05	**8c**	Weelabarrabak Shire Council borrowed \$3 000 000 at the beginning of 2005. The annual interest rate is 12%. Each year, interest is calculated on the balance at the beginning of the year and added to the balance owing. The debt is to be repaid by equal annual repayments of \$480 000, with the first repayment being made at the end of 2005. Let A_n be the balance owing after the n-th repayment. (i) Show that $A_2 = (3 \times 10^6)(1.12)^2 - (4.8 \times 10^5)(1 + 1.12)$. (ii) Show that $A_n = 10^6[4 - (1.12)^n]$. (iii) In which year will Weelabarrabak Shire Council make the final repayment?	**1** **2** **2**

i. Let $A_1 = 3\,000\,000 \times 1.12 - 480\,000$
$= (3 \times 10^6) \times 1.12 - (4.8 \times 10^5)$

$A_2 = A_1 \times 1.12 - (4.8 \times 10^5)$
$= [(3 \times 10^6) \times 1.12 - (4.8 \times 10^5)] \times 1.12 - (4.8 \times 10^5)$
$= (3 \times 10^6) \times 1.12^2 - (4.8 \times 10^5)(1 + 1.12)$

ii. $A_3 = (3 \times 10^6) \times 1.12^3 - (4.8 \times 10^5)(1 + 1.12 + 1.12^2)$
$\therefore A_n = (3 \times 10^6) \times 1.12^n - (4.8 \times 10^5)(1 + 1.12 + 1.12^2 + \ldots + 1.12^{n-1})$

Now, for $1 + 1.12 + 1.12^2 + \ldots + 1.12^{n-1}$, $a = 1$, $r = 1.12$ and use

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1(1.12^n - 1)}{1.12 - 1}$$

$$= \frac{1(1.12^n - 1)}{0.12}$$

$$\therefore A_n = (3 \times 10^6) \times 1.12^n - (4.8 \times 10^5) \times \frac{1(1.12^n - 1)}{0.12}$$
$= (3 \times 10^6) \times 1.12^n - 40 \times 10^5 \times (1.12^n - 1)$
$= (3 \times 10^6) \times 1.12^n - (4 \times 10^6) \times (1.12^n - 1)$
$= (3 \times 10^6) \times 1.12^n - (4 \times 10^6) \times 1.12^n + (4 \times 10^6)$
$= -1 \times 10^6 \times 1.12^n + 4 \times 10^6$
$= 10^6(4 - 1.12^n)$

iii. Let $A_n = 0$ $\quad 10^6(4 - 1.12^n) = 0$
$4 - 1.12^n = 0$
$1.12^n = 4$
$\log 1.12^n = \log 4$
$n \log 1.12 = \log 4$
$$n = \frac{\log 4}{\log 1.12}$$
$= 12.23251075$
$\therefore$ during 2017 as loan was at beginning of 2005

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Better responses to this part used clear and logical setting out.

(i) The better responses involved candidates writing down an expression for A_1 and then an expression for A_2 in terms of A_1 before obtaining the desired result.

(ii) Many candidates could write down the general expression for A_n but were unable to simplify to the desired result. Common errors included writing the last term of the geometric series as 1.12^n instead of 1.12^{n-1}, and not being able to simplify the term $4.8\times10^5\left(\frac{1.12^n-1}{0.12}\right)$.

(iii) An answer to this question could be obtained algebraically or by a trial and error substitution into the given formula. Both methods were attempted. Common errors made by candidates in this part included not recognising the need to solve $A_n = 0$, not being able to solve $1.12^n = 4$, and not being able to correctly interpret their answer in terms of the question.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/