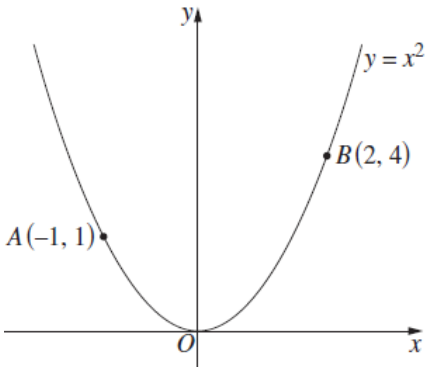
10 **7b** The parabola shown in the diagram is the graph $y = x^2$. The points $A(-1, 1)$ and $B(2, 4)$ are on the parabola.

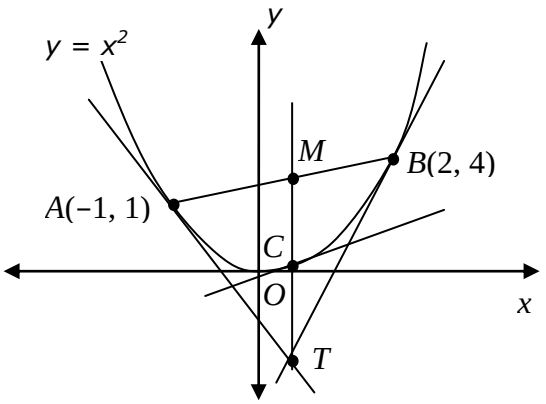
(i) Find the equation of the tangent to the parabola at A. **2**

(ii) Let M be the midpoint of AB. There is a point C on the parabola such that the tangent at C is parallel to AB. Show that the line MC is vertical. **2**

(iii) The tangent at A meets the line MC at T. Show that the line BT is a tangent to the parabola. **2**



State Mean:
1.59/2
0.74/2
0.57/2



(i) $y = x^2$

$\frac{dy}{dx} = 2x$

At $x = -1$, $\frac{dy}{dx} = -2$

using $(-1, 1)$ and $m = -2$,

$y - y_1 = m(x - x_1)$

$y - 1 = -2(x - (-1))$

$y - 1 = -2(x + 1)$

$y - 1 = -2x - 2$

$2x + y + 1 = 0$

$\therefore$ eqn of tangent is $2x + y + 1 = 0$

(ii) Using $A(-1, 1)$ and $B(2, 4)$,

$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-1 + 2}{2}, \frac{1 + 4}{2}\right)$

$= \left(\frac{1}{2}, \frac{5}{2}\right)$

$= (0.5, 2.5)$

$\therefore$ co-ords of M *is* $(0.5, 2.5)$

Now, using $A(-1, 1)$ and $B(2, 4)$,

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3}{3}$

$= 1$

$\therefore$ gradient of tangent at M is 1

Let $\frac{dy}{dx} = 2x = 1$

$x = \frac{1}{2}$

$\therefore$ As M and C both have x-value of $\frac{1}{2}$, then MC is vertical.

(iii) tangent at A is $2x + y + 1 = 0$:

At T, subs $x = \frac{1}{2}$:

$2 \times \frac{1}{2} + y + 1 = 0$

$1 + y + 1 = 0$

$y = -2$

$\therefore T(\frac{1}{2}, -2)$

Now, using $T(\frac{1}{2}, -2)$ and $B(2, 4)$,

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4 + 2}{2 - \frac{1}{2}}$

$= \frac{6}{1\frac{1}{2}}$

$= 4$

$\therefore$ gradient of BT is 4.

But, gradient of tangent at B:

$y = x^2$

$\frac{dy}{dx} = 2x$

At $x = 2$, $\frac{dy}{dx} = 4$

As both have gradient of 4, then BT is tangent to the parabola.

$= \frac{4-1}{2+1}$	

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) The most common errors were: using the gradient of AB instead of the gradient of the tangent; incorrectly simplifying the equation $y-1=2(x+1)$; finding the gradient of the tangent but not then finding the equation.

(ii) Many candidates could find the midpoint and gradient of AB but the key step in a better response in this part was the realisation that the gradient of AB had to be equated to the gradient function of the parabola.

Common errors included: finding the point C by using the fact that it is vertically underneath C rather than the fact that the gradient of the tangent had equal the gradient of AB; confusing being vertical with being perpendicular, for example claiming the slope of $MC=-1$ by using $m_1m_2=-1$; and not giving any reason at all why MC is vertical.

(iii) The key step in obtaining a better response to this part was knowing that the equation of MC is $x=\frac{1}{2}$.

Source: **http://www.boardofstudies.nsw.edu.au/hsc_exams/**