Topic 1
Calculator, arithmetic

THEMES

1 **Calculator usage**
2 **Recurring decimals**
3 **Surds**
4 **Indices**
5 **Scientific notation, significant figures**

FORMULA TEST

1 $\sqrt{a}\sqrt{b} =$

2 $\dfrac{\sqrt{a}}{\sqrt{b}} =$

3 $\left(\sqrt{a}\right)^2 =$

4 $a^m \times a^n =$

5 $\dfrac{a^m}{a^n} =$

6 Express $\sqrt{a}$ in index form.

7 Express $\sqrt[n]{a}$ in index form.

8 Express $\dfrac{1}{a}$ in index form.

9 Express $\dfrac{1}{a^n}$ in index form.

10 Express $\dfrac{1}{\sqrt[n]{a}}$ in index form.

11 $\left(a^m\right)^n =$

12 $(ab)^n =$

The following formulas and results are likely to be used in solving questions in this topic.

1 $\sqrt{a}\sqrt{b} = \sqrt{ab}$

2 $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

3 $\left(\sqrt{a}\right)^2 = a$

4 $a^m \times a^n = a^{m+n}$

5 $\dfrac{a^m}{a^n} = a^{m-n}$

6 $\sqrt{a} = a^{\frac{1}{2}}$

7 $\sqrt[n]{a} = a^{\frac{1}{n}}$

8 $\dfrac{1}{a} = a^{-1}$

9 $\dfrac{1}{a^n} = a^{-n}$

10 $\dfrac{1}{\sqrt[n]{a}} = a^{-\frac{1}{n}}$

11 $\left(a^m\right)^n = a^{mn}$

12 $(ab)^n = a^n b^n$

CALCULATOR, ARITHMETIC
Examination questions

1 Find $5^{3.5}$ correct to two decimal places.

2 Find, correct to two decimal places, the value of
$$\frac{4\cdot7\times5\cdot4}{6\cdot9+3\cdot8}$$

3 Find, correct to two decimal places, the value of
$$\frac{(2\cdot34)^3}{6\cdot91-2\cdot97}$$

4 Find, correct to 2 decimal places:

(i) $(2\cdot6)^4$ (ii) $\dfrac{5\cdot7-6\cdot9}{5\cdot7\times2\cdot4}$

5 Calculate $\dfrac{169\cdot2}{13\cdot4\times5\cdot8}$ correct to 2 decimal places.

6 Calculate $\dfrac{\sqrt{5\cdot7+6\cdot8}}{9\cdot4}$. Round off your answer to one decimal place.

7 Find the value of $\dfrac{3\cdot24}{\sqrt{6\cdot12-1\cdot64}}$. Give your answer correct to 2 decimal places.

8 Find the value of $17^{-0.6}$ to two decimal places.

9 Find the value of $13^{-1.3}$ correct to 2 significant figures.

10 Given that $t^3 = 2000$, find t, rounded off correct to the nearest whole number.

11 If $S = 2\pi r(r+h)$, find S, rounded off correct to one decimal place, when $r = 1\cdot400$ and $h = 3\cdot705$.

12 Given that $I = \dfrac{E}{R-2r}$ and $E = 0\cdot7$, $R = 1\cdot3$, and $r = -0\cdot6$, find I.

13 Given that $V = \dfrac{1}{3}\pi r^2 h$ and that $r > 0$, find r if $V = 3000$ and $h = 15$.
Give your answer correct to one decimal place.

14 The length, ℓ, of the base of a square pyramid of volume V and perpendicular height h, is given by
$$\ell = \sqrt{\frac{3V}{h}}.$$
Find ℓ, correct to two decimal places, if $V = 850$ and $h = 6\cdot54$.

15 If $S = \dfrac{a}{1-r}$, find the value of a when $S = 90$ and $r = \dfrac{1}{3}$.

16 The volume V of a sphere is given by $V = \dfrac{4}{3}\pi r^3$. If a sphere has a volume of 15 cm³, find the radius correct to two decimal places.

17 Find the value of $2\pi\sqrt{\dfrac{\ell}{g}}$ if $\ell = 3\cdot1$ and $g = 9\cdot8$.
Give your answer to 2 significant figures.

18 (i) Rationalise the denominator of $\dfrac{3}{3-\sqrt{2}}$.

(ii) Find integers a and b such that
$$\frac{3}{3-\sqrt{2}} = a+\sqrt{b}.$$

19 Express $\dfrac{1}{5-\sqrt{3}}$ with a rational denominator.

20 Rationalise the denominator of $\dfrac{3}{\sqrt{5}-2}$.

21 A bookshop increases the price of a book by 15% to a new selling price of $42·55. What was the selling price of the book before this increase?

22 The price of a pair of shoes for sale at $56·00 is to increase by 20%. What will be the new price of the shoes?

23 The local Council increased municipal rates by $5\frac{1}{2}$%. The new rate for a property is $1865. What was the previous rate for this property? Give your answer correct to the nearest dollar.

24 At a hardware sale, all items are to be sold at a discount of $17\frac{1}{2}$% off the marked price. What is the cost of an item with a marked price of $27·60?

25 Find the average of 76, 28, 81, 41, 64, 58, 39. Give your answer correct to one decimal place.

26 The speed of light is 299 725 kilometres per second. Write this number correct to the nearest hundred.

27 The distance from the Sun to the Earth is 149 492 000 km. Write this in scientific notation, correct to 3 significant figures.

28 A particular shadecloth cuts out 20% of the light and lets through the remaining 80%.
(i) Show that two layers of the shadecloth let through 64% of the light.
(ii) How many layers of the shadecloth are required to cut out at least 95% of the light?

29 The value (V) of a motor vehicle after a period of n years is given by the formula $V = V_O(1-R)^n$ where V_O is the original value of the vehicle and R is the annual percentage rate of depreciation. A vehicle bought 4 years ago for $18 000 was sold for $10 000. Calculate the annual rate of depreciation of this vehicle. Give your answer to the nearest whole number.

30 The volume of one litre of a liquid solvent decreases to V litres after n days according to the formula $V = (1-r)^n$. Find the value of r (to 3 significant figures) if $V = 0\cdot76$ after twenty days.

CALCULATOR, ARITHMETIC
Worked solutions to examination questions

1 $5^{3.5} = 279.508\,49\ldots = 279.51$ (2 d.p.)

2 $\dfrac{4.7 \times 5.4}{6.9 + 3.8} = 2.371\,962\ldots = 2.37$ (2 d.p.)

3 $\dfrac{(2.34)^3}{6.91 - 2.97} = 3.252\,006\ldots = 3.25$ (2 d.p.)

4 (i) $2.6^4 = 45.6976 = 45.70$ (2 d.p.)

    (ii) $\dfrac{5.7 - 6.9}{5.7 \times 2.4} = -0.087\,719\ldots = -0.09$ (2 d.p.)

5 $\dfrac{169.2}{13.4 \times 5.8} = 2.177\,045\ldots = 2.18$ (2 d.p.)

6 $\dfrac{\sqrt{5.7 + 6.8}}{9.4} = 0.376\,12\ldots = 0.4$ (1 d.p.)

7 $\dfrac{3.24}{\sqrt{6.12 - 1.64}} = 1.530\,756\ldots = 1.53$ (2 d.p.)

8 $17^{-0.6} = 0.182\,696\ldots = 0.18$ (2 d.p.)

9 $13^{-1.3} = 0.035\,634\ldots = 0.036$ (2 sig. figs.)

10 $t^3 = 2000 \quad \therefore \quad t = \sqrt[3]{2000}$
$$= 12.599\,21\ldots$$
$$\doteqdot 13 \text{ (to nearest whole number)}$$

11 $S = 2\pi r(r + h)$
$$= 2 \times \pi \times 1.4(1.4 + 3.705)$$
$$= 44.9059\ldots$$
$$\doteqdot 44.9 \text{ (to 1 d.p.)}$$

12 $I = \dfrac{E}{R - 2r} = \dfrac{0.7}{1.3 - 2(-0.6)} = 0.28$

13 $V = \dfrac{1}{3}\pi r^2 h$
$$3V = \pi r^2 h$$
$$r^2 = \frac{3V}{\pi h}$$
$$r = \sqrt{\frac{3V}{\pi h}}$$
$$= \sqrt{\frac{3 \times 3000}{\pi \times 15}}$$
$$= 13.819\,76$$
$$\doteqdot 13.8 \text{ (to 1 d.p.)}$$

14 $\ell = \sqrt{\dfrac{3V}{h}}$
$$= \sqrt{\frac{3 \times 850}{6.54}}$$
$$= 19.746\,09\ldots$$
$$\doteqdot 19.75 \text{ (to 2 d.p.)}$$

Calculator techniques involving division:

(a) If the denominator is a product pair,

for example $\dfrac{36.7}{4.7 \times 2.9}$ then use $\boxed{\div}$, $\boxed{\div}$

viz. $36.7 \boxed{\div} 4.7 \boxed{\div} 2.9 =$

(b) If the denominator involves a few different operations and the numerator is a single term (or product),

for example $\dfrac{3.4 \times 2.6}{4.2^2 + \sqrt{1.8}}$

then perform the denominator first, divided by the numerator, then use the reciprocal key $\boxed{\frac{1}{x}}$ to 'turn it upside down' again.

$4.2 \boxed{x^2} + \boxed{\sqrt{}} \ 1.8 \boxed{\div} 3.4 \boxed{\div} 2.6 \boxed{=} \boxed{\frac{1}{x}}$

---

10 Make t the subject by finding the cube root.

13 Make r the subject first (algebraically), then substitute values.
Note:
— with equations, clear fractions first
— 'undo' operations by the reverse: $r^2 \Rightarrow \sqrt{}$

15 $S = \dfrac{a}{1-r}$

$90 = \dfrac{a}{1-\frac{1}{2}} = \dfrac{a}{\frac{1}{2}}$

$\therefore\ a = 90 \times \dfrac{1}{2} = 45$

16 $V = \dfrac{4}{3}\pi r^3$

$3V = 4\pi r^3$

$r^3 = \dfrac{3V}{4\pi}$

$r = \sqrt[3]{\dfrac{3V}{4\pi}}$

$\quad = \sqrt[3]{\dfrac{3 \times 15}{4 \times \pi}}$

$\quad = 1{\cdot}529\,915\ldots \doteqdot 1{\cdot}53$ (to 2 d.p.)

17 $2\pi\sqrt{\dfrac{\ell}{g}} = 2\pi\sqrt{\dfrac{3{\cdot}1}{9{\cdot}8}}$

$\qquad\qquad = 3{\cdot}533\,84\ldots \doteqdot 3{\cdot}5$ (to 2 sig. figs.)

18   (i) $\dfrac{3}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = \dfrac{3(3+\sqrt{2})}{9-2} = \dfrac{9+3\sqrt{2}}{7}$

> **18** Multiply 'top and bottom' by the conjugate.

(ii) $\dfrac{3}{3-\sqrt{2}} = \dfrac{9+3\sqrt{2}}{7}$ from (i)

$\qquad\qquad = \dfrac{9}{7} + \dfrac{3}{7}\sqrt{2}$

$\qquad\qquad = \dfrac{9}{7} + \sqrt{\dfrac{9}{49}}\,\sqrt{2}$

$\qquad\qquad = \dfrac{9}{7} + \sqrt{\dfrac{18}{49}}$

$\qquad\qquad = a + \sqrt{b} \qquad \therefore\ a = \dfrac{9}{7},\ b = \dfrac{18}{49}$

19 $\dfrac{1}{5-\sqrt{3}} \times \dfrac{5+\sqrt{3}}{5+\sqrt{3}} = \dfrac{5+\sqrt{3}}{25-3} = \dfrac{5+\sqrt{3}}{22}$

20 $\dfrac{3}{\sqrt{5}-2} \times \dfrac{\sqrt{5}+2}{\sqrt{5}+2} = \dfrac{3(\sqrt{5}+2)}{5-4} = 3(\sqrt{5}+2)$

21   Let the original price be represented by 100%

$\therefore$ new price $= 115\% = \$42{\cdot}55$

$\qquad\qquad 1\% = \dfrac{\$42{\cdot}55}{115}$

$\qquad\quad 100\% = \dfrac{\$42{\cdot}55}{115} \times 100$

$\qquad\qquad\quad = \$37{\cdot}00$

$\therefore$ original price is $37·00

> **21** Questions of this type are best solved using the 'unitary method'.

22   Let the original price be represented by 100%

$\therefore$ new price $= 120\%$ of $56·00

$\qquad\qquad = 1{\cdot}2 \times \$56{\cdot}00$

$\qquad\qquad = \$67{\cdot}20$

23 Let the old rate be represented by 100%

$\therefore$ new rate $= 105\frac{1}{2}\% = \$1865$

$\therefore$ $\qquad\qquad 1\% = \dfrac{\$1865}{105 \cdot 5}$

$\qquad\qquad 100\% = \dfrac{1865}{105 \cdot 5} \times 100$

$\qquad\qquad\qquad = \$1767 \cdot 77 \doteqdot \$1768$ to nearest dollar

24 Discounted price $= 100\% - 17\frac{1}{2}\% = 82\frac{1}{2}\%$

$\therefore$ Sale price $= 82\frac{1}{2}\%$ of $\$27 \cdot 60$

$\qquad\qquad = 0 \cdot 825 \times \$27 \cdot 60$

$\qquad\qquad = \$22 \cdot 77$

25 Average $= \dfrac{76 + 28 + 81 + 41 + 64 + 58 + 39}{7}$

$\qquad\qquad = 55 \cdot 2857 \ldots$

$\qquad\qquad \doteqdot 55 \cdot 3$ (to 1 d.p.)

26 299 725 km/s $\approx$ 299 700 km/s (to nearest hundred)

27 Distance $= 149\ 492\ 000$ km

$\qquad\qquad = 1 \cdot 494\ 92 \times 10^8$ km

$\qquad\qquad \doteqdot 1 \cdot 49 \times 10^8$ km (to 3 sig. figs.)

28 (i) Each successive layer lets through 80% of light.

$\therefore$ 2 layers let through $0 \cdot 8 \times 0 \cdot 8 = 0 \cdot 64 = 64\%$

(ii) At least 95% of light cut out is equivalent to
at most 5% of light let through.
Let n be the number of layers required

$\therefore$ $\quad (0 \cdot 8)^n \leq 5\%$

i.e. $\quad (0 \cdot 8)^n \leq 0 \cdot 05$

$\qquad\qquad n = 14$ by calculator

29 $\qquad V = V_O(1 - R)^n$

$10\ 000 = 18\ 000(1 - R)^4$

$\dfrac{10}{18} = (1 - R)^4$

$1 - R = \sqrt[4]{\dfrac{10}{18}}$

$\qquad\quad = 0 \cdot 863$

$\therefore \quad r = 1 - 0 \cdot 863$

$\qquad\quad = 0 \cdot 137$

$\therefore$ annual rate of depreciation $(R) = 13 \cdot 7\% \approx 14\%$ (to nearest whole number)

30 $\qquad V = (1 - r)^n$

$0 \cdot 76 = (1 - r)^{20}$

$1 - r = \sqrt[20]{0 \cdot 76}$

$\qquad\quad = 0 \cdot 986\ 37 \ldots$

$\therefore \quad r = 1 - 0 \cdot 986\ 37 \ldots$

$\qquad\quad = 0 \cdot 136\ 28$

$\qquad\quad \approx 0 \cdot 136$ (to 3 sig. figs.)

---

28 (ii) Think about the percentage
of light let through.
Using a calculator,
$0 \cdot 8 \times 0 \cdot 8 \times 0 \cdot 8 \times \ldots$ until
you reach a value $\leq 0 \cdot 05$

Note this could have been solved
using logarithms:

$\qquad (0 \cdot 8)^n \leq 0 \cdot 05$

$\qquad n \log 0 \cdot 8 \leq \log 0 \cdot 05$

$\therefore \quad n \geq \dfrac{\log 0 \cdot 05}{\log 0 \cdot 8}, \ (\log 0 \cdot 8 < 0)$

$\qquad\quad \geq 13 \cdot 42 \ldots$

i.e. $n = 14$