2016 16 a A particle moves in a straight line.

Its velocity v ms^{-1} at time t seconds is given by $v = 2 - \frac{4}{t+1}$.

(i) Find the initial velocity. **1**

(ii) Find the acceleration of the particle when the particle is stationary. **2**

(iii) By considering the behavior of v for large t, sketch a graph of v against t for $t \geq 0$, showing any intercepts. **2**

(iv) Find the exact distance travelled by the particle in the first 7 seconds. **3**

(i) Let $t = 0$:

$\therefore v(0) = 2 - \frac{4}{1}$

$= -2$

$\therefore$ the initial velocity is -2 ms^{-1}.

State Mean: **0.93**

(ii) Let $v = 0$:

$\therefore 2 - \frac{4}{t+1} = 0$

$\frac{4}{t+1} = 2$

$2t + 2 = 4$

$2t = 2$

$t = 1$

As $v = 2 - 4(t + 1)^{-1}$

$\therefore \ a = 4(t + 1)^{-2}.1$

$= \frac{4}{(t+1)^2}$

Subs t = 1:

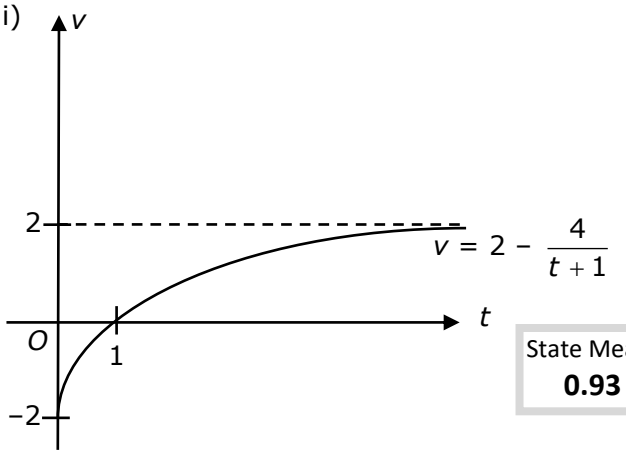
$a(1) = \frac{4}{(1+1)^2}$

$= 1$

$\therefore$ the acceleration is 1 ms^{-2}.

State Mean: **1.23**

(iii)



State Mean: **0.93**

(iv) $d = \left|\int_0^1 \left(2 - \frac{4}{t+1}\right) dt\right| + \int_1^7 \left(2 - \frac{4}{t+1}\right) dt$

$= \left|\left[2t - 4\ln(t+1)\right]_0^1\right| + \left[2t - 4\ln(t+1)\right]_1^7$

$= |2 - 4\ln 2 - 0| + 2(7) - 4\ln(7 + 1)$

$- [2(1) - 4\ln(1 + 1)]$

Now, as $|2 - 4\ln 2| = 4\ln 2 - 2$,

$d = 4\ln 2 - 2 + 14 - 4\ln 8 - 2 + 4\ln 2$

$= 8\ln 2 + 10 - 4\ln 8$

$= 8\ln 2 + 10 - 4\ln 2^3$

$= 8\ln 2 + 10 - 12\ln 2$

$= 10 - 4\ln 2$

$\therefore$ the distance is $(10 - 4\ln 2)$ metres.

State Mean: **0.79**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

BOSTES: Notes from the Marking Centre

This information is released by BOSTES in late Term 1 2017.