](#)

- 09 3b** The circle in the diagram has centre N . The line LM is tangent to the circle at P .

[Solution](#)

- (i) Find the equation of LM in the form $ax + by + c = 0$. **2**
- (ii) Find the distance NP . **2**
- (iii) Find the equation of the circle. **1**

- 09 5a** In the diagram, the points A and C lie on the y -axis and the point B lies on the x -axis. The line AB has equation $y = \sqrt{3}x - 3$. The line BC is perpendicular to AB .

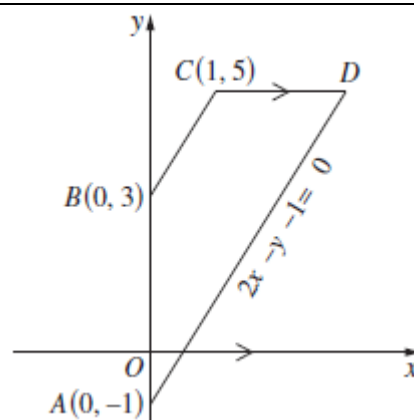


- (i) Find the equation of the line BC . 2
- (ii) Find the area of the triangle ABC . 2

- 08 2b** Let M be the midpoint of $(-1, 4)$ and $(5, 8)$.
Find the equation of the line through M with gradient $-\frac{1}{2}$. 2

[Solution](#)

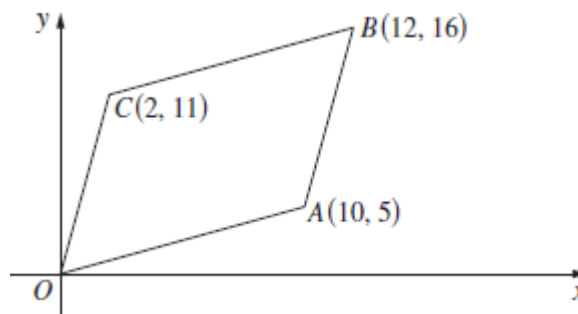
- 08 3a** In the diagram $ABCD$ is a quadrilateral. The equation of the line AD is $2x - y - 1 = 0$.
- (i) Show that $ABCD$ is a trapezium by showing BC is parallel to AD . 2
- (ii) The line CD is parallel to the x -axis. Find the co-ordinates of D . 1
- (iii) Find the length of BC . 1
- (iv) Show that the perpendicular distance from B to AD is $\frac{4}{\sqrt{5}}$. 2
- (v) Hence, or otherwise, find the area of the trapezium $ABCD$. 2

[Solution](#)

- 07 1f** Find the equation of the line that passes through the point $(-1, 3)$ and is perpendicular to $2x + y + 4 = 0$. 2

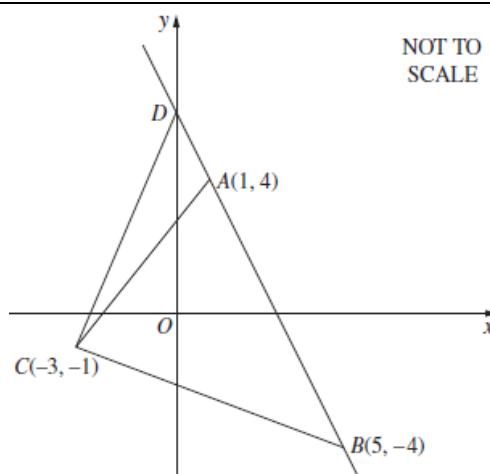
[Solution](#)

- 07 3a** In the diagram, A , B and C are the points $(10, 5)$, $(12, 16)$ and $(2, 11)$ respectively. Copy or trace this diagram into your writing booklet.
- (i) Find the distance AC . 1
- (ii) Find the midpoint of AC . 1
- (iii) Show that $OB \perp AC$. 2
- (iv) Find the midpoint of OB and hence explain why $OABC$ is a rhombus. 2
- (v) Hence, or otherwise, find the area of $OABC$. 1

[Solution](#)

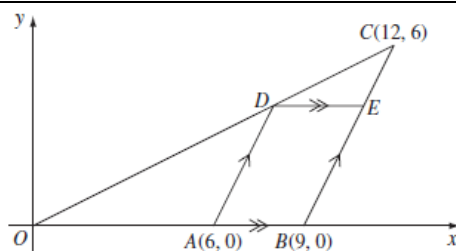
- 06 3a** In the diagram, A , B and C are the points $(1, 4)$, $(5, -4)$ and $(-3, -1)$ respectively. The line AB meets the y -axis at D .

- Show that the equation of the line AB is $2x + y - 6 = 0$.
- Find the coordinates of the point D .
- Find the perpendicular distance of the point C from the line AB .
- Hence, or otherwise, find the area of the triangle ADC .

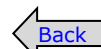
[Solution](#)**2****1****1****2**

- 05 3c** In the diagram, A , B and C are the points $(6, 0)$, $(9, 0)$ and $(12, 6)$ respectively. The equation of the line OC is $x - 2y = 0$. The point D on OC is chosen so that AD is parallel to BC . The point E on BC is chosen so that DE is parallel to the x -axis.

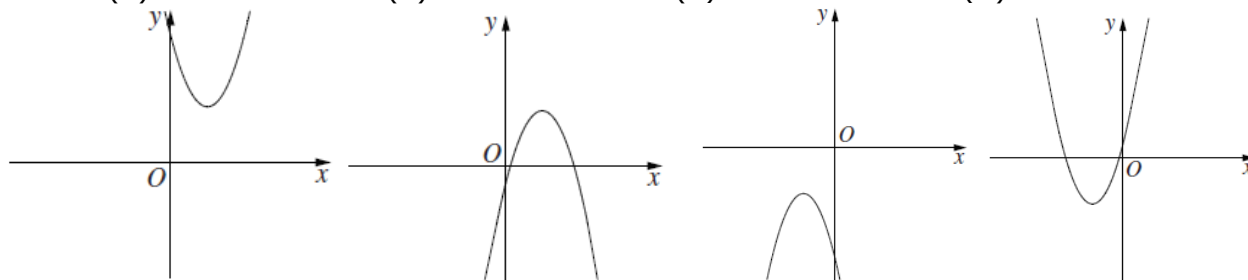
- Show that the equation of the line AD is $y = 2x - 12$.
- Find the coordinates of the point D .
- Find the coordinates of the point E .
- Prove that $\triangle OAD \parallel \triangle DEC$.
- Hence, or otherwise, find the ratio of the lengths AD and EC .

[Solution](#)**2****2****1****2****1**

The Quadratic Polynomial and the Parabola



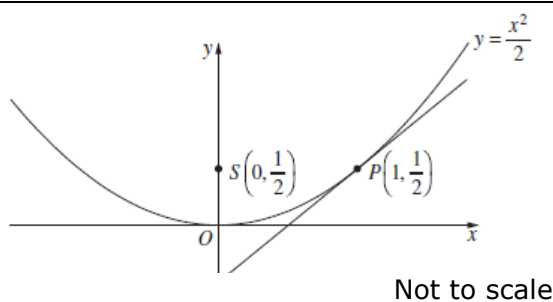
- 16 3** Which diagram best shows the graph of the parabola $y = 3 - (x - 2)^2$? **1** [Solution](#)
- (A) (B) (C) (D)



- 16 13** Consider the parabola $x^2 - 4x = 12y + 8$. **2** [Solution](#)
- b** (i) By completing the square, or otherwise, find the focal length of the parabola. **2**
- (ii) Find the coordinates of the focus. **1**

- 15 12** For what values of k does the quadratic equation $x^2 - 8x + k = 0$ have real roots? **2** [Solution](#)
- d**

- 15 12** The diagram shows the parabola **1** [Solution](#)
- e** $y = \frac{x^2}{2}$ with focus $S(0, \frac{1}{2})$. A tangent
- to the parabola is drawn at $P(1, \frac{1}{2})$.
- (i) Find the equation of the tangent at the point P . **2**
- (ii) What is the equation of the directrix of the parabola? **1**
- (iii) The tangent and directrix intersect at Q . Show that Q lies on the y -axis. **1**
- (iv) Show that $\triangle PQS$ is isosceles. **1**



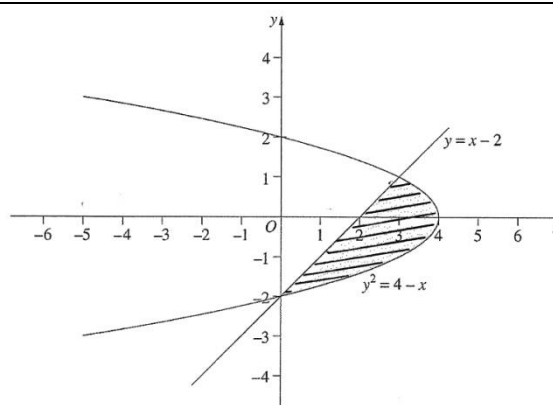
- 14 14** The roots of the quadratic equation $2x^2 + 8x + k = 0$ are α and β . **2** [Solution](#)
- b** (i) Find the value of $\alpha + \beta$. **1**
- (ii) Given that $\alpha^2\beta + \alpha\beta^2 = 6$, find the value of k . **2**

- 13 7** A parabola has focus $(5, 0)$ and directrix $x = 1$. **1** [Solution](#)
- What is the equation of the parabola?
- (A) $y^2 = 16(x - 5)$ (B) $y^2 = 8(x - 3)$ (C) $y^2 = -16(x - 5)$ (D) $y^2 = -8(x - 3)$

- 12 3** The quadratic equation $x^2 + 3x - 1 = 0$ has roots α and β . **1** [Solution](#)
- What is the value of $\alpha\beta + (\alpha + \beta)$?
- (A) 4 (B) 2 (C) -4 (D) -2

- 12 8** The diagram shows the region enclosed by $y = x - 2$ and $y^2 = 4 - x$. Which of the following pairs of inequalities describes the shaded region in the diagram?

- (A) $y^2 \leq 4 - x$ and $y \leq x - 2$
 (B) $y^2 \leq 4 - x$ and $y \geq x - 2$
 (C) $y^2 \geq 4 - x$ and $y \leq x - 2$
 (D) $y^2 \geq 4 - x$ and $y \geq x - 2$

**1** [Solution](#)

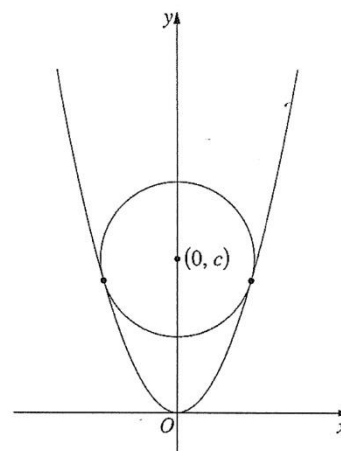
- 12 11** Find the coordinates of the focus of the parabola $x^2 = 16(y - 2)$.

2 [Solution](#)**e**

- 12 16** The circle $x^2 + (y - c)^2 = r^2$, where $c > 0$ and $r > 0$, lies inside the parabola $y = x^2$. The circle touches the parabola at exactly two points located symmetrically on opposite sides of the y -axis, as shown in the diagram.

[Solution](#)

- (i) Show that $4c = 1 + 4r^2$.
 (ii) Deduce that $c > \frac{1}{2}$.

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- 11 2a** The quadratic equation $x^2 - 6x + 2 = 0$ has roots α and β .

[Solution](#)

- (i) Find $\alpha + \beta$.
 (ii) Find $\alpha\beta$.
 (iii) Find $\frac{1}{\alpha} + \frac{1}{\beta}$.

1**1****1**

- 11 3b** A parabola has focus $(3, 2)$ and directrix $y = -4$. Find the coordinates of the vertex.

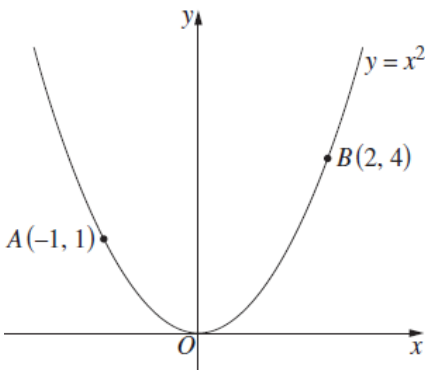
2 [Solution](#)

- 11 6b** A point $P(x, y)$ moves so that the sum of the square of its distance from each of the points $A(-1, 0)$ and $B(3, 0)$ is equal to 40. Show that the locus of $P(x, y)$ is a circle, and state its radius and centre.

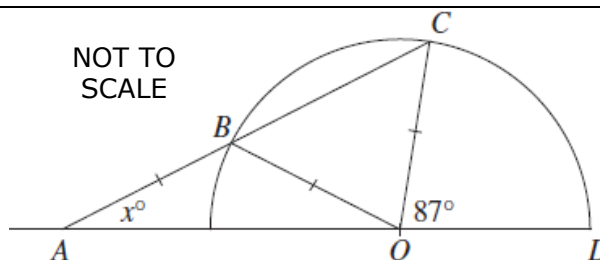
3 [Solution](#)

- 10 2b** Solve the inequality $x^2 - x - 12 < 0$.

2 [Solution](#)

10	7b	The parabola shown in the diagram is the graph $y = x^2$. The points $A(-1, 1)$ and $B(2, 4)$ are on the parabola. (i) Find the equation of the tangent to the parabola at A . (ii) Let M be the midpoint of AB . There is a point C on the parabola such that the tangent at C is parallel to AB . Show that the line MC is vertical. (iii) The tangent at A meets the line MC at T . Show that the line BT is a tangent to the parabola.		Solution
			2	
			2	
			2	
09	4b	Find the values of k for which the quadratic equation $x^2 - (k + 4)x + (k + 7) = 0$ has equal roots.	3	Solution
08	4c	Consider the parabola $x^2 = 8(y - 3)$. (i) Write down the coordinates of the vertex. (i) Find the coordinates of the focus. (iii) Sketch the parabola.	1 1 1	Solution
07	7a	(i) Find the coordinates of the focus, S , of the parabola $y = x^2 + 4$. (ii) The graphs of $y = x^2 + 4$ and the line $y = x + k$ have only one point of intersection, P . Show that the x -coordinate of P satisfies $x^2 - x + 4 - k = 0$. (iii) Using the discriminant, or otherwise, find the value of k . (iv) Find the coordinates of P . (v) Show that SP is parallel to the directrix of the parabola.	2 1 1 2 1	Solution
06	7a	Let α and β be the solutions of $x^2 - 3x + 1 = 0$. (i) Find $\alpha\beta$. (ii) Hence find $\alpha + \frac{1}{\alpha}$.	1 1	Solution
06	7c	(i) Write down the discriminant of $2x^2 + (k - 2)x + 8$, where k is a constant. (ii) Hence, or otherwise, find the values of k for which the parabola $y = 2x^2 + kx + 9$ does not intersect the line $y = 2x + 1$.	1 2	Solution
06	9a	Find the coordinates of the focus of the parabola $12y = x^2 - 6x - 3$.	2	Solution
05	1f	Find the coordinates of the focus of the parabola $x^2 = 8(y - 1)$.	2	Solution

- 16 12** The diagram shows a semicircle with centre O . It is given that $AB = OB$, $\angle COD = 87^\circ$ and $\angle BAO = x^\circ$.
- b**
- (i) Show that $\angle CBO = 2x^\circ$, giving reasons.
- (ii) Find the value of x , giving reasons.



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- 16 15** Maryam wishes to estimate the height, h metres, of a tower, ST , using a square, $ABCD$, with side length 1 metre.

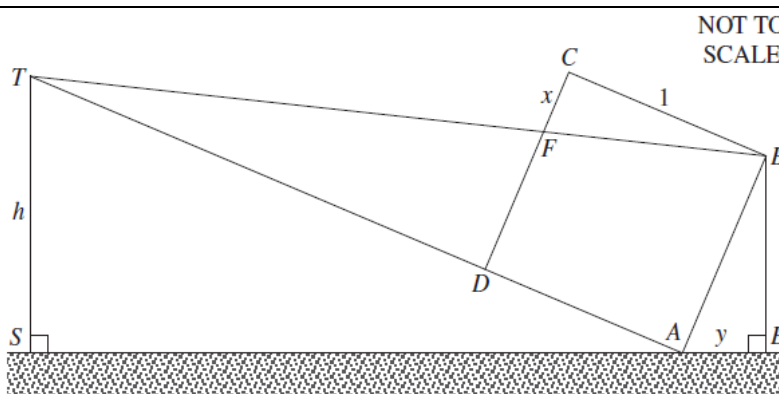
She places the point A on the horizontal ground and ensures that the point D lies on the line joining A to the top of the tower T .

The point F is the intersection of the line joining B and T and the side BC . The point E is the foot of the perpendicular from B to the ground. Let CF have length x metres and AE have length y metres.

Copy and trace the diagram into your writing booklet.

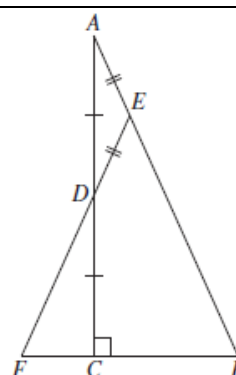
- (i) Show that $\triangle FCB$ and $\triangle BAT$ are similar.
- (ii) Show that $\triangle TSA$ and $\triangle AEB$ are similar.
- (iii) Find h in terms of x and y .

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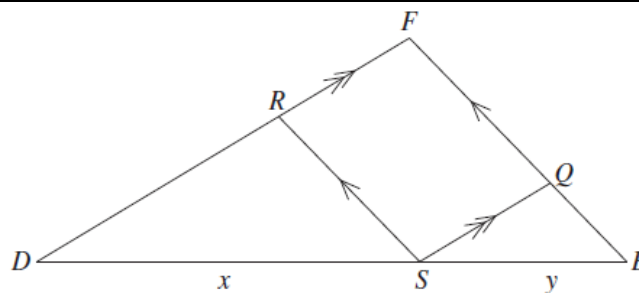
- 15 15** The diagram shows $\triangle ABC$ which has a right angle at C . The point D is the midpoint of the side AC . The point E is chosen on AB such that $AE = ED$. The line segment ED is produced to meet the line BC at F . Copy or trace the diagram into your writing booklet.

- (i) Prove that $\triangle ACB$ is similar to $\triangle DCF$.
- (ii) Explain why $\triangle EFB$ is isosceles.
- (iii) Show that $EB = 3AE$.



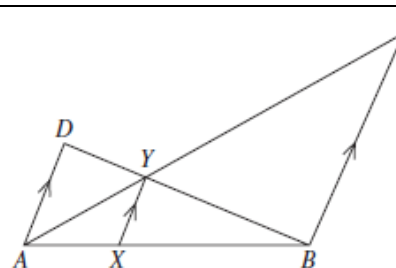
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- 14 15** In $\triangle DEF$, a point S is chosen on the side DE . The length of DS is x , and the length of ES is y . The line through S parallel to DF meets EF at Q . The line through S parallel to EF meets DF at R . The area of $\triangle DEF$ is A . The areas of $\triangle DSR$ and $\triangle SEQ$ are A_1 and A_2 respectively.


[Solution](#)

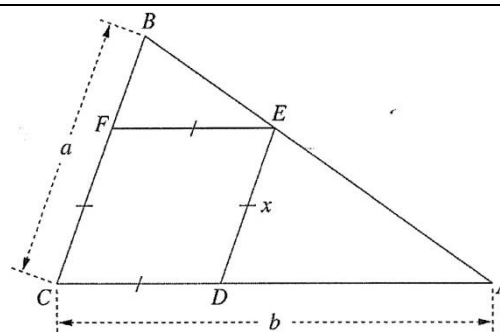
- (i) Show that $\triangle DEF$ is similar to $\triangle DSR$. 2
- (ii) Explain why $\frac{DR}{DF} = \frac{x}{x+y}$. 1
- (iii) Show that $\sqrt{\frac{A_1}{A}} = \frac{x}{x+y}$. 2
- (iv) Using the result from part (iii) and a similar expression for $\sqrt{\frac{A_2}{A}}$, deduce that 2
- $$\sqrt{A} = \sqrt{A_1} + \sqrt{A_2}.$$

- 13 16** The diagram shows triangles ABC and ABD with AD parallel to BC . The sides AC and BD intersect at Y . The point X lies on AB such that XY is parallel to AD and BC .


[Solution](#)

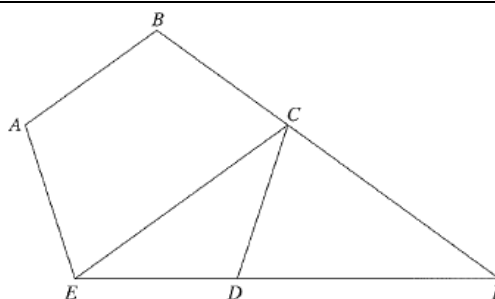
- (i) Prove that $\triangle ABC$ is similar to $\triangle AXY$. 2
- (ii) Hence, or otherwise, prove that 2
- $$\frac{1}{XY} = \frac{1}{AD} + \frac{1}{BC}.$$

- 12 16** The diagram shows a triangle ABC with sides $BC = a$ and $AC = b$. The points D , E and F lie on the sides AC , AB and BC , respectively, so that $CDEF$ is a rhombus with sides of length x .


[Solution](#)

- (i) Prove that $\triangle EBF$ is similar to $\triangle AED$. 2
- (ii) Find an expression for x in terms of a and b . 2

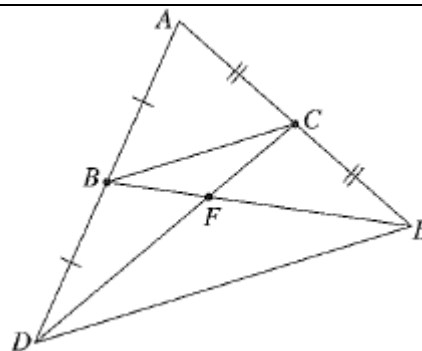
- 11 6a** The diagram shows a regular pentagon $ABCDE$. Sides ED and BC are produced to meet at P . Copy or trace the diagram into your writing booklet.


[Solution](#)

- (i) Find the size of $\angle CDE$. 1
- (ii) Hence, show that $\triangle EPC$ is isosceles. 2

- 11 9a** The diagram shows $\triangle ADE$, where B is the midpoint of AD and C is the midpoint of AE . The intervals BE and CD meet at F .

- (i) Explain why $\triangle ABC$ is similar to $\triangle ADE$.
(ii) Hence, or otherwise, prove that the ratio $BF:FE = 1:2$.

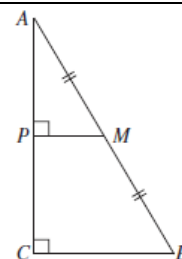

[Solution](#)

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- 09 4c** In the diagram, $\triangle ABC$ is a right-angled triangle, with the right angle at C .

The midpoint of AB is M , and $MP \perp AC$.

- (i) Prove that $\triangle AMP$ is similar to $\triangle ABC$.
(ii) What is the ratio of AP to AC ?
(iii) Prove that $\triangle AMC$ is isosceles.
(iv) Show that $\triangle ABC$ can be divided into two isosceles triangles.
(v) Copy or trace this triangle into your writing booklet and show how to divide it into four isosceles triangles.


[Solution](#)

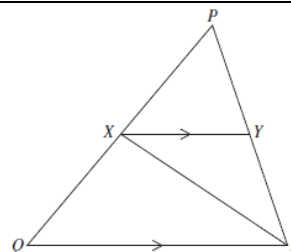
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- 08 4a** In the diagram, XR bisects $\angle PRQ$ and $XY \parallel QR$.

Copy or trace the diagram into your writing booklet.

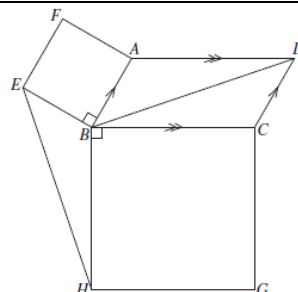
Prove that $\triangle XYR$ is an isosceles triangle.



2 [Solution](#)

- 08 8b** In the diagram, $ABCD$ is a parallelogram and $ABEF$ and $BCGH$ are both squares. Copy or trace the diagram into your writing booklet.

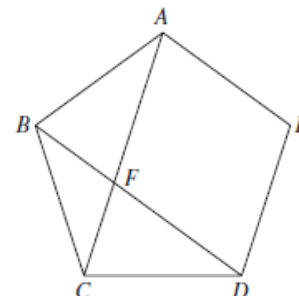
- (i) Prove that $CD = BE$.
(ii) Prove that $BD = EH$.


[Solution](#)

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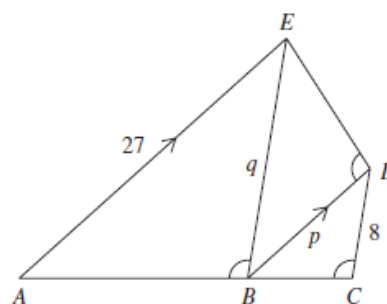
- 07 5a** In the diagram, $ABCDE$ is a regular pentagon. The diagonals AC and BD intersect at F . Copy or trace this diagram into your writing booklet.

- (i) Show that the size of $\angle ABC$ is 108° .
(ii) Find the size of $\angle BAC$.
Give reasons for your answer.
(iii) By considering the sizes of angles, show that $\triangle ABF$ is isosceles.


[Solution](#)

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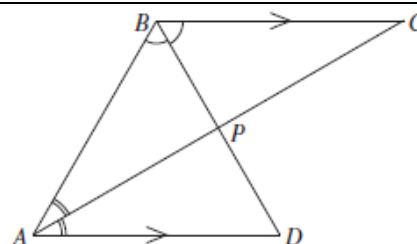
- 07 8b** In the diagram, AE is parallel to BD , $AE = 27$, $CD = 8$, $BD = p$, $BE = q$ and $\angle ABE$, $\angle BCD$ and $\angle BDE$ are equal. Copy or trace this diagram into your writing booklet.
- (i) Prove that $\triangle ABE \parallel \triangle BCD$.
- (ii) Prove that $\triangle EDB \parallel \triangle BCD$.



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[Solution](#)

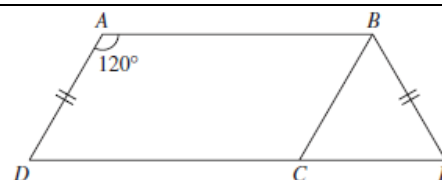
- 06 6a** In the diagram, AD is parallel to BC , AC bisects $\angle BAD$ and BD bisects $\angle ABC$. The lines AC and BD intersect at P . Copy or trace the diagram into your writing booklet.
- (i) Prove that $\angle BAC = \angle BCA$.
- (ii) Prove that $\triangle ABP \cong \triangle CBP$.
- (iii) Prove that $ABCD$ is a rhombus.



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[Solution](#)

- 05 5b** The diagram shows a parallelogram $ABCD$ with $\angle DAB = 120^\circ$. The side DC is produced to E so that $AD = BE$. Copy or trace the diagram into your writing booklet.
- Prove that $\triangle BCE$ is equilateral.

**3**[Solution](#)

The Tangent to a Curve and the Derivative of a Function



16	11 b	Differentiate $\frac{x+2}{3x-4}$.	2	Solution
15	12 c	Find $f'(x)$, where $f(x) = \frac{x^2+3}{x-1}$.	2	Solution
14	11 c	Differentiate $\frac{x^3}{x+1}$.	2	Solution
13	11 b	Evaluate $\lim_{x \rightarrow 2} \frac{x^3-8}{x^2-4}$.	2	Solution
12	11 c	Find the equation of the tangent to the curve $y = x^2$ at the point where $x = 3$.	2	Solution
11	2c	Find the equation of the tangent to the curve $y = (2x+1)^4$ at the point where $x = -1$.	3	Solution
10	8d	Let $f(x) = x^3 - 3x^2 + kx + 8$, where k is a constant. Find the values of k for which $f(x)$ is an increasing function.	2	Solution
09	1d	Find the gradient of the tangent to the curve $y = x^4 - 3x$ at the point $(1, -2)$.	2	Solution
09	6c	<p>The diagram illustrates the design for part of a roller-coaster track. The section RO is a straight line with slope 1.2 and the section PQ is a straight line with slope -1.8. The section OP is a parabola $y = ax^2 + bx$. The horizontal distance from the y-axis to P is 30 m. In order that the ride is smooth, the straight sections must be tangent to the parabola at O and at P.</p> <p>(i) Find the values of a and b so that the ride is smooth.</p> <p>(ii) Find the distance d, from the vertex of the parabola to the horizontal line through P, as shown on the diagram.</p>	3 2	Solution
09	8a	<p>The diagram shows the graph of a function $y = f(x)$.</p> <p>(i) For which values of x is the derivative, $f'(x)$, negative?</p> <p>(ii) What happens to $f'(x)$ for large values of x?</p> <p>(iii) Sketch the graph of $y = f'(x)$.</p>	1 1 2	Solution
08	2a	Differentiate with respect to x : (i) $(x^2 + 3)^9$	2	Solution
05	2b	Differentiate with respect to x : (ii) $\frac{x^2}{x-1}$	2	Solution

