07	**8a**	One model for the number of mobile phones in use worldwide is the exponential growth model, $N = Ae^{kt}$, where N is the estimate for the number of mobile phones in use (in millions), and t is the time in years after 1 January 2008. (i) It is estimated that at the start of 2009, when $t = 1$, there will be 1600 million mobile phones in use, while at the start of 2010, when $t = 2$, there will be 2600 million. Find A and k. (ii) According to the model, during which month and year will the number of mobile phones in use first exceed 4000 million?	**3** **2**

(i) $N = Ae^{kt}$

When $t = 1$ and $A = 1600$:

$1600 = Ae^{k}$ ……… **1**

When $t = 2$ and $A = 2600$:

$2600 = Ae^{2k}$ ……… **2**

2 ÷ **1**: $\frac{2600}{1600} = e^{k}$

$e^{k} = \frac{13}{8}$

Taking logs of both sides:

$k = \log_e \frac{13}{8}$

Subs into **1**: $Ae^{k} = 1600$

$A = \frac{1600}{e^{k}}$

$= 1600 \div \frac{13}{8}$

$= \frac{12800}{13}$

$\therefore A = \frac{12800}{13}$ and $k = \log_e \frac{13}{8}$

(ii) $N = \frac{12800}{13}e^{kt}$

When $N = 4000$:

$4000 = \frac{12800}{13}e^{kt}$

$\frac{52000}{12800} = e^{kt}$

$e^{kt} = \frac{65}{16}$

Taking logs of both sides:

$kt = \log_e \frac{65}{16}$

$t = \log_e \frac{65}{16} \div k$

$= \log_e \frac{65}{16} \div \log_e \frac{13}{8}$

$= 2.887283175 \ldots$

Time is 2 years + 0.887283175 × 12 mths

= 2 years + 10.6473981 months

∴ First exceed in November 2010.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) This part of the question required the candidate to substitute the given information into the equation $N = Ae^{kt}$ to produce two equations. These equations then had to be solved simultaneously to find the values for k and A. Dividing the two equations to eliminate A seemed to be the most successful approach. Further, candidates who wrote their value of k in decimal form rather than exact form had less trouble in using that value to subsequently find A.

Common problems in this part included:

- Just picking a value for A, for example $A = 1600$, or $A = 600$.
- Attempting to eliminate A by subtracting the two equations, for example $2600 - 1600 = Ae^{2k} - Ae^{k} = e^{k}$.
- Careless setting out so that $\ln\frac{1600}{A}$ became $\frac{\ln 1600}{A}$.
- Using base 10 logarithms instead of natural logarithms.

(ii) The most common errors in this part were using wrong units (eg putting $N = 4\ 000\ 000\ 000$ but leaving A as 935), mishandling the exponential term by letting $e^{\ln\left(\frac{13}{8}\right)t} = \frac{13t}{8}$, and not being able to interpret the value of t (eg interpreting $t = 2.887$ as corresponding to the year 2010 but in the month of August).

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/