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11	**8c** State Mean: **1.09/2** **0.34/1** **1.47/3**	When Jules started working she began paying \$100 at the beginning of each month into a superannuation fund. The contributions are compounded monthly at an interest rate of 6% per annum. She intends to retire after having worked for 35 years. (i) Let \$$P$ be the final value of Jules's superannuation when she retires after 35 years (420 months). Show that \$$P$ = \$143 183 to the nearest dollar. (ii) Fifteen years after she started working Jules read a magazine article about retirement, and realized that she would need \$800 000 in her fund when she retires. At the time of reading the magazine article she had \$29 227 in her fund. For the remaining 20 years she intends to work, she decides to pay a total of \$$M$ into her fund at the beginning of each month. The contributions continue to attract the same interest rate of 6% per annum, compounded monthly. At the end of n months after starting the new contributions, the amount in the fund is \$$A_n$. (1) Show that $A_2 = 29\ 227 \times 1.005^2 + M(1.005 + 1.005^2)$. (2) Find the value of M so that Jules will have \$800 000 in her fund after the remaining 20 years (240 months).	**2** **1** **3**

(i) 6% pa = 0.5% per month

$$P = 100 \times 1.005^{420} + 100 \times 1.005^{419} + \ldots + 100 \times 1.005$$
$$= 100[1.005 + 1.005^2 + \ldots + 1.005^{420}]$$

Using $a = 1.005$, $r = 1.005$, $n = 420$,

$$P = 100 \times \frac{1.005(1.005^{420} - 1)}{1.005 - 1}$$
$$= 143\ 183.385$$

$\therefore$ \$143 183 to nearest dollar

(1) $A_1 = (29\ 227 + M) \times 1.005$

$= 29\ 227 \times 1.005 + 1.005M$

$A_2 = [(29\ 227 \times 1.005 + 1.005M) + M] \times 1.005$

$= 29\ 227 \times 1.005^2 + 1.005^2M + 1.005M$

$= 29\ 227 \times 1.005^2 + M(1.005 + 1.005^2)$

(2) $A_{240} = 29\ 227 \times 1.005^{240} + M(1.005 + 1.005^2 + \ldots + 1.005^{240})$

Using $a = 1.005$, $r = 1.005$, $n = 240$,

$$800\ 000 = 29\ 227 \times 1.005^{240} + M \times \frac{1.005(1.005^{240} - 1)}{1.005 - 1}$$

$800\ 000 = 96\ 747.34621 + 464.3510996M$

$464.3510996M = 703\ 252.6538$

$M = 1514.48$ (2 dec places)

$\therefore$ she needs to contribute \$1514.48 each month.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) In better responses, candidates were able to establish the series or recognise a superannuation problem and substitute into a formula for the sum of n terms. Common errors included introducing an extra \$100 term, interpreting the value for n incorrectly (with many not using 240 (months) but 20 (years)), using an incorrect formula for the sum of a geometric series, writing the first term of the series as 1 and not 1.005 or not converting to a (correct) monthly interest rate.

(ii) (1) Many candidates recognised the pattern to write $A_1 = 29227 \times 1.005 + M(1.005)$ then $A_2 = A_1 \times 1.005 + M(1.005)$ (or another form) before producing the required expression.

Common errors included writing an incorrect expression for A_1 or not using $M(1.005)$ in the calculations.

(2) Generally candidates were able to use the pattern and process provided in the previous part to develop a pattern for A_{240}. Common errors included misunderstanding the intent of the question and treating it as a time-payment question where $A_{240} = 0$ and solving to find M, difficulty in solving the equation $800\ 000 = 29227 \times 1.005^{240} + M \times \frac{1.005\left(1.005^{240} - 1\right)}{1.005 - 1}$, calculating $800\ 000 \div \left(29227 \times 1.005^{240}\right)$ instead of $800\ 000 - 29227 \times 1.005^{240}$, incorrectly determining the number of terms in the sum and using an incorrect formula for for S_n.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/