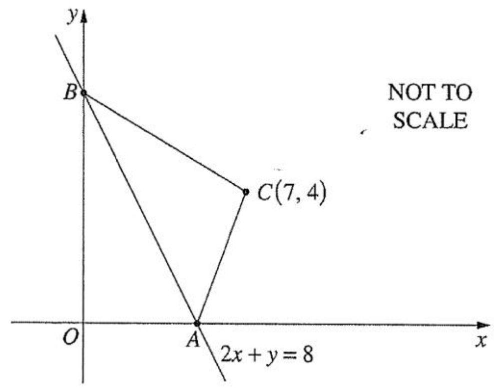
Want more revision exercises? Get **MathsFit** - *New from projectmaths.*

12 **13a** The diagram shows a triangle ABC. The line $2x + y = 8$ meets the x and y axes at the points A and B respectively. The point C has coordinates $(7, 4)$.



(i) Calculate the distance AB. **2**

(ii) It is known that $AC = 5$ and $BC = \sqrt{65}$. (Do NOT prove this.) Calculate the size of $\angle ABC$ to the nearest degree. **2**

(iii) The point N lies on AB such that CN is perpendicular to AB. Find the coordinates of N. **3**

(i) $A(x, 0)$: subs $y = 0$ in $2x + y = 8$

$$2x = 8$$
$$x = 4$$

$B(0, y)$: subs $x = 0$ in $2x + y = 8$

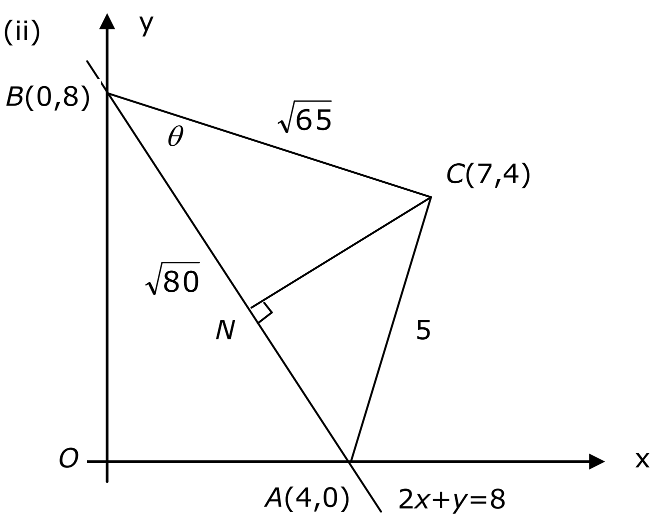
$$y = 8$$

$$\text{distance} = 4^2 + 8^2 = 16 + 64 = 80$$

$$AB = \sqrt{80}$$

$\therefore$ distance is $\sqrt{80}$ units

(ii)



Let $\angle ABC = \theta$

$$\text{Using } \cos\theta = \frac{(\sqrt{65})^2 + (\sqrt{80})^2 - 5^2}{2 \times \sqrt{65} \times \sqrt{80}}$$

$$= \frac{120}{144.222051..}$$

$$= 0.83205\ ...$$

$$\theta = 33.69006751\ ...$$

$\therefore\ \angle ABC = 34^\circ$

State Mean:
1.79/2
1.14/2
1.60/3

(iii) Gradient of $2x + y = 8$ is -2

$\therefore$ gradient of perpendicular $= \frac{1}{2}$

equation of CN:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{1}{2}(x - 7)$$

$$2y - 8 = x - 7$$

$$x - 2y = -1$$

Solve simultaneously:

$$2x + y = 8 \quad \text{............} 1$$

$$x - 2y = -1 \quad \text{............} 2$$

$$2 \times 2 \quad 2x - 4y = -2 \quad \text{..........} 3$$

$$1 - 3 \quad 5y = 10$$

$$y = 2$$

Subs in 1

$$2x + 2 = 8$$
$$2x = 8 - 2$$
$$2x = 6$$
$$x = 3 \qquad \therefore N(3, 2)$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) In most responses, candidates used the distance formula or Pythagoras' theorem to find the distance AB. In weaker responses, errors included carelessly swapping the coordinates of A and B or incorrectly calculating the x and y intercepts of the line AB.

(ii) In most responses, candidates recognised that this part required the use of the cosine rule to find $\angle ABC$. In better responses, candidates correctly substituted into the formula $\cos B = \dfrac{a^2+c^2-b^2}{2ac}$ and rounded their answer to the nearest degree.

In responses where candidates stated the formula as $b^2 = a^2 + c^2 - 2ac\cos B$ errors were made when changing the subject to In other responses, common errors included not rounding to the nearest degree, inconsistent substitution into the cosine rule, finding the angle in radians or assuming that ΔABC was right-angled.

In a few responses, candidates successfully found the size of $\angle ABC$ by finding the perpendicular distance CN and then using the sine rule or right-angled trigonometry.

(iii) In better responses, candidates found the equation CN and then solved it with the equation for AB to find the coordinates of N. In weaker responses, candidates provided as incorrect equation for CN often resulting from using the reciprocal gradient of AB instead of the negative reciprocal. Algebraic and arithmetic errors were common.

In a few responses, candidates used the perpendicular distance formula to find the distance CN, then solved a distance equation with the equation of AB to find N. This method required many algebraic steps and often proved too difficult to complete.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/